This paper considers a relevant issue related to the influence exerted by the fuzziness in linear dynamic system parameters on its stability. It is known that the properties of automated control systems can change under the influence of parametric disturbances. To describe the change in such properties of the system, the concept of roughness is used.

It should be noted that taking into consideration the fuzziness in the parameters of mathematical models could make it possible at the design stage to assess all the risks that may arise as a result of an uncontrolled change in the parameters of dynamic systems during their operation. To prevent negative consequences due to variance in the parameters of mathematical models, automated control systems are designed on the basis of the requirement for ensuring a certain margin of stability of the system in terms of its amplitude and phase. At the same time, it remains an open question whether such a system would satisfy the conditions of roughness.

Parameters of the mathematical model of a system are considered as fuzzy quantities that have a triangular membership function. This function is inconvenient for practical use, so it is approximated by the Gaussian function. That has made it possible to obtain formulas for calculating the characteristic polynomial and the transfer function of the open system, taking into consideration the fuzziness of their parameters.

When investigating the system according to Mikhailov's criterion, it was established that the dynamic system retains stability in the case when the parameters of the characteristic equation are considered as fuzzy quantities. It has been determined that the quality of the system significantly deteriorated in terms of its stability that could make it enter a non-steady state. When using the Nyquist criterion, it was established that taking into consideration the fuzziness in the parameters of the transfer function did not affect the stability of the closed system but there was a noticeable decrease in the system stability reserve both in terms of phase and amplitude. The relative decrease in the margin of stability for amplitude was 16 %, and for phase – 17.4 %

Keywords: mathematical model, stability, fuzziness, membership function, transfer function, dynamic system

DETERMINING THE EFFECT OF FUZZINESS IN THE PARAMETERS OF A LINEAR DYNAMIC SYSTEM ON ITS STABILITY

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1. Introduction

Due to a series of factors, the parameters of linearized mathematical models of real objects are determined with a certain inaccuracy, which can cause the loss of stability by the systems of automated control over such objects. To prevent negative consequences from a variance in the parameters of mathematical models, automated control systems are designed on the basis of the requirement for ensuring a certain margin of stability in the system in terms of its amplitude and phase.

When designing automated control systems (ACSs), we take into consideration that information about the parameters of a linear (linearized) model is inaccurate. Let a_i be some parameter of the model, and $a_i^{(0)}$ – its nominal value. Then $a_i = a_i^{(0)} + \Delta a_i$, where Δa_i is the parametric disturbance.

The nature of a parametric disturbance may be different. It may relate to the assumptions when building a mathematical model of the system, or errors that arise as a result of model linearization. As well as changing the modes of operation of the control object, aging of system elements, errors in

measuring technological parameters, their variations within technological tolerances, etc.

Parametric disturbances can cause significant changes in ACS properties. To characterize the change of such properties, the concept of roughness is used. An automated control system is termed rough [1] relative to the parameters of a_i if the system maintains asymptotic stability when changing the $a_i^{(0)}$ parameters by a value of Δa_i.

It is important at the system's design stage to assess all the risks that may arise as a result of uncontrolled changes in the parameters of dynamic systems during their operation. This task can be solved by taking into consideration the fuzziness in the parameters of mathematical models.

2. Literature review and problem statement

Modern scientific literature [2–9] considers many issues related to linear dynamic systems. These are features of their mathematical notation, behavior, stability issues, analysis of

special modes of such systems in the presence of persistent fluctuations and self-oscillations.

Work [2] reports the systematization of mathematical models of control systems in the form of transfer functions. It follows from that work that the transfer function in a standard-factorized form combines the convenience of analysis and synthesis of automated control systems. The author of [3] illustrates the use of transfer functions to describe linear dynamic systems. It should be noted that despite the convenience of models in the form of transfer functions, there is an issue related to taking into consideration the subtle features of control processes. Study [4] applied a series of provisions from the theory of fuzzy sets to describe the nonlinear characteristics of backlash connections, which show the effectiveness of using a fuzzy set theory to study the stability of dynamic systems. Work [5] suggests an improved method for investigating the stability of a nonlinear fuzzy logic control system that employs the Lyapunov function. However, it remains an open question to investigate the fuzziness in the parameters of the transfer function.

A detailed numerical study into the system stability using the Lyapunov criterion is carried out in [6]; however, insufficient attention is paid to the analysis of the stability margin of the dynamic system. Work [7] reports a study into the models of linear parametric systems of ordinary differential equations with variable measurability of phase space. As a result, it was established that, based on the methods of practical stability and conditions, parameters can be evaluated to analyze the sensitivity of systems with variable dimensionality of phase space. The authors of [8] proposed a method for analyzing the stability of automated systems with logic control devices when exposed to parametric disturbances, which makes it possible to determine the boundaries of the stability areas of systems in the space of increase in their parameters. However, insufficient attention is paid to the effect exerted on the system stability by uncontrolled changes in the parameters of dynamic systems during their operation.

Our analysis of literary sources has revealed that the issue related to studying the influence of fuzzy parameters of the transfer function on the stability indicators of the dynamic system remains open.

3. The aim and objectives of the study

The purpose of this work is to determine the effect of fuzziness in the parameters of the transfer function of the dynamic system on its stability.

To accomplish the aim, the following tasks have been set:
– to carry out the process of approximation of the triangular membership function by the Gaussian function and calculate the error;
– to derive a formula for calculating the characteristic polynomial and the transfer function of the dynamic system taking into consideration the fuzziness of its parameters.

4. Materials and methods to study a dynamic system taking into consideration the fuzziness of its parameters

During the study, we have applied methods from the mathematical theory of fuzzy sets and fuzzy logic to derive a transfer function of the open system taking into consideration the fuzziness in its parameters.

Numerical methods to approximate a triangle membership function with the Gaussian function to simplify further mathematical operations. A golden-section method was used to find a minimum of the function. The dependence coefficients were determined by a least-square method.

We applied methods from automated control theory, specifically the Nyquist and Mikhailov criteria, to study the stability of the system taking into consideration the fuzziness in the parameters of the transfer function.

5. Results of studying a fuzziness factor in the mathematical description of an object

5. 1. Approximating a triangular function by the Gaussian membership function

When designing automated control systems, the issue of inaccuracies in the parameters of the mathematical model is conventionally resolved by selecting certain values of a reserve for amplitude and phase. Another way to take into consideration inaccuracies in the parameters of the mathematical models of linearized systems is to set an M-indicator [9], which characterizes the shortest distance from the point $(-1, j0)$ on the complex plane to the point of contact with the amplitude-phase characteristic of the open system of the $1/M$ radius circle with the center at the point $(-1, j0)$ (Fig. 1).
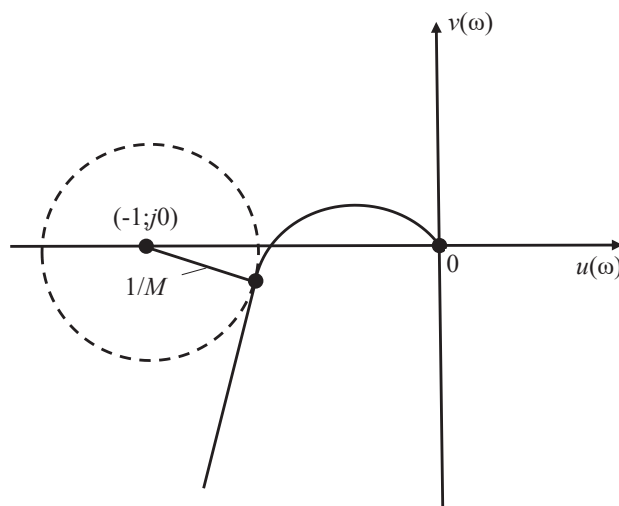

Fig. 1. Geometric interpretation of M-indicator

Assume the amplitude-phase characteristic $W(j\omega) = u(\omega)+jv(\omega)$ of an open system is known. Then the M-indicator is calculated from the following formula given in [1]:

$$M = \max_{\omega} \left| \frac{1}{1+W(j\omega)} \right| = \frac{1}{\min_{\omega} \left| 1+W(j\omega) \right|}.$$

In order for the system to be stable when its parameters vary, the M-indicator is chosen from the condition: $1.4 \le M \le 2$. The choice of this value for the M-indicator has no theoretical justification and is based on the empirical experience of many researchers.

Let the characteristic polynomial of the system (closed or open) be as follows:

$$Q(p) = \sum_{i=0}^{n} a_i p^{n-i}. \tag{1}$$

We shall consider the parameters of the characteristic equation to be fuzzy quantities with a triangular membership function [10].

To take into consideration the fuzziness factor when mathematically describing objects, it is necessary to perform certain arithmetic operations on fuzzy quantities. The process of performing arithmetic operations (addition, subtraction, multiplication, and division) becomes possible if fuzzy numbers are defined as numbers of the (L-R) type.

Let x be a fuzzy quantity of the (L-R)-type. Then its membership function can be represented as a composition of L and R functions [10]:

$$\mu_{L-R}(z) = \begin{cases} L\left(\dfrac{a_x - x}{\alpha_L}\right), x \le a_x, \\ R\left(\dfrac{x - a_x}{\alpha_R}\right), x > a_x, \end{cases}$$

where $\alpha_L > 0$, $\alpha_R > 0$ is the left-hand and right-hand fuzzy coefficients; z_0 is the modal value of a fuzzy number.

Thus, a fuzzy number of the (L-R)-type is uniquely determined by the three parameters $\langle a_x, \alpha_L, \alpha_R \rangle$.

Note that the triangular membership function, which is symmetrical relative to a_x, is a function of the (L-R)-type. This function is inconvenient for practical use because it is not differentiated at some points in the definition area.

Therefore, the following triangular membership function:

$$\mu(x) = \begin{cases} \dfrac{2}{\Delta}(x - a_x) + 1, x \in [a_x - \Delta/2; a_x], \\ -\dfrac{2}{\Delta}(x - a_x) + 1, x \in [a_x; a_x + \Delta/2] \end{cases} \tag{2}$$

is to be approximated by the following Gaussian function:

$$\mu_G(x) = \exp\left(-\frac{(x - a_x)^2}{2\alpha^2}\right), \tag{3}$$

where Δ is the uncertainty interval of a fuzzy quantity x; $\mu(a_x) = \mu_G(a_x) = 1$; α is the concentration coefficient of a fuzzy quantity x.

Since functions (2), (3) are monotonic at each interval that determines $x \in [a_x - \Delta/2; a_x]$ and $x \in [a_x; a_x + \Delta/2]$, then, when they are approximated, such functions would have no more than two common points. The first one is determined by the value a_x, and the second is when $x = x_a$. At this value of x, the following ratio holds:

$$\mu(x_a) = \mu_G(x_a) = \theta. \tag{4}$$

It is obvious that the value of a_x does not affect the shape of membership functions (2) and (3), but only determines their position on the abscissa axis. Therefore, the a_x value does not affect the accuracy of the approximation of function (2) by function (3). Assume $a_x = 0$. Then formulas (2), (3) take the following form:

$$\mu(x) = \begin{cases} \dfrac{2}{\Delta}x + 1, x \in [-\Delta/2; 0], \\ -\dfrac{2}{\Delta}x + 1, x \in [0; \Delta/2] \end{cases} \tag{5}$$

and

$$\mu_G(x) = \exp\left(-\frac{x^2}{2\alpha^2}\right). \tag{6}$$

Given that functions (5), (6) are symmetric relative to the coordinate origin, the approximation is to be carried out in the interval of values $x \in [0; \Delta/2]$.

Find $\mu(x_a)$ from equation (5) at the interval of values $x \in [0; \Delta/2]$. We obtain:

$$\mu(x_a) = -\frac{2}{\Delta}x_a + 1.$$

Once condition (4) is considered, we obtain:

$$\theta = -\frac{2}{\Delta}x_a + 1.$$

Find from the last equation:

$$x_a = \frac{(1 - \theta)\Delta}{2}. \tag{7}$$

Taking into consideration the value of x_a, which is determined from formula (7), the membership function (6) is as follows:

$$\mu_G(x_a) = \exp\left(-\frac{(1 - \theta)^2 \Delta^2}{8\alpha^2}\right).$$

Since $\mu_G(x_a) = \Theta$, then:

$$\exp\left(-\frac{(1 - \theta)^2 \Delta^2}{8\alpha^2}\right) = \theta.$$

Hence, we find:

$$\alpha^2 = -\frac{(1 - \theta)^2 \Delta^2}{8\ln\theta}, \tag{8}$$

where $0 < \Theta < 1$.

The analysis of formula (8) reveals that the concentration coefficient α to membership function (3) depends on the basis Δ of the triangular membership function and on the value of the ordinate, which is determined by the intersection point between membership functions (2) and (3) when $x \in [a_x - \Delta/2; a_x]$.

Since Δ is an *a priori* known quantity, the accuracy of the approximation of function (2) by function (3) would depend on the value of the ordinate Θ.

The accuracy of the approximation is determined as the sum of the squares of the deviation of the ordinates of function (6) from the corresponding ordinates of function (5):

$$E = \sum_{i=1}^{N}\left(\mu(x_i) - \mu_G(x_i)\right)^2. \tag{9}$$

where $x_i \in [0; \Delta/(2iT)]$; T is a sample step; n is the number of ordinates of the function $\mu(x)$ on segment $x \in [0; \Delta/2]$.

The parameter Θ is selected from the condition of a minimum for expression (9). To this end, we substitute in ratio (9)

the value of $\mu_G(x_i)$, which is determined from formula (6). In this case, we take into consideration the value of the quantity α^2 according to formula (8). As a result, we obtain:

$$E(\theta) = \sum_{i=1}^{n} \left(\mu_i - \exp\left(\frac{4x_i^2 \ln\theta}{(1-\theta)^2 \Delta^2} \right) \right)^2, \tag{10}$$

where $\mu_i = \mu(x_i)$.

The $E(\Theta)$ function is nonlinear; the Θ value, which minimizes (10), can only be found by a numerical method. Since known numerical methods yield only a local minimum [11] at a certain Θ change interval, we build a chart of the $E(\Theta)$ function (Fig. 2).
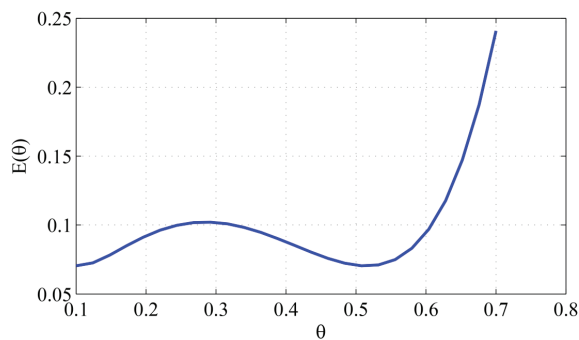

Fig. 2. Dependence of approximation error on the Θ value

The chart built for $\Delta=0.5$ shows that function (10) reaches its smallest value on the segment. To find a minimum of function (10), we use a golden-section method [12].

The following program settings were selected:
– a starting point for finding the local minimum interval, 0.4;
– an error of searching for a minimum of function (10), 10^{-6};
– the uncertainty interval of a fuzzy quantity, 0.5.

The result is the following solution to the problem:

$$\Theta^* = 0.5152; \ E(\alpha^*) = 0.703.$$

Fig. 3 illustrates the process of approximating function (5) by function (6).
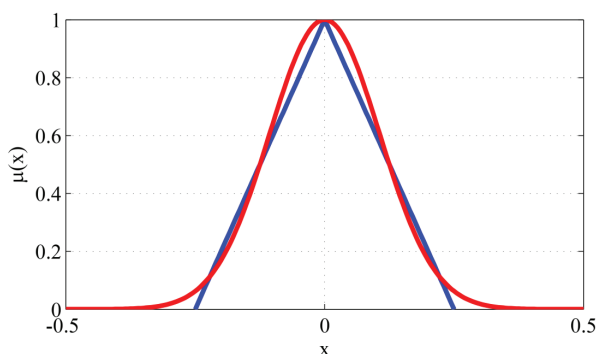

Fig. 3. Approximation of a triangular function by the Gaussian membership function

The analysis of Fig. 3 reveals that the value of Θ^* almost does not depend on the value of the uncertainty interval Δ, and the $E^*(\Delta)$ quantity is a monotonically ascending function that has a linear character.

A least-square method was applied to derive the following dependence coefficients:

$$E^*(\Delta) = \alpha_0 + \alpha_1 \Delta. \tag{11}$$

As a result, we obtained: $\alpha_0 = 0.0026$ and $\alpha_0 = 0.1355$.

In Fig. 4, «o» symbols mark the values derived from solving the problem to minimize function (10) («experimental» data); a solid line is constructed according to equation (11). In fact, we observe complete convergence between the «experimental» and estimation data, as evidenced by the value of the approximation error, calculated as the sum of squares of deviations of estimated values from the corresponding «experimental» data. The error of approximation was calculated according to the following formula:

$$\delta = \sum_{i=1}^{N} \left(E_i^* - E^*(\Delta_i) \right)^2$$
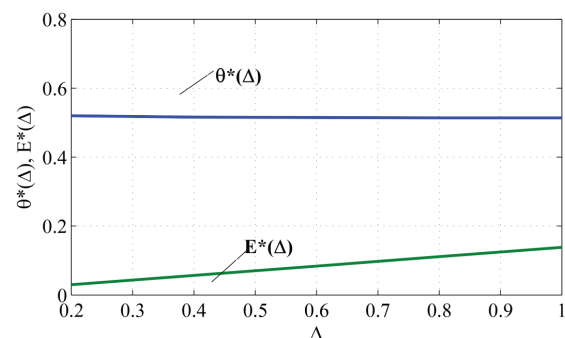
at $N=5$, it is $\Delta = 5.9 \cdot 10^{-8}$.

Fig. 4. Dependence of $\Theta^* = (\Delta)$ and $E^*(\Delta)$ on a change in the value of Δ

It should be noted that the authors of work [13] selected, without justification, based on intuitive considerations, the value $\Theta=0.5$. As it follows from Table 1, the value $\Theta=0.5$ does not differ much from the values of Θ^*, which were obtained from solving the minimization problem (9).

5. 2. Calculating the characteristic polynomial and transfer function of an open system taking into consideration the fuzziness of its parameters

In formula (1), the complex variable p is an explicit quantity. Since the parameters of the characteristic equation $a_i, \ i = \overline{0,n}$ are the fuzzy quantities, the polynomial $Q(p)$ is also a fuzzy quantity.

When performing the operations of adding Gaussian fuzzy numbers, as well as multiplying a Gaussian fuzzy number by an explicit quantity, the result is a Gaussian fuzzy number [7].

Thus, there is every reason to believe that $\hat{Q}(p)$ is a fuzzy quantity with the following membership function:

$$\mu(Q) = \exp\left(-\frac{(\hat{Q} - a_q)^2}{2\alpha_q^2} \right), \tag{12}$$

where a_q, α_q is the modal value and the concentration coefficient of a fuzzy quantity Q.

To find the a_q and α_q parameters, and a membership function (12), the following operations should be performed over

fuzzy numbers: adding fuzzy numbers and multiplying a fuzzy number by an explicit quantity.

Based on the rules for performing arithmetic operations involving fuzzy numbers [10], we adapt them for the case of Gaussian membership functions (3). Then any fuzzy number would be characterized by two parameters – a modal value and a fuzzy coefficient.

Work [14] proves that the operation of calculating the sum of fuzzy numbers and multiplying a fuzzy number by an explicit number is carried out using the following formula:

$$a_q = \sum_{i=0}^{n} \varphi_i \hat{a}_i, \quad \alpha_q = \sum_{i=0}^{n} \varphi_i \alpha_i. \tag{13}$$

For the case under consideration: $\varphi_i = p^{n-i}, i = \overline{0,n}.$

Additionally, we assume that the necessary conditions for the stability of a linear (linearized) dynamic system are met, that is, $a_i > 0, i = \overline{0,n}.$

Let γ be a slice for membership function (12). Then:

$$\exp\left(-\frac{\left(\hat{Q} - a_q\right)^2}{2\alpha_q^2}\right) = \gamma_q,$$

where $0 < \gamma_q \leq 1.$

Find from the last equation:

$$\hat{Q} = a_q + \alpha_q \sqrt{\ln\frac{1}{\gamma_q^2}}.$$

The value of the γ slice determines the degree of «blur» of a fuzzy quantity $\hat{Q}$. With an increase in the value of γ_q, the fuzziness of the function $\hat{Q}$ decreases, and, at $\gamma_q = 1$, the function becomes an explicit quantity and, conversely, with a decrease in γ_q, the uncertainty increases in the estimation of the parameters of characteristic polynomial (1).

If we take into consideration a_q and α_q, which are determined by ratios (13), we obtain a characteristic polynomial of the system, provided that the parameters of dependence (1) are treated as fuzzy quantities. Thus:

$$\hat{Q}(p) = \sum_{i=0}^{n} \hat{a}_i p^{n-i} + \sqrt{\ln\frac{1}{\gamma_q^2}} \sum_{i=0}^{n} \alpha_i p^{n-i},$$

or

$$\hat{Q}(p) = \sum_{i=0}^{n} \left(\hat{a}_i + a_\gamma \alpha_i\right) p^{n-i}, \tag{14}$$

where

$$a_\gamma = \sqrt{\ln\frac{1}{\gamma_q^2}}.$$

Since all parameters a_i of characteristic polynomial (1) are interpreted as fuzzy quantities with a triangular membership function (2), which are approximated by exponential function (3), then $\alpha_i, i = \overline{0,n}$ is to be calculated from formula (7):

$$\alpha_i = \eta \Delta_{q,i}, \quad i = \overline{0,n}, \tag{15}$$

where

$$\eta = \left(1-\theta\right)\left(8\ln\frac{1}{\theta}\right)^{-1/2}.$$

Taking into consideration formula (15), characteristic equation (14) takes the following form:

$$\hat{Q}(p) = \sum_{i=0}^{n} \left(\hat{a}_i + A_\gamma \Delta_{q,i}\right) p^{n-i}, \tag{16}$$

where $A_\gamma = a_\gamma \cdot \eta.$

If we take into consideration the values of a_γ and η, then

$$A_\gamma = \frac{1}{2}\left(1-\theta\right)\sqrt{\frac{\ln\gamma_q}{\ln\theta}}.$$

Let the transfer function of the dynamic system be assigned:

$$W(p) = \frac{R(p)}{Q(p)}, \tag{17}$$

where $R(p)$, $Q(p)$ are the polynomials of powers m and n, respectively ($m \leq n$).

Assume that the b_j coefficients of the polynomial

$$R(p) = \sum_{j=0}^{m} b_j p^{m-j}$$

are the fuzzy numbers with a triangular membership function, which is approximated by the Gaussian function (3). Then, based on the results obtained for the polynomial $Q(p)$, we obtain:

$$b_r = \sum_{j=0}^{m} \hat{b}_j p^{m-j}, \quad \alpha_r = \sum_{j=0}^{m} \alpha_{r,j} p^{m-j}, \tag{18}$$

where $\alpha_{r,j} = \eta \Delta_{r,j}.$

By analogy to formula (17), record:

$$\hat{R}(p) = \sum_{j=0}^{m} \left(\hat{b}_j + A_\gamma \Delta_{r,j}\right) p^{m-j}. \tag{19}$$

Since (16) and (19) are the fuzzy quantities, transfer function (18) is also a fuzzy quantity with the following membership function:

$$\mu(W) = \exp\left(-\frac{\left(\hat{W} - a_w\right)^2}{2\alpha_w^2}\right). \tag{20}$$

Find the modal value a_w and the amount of blur a_w. If we have a ratio between two fuzzy quantities [10], then:

$$a_w = \frac{b_r}{a_q}; \quad \alpha_w = \frac{a_q \alpha_r + b_r \alpha_q}{a_q^2}, \tag{21}$$

where a_q, b_r, α_q and α_r are calculated from formulas (15), (18).

If one sets a γ_w slice that characterizes the degree of fuzziness of the parameters of the transfer function of the dynamic system, then we find from equation (20):

$$\hat{W} = a_w + \alpha_w \sqrt{\ln\frac{1}{\gamma_w^2}}.$$

Considering formulas (22), we obtain:

$$\hat{W} = \frac{b_r}{a_q} + a_{w,\gamma}\frac{a_q \alpha_r + b_r \alpha_q}{a_q^2},$$

where $a_{w,\gamma} = \sqrt{\ln\frac{1}{\gamma_w^2}}.$

Taking into consideration the values of a_q, b_r, α_q, and α_r, which are calculated from formulas (15) and (18), we obtain:

$$\widehat{W} = \frac{\sum_{j=0}^{m} \widehat{b}_j p^{m-j}}{\sum_{i=0}^{n} \widehat{a}_i p^{n-i}} +$$

$$+ A_{w,\gamma} \frac{\sum_{i=0}^{n} \widehat{a}_i p^{n-i} \sum_{j=0}^{m} \Delta_{r,j} p^{m-j} + \sum_{j=0}^{m} \widehat{b}_j p^{m-j} \sum_{i=0}^{n} \Delta_{q,i} p^{n-i}}{\left(\sum_{i=0}^{n} \widehat{a}_i p^{n-i} \right)^2}, \qquad (22)$$

where

$$A_{w,\gamma} = \frac{1}{2}(1-\theta)\sqrt{\frac{\ln\gamma_w}{\ln\theta}}.$$

For the case when $\gamma_w=1$, the value $A_{w,\gamma}=0$, and then we come to the deterministic (classical) problem of determining the stability of the dynamic system.

The second term in formula (22) is a kind of «penalty», incurred as a result of taking into consideration the fuzziness in the parameters of the transfer function of the dynamic system.

The characteristic equation of the dynamic system is as follows:

$$Q(p) = a_0 p^3 + a_1 p^2 + a_2 p + a_3, \qquad (23)$$

where $a_0=2.4$, $a_1=4.0$, $a_2=2.5$, $a_3=3.0$.

For each a_i, $i=\overline{0,3}$ parameter, the following uncertainty intervals have been defined: $\Delta_{q,0}=0.45$, $\Delta_{q,1}=0.11$, $\Delta_{q,2}=0.10$, $\Delta_{q,3}=0.31$.

For the γ-slices, $\gamma_q=1$ (parameters of the characteristic equation of an explicit quantity) and $\gamma_q=0.15$, we built the charts of Mikhailov hodograph (Fig. 5).

Our analysis of Fig. 5 shows that the dynamic system maintains stability even when the parameters of characteristic equation (23) are considered as fuzzy quantities. However, the quality of the system, in terms of its stability, has significantly deteriorated; with further degradation of its parameters, the system can enter a non-steady state.

Next, assume an open system is represented by the following transfer function:

$$W(p) = \frac{b_0 p + b_1}{p(a_0 p^2 + a_1 p + a_2)}, \qquad (24)$$

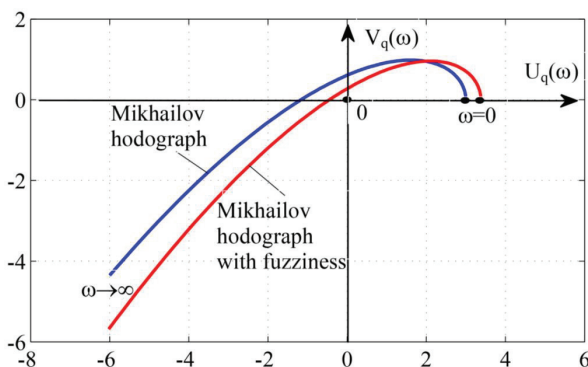where $a_0=4.0$, $a_1=2.5$, $a_2=3.0$, $b_0=2.0$, $b_1=1.4$.

Fig. 5. Mikhailov hodographs

The parameters of transfer function (25) were treated as fuzzy quantities with the following intervals of uncertainty: $\Delta_{q,0}=0.21$, $\Delta_{q,1}=0.4$, $\Delta_{q,2}=0.31$, $\Delta_{r,0}=0.26$, $\Delta_{r,1}=0.19$.

Employing the software written in the MATLAB programming environment, the Nyquist hodographs were constructed (Fig. 6) for two cases: $\gamma_w=1$ (the parameters of transfer function (24) are the explicit numbers); $\gamma_w=0.25$.
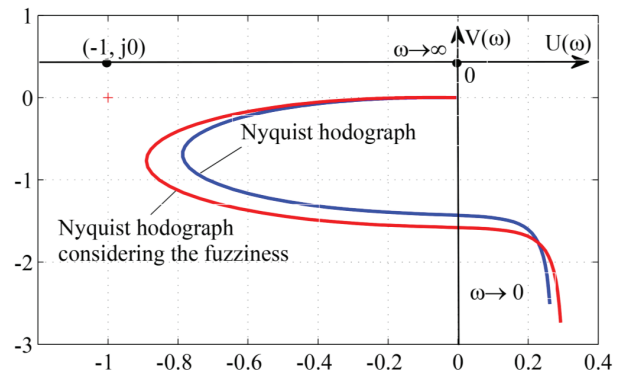

Fig. 6. Nyquist hodographs

Fig. 6 shows that a closed system would be stable at $\gamma_w=1$ and while the fuzzy parameters of transfer function (24) are accounted for. To assess the effect of fuzziness on the system's roughness, the system's stability reserves for amplitude and phase were determined. As a result, the following values were obtained:

– amplitude stability margin, $Gm=21.9$ dB; phase stability margin, $Pm=38.4$ degrees;

– $\gamma_w=0.25$: amplitude stability margin, $Gm=18.4$ dB; phase stability margin, $Pm=31.7$ degrees.

Taking into consideration the fuzziness in the parameters of transfer function (24) did not affect the stability of the closed system; however, there was a noticeable decrease in the system stability reserve both in phase and amplitude. The relative decrease in the margin of stability for amplitude was 16 %, and for phase – 17.4 %.

Thus, taking into consideration the fuzziness in the parameters of mathematical models would make it possible at the design stage to assess all the risks that may arise as a result of an uncontrolled change in the parameters of dynamic systems during their operation.

6. Discussion of results of studying the system stability when considering the fuzziness in the parameters of the transfer function

Our analysis of literary sources allows us to conclude that the parameters of the mathematical models of systems that are represented in the form of transfer functions can be determined with a certain accuracy only. There is always a danger of loss of stability by the system due to system degradation, which is manifested in the change of its parameters. To prevent such an event, systems are designed with a certain margin of stability for amplitude and phase. However, at the design stage, it is important to assess the risks that may arise in the case of inadequate knowledge of the parameters of the transfer function. To solve this problem, the parameters of the transfer function are proposed to be considered as fuzzy quantities with a triangular membership function (2), which

is inconvenient for practical use. Therefore, it is proposed to approximate the triangular membership function by the Gaussian function (3). As a result, formula (8) has been derived, which makes it possible to express the concentration coefficient through the uncertainty interval of the fuzzy quantity. Formula (8) is key in further research because it allows the application of fuzzy mathematics of the (L-R)-type.

That has made it possible to obtain meaningful results from studying systems for stability using the criteria by Mikhailov and Nyquist when considering the fuzziness in the parameters of the transfer function. An important result of our research is to identify the fact that the fuzziness in the parameters of the system's transfer function significantly affects its stability. A numerical experiment has shown that the phase and amplitude stability margin decreased by 17.4 % and 16 %, respectively.

In contrast to methods based on the stochastic theory of system stability, the reported method does not require knowledge of those laws that govern the distribution of parameters of dynamic systems' models that are is quite problematic to obtain in practice. It would suffice for each parameter to specify the interval of fuzziness only, which is not difficult to determine based on the practical experience of the researcher.

7. Conclusions

1. We have approximated a triangular function by the Gaussian membership function. A least-square method was used to determine the coefficients of dependence that equal $\alpha_0=0.0026$ and $\alpha_0=0.1355$. The error of approximation is $\Delta=5.9\cdot10^{-8}$.

2. The formulas were built for calculating the characteristic polynomial and the transfer function of an open system, taking into consideration the fuzziness of their parameters. That has made it possible to establish that taking into consideration the fuzziness in the parameters of the transfer function did not affect the stability of the closed system. However, there was a noticeable decrease in the system stability reserve both in terms of phase and amplitude. The relative decrease in the margin of stability for amplitude was 16 %, and for phase – 17.4 %.

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