Mathematical modeling and computer simulation methods have been used to investigate the extent of influence exerted by bio-clogging on the dynamics of excess head scattering in the soil massif. To this end, the classical equation of filtration consolidation has been modified for the case of variable porosity resulting from changes in the biomass. The numerical solution to the constructed mathematical model in the form of a nonlinear boundary problem was derived by a finite-element method. Numerical experiments were carried out and their analysis was performed. Specifically, this paper shows the charts of pressure differences in the soil array when neglecting bio-clogging and when estimating the effects exerted by bio-clogging at specific points in time. The numerical experiments demonstrated that in two years after the onset of the consolidation process in the neighborhood of the lower limit of the examined soil mass with a thickness of 10 meters, excess heads fall from the initial value of 10 m to 4 m. The greatest impact from the clogging of pores by microorganisms is revealed in the neighborhood of an upper limit. At a depth of 1 m, at t=180 days, the pressure difference reaches 2.4 m. This is about 200 % of the pressure distribution without taking into account the effects of bio-clogging. Over time, the effect of bacteria on the distribution of pressures in the neighborhood of the upper boundary decreases. However, this effect extends to the entire soil mass, up to the lower limit. Thus, at t=540 days, at the lower limit, the effect of bio-clogging leads to that excess heads are 1.8 m greater than for the case of pure water filtration (a relative increase of about 80 %).

Bio-clogging processes are intensified as a result of the development of microorganisms when organic chemicals enter the porous environment. Therefore, from a practical point of view, studying them is especially relevant for household waste storage facilities and the stability of their soil bases. It is advisable to undertake research by using the methods of mathematical modeling and computer simulation

Keywords: excess heads, bio-clogging, organic waste, finite element method, filtration consolidation

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CONSTRUCTING A MATHEMATICAL MODEL AND STUDYING NUMERICALLY THE EFFECT OF BIO-CLOGGING ON SOIL FILTRATION CONSOLIDATION

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1. Introduction

The problem of soil compaction has remained acute up to now and is likely to become more relevant due to the intensification of construction work. Applying external loads on the soil causes the emergence of excess heads in the porous liquid of the porous medium. Therefore, the compaction dynamics can be monitored through the dynamics of scattering of these excess heads. The presence of excess heads, for example, in the porous liquid of fully saturated soil, means the incompleteness of its (soil) consolidation. This is described as part of the so-called soil consolidation filtration theory, which was first reflected in the mathematical models by Karl Terzagi and Victor Florin [1]. The main provisions of the theory developed in the 1930s have been used to this day. Monitoring, as well as the parameters of filtration consolidation of soils, is a mandatory element in the State building codes and rules [2, 3]. All construction works are regulated by such official documents equated to the status of laws.

The dynamics of dissipation of excess heads over time, and, consequently, the dynamics of the compaction process, are determined by the filtration characteristics of a porous environment. The higher the filtration coefficient that is included in Darcy's law, the greater the rate of dissipation of excess heads. As a result, the soil enters a stabilized state faster. If some factors affect the reduction of the filtration coefficient (for example, the clogging of pores, an increase in the viscosity of the pore fluid, etc.), that then increases the time of dissipation of excess heads and the time of stabilization of the soil base of the structure.

Applied loads can include not only civilian or industrial buildings but also waste storage (both liquid and solid). If there are organic wastes in storage facilities, the development of microorganisms becomes one of the factors influencing the soil porosity, and, therefore, the filtration coefficient. The consequence of the development of microorganisms is the bio-clogging of pores in a porous environment. The presence of microorganisms in the soil cannot lead to additional excess heads but indirectly, due to a decrease in the filtration coefficient, leads to a slowdown in the dissipation of such pressures. Their (excess heads) presence leads to uneven soil subsidence in the bases of civil and industrial structures. Examples of field observations of uneven soil subsidence as a result of its filtration consolidation are given in paper [4] (research in Busan, South Korea, from 2007 to 2019). Paper [5] noted that the observed uneven soil subsidence due to consolidation was greater in industrial areas than in areas with civil buildings.

The authors of [6] investigated, among other factors, the impact of the development of microbes as one of the factors of denitrification in agricultural production. The cited work shows that bio-processes in porous environments, including the development of microorganisms, are important; studying them is an urgent task. This is evidenced by results reported in [7]. Paper [8] shows that filtration consolidation is a process that can lead to uneven soil subsidence in the foundations of civil and industrial structures.

Bio-clogging processes, as a result of the development of microorganisms, are intensified when organic chemicals enter the porous environment. Therefore, from a practical point of view, studying them is especially relevant for household waste storage facilities and the stability of their soil bases. And it is advisable to carry them out using the methods of mathematical modeling and computer simulation.

But, in fact, the compaction of the porous environment is also associated with a change in porosity due to the imposed loads. That is, if the porous environment falls into the zone of influence of organic waste, then, unlike the classic case, at least two factors affect the dynamics of changes in porosity – applied loads and microorganisms. Investigating this case requires the modification of the classical approach to the study of soil consolidation by mathematical modeling and computer simulation methods.

2. Literature review and problem statement

The studies of bio-clogging processes, the development of bio-films, and their impact on the parameters of porous environments are reported in detail in [9, 10]. However, the cited works do not build mathematical models of the specified processes; only experimental studies are described. The authors of [11, 12] derived some equations that are components of the mathematical model of the influence of microorganisms on filtration processes in soil environments. However, neither a general mathematical model of those biological processes nor solutions are given in the cited works. Also, now there are quite a lot of works where these effects are taken into consideration in mathematical and computer models in porous environments [13, 14].

However, among such models, there are no mathematical models of the impact of bio-clogging on the processes of dissipation of excess heads in the soils arising from the imposed external loads. Similarly, current studies of filtration consolidation processes by mathematical modeling and computer simulation methods [15–17] disregard the effects of bio-clogging. However, bio-clogging is associated with a decrease in the volume of pores, which, in turn, can affect the dynamics of consolidation processes that are also associated with changes in the volume of pore space. For a more detailed study, refutation or confirmation of the hypothesis suggested, more detailed studies of interrelated bio-clogging and consolidation processes are required.

Paper [18] investigated the processes of soil consolidation under the influence of inorganic chemicals. The peculiarity of the presence of inorganic chemical solutions in the pores of clay soils (for them the problem of consolidation is acute) is pronounced osmotic flows and the dependence of the filtration coefficient on their (chemicals) concentration. The latter feature is associated with the presence in clays of a large amount of bound water and filtration of electrolytes as a solution of an inorganic chemical. In addition, work [19] studied the influence of salinity-desalination processes on the dynamics of dissipation of excess heads. However, the cited works do not consider the peculiarities of soil consolidation in the case of propagation of organic substances. In the presence of organic substances in the soil, the development of microorganisms is intensified (which is fundamentally impossible in the presence of inorganic chemicals) and, as a result, bio-clogging of porous material occurs.

Bio-clogging leads to a change in the hydrophysical parameters of the porous environment. Specifically, the conductivity of the environment decreases, which is mathematically reflected by the reduction in the filtration coefficient. Examples of such studies confirming the above statement are given in works [20, 21]. In work [20], for example, it was established that the value of the soil filtration coefficient decreased by an order of magnitude by day 540 from the beginning of the experiment, while in [21] – already by day 12. A detailed overview of scientific papers on the effect of bio-clogging on the value of the filtration coefficient is given in [22]. Also, work [22] reported the corresponding mathematical model and studied the effect of bio-clogging on the value of pressure jumps in the geo barrier. The impact on the excess heads in the entire soil massif has not been investigated there. In addition, a classical equation is used as a consolidation equation. At the same time, work [23] proposes a methodology for deriving the consolidation equation taking into consideration the dynamic change in various impact factors. This procedure is also used in the study of cleaning processes in bio plateau filters. Article [24] focuses on the mathematical modeling of dynamic filtration consolidation processes in saturated geoporous environments based on the biparabolic equation of evolution and its fractional-differential analog. Work [25] reports a solution to the problem of compaction of the soil with a threshold gradient, which can be expressed by elementary functions. However, such an impact factor as bio-clogging is not taken into consideration.

Thus, bio-clogging, as one of the consequences of the development of microorganisms, leads to a change in the hydrological parameters of the soil in the bases of organic waste storage facilities. However, available mathematical models of soil compaction do not take into consideration such changes. That is why it is justified to conduct studies that would cover the issues on the assessment of the impact of microorganisms in soil pores on the dynamics of dissipation of excess heads. To this end, it is necessary to improve the mathematical model of filtration consolidation by taking into consideration the effect of bio-clogging in the pressure equation; to find numerical finite-element solutions to the corresponding nonlinear boundary problem for a system of differential equations; to conduct a series of numerical experiments and analyze them.

3. The aim and objectives of the study

The purpose of this study is to quantitatively assess the impact of the presence of microorganisms in soil pores on the dynamics of dissipation of excess heads in the soil massif, which is in a state of filtration consolidation. This would make it possible to draw conclusions about the degree of influence of bio-clogging on the compaction process.

To accomplish the aim, the following tasks have been set:

– to build a mathematical model of filtration consolidation taking into consideration the effect of bio-clogging;

– to find numerical finite-element solutions to the corresponding nonlinear boundary problem for a system of differential equations;

– to conduct a series of numerical experiments and analyze the results.

4. The study materials and methods

Our derivation of the modified filtration consolidation equation for the case of the effect of bio-clogging is based on the results reported in work [23]. In it, the generalized equation of filtration consolidation, taking into consideration the influence of technogenic factors, takes the following form

$$\frac{\partial e}{\partial t}+\frac{\partial e}{\partial \Theta}\cdot\frac{\partial \Theta}{\partial t}+\sum_{i=2}^{n}\frac{\partial e}{\partial s_i}\cdot\frac{\partial s_i}{\partial t}+e\left(\frac{1}{\rho_p}\frac{d\rho_p}{dt}-\frac{1}{\rho_m}\frac{d\rho_m}{dt}\right)=$$
$$=(1+e)\left(\nabla\cdot\left(k_h(t,S)\nabla h\right)-\nabla\cdot F_{osm}\right), \qquad (1)$$

where:

– e – soil porosity coefficient;

– Θ – the sum of the main stresses in the skeleton of a porous environment;

– $S=(\Theta, s_2, ..., s_n)$ – impact factors influencing a change in porosity and filtration coefficient k_h;

– F_{osm} – osmotic flow;

– ρ_p, ρ_m – water density and solid, water-solute components density;

– h – excess heads in the porous liquid.

The following assumptions have been accepted. First, a one-dimensional case with one spatial variable x is considered. Second, suppose that a chemical solution of organic matter enters the soil pores. Such chemical solutions may be the result of impregnation or surface effluents from waste storage facilities. The possibility of such a scenario in the case of disposal of solid municipal waste is indicated, for example, in review paper [26]. The possibility of such filtration flows is also mentioned in work [27], which investigates the BIOLEACH model for assessing the production of biogas in real time.

For organic substances, osmotic flows are not characteristic; $F_{osm}\equiv 0$. Also, solid water-solute components are absent; $d\rho_m/dt\equiv 0$. The concentration of the chemical organic solution is small; $d\rho_p/dt\equiv 0$. Besides, among the factors of influence, we would consider only bio-clogging, that is, a change in the biomass of microorganisms in a porous environment.

Numerical methods, namely a finite-element method (FEM), were used to find an approximate solution to the corresponding nonlinear boundary problem. FEM makes it possible to sample the problem for spatial variables. As a result, we obtain a Cauchy problem for a system of first-order nonlinear differential equations relative to unknown functions that depend only on time. To find a solution to the Cauchy problem, time-based sampling techniques are used. In the numerical experiments, piecewise quadratic functions were used as basic FEM functions while a completely implicit linearized difference scheme was applied as a sampling scheme in time.

Glucose solution was taken as an organic substance for our numerical experiments. Data from the scientific literature were employed in the dependences of the filtration coefficient on porosity and to convert the biomass to volume.

5. Results of investigating the problem of filtration consolidation, taking into consideration the effect of bio-clogging, using a finite-element method

5. 1. Construction of a mathematical model of the compaction of saturated soil, taking into consideration the effect of bio-clogging

Considering the accepted assumptions and neglecting the creeping soil skeleton, equation (1) produces the following:

$$\gamma a\frac{\partial h}{\partial t}+\frac{\partial e}{\partial B}\frac{dB}{dt}=(1+e)\frac{\partial}{\partial x}\left(k(e)\frac{\partial h}{\partial x}\right), \qquad (2)$$

where a is the compression coefficient of the soil in compression dependence (for a one-dimensional case, $e=-a\Theta+e_0$); B – the biomass of microorganisms in the soil (reduced to the unit of the volume of the porous environment).

The process of filtration consolidation for the case of the distribution of organic chemicals is investigated in the soil layer with a total thickness of l (Fig. 1).

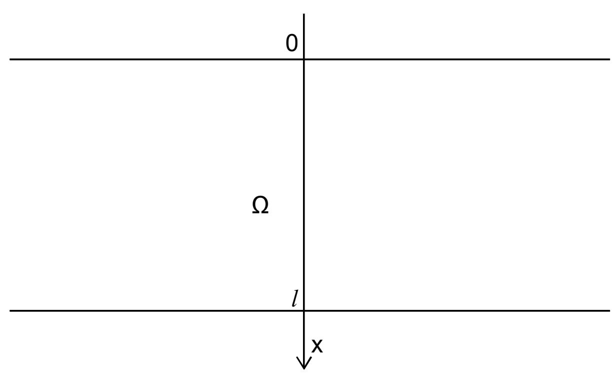

Fig. 1. Examined soil layer

A mathematical model would include the equation above (2) of filtration consolidation. Also, its components should be the equation for the transfer of organic chemicals in the porous liquid of a porous medium and the equation of the dynamics of biomass of bacteria in a porous environment based on the Mono equation [22]. As a result, we obtain the following boundary problem:

$$\gamma a\frac{\partial h}{\partial t}+\frac{\partial e}{\partial B}\frac{dB}{dt}=(1+e)\frac{\partial}{\partial x}\left(k(e)\frac{\partial h}{\partial x}\right),$$

$$x\in\Omega,\ \ t>0, \tag{3}$$

$$h(x,t)\big|_{x=0}=\bar{h}_0(t),\ \ t\geq 0, \tag{4}$$

$$u(x,t)\big|_{x=l}=\left(-k(e)\frac{\partial h}{\partial x}\right)\bigg|_{x=l}=0,\ t\geq 0, \tag{5}$$

$$h(x,0)=h_0(x),\ x\in\Omega, \tag{6}$$

$$n\frac{\partial c}{\partial t}=\frac{\partial}{\partial x}\left(nD\frac{\partial c}{\partial x}\right)-u\frac{\partial c}{\partial x}-\frac{\mu_{\max}}{Y}B\left(1-\frac{B}{B_{\max}}\right)\frac{c}{k_c+c},$$

$$x\in\Omega,\ t>0, \tag{7}$$

$$c(x,t)\big|_{x=0}=\bar{c}_0(t),\ t\geq 0, \tag{8}$$

$$q_c(x,t)\big|_{x=l}=-nD\frac{\partial c}{\partial x}\bigg|_{x=l}=0,\ \ t\geq 0, \tag{9}$$

$$c(x,0)=c_0(x),\ \ x\in\Omega, \tag{10}$$

$$\frac{\partial B}{\partial t}=\mu_{\max}B\left(1-\frac{B}{B_{\max}}\right)\frac{c}{k_c+c}-\mu_{dec}B,$$

$$x\in\Omega,\ \ t>0, \tag{11}$$

$$\mathrm{B}(x,0)=B_0(x),\ \ x\in\Omega, \tag{12}$$

where:

– h – excess head;

– k – filtration coefficient that depends on the porosity coefficient e;

– u – filtration speed;

– n – soil porosity;

– q_c – a flow of chemicals;

– μ_{dec} – a biomass decay coefficient;

– c – the concentration of the porous solution of organic chemical;

– D – an organic matter diffusion coefficient;

– $\mu_{\max}$ – the maximum level of biomass growth;

– B – the concentration of bacteria biomass in a porous environment;

– $B_{\max}$ – the maximum possible concentration of biomass of bacteria in a porous environment;

– k_c – a parameter in the equation of the biomass dynamics at which the level of biomass growth equals half the maximum;

– Y – bacterial output.

5. 2. Numerical solution to a filtration consolidation problem taking into consideration the effect of bio-clogging using a finite-element method

Similarly to [28], let H_0 be the space of vector-functions $\{s_1(x);\ s_2(x);\ s_3(x)\}$, whose each component in the interval (0;1) belongs to the Sobolev space $W_2^1(\Omega)$, and the first two acquire zero values at the ends of the segment [0,l], where the functions $h(x,t)$ and $c(x,t)$ are given boundary conditions of the first kind, respectively. Let H be the space of functions $\{\upsilon_1(x,t);\ \upsilon_2(x,t);\ \upsilon_3(x,t)\}$, whose each component is integrated with a square along with its first derivatives in the interval (0; 1)

$$\frac{\partial v_i}{\partial t},\ \ \frac{\partial v_i}{\partial x},\ \ i=\overline{1,3},\ \forall t\in(0;T].$$

Moreover, the first two satisfy the boundary conditions of the first kind, as well as the functions $h(x,t)$, $c(x,t)$, respectively. Here, $T>0$.

Take $\{s_1(x);\ s_2(x);\ s_3(x)\}$ belongs to H_0. Multiply equation (3) and the initial condition (6) by $s_1(x)$, integrating them in the segment [0, l]; we obtain

$$\int_0^l \gamma a\frac{\partial h}{\partial t}s_1(x)\mathrm{d}x+\int_0^l\frac{\partial e}{\partial B}\frac{\partial B}{\partial t}s_1(x)\mathrm{d}x+$$
$$+\int_0^l(1+e)k(e)\frac{\partial h}{\partial x}\frac{ds_1(x)}{dx}\mathrm{d}x=0,\qquad \forall t\in(0;T], \tag{13}$$

$$\int_0^l h(x,0)s_1(x)\mathrm{d}x=\int_0^l h_0(x)s_1(x)\mathrm{d}x,\ \ \forall t\in(0;T]. \tag{14}$$

Similarly, for the concentration of the organic chemicals and biomass, we obtain

$$\int_0^l n\frac{\partial c}{\partial t}s_2(x)\mathrm{d}x+\int_0^l nD\frac{\partial c}{\partial x}\frac{ds_2(x)}{dx}\mathrm{d}x+$$
$$+\int_0^l u\frac{\partial c}{\partial x}s_2(x)\mathrm{d}x+\qquad \forall t\in(0;T], \tag{15}$$
$$+\int_0^l\frac{\mu_{\max}}{Y}\left(1-\frac{B}{B_{\max}}\right)B\frac{c}{k_c+c}s_2(x)\mathrm{d}x=0,$$

$$\int_0^l c(x,0)s_2(x)\mathrm{d}x=\int_0^l c_0(x)s_2(x)\mathrm{d}x,\ \forall t\in(0;T], \tag{16}$$

$$\int_0^l\frac{\partial B}{\partial t}s_3(x)\mathrm{d}x-$$
$$-\int_0^l\left(\mu_{\max}\left(1-\frac{B}{B_{\max}}\right)\frac{c}{k_c+c}-\mu_{dec}\right)Bs_3(x)\mathrm{d}x=0,$$
$$\forall t\in(0;T], \tag{17}$$

$$\int_0^l B(x,0)s_3(x)\mathrm{d}x=\int_0^l B_0(x)s_3(x)\mathrm{d}x,\ \ \forall t\in(0;T]. \tag{18}$$

Definition 1. The function $\{h(x,t);\ c(x,t);\ B(x,t)\}$ that belongs to H, which, for any $\{s_1(x);\ s_2(x);\ s_3(x)\}$, satisfies the integrated ratios (13) to (18), is termed a generalized solution to boundary problem (3) to (12).

An approximate generalized solution to boundary problem (3) to (12) is to be found in the following form:

$$h(x,t)=\sum_{i=1}^N h_i(t)\phi_{i1}(x),\ \ c(x,t)=\sum_{i=1}^N c_i(t)\phi_{i2}(x),$$

$$B(x,t)=\sum_{i=1}^N B_i(t)\phi_{i3}(x), \tag{19}$$

where $h_i(t)$, $c_i(t)$, $B_i(t)$ are the unknown coefficients that depend only on time. If we consider (19) as an approximate finite-element solution, then the functions $\varphi_{i1}(x)$, $\varphi_{i2}(x)$, $\varphi_{i3}(x)$ are the polynomial basis functions with a finite carrier. Similarly to [22], in order to simplify further considerations, and in accordance with the numerical experiments, we shall assume that for the approximate search for $h(x, t)$, $c(x, t)$, and $B(x, t)$ the same grid of finite elements is used. In this case, $\varphi_{i1}(x)\equiv\varphi_{i2}(x)\equiv\varphi_{i3}(x)$, and, in order to avoid double indexing, we denote the specified basis functions through $\varphi_i(x)$, $i=\overline{1,N}$.

Next, given the weak statement of (13) to (18) of problem (3) to (12), taking into consideration (19), we obtain

$$M_1\cdot\frac{dH}{dt}+M_{13}\cdot\frac{dB}{dt}+L_1(B)\cdot H(t)=0, \qquad (20)$$

$$M_1^{'}\cdot H^{(0)}=F_1^{'}, \qquad (21)$$

$$M_2(B)\cdot\frac{dC}{dt}+L_2(H,B)\cdot C(t)=0, \qquad (22)$$

$$M_2^{'}\cdot C^{(0)}=F_2^{'}, \qquad (23)$$

$$M_3\cdot\frac{dB}{dt}+L_3(C,B)\cdot B(t)=0, \qquad (24)$$

$$M_3^{'}\cdot B^{(0)}=F_3^{'}, \qquad (25)$$

Where

$$H=\left(h_i(t)\right)_{i=1}^{N},\ \ C=\left(c_i(t)\right)_{i=1}^{N},\ \ B=\left(b_i(t)\right)_{i=1}^{N},$$

$$H^{(0)}=\left(h_i(0)\right)_{i=1}^{N},\ \ C^{(0)}=\left(c_i(0)\right)_{i=1}^{N},\ \ B^{(0)}=\left(b_i(0)\right)_{i=1}^{N},$$

$$M_k=\left(m_{ij}^{(k)}\right)_{i,j=1}^{N},\ \ L_k=\left(l_{ij}^{(k)}\right)_{i,j=1}^{N},\ \ F_k=\left(f_i^{(k)}\right)_{i=1}^{N},$$

$$F_k^{'}=\left(f_i^{'(k)}\right)_{i=1}^{N},\ \ M_k^{'}=\left(m_{ij}^{'(k)}\right)_{i,j=1}^{N},$$

$$m_{ij}^{'(k)}=\int_0^l\varphi_i\varphi_j\,\mathrm{d}x,\ \ k=\overline{1,3},\ \ M_{13}=\left(m_{ij}^{(13)}\right)_{i,j=1}^{N},$$

$$m_{ij}^{(1)}=\int_0^l\gamma a\phi_i\phi_j\,\mathrm{d}x,\ \ f_{ij}^{'(1)}=\int_0^l h_0\varphi_i\,\mathrm{d}x,$$

$$l_{ij}^{(1)}=\int_0^l(1+e)k(e)\frac{d\phi_i}{dx}\frac{d\phi_j}{dx}\,\mathrm{d}x,$$

$$m_{ij}^{(13)}=\int_0^l\frac{\partial e}{\partial B}\varphi_i\varphi_j\,\mathrm{d}x,$$

$$m_{ij}^{(2)}=\int_0^l n\phi_i\phi_j\,\mathrm{d}x,\ \ f_{ij}^{'(2)}=\int_0^l c_0\varphi_i\,\mathrm{d}x,$$

$$l_{ij}^{(2)}=\int_0^l nD\frac{d\phi_j}{dx}\frac{d\phi_i}{dx}\,\mathrm{d}x+\int_0^l u\frac{d\phi_j}{dx}\phi_i\,\mathrm{d}x+$$
$$+\int_0^l\frac{\mu_{\max}}{Y}\left(1-\frac{B}{B_{\max}}\right)\frac{B}{k_c+c}\phi_j\phi_i\,\mathrm{d}x,$$

$$m_{ij}^{(3)}=\int_0^l\phi_i\phi_j\,\mathrm{d}x,\ \ f_{ij}^{'(3)}=\int_0^l B_0\varphi_i\,\mathrm{d}x,$$

$$l_{ij}^{(3)}=-\int_0^l\left(\mu_{\max}\left(1-\frac{B}{B_{\max}}\right)\frac{c}{k_c+c}-\mu_{dec}\right)\phi_j\phi_i\,\mathrm{d}x.$$

The system of equations (20) to (25) is a Cauchy problem for the system of nonlinear differential equations of the first order. Finding a solution to it also requires the use of appropriate sampling schemes, which are described in detail in works [22, 29].

5. 3. Results of the numerical experiments to solve the problem of filtration consolidation taking into consideration the effect of bio-clogging

In the model problem, we considered a layer of soil with a thickness of l=10 m. The step on the variable x was 0.02 m. The step on the time τ=1 day, a=5.12·10^{-5} m^2/H is the coefficient of soil compressibility, γ=10^4 H/m^3 is the specific weight of the pore solution. The initial distribution of heads is $h_0(x)$=10 m, which corresponds to the application to the soil surface of the corresponding load – wastes that are a source of organic matter. At the upper limit, an unhindered outflow of pore fluid is provided, and there is no drainage at the lower limit. Silt Loam [30] was considered as the soil at: k_0=0.108 m/day, n_0=0.45, where the index "0" means the initial values. Also, $e_0=n_0/(1-n_0)$.

Glucose solution [22] was considered an organic solution. The initial distribution of glucose concentration in the porous water is $c_0(x)$=1 mM. The glucose diffusion coefficient is D=0.5184·10^{-4} m^2/day.

The following dependence from [31] is used for the dependence of filtration coefficient on porosity:

$$\frac{k}{k_0}=\left(\frac{n_0-n_B}{n_0}\right)^{19/6},$$

where n_B is the volume of biomass per unit of the volume of the porous medium. To convert the concentration of biomass to volume, we accept the assumption that the biovolume is 80 % water, and the remaining 20 % of the dry mass is 50 % carbon (bio-carbon) [32]. In other words,

$$n_B=\frac{B}{0.8\rho_w+0.2\cdot{}^{\rho_c}\!/_2},$$

where ρ_w is the density of water; ρ_c – density of bio-carbon. In the numerical experiments, ρ_c=100 kg/m^3. The parameters of the biomass kinetics are as follows: $\mu_{\max}$=0.2 day^{-1}, $B_{\max}$=100 kg/m^3, μ_{dec}=0.01 day^{-1}, k_c=0.06 mM, Y=50. Given the dependence $e=n/(1-n)$, we obtain

$$\frac{\partial e}{\partial B}=\frac{\partial e}{\partial n}\cdot\frac{\partial n}{\partial B}=-\frac{1}{(1-n)^2}\frac{1}{0.8\rho_n+0.2\frac{\rho_c}{2}},$$

where $n=n_0-n_B$.

Piecewise quadratic functions were used as the FEM basis functions. The results of our numerical experiments are shown in Fig. 2–4.

Fig. 2–4 shows, as an example, the results from the numerical experiments at t=180 days, 360 days, 540 days, and 720 days.

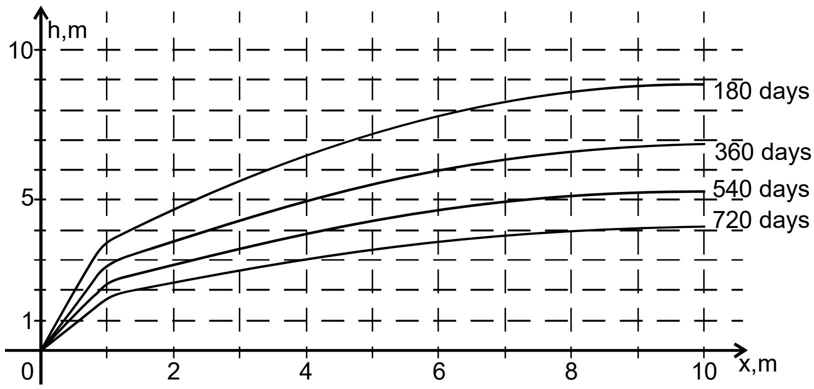

Fig. 2. Distribution of excess heads under the influence of bio-clogging

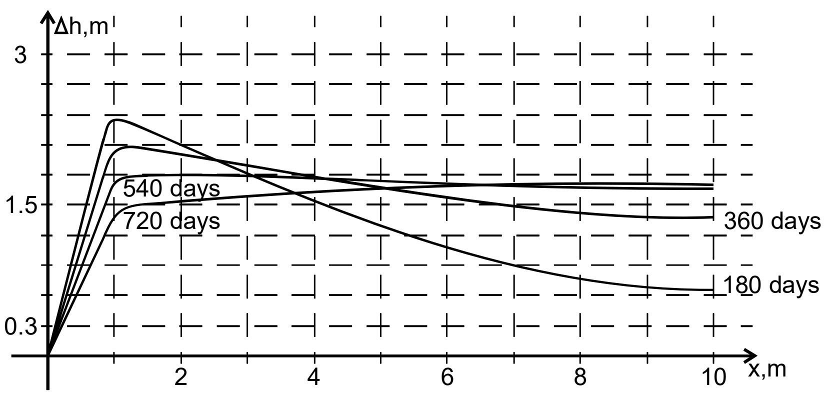

Fig. 3. Distribution of excess head difference considering and disregarding the effect of bio-clogging

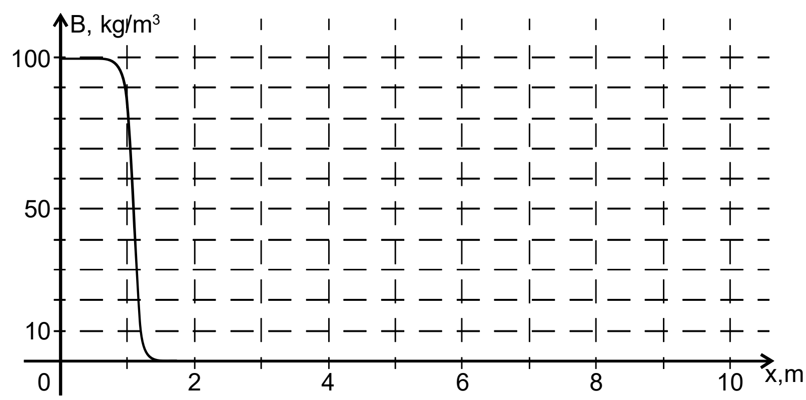

Fig. 4. The distribution of bacteria biomass at a time of 720 days

6. Discussion of results of solving a filtration consolidation problem taking into consideration the effect of bio-clogging

Fig. 2 shows that in two years (720 days) after the onset of the consolidation process in the lower boundary of the soil massif, excess heads fall from the initial value of 10 m to 4 m. The effect of bio-clogging itself is best traced from Fig. 3. It shows the charts of excess head differences in case of taking into consideration the effects of bio-clogging and without taking them into consideration, respectively. Therefore, positive values of head differences in Fig. 3 mean that when bio-clogging is taken into consideration, the heads dissipate slower than in case of neglecting such a phenomenon. Slowing down the head dissipation physically means unfinished consolidation of the soil mass. The greatest impact exerted by the clogging of pores by microorganisms is in the neighborhood of an upper limit. This is logical because nutrients enter the soil from the upper limit and it is at a shallow depth (Fig. 4) that intensive development of bacteria biomass occurs. Fig. 3 shows that at a depth of 1 m, at t=180 days, the head difference reaches 2.4 m. That is about 200 % of the head distribution without taking into account the effect of bio-clogging. The insignificant depth of penetration of bacteria biomass is explained by the nature of the spread of organic substances. Fig. 2 shows that the filtration rate of the pore fluid would be directed towards the upper limit (in the direction of the head anti-gradient). Therefore, the main role in the spread of organic matter in the soil would belong to the diffusion component. Given the low diffusion coefficient of the glucose solution, organic matter would penetrate the soil base only by 2 meters in two years. And that affects the dynamics of the development of microorganisms, the main role for which belongs to the presence of nutrients. Over time, the effect of bacteria on the distribution of heads in the neighborhood of the upper boundary decreases. However, this effect extends to the entire soil mass, up to the lower limit (charts in Fig. 3 at t=540 days, t=720 days). Thus, at t=540 days, at the lower limit, the effect of bio-clogging leads to the fact that excess heads are 1.8 m greater than for the case of pure water filtration (a relative increase of about 80 %).

From a physical point of view, bio-clogging, which means a decrease in the volume of pores, should lead to a slowdown in the dissipation of excess heads (after all, the filtration coefficient also decreases). Such conclusions are confirmed by the results from the numerical experiments (for example, Fig. 3). The distribution of the head difference in Fig. 3 actually confirms the physics of the process. However, to quantify the impact, one needs to find solutions to the refined mathematical model. An example of the numerical experiment shows that exposure to bio-clogging causes changes in heads, by 80–200 %, compared to pure water filtration. Despite the spread of microorganisms only to a depth of 2 meters, as shown by charts in Fig. 3, the effect of bio-clogging in 1.5–2 years extends to heads to a depth of 10 meters.

However, for the possibility of carrying out such calculations for actual conditions, the hydrological characteristics of soils and the parameters of microorganism growth dynamics should be known. This study could be advanced by increasing the dimensionality of the problem, improving the models of the development of microorganisms, and proposing specific engineering solutions to reduce the effect of bio-clogging on the excess heads in the soil.

7. Conclusions

1. We have improved the mathematical model of filtration consolidation during the filtration of organic matter in the soil array, taking into consideration the effect of bio-clogging. At the same time, the equation of filtration consolidation has been modified taking into consideration the impact exerted on a change in porosity by the dynamics of biomass development. The Mono equation was used as an equation for the dynamics of bacteria biomass in a porous environment.

2. The numerical solutions to the corresponding nonlinear boundary problem were found by using a finite-element method. Computer simulation has shown that the presence of microorganisms in the soil's pores significantly affects the dynamics of dissipation of excess heads. Specifically, bio-clogging (based on the solution to the model problem) leads to a relative slowdown in the dissipation of heads by 80–200 % compared to the case of neglecting the microorganisms. Such influences should be taken into consideration in the studies of landslides that are exposed to organic chemicals because the presence of excess heads could lead to the intensification of shear processes. Estimating the extent of such impacts on landslide events is the essence of a separate task of research in the mechanics of porous environments.

3. The results of our experiments showed the adequacy of the improved mathematical model to the processes studied – a slowdown in the dissipation of pressures under the influence of bio-clogging. In practice, our research could be used in the preparation of design documentation and structural solutions for organic waste storage facilities. The development of microorganisms and the process of bio-clogging of pores of a porous environment with biomass leads to a slowdown in the dissipation of excess heads. After all, excess heads are a negative phenomenon in general, which leads to disrupting the stability of the soil foundations of buildings.

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