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DETERMINING THE WIDTH OF A LAYER CUT WITH SAWS WITH MULTIDIRECTIONAL TEETH

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To predict the workability of a tool structure at the design stage, it is necessary to calculate the parameters of the cut layer when this tool is used because the cut layer's size determines the strength and dynamic characteristics of the cutting process.

It is known that the size and shape of the cut layer are affected by the allowance cutting scheme embedded in the tool design. Therefore, the parameters of the cut layer with the tool must be investigated taking into consideration the actual shapes and location of the cutting edges of the tool teeth and the cutting scheme with individual teeth.

Existing analytical dependences on determining the thickness of the cut layer do not take into consideration the group arrangement of the teeth, which have a different shape and location of their cutting edges. Therefore, a procedure for determining the thickness of the cut layer analytically has been proposed, using the example of circular saws with multidirectional teeth while taking into consideration the patterns in the arrangement of the cutting edges of individual teeth and the real movements of the tool during its operation.

The proposed procedure makes it possible to determine the parameters of the layer cut with the tool at both constant and progressive allowance cutting schemes. One can also specify the parameters of the cut layer at any time of the tool's operation and analyze the change in the shape of the slice in time.

Based on the analysis of the parameters of the cut layer, it has been established that saws with multidirectional teeth do not work with the entire width of the cutting edge but only in its part, whose share does not exceed 55 % of the width of the tool.

The procedure reported here could be used to determine the loading of the cutting tool part with a more complex cutting scheme, which also includes tools that are operated by the form-generating method

Keywords: thickness of the cut layer, circular saw, allowance cutting scheme, cutting edge, cutting edge shape, multidirectional teeth

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1. Introduction

When designing new or improving existing tool structures, the task of predicting their workability arises at the design stage. The best method for establishing workability is laboratory and industrial research, which also makes it possible to determine the impact of the design parameters of the tool and machining modes on the quality characteristics of a workpiece. However, conducting such research in full requires significant economic and time costs.

It is possible to reduce the cost of research using computer simulation methods that make it possible to investigate the tool's performance in the process of its operation and evaluate the characteristics of the machined surfaces. Such methods, although somewhat simplified due to certain assumptions, nevertheless reflect the relationship of the parameters under study and make it possible to determine the further direction of research.

The construction of models for cutting and forecasting the tool workability requires an accurate description of the characteristics of the cutting process, which are significantly influenced by the parameters of the cut layer. The parameters

of the cut layer determine the force loads on the tool, affect the power of the heat generation in the cutting area. Changing the area of the cut layer in time causes a corresponding change in cutting efforts, introduces an additional dynamic component in the cutting process, which negatively affects the quality of the machined surface and the tool stability.

Determining the parameters of the cut layer is a relevant task, the solution of which could make it possible already at the stage of designing the tool to predict the parameters of the cutting process, to analyze the possibility of reducing vibrations by making changes to the tool design.

2. Literature review and problem statement

Works [1–3] report a general approach to determining the layer cut with the tool and specify those factors that influence its formation. In addition, they give analytical dependences for determining the parameters of the layer cut with various tools, which include the cross-section, thickness, width, and length of the cut layer [4]. However, when determining the parameters of the layer cut with cutter

blades, which include circular saws, simplifications are accepted. The process of cutting with cutter blades includes two movements: the main one is the rotation of the cutter around the axis, and the movement of the feed – the translational motion. Thus, the trajectory of the teeth of the cutter would be a cycloid. In works [1–3], the trajectory of the teeth of the tool is considered in the form of a circle arc, which greatly simplifies the calculations. However, the tools working on a single scheme of cutting allowances are considered, in which the cutting edge of each tooth, in shape, size, and position, does not differ from other teeth. In works [1–3], in addition, it is noted that the parameters of the cut layer characterize the loading of the cutting edges of the tool and, together with the cutting speed, determine the strength, dynamic, and thermal characteristics of the cutting process.

The studies reported in [5, 6] show that the thickness of the cut layer is a defining factor that affects the cutting forces during milling. In [5], the impact of the thickness of the cut layer on the medium and instant cutting forces was investigated. However, the calculations of the thickness of the cut layer were also carried out according to approximate dependences, similar to works [1–3]. Paper [6] reports a procedure of modeling the cutting forces during milling based on determining the thickness of the cut layer accurately. In addition, unlike previous works, cycloidal movement is adopted as the trajectory of the teeth of the cutter. However, a standard cutter is considered, in which all the teeth have the same shape and arrangement of the cutting edge.

In addition, papers [7, 8] show that the dynamic characteristics of the cutting process are also affected by the nature of the change in the thickness of the cut layer [7] and the rate of its growth and descent [8]. However, the cited papers investigated only those tools that operate under a single scheme of allowance cutting.

The influence of the parameters of the cutting modes, the cutting strategy, and the cut layer on the force characteristics of the cutting process is addressed in a series of works [9–11]. They describe various approaches to clarifying the models of chipping and their impact on the cutting process at sharpening [9], milling [10], and micro-cutting [11]. Various strategies, cutting modes, and assumptions for tools with a single allowance cutting scheme are considered.

Paper [12] describes a method of calculating the thickness of the cut layer during end milling using circular interpolation compared to a linear one. Work [13] modeled the trajectory of the end milling tooth for linear, arc, and profile milling and derived dependences on the thickness of the cut layer, the angles of entry and exit of the tool teeth from a workpiece. In the above works, the dependences of determining the parameters of the cut layer were clarified, in accordance with the trajectories of the tools. However, standard tool designs are considered, which operate according to a single allowance cutting scheme.

In order to determine the parameters of the cut layer with higher accuracy, a new approach was suggested in work [14]. The authors proposed, instead of solving the systems of equations of tooth trajectories by a numerical method, approximating them by a Taylor series, thereby simplifying the acquired dependences.

Existing procedures, while taking into consideration the actual trajectory of the teeth of the cutter, however, are intended for tools that work under a constant scheme of cutting allowance. The teeth of such tools have the same shape, size, and arrangement, as, for example, in circular saws [15].

Along with this, there are tools operated under a group allowance cutting scheme in which the shape and/or dimensions of the cutting edge of the teeth in the group differ from each other.

These tool designs include circular saws [16], the teeth of which are grouped into two (Fig. 1, *a*) or more (Fig. 1, *b*). According to studies [2, 3], the nature of the formation of the cut layer and its parameters for such tools differs from tools with a single allowance cutting scheme.

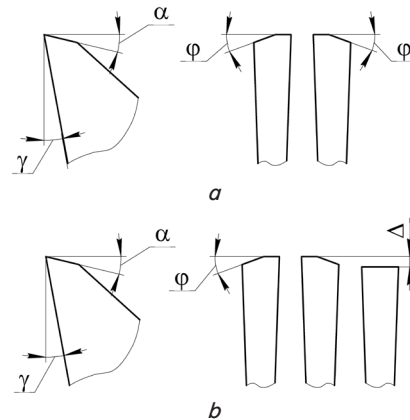


Fig. 1. The shape of saw teeth [16]:
a – a group of two teeth; *b* – a group of three teeth

Using the group scheme of the arrangement of the teeth, as noted in works [1–3], reduces the load on each individual tooth and on the tool in general. In addition, this allowance cutting scheme could reduce the cutting forces [1–3]. However, the characteristics of the cut layer for such tools are not determined, which is associated with the difficulties of calculations.

Study [17] was conducted to determine the effect of the shape of the saw teeth on the cutting speed with the tool; it showed that by changing the shape of the teeth, it is possible to achieve a decrease in the vibrations of the saw during its operation, as well as greater dynamic stability. However, the study considered only those saws in which the teeth were of the same shape and size. A group scheme of teeth arrangement was not investigated.

The cutting edge of saw teeth (Fig. 1) is determined by the arrangement of the front and rear surfaces of the teeth [4]. The front surface is arranged at the front angle γ , the rear surface is arranged at two angles: the rear angle α and the angle of the bevel; the rear surface is, accordingly, the surface of the general position. Thus, the cutting edge of such cutters would be the line of the general position. The position of the cutting edge would be determined by the angle in the plan and the angle of inclination of the edge. In addition, the teeth are alternately beveled [16], that is, alternate one by one.

This arrangement of the saw teeth leads to that the saws described in [16], unlike the saws applied in [15], operate in the conditions of oblique cutting. And, as noted in works [1–3], the oblique cutting leads to a decrease in the thickness of the cut layer, the force and dynamic characteristics of the cutting process.

The effect of oblique cutting on the cutting process was investigated in paper [18]. Experimentally, it has been established that increasing the angle of inclination of the edge λ reduces the thickness of the cut layer.

Thus, the arrangement of the cutting edge at the angle of inclination of the edge λ helps improve the cutting conditions.

However, the group scheme of cutting the allowance was not investigated since this arrangement of the teeth (two in the group (Fig. 1, *a*) and three (Fig. 1, *b*)) causes difficulties in determining the parameters of the cut layer.

Given the fact that each saw tooth (Fig. 1, *a*) accepts different values of angles in the plan φ and the edge inclination λ , each tooth would have its own cutting surface during the cutting process. This surface would be different from the cutting surface of the preceding tooth. Therefore, determining the thickness of the cut layer as the length of the normal to the cutting surface, limited by the position of the cutting surfaces of the two adjacent teeth, would be incorrect.

Therefore, when determining the load on saw edges, there is the task of determining the dimensions of the cross-section of the cut layer, that is, the thickness of the slices and the active length of the cutting edge, which are dependent on angles in the plan φ and the edge inclination λ .

The general procedure for determining the thickness of the cut layer a at any point of the cutting edge of any tool analytically is reported in work [19]; the dependence for calculation takes the following form (1):

$$a = \frac{(\bar{N} \cdot \bar{S})}{\bar{N}}, \quad (1)$$

where N is the vector of the normal to the cutting surface at the examined point; S is the feed vector.

Based on the above dependence (1), it is possible for each tool tooth to calculate the thickness of the cut layer at any point in its cutting edge at any time of the tool operation.

However, studies [10–14, 18, 19] do not show how the shape of the cutting edge of the tool tooth affects the parameters of the cut layer, and how the group arrangement of the tool teeth should be taken into consideration.

Paper [20] reports a procedure for determining the parameters of the cut layer of circular saws that operate under a group scheme of cutting the allowance, by using CAD systems. It is shown that the thickness of the layer cut with a tooth is determined by the location of the cutting edges of the investigated tooth and the preceding tooth since the shapes of the teeth cutting surfaces are different. However, although this procedure is demonstrative, it is unsuitable for the automation of calculations.

Our review of works [1–19] has shown that the existing procedures for determining the thickness of the cut apply only to those tools that operate according to a single cutting scheme; in addition, some of them are approximate. Work [20] is the only one that shows determining the thickness of the cut layer by using CAD systems, for tools whose teeth are assembled in groups. However, this procedure cannot be used to automatically calculate the parameters of a cut layer. In other words, there is an issue related to determining the parameters of the cut layer for tools with a group cutting scheme.

3. The aim and objectives of the study

The aim of this work is to determine the parameters of the layer cut with circular saws that operate according to the group scheme of cutting the allowance. That would make it possible to build accurate models of cutting processes involving saws with multidirectional teeth. In addition, the appropriate analysis could reduce the dy-

namic component of the cutting process and, accordingly, improve the durability of the tool and the quality of machined surfaces. To accomplish the aim, the following tasks have been set:

- to devise a procedure for determining the parameters of the cut layer for circular saws with multidirectional teeth;
- to calculate the parameters of the layer cut with circular saws with multidirectional teeth.

4. Materials and methods to study the parameters of a layer cut with a circular tool working under a group scheme of cutting the allowance

We determined the parameters of a layer cut with the tool taking into consideration the kinematics of the cutting process by using the symbolic computing system. The application of such an approach is due to the complexity of the mathematical notation of the cutting process by a tool with a group allowance cutting scheme and a large number of interrelated equations. In addition, symbolic computing systems are clear and make it possible to easily perform symbolic differentiation, integration, the substitution of expressions, and simplification of formulas.

The result of considering an estimation scheme, which reflects the kinematics of the cutting process and determining the required vectors, is a system of equations solving which produces the thickness of the cut layer. Since the equations included in the system are defined in a symbolic form, the problem is solved in the system of automated design Mathcad (PTC Inc., USA), which makes it possible to make calculations for a large array of estimation points and positions.

5. Determining the parameters of a layer cut with saws with multidirectional teeth

5.1. Procedure to analytically determine the parameters of a layer cut with circular saws with multidirectional teeth

According to [15, 16], circular saw teeth for metal machining may take different shapes of cutting edges arranged in the axial plane: straight and polygonal – with different arrangement and alternation of faces on the teeth.

Such a variety of shapes of the cutting edges of circular cutters is due to the complicated conditions of the process of metal cutting; the complicated design of cutting teeth is aimed at increasing the durability of the tool and the machining performance.

One of the saw structures aimed at reducing the force characteristics and increasing the dynamic stability of the tool during the cutting process is a circular saw with multidirectional teeth. The structure of such a saw for cutting metals consists of two teeth in a group, the cutting edges of which are located at sign-different angles in the plan φ and the edge angles of inclination λ (Fig. 2).

To solve the task of determining the thickness of the cut layer, the following estimation scheme is proposed (Fig. 3).

The $S(X, Y, Z)$ coordinate system is stationary (Fig. 3). The cutter rotation axis is always parallel to the Z axis and moves in the XZ plane in the direction of the feed S_0 (feed to the spin of the saw); if the feed is forward – in the opposite direction to the X axis, for an oncoming feed – in the direction of the X axis.

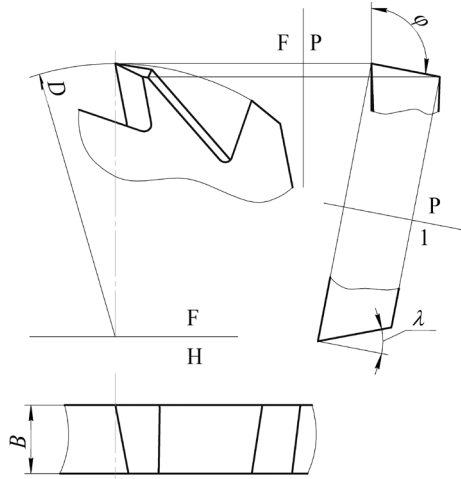


Fig. 2. The examined saw teeth shape

The spatial position of a sawtooth when rotating it around the tool axis is set by the angle of rotation t ; the tool is rotated counterclockwise.

The position of the cutting edge of a sawtooth is set by the main angle in the plan φ and the angle of inclination of the cutting edge λ . The position of the investigated point M on the cutting edge is set by the parameter u , which corresponds to the Z coordinate of this point.

The upper plane of a workpiece is located relative to the tool axis, at a distance of h (Fig. 3).

To determine the thickness of the cut layer, we first derive an equation of the cutting edge of the tool with multidirectional teeth arranged chequerwise.

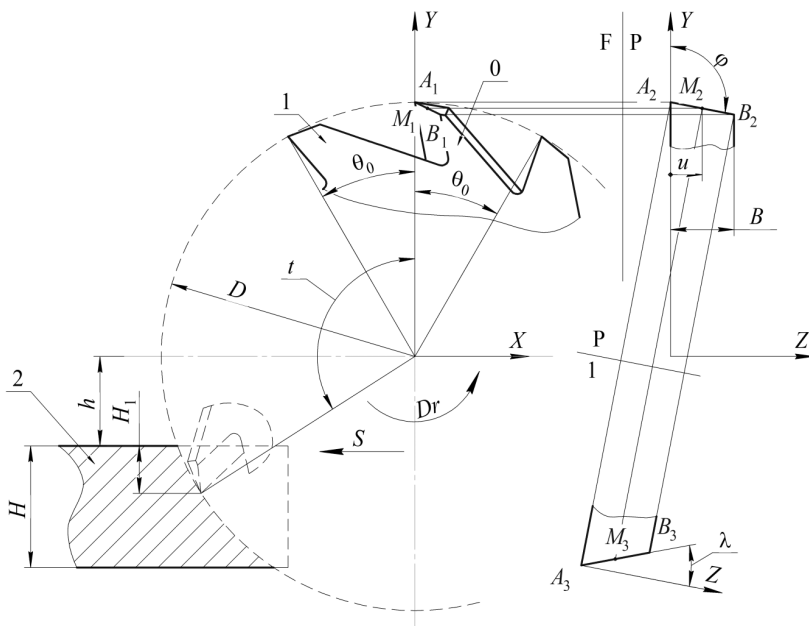


Fig. 3. Estimation scheme for determining the thickness of a layer cut with a saw with multidirectional teeth: D – the outer diameter of the saw, B – saw width, θ_0 – the angular step of the teeth in the middle cross-section of the saw, AB – cutting edge boundary points, M – the cutting edge point under study, h – the position of a workpiece relative to the saw axis, H – the height of a workpiece, H_1 – the position of the investigated tooth relative to the upper plane of a workpiece; 0 – tooth number 0 (studied tooth), 1 – tooth number 1 (preceding tooth), 2 – workpiece

In accordance with the scheme in Fig. 3, the equation of a cutting edge in a vector form is described by the following dependence (2):

$$K(u, n) = \begin{pmatrix} \frac{-u \cdot (-1)^n + B \cdot \frac{(-1)^n - 1}{2} \cdot \text{tg } \lambda}{\sin(\varphi \cdot (-1)^n)} \\ \frac{D}{2} - \left(-u \cdot (-1)^n + B \cdot \frac{(-1)^n - 1}{2} \right) \cdot \text{ctg } \varphi \\ u \\ 1 \end{pmatrix}, \quad (2)$$

where u is the parameter responsible for the position of the point under study on the cutting edge (Fig. 3); n is the number of the tooth under study, which executes cutting (teeth numbering begins with 0 (Fig. 3)).

Derive the equation of the cutting surface formed by such a cutting edge (2) when it moves during operation.

To this end, we first write down the matrix of affine transformation, describing the rotational motion of the tool (the main cutting movement) and, at the same time, the translational movement (feed movement – S_0) applied to its axis:

$$m_i(t, n) = \begin{pmatrix} \cos(t + \theta \cdot n) & -\sin(t + \theta \cdot n) & 0 & S_0 \cdot t \\ \sin(t + \theta \cdot n) & \cos(t + \theta \cdot n) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad (3)$$

where t is the motion parameter corresponding to the angle of rotation of the tool around its axis (Fig. 3); S is the parameter of translational displacement, which corresponds to the movement of the cutter along the X axis (Fig. 3).

The movement parameter t is limited to the angle of contact of the investigated tooth with a machined workpiece and is within $t_{initial} \leq t \leq t_{end}$ where the initial value of the contact angle $t_{initial}$ is determined from the dependence $P_{nY}(u, t, n) = h$, and the end value of t_{end} – from the dependence $P_{nY}(u, t, n) = h - H$.

The displacement parameter S is defined by the value and the sign. The sign of parameter S is determined by the direction of movement of the tool: in the case of oncoming feed – « $+S$ », in the case of forwarding feed « $-S$ ».

The value of the displacement parameter is related to the amount of feed of the tool at rotation S_0 or the value of the feed to tooth S_z and the number of teeth z , and is determined from the following dependence (4):

$$S = \frac{S_0}{2 \cdot \pi} = \frac{S_z \cdot z}{2 \cdot \pi}. \quad (4)$$

The angular step of the teeth θ_0 in the middle cross-section of the saw is propor-

tional to the number of teeth z and is determined by the following dependence (5):

$$\theta_0 = \frac{2\pi}{z}. \tag{5}$$

Thus, the equation of the cutting surface of any tool tooth during its operation, that is, when rotating it around the saw axis and the translational motion with the feed value of S_0 or S_Z , is determined from the following dependence (6):

$$R(u,t,n) = m_t(t,n) \cdot K(u,n). \tag{6}$$

To find the thickness of the cut layer a at the examined point of the cutting edge, we shall write a vector of the normal to the cutting surface.

Knowing the cutting surface equation (4), at a fixed value of the parameter n (the number of the tooth under study), the normal to the cutting surface would be determined as the vector product of the cutting surface equation from partial derivatives according to the independent parameters u, t .

A vector of the normal must be directed to the body of a workpiece. For convenience, the normal equation is written in a single form (7):

$$V(u,t,n) = \frac{\partial}{\partial t} R(u,t,n); \quad \tau(u,t,n) = \frac{\partial}{\partial u} R(u,t,n);$$

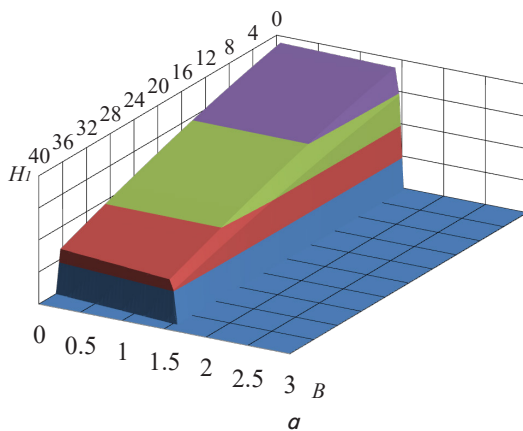
$$NP_n(u,t,n) = \frac{-(V(u,t,n) \times \tau(u,t,n)) \cdot (-1)^n}{|V(u,t,n)| \cdot |(\tau(u,t,n))|}. \tag{7}$$

The thickness of the cut layer would be determined as the smallest positive value of the parameter a when crossing the normal NP_n with the cutting surfaces of individual teeth, the cutting of which occurred earlier, that is, those teeth for which the parameter n accepts a larger value than that of the tooth under study.

In the case where the thickness of the cut layer a is negative, the material is not cut.

The equation of single normal $NP0_n$ to the cutting surface (7) at the cutting edge point under study is determined from the following dependence (8):

$$NP0_n(u,t,n,l) = \begin{pmatrix} R_x(u,t,n) + NP_{nx}(u,t,n) \cdot l \\ R_y(u,t,n) + NP_{ny}(u,t,n) \cdot l \\ R_z(u,t,n) + NP_{nz}(u,t,n) \cdot l \end{pmatrix}, \tag{8}$$



where R_x, R_y, R_z are the components, along the corresponding coordinate axes, of the cutting surface of the studied tooth (6); NP_x, NP_y, NP_z are the components, along the corresponding coordinate axes, of the vector of the normal to the cutting surface of the studied tooth (7); l is the parameter corresponding to the length of the normal to the cutting surface of the tooth under study.

The thickness of a cut layer is determined as the minimum value of the parameter l from the following dependences (9):

$$a = \min(l_1, l_2, l_3, l_4) \Leftarrow \begin{cases} P_n(u_1, t_1, n+1) = NP_{n0}(u, t, n, l_1); \\ P_n(u_2, t_2, n+2) = NP_{n0}(u, t, n, l_2); \\ h = NP_{n0}(u, t, n, l_3); \\ h - H = NP_{n0}(u, t, n, l_4), \end{cases} \tag{9}$$

where u_1, t_1 are the independent parameters describing the cutting surface of the preceding tooth that participated in the process of cutting earlier than the studied one; u_2, t_2 are the independent parameters describing the cutting surface of the second preceding tooth that participated in the process of cutting earlier than the studied one.

The last two equations in dependences (9) are equations of the intersection of the normal to the cutting surface with the boundary surfaces of a workpiece (Fig. 3).

5. 2. Calculation of the parameters of a layer cut with circular saws with multidirectional teeth, in time and along the edge

Based on the above scheme (Fig. 3) and the resulting dependences (1) to (9), we calculated the parameters of the cut layer a for a circular saw with multidirectional teeth. The saw dimensions are: diameter $D=200$ mm, width $B=3$ mm, the number of teeth $z=64$, tooth geometry: $\varphi=115^\circ, \lambda=-20^\circ$. The cutting modes: a forward feed $S_2=0.05$ mm/tooth. The workpiece parameters (Fig. 3): the workpiece height is $H=40$ mm, the workpiece position relative to a saw axis is $h=50$ mm.

The results of calculating the distribution of the thickness of the cut layer a , depending on the location of the examined saw tooth on the cutting path and the width of the tool B , are shown in Fig. 4.

Changing the shape of the cut layer of the material with a sawtooth, depending on the cutting path and the parameter t , is shown in Fig. 5. The study conditions correspond to those for Fig. 4.

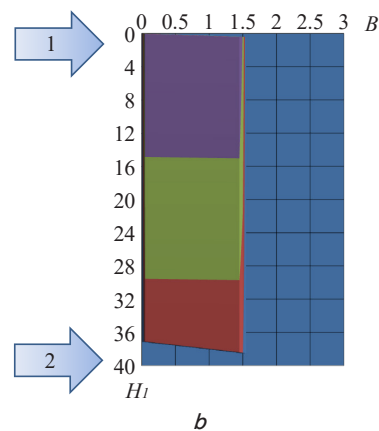


Fig. 4. The cut layer thickness distribution: a – depending on the location of the tooth, on the cutting path; b – by the tool width: 1 – cutting into a workpiece; 2 – leaving a workpiece

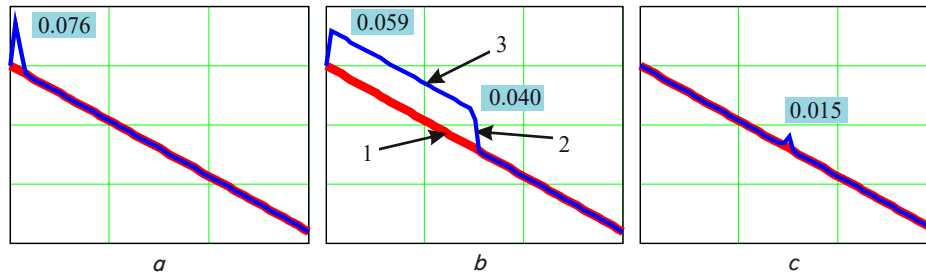


Fig. 5. The shape and thickness of a layer of the material cut with a tooth of the saw with multidirectional teeth, depending on the cutting path: *a* – entering a workpiece; *b* – in the middle of a workpiece; *c* – leaving a workpiece; 1 – cutting edge, 2 – the boundary of the cut layer formed by the preceding tooth, 3 – the boundary of the cut layer formed by the second preceding tooth

Fig. 5 shows the mechanism that forms a layer cut with a separate circular saw tooth with multidirectional teeth during operation. One can see that the shape and parameters of the layer cut with a saw tooth are affected by the preceding two teeth during its operation.

6. Discussion of results of studying the thickness of a layer cut with circular saws with multidirectional teeth

The calculation results have demonstrated the following:

- the thickness of a layer cut with a sawtooth is not constant both in the cutting path, by the height of a workpiece, and by the width of the saw (Fig. 4, *a*); the maximum value is $a_{\max}=0.076$ mm (Fig. 4, *a*);

- the width of the cut layer (Fig. 4, *a*) is smaller than the width of the sawtooth, unlike circular saws, in which the edge angle of inclination $\lambda=0$; the largest width is 0.55 B.

The calculation results agree with the findings reported in work [20], which were obtained on the basis of a graphical method for determining the parameters of the cut layer using computer systems of 3D design.

Such results are explained by the spatial arrangement of the cutting edge – at an angle in the plan φ and the edge angle of inclination λ [1–3]. In addition, this distribution of the thickness of the cut layer leads to a smoother increase in cutting force, which has a positive effect on the durability of the tool [5, 6] and its dynamic characteristics [7, 8].

The study analysis (Fig. 4, *b*) has shown that, due to the location of the cutting edge of the saw teeth at the angle of inclination of the edge $\lambda \neq 0$, the cutting into a workpiece occurs relatively smoothly, not by the entire width of the cutting edge of the tooth at once. A similar pattern is observed when the tooth comes out of a workpiece (Fig. 4, *b*) – the cutting edge smoothly comes out of the workpiece, unlike circular saws, in which the angle of inclination of the edge $\lambda=0$.

This nature of cutting the tooth into a workpiece and leaving it causes a smooth growth and decrease in the load on the tooth, without sharp impacts. This nature of changing loads on the tool tooth has a positive effect on the force [1–3, 5, 6] and dynamic [7, 8, 18] characteristics of the cutting process.

Our analysis of the layer (Fig. 5) cut with a sawtooth with multidirectional teeth has shown that the shape, thickness, and width of the cut layer are affected by:

- the parameters of a workpiece and its position relative to the axis of the saw (Fig. 3, 4);
- the geometrical parameters of the tooth (2) and cutting modes (3), (4);

- the position of the preceding tooth, which worked earlier than the one under study;

- the position of the second preceding tooth, located similarly to the investigated tooth in one tooth.

Based on the analysis of the cut layer (Fig. 4, 5), the shape and size of the cut layer (Fig. 5) are formed by the following:

- at the time of cutting into a workpiece – the preceding tooth (Fig. 5, *a*);

- in the process of cutting – the second preceding tooth determines the thickness, and the preceding tooth – only the width of the cut layer (Fig. 5, *b*);

- at the time the tooth leaves a workpiece – only the preceding tooth (Fig. 5, *c*).

The derived dependences make it possible to determine the parameters of a cut layer when cutting off rectangular-cross-section workpieces with saws during a constant cutting. To determine the thickness of the cut layer when cutting off workpieces of another cross-section, one must set a mathematical notation of the limiting surfaces of the workpiece.

According to our procedure, one could define the parameters of a layer cut with any tool under an arbitrary scheme of allowance cutting. To this end, it is necessary to write down, for each tooth, an equation of cutting edges $K(2)$, derive the equation of single normals $NP0_n(8)$ to the cutting surface, and build a system of equations (9).

Using this procedure, one could define the parameters for a layer cut with each individual tooth of a multi-blade tool during constant cutting.

The above procedure does not take into consideration another specific shape of a workpiece and the steps of cutting the tool into and out of the workpiece.

7. Conclusions

1. This paper, based on the consideration of the kinematics of the process, the structural and geometric features of circular saws with multidirectional teeth, has described a procedure to analytically determine the parameters of a cut layer. The procedure implies that determining the parameters of the cut layer considers not only the kinematics of the cutting process but also the specific shape and arrangement of each individual tool tooth.

The above procedure makes it possible to establish the parameters of a cut layer along the cutting edge of the circular saw tooth at any period when cutting off rectangular workpieces at the specified location of the workpiece relative to the saw.

2. Based on the calculation of the parameters of the cut layer, it has been established that:

– the thickness of the cut layer is not constant both along the cutting edge and cutting path, depending on the geometry of the tool teeth, the cutting kinematics, the shape and position of a workpiece;

– the thickness of the cut layer smoothly increases when the tool enters the workpiece and smoothly decreases when it leaves it, which positively affects the cutting process;

– the width of cutting with a separate tooth is not a constant value; the maximum value is 55 % of the width of the tool cutting.

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