Material particles interact with the working moving surfaces of machines in various technological processes. Mechanics considers a technique to describe the movement of a point and decompose the speed and acceleration into single unit vectors of the accompanying trajectory trihedron for simple movement. The shape of the spatial curve uniquely sets the movement of the accompanying Frenet trihedral as a solid body. This paper has considered the relative movement of a material particle in the static plane of the accompanying Frenet trihedron, which moves along a flat curve with variable curvature. Frenet formulas were used to build a system of differential equations of relative particle movement. In contrast to the conventional approach, the chosen independent variable was not the time but the length of the arc of the guide curve along which the trihedron moves. The system of equations has been built in the projections onto the unit vectors of the moving trihedron; it has been solved by numerical methods. The use of the accompanying curve trihedron as a moving coordinate system makes it possible to solve the problems of the complex movement of a point. The shape of the curve guide assigned by parametric equations in its length function determines the portable movement of the trihedron and makes it possible to use Frenet formulas to describe the relative movement of a point in the trihedron system. This approach enables setting the portable movement of the trihedron osculating plane along a curve with variable curvature, thereby revealing additional possibilities for solving problems on a complex movement of a point at which rotational motion around a fixed axis is a partial case. The proposed approach has been considered using an example of the relative movement of cargo in the body of a truck moving along the road with a curvilinear axis of variable curvature. The charts of the relative trajectory of cargo slip and the relative speed for the predefined speed of the truck have been constructed

Keywords: accompanying trihedron, guide curve, slip trajectory, movement speed, friction coefficient

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THE POSSIBILITY TO APPLY THE FRENET TRIHEDRON AND FORMULAS FOR THE COMPLEX MOVEMENT OF A POINT ON A PLANE WITH THE PREDEFINED PLANE DISPLACEMENT

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1. Introduction

The interaction between the particles of technological material and the working moving surfaces of machines occurs in various manufacturing processes. During such interaction, the particles are forced to slide on the surface in a certain way in relative motion and follow another trajectory in absolute motion. The absolute trajectory is the geometric sum of the relative motion of the particle's slip and the portable movement of the surface. To add these movements, it is convenient to use two coordinate systems: moving, relative to which the relative movement of the particle is executed, and motionless, relative to which the portable movement of the surface and the absolute movement of the particle are executed. In mechanics, there is a well-known technique to describe the movement of a point and decompose the speed and acceleration into unit vectors of the trihedron accompanying the trajectory; however, that applies to simple movement.

The application of the Frenet formulas and trihedron in the complex movement of a point on a plane makes it possible to solve these problems in a new way.

2. Literature review and problem statement

The study of the movement of different environments sometimes is reduced to investigating a separate particle. The movement of a single particle or material point cannot be identified with the movement of a body or technological material consisting of individual particles. However, based on the movement of a separate particle, regularities can be identified that can, to some extent, be transferred to the body or material, or identify the area of further research. Thus, the authors of paper [1] considered the scattering of mineral fertilizers by a centrifugal working device on the example of a separate particle, which made it possible to determine the effect of airflow velocity on the resulting distribution of granules under different modes. Work [2] reported a simulation of such a process. The movement of a grain mixture particle during aspirational separation was studied in paper [3] involving the construction of differential equations of particle movement. The effect of oscillating motion on the effectiveness of separation of mixtures is revealed in [4]; however, the authors considered only the oscillatory movement caused by axial displacements. The theory of dispersion of particles of mineral fertilizers is highlighted in monograph [5]. Studies of body movement in some cases can also be reduced to a particle. That applies to the case when the forces of inertia induced by the rotation of a body can be neglected due to the small angular velocities of their rotation [6]. The above works [1–6] investigated the issues related to complex particle movement in detail. For them, it is common that time acts as an independent variable.

A close problem statement is the movement of a particle on a rough horizontal plane, which rotates around a vertical axis [7]. Its solution is known and comes down to a system of differential equations, which must be solved by numerical methods. However, the issues of the complex movement of a point on a plane, the point of which, during portable motion, executes not a circle but a flat curve of variable curvature, remained to be studied. In this case, there may be difficulties of cumbersome analytical notation of the relative movement of the particle. An option to overcome these difficulties may be to use the Frenet trihedron and formulas, which were used in the study of the movement of a particle on the inner surface of a spherical segment orbiting a vertical axis, considered in paper [8], as well as on the outer surface of a horizontal cylinder with an oscillatory motion [9].

3. The aim and objectives of the study

The aim of this work is to show the possibility of using the Frenet trihedron and formulas to analytically describe the complex movement of a material point on its plane, whose portable movement is set by a flat guide curve along which the trihedron moves.

To accomplish the aim, the following tasks have been set:

– to build a system of differential equations of relative particle movement using the Frenet formulas;

– to consider the proposed approach using an example of the relative movement of cargo in the body of a truck that moves along the road with a curvilinear axis of variable curvature;

– to solve the constructed system of differential equations by numerical methods;

– to build the charts of the relative trajectory of the slip of cargo in the body of a truck that moves along a road with a curvilinear axis of variable curvature.

4. Materials and methods to study the possibility of applying the Frenet trihedron in the complex movement of a point

Since the shape of a spatial curve uniquely sets the movement of the accompanying Frenet trihedron, the movement of each of its faces is uniquely assigned in this way. If the guide curve is spatial, then the pattern of trihedron movement depends on two differential characteristics of the curve: its curvature and torsion. We have considered a simplified version for a flat curve whose torsion is zero. In Fig. 1, the plane μ hosts the curve C_e, along which the accompanying trihedron $\bar{\tau}$, $\bar{n}$, $\bar{b}$. moves. In this case, the osculating plane of the trihedron, formed by the unit vectors of the tangent $\bar{\tau}$ and the main normal $\bar{n}$, coincides with the plane of the curve guide itself (in Fig. 1, 2, the trihedron osculating plane is shaded).

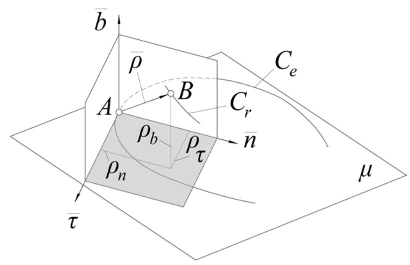

Fig. 1. Graphic illustration to the construction of the vector equation of the movement of point B as a geometric sum of the portable movement of the trihedron and the relative movement of the point in the trihedron: the accompanying trihedron $\bar{\tau}$, $\bar{n}$, $\bar{b}$ of the guide flat curve C_e with the marked coordinates of point B in its system

When the trihedron moves along the guide curve C_e at speed V_e, the osculating plane would rotate as well. The angle of its rotation α is determined by the angle of rotation of the tangent $\bar{\tau}$. When the trihedron moves along the curve to the length of its arc Δs, the tangent returns at angle $\Delta\alpha$. It is known that the boundary of the ratio $\Delta\alpha/\Delta s$ when Δs tends to zero is the value of the curvature k of the curve. On the other hand, the boundary of $\Delta\alpha/\Delta t$, where Δt the time during which the rotation took place, produces the value of the angular velocity ω of the rotation of static plane. It is obvious that there is a relationship between the curvature k of the curve and the angular velocity ω of the osculating plane rotation. It can be found:

$$\omega = \frac{d\alpha}{dt} = \frac{d\alpha}{ds} \cdot \frac{ds}{dt} = kV_e. \quad (1)$$

It follows from dependence (1) that at constant values of the speed of movement of the trihedron and the curvature of the guide curve (that is, a circle), the angular velocity of the osculating plane rotation is also a constant value.

The absolute movement of a particle consists of two movements – the portable movement of the Frenet trihedron and the relative movement of the particle in the trihedron system. Let the particle be located at a certain distance ρ from the vertex of the trihedron. Then its position in a vector form will be written as follows (Fig. 2):

$$\bar{R} = \bar{r} + \bar{\rho}, \tag{2}$$

where $\bar{R}$ is the radius-vector of the particle position in a stationary $OXYZ$ system $\bar{r}$ is the radius-vector of the point on the curve in which the vertex A of the trihedron is located; $\bar{\rho}$ is the radius-vector of the particle position in the trihedron system.

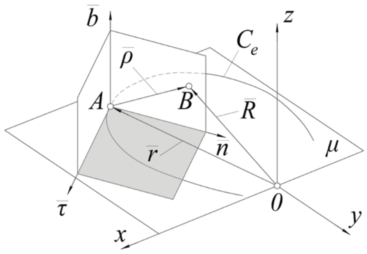

Fig. 2. Diagram showing the construction of the vector equation of the position of point B, taking into consideration two systems: $\bar{\tau}$, $\bar{n}$, $\bar{b}$ and $OXYZ$

We assume that the ρ_τ, ρ_n and ρ_b coordinates in the trihedron system are variable and dependent on its position on the curve C_e, that is, dependent on the length of the arc s of the guide curve. In this case, when the trihedron moves along the curve, point B would move in a certain way within its system, executing the relative trajectory C_r (Fig. 1). We shall rewrite expression (2) taking into consideration the decomposition of the vector $\bar{\rho}$ into the unit vectors of the trihedron:

$$\bar{R} = \bar{r} + \bar{\tau}\rho_\tau + \bar{n}\rho_n + \bar{b}\rho_b. \tag{3}$$

To find the absolute velocity of a particle V_a, one needs to differentiate the vector equation (3) for time t. The use of the Frenet trihedron makes it possible to apply Frenet formulas that are widely known in differential geometry. That makes it possible to simply find the derivatives from the unit vectors of the trihedron in projections onto the same unit vectors. However, in this case, the independent variable must be the length of the arc s – the path traveled by the vertex of the trihedron when moving along the curve C_e. Adopting s as an independent variable, we find the relationship between the absolute velocity V_a and the derivative from the vector $\bar{R}$ for variable s:

$$V_a = \frac{d\bar{R}}{dt} = \frac{d\bar{R}}{ds}\frac{ds}{dt} = V_e\frac{d\bar{R}}{ds}. \tag{4}$$

Thus, to derive the expression of absolute velocity V_a, one needs to multiply the velocity V_e of the portable movement of the trihedron along the curve C_e by the derivative from expression (3). Differentiate (3) for variable s considering $\rho_\tau=\rho_\tau(s)$, $\rho_n=\rho_n(s)$ and $\rho_b=\rho_b(s)$:

$$\frac{d\bar{R}}{ds} = \frac{d\bar{r}}{ds} + \left(\frac{d\bar{\tau}}{ds}\rho_\tau + \bar{\tau}\frac{d\rho_\tau}{ds}\right) + \left(\frac{d\bar{n}}{ds}\rho_n + \bar{n}\frac{d\rho_n}{ds}\right) + \left(\frac{d\bar{b}}{ds}\rho_b + \bar{b}\frac{d\rho_b}{ds}\right). \tag{5}$$

The derivatives $\frac{d\bar{r}}{ds}, \frac{d\bar{\tau}}{ds}, \frac{d\bar{n}}{ds}, \frac{d\bar{b}}{ds}$, according to Frenet formulas, are represented by the projections onto the unit vectors of a trihedron through the curvature k and the torsion σ of the guide curve. The guide curve C_e is a flat curve, so the torsion is $\sigma=0$. Frenet formulas, in this case, are simplified; they take the following form [10]:

$$\dot{\bar{r}} = \bar{\tau};\quad \dot{\bar{\tau}} = k\bar{n};\quad \dot{\bar{n}} = -k\bar{\tau};\quad \dot{\bar{b}} = 0. \tag{6}$$

The implementation of procedures (1) to (6) allowed us to proceed to the construction of a system of differential equations of the relative motion of a particle on a plane using Frenet formulas.

5. The results of studying the possibility of using the Frenet trihedron in the complex movement of a point

5. 1. Constructing a system of differential equations of the relative movement of a particle on a plane using Frenet formulas

Considering the fulfillment of condition $\rho_b = \dot{\rho}_b = 0$, related to the movement of a point in the horizontal (that is, osculating) plane of the trihedron, and substituting the expressions of derivatives (6) to (5), after grouping the projections by unit vectors $\bar{\tau}$, $\bar{n}$, $\bar{b}$, we obtained:

$$\dot{\bar{R}} = \bar{\tau} + \bar{n}k\rho_\tau + \bar{\tau}\dot{\rho}_\tau - \bar{\tau}k\rho_n + \bar{n}\dot{\rho}_n = \\ = \bar{\tau}\left(1 + \dot{\rho}_\tau - k\rho_n\right) + \bar{n}\left(\dot{\rho}_n + k\rho_\tau\right). \tag{7}$$

Differentiating the absolute velocity (4) for time t, on the condition V_e=const, the expression of absolute acceleration w is derived:

$$w = \frac{d}{dt}\left(V_e\frac{d\bar{R}}{ds}\right) = V_e^2\frac{d^2\bar{R}}{ds^2}. \tag{8}$$

The second derivative $\ddot{\bar{R}}$ was found from differentiating the vector expression (7) for variable s using the Frenet formulas (6), taking into consideration the fact that the curvature $k=k(s)$ is a variable quantity; the resulting expressions are grouped by the unit vectors of the trihedron:

$$\ddot{\bar{R}} = \dot{\bar{\tau}}\left(1 + \dot{\rho}_\tau - k\rho_n\right) + \bar{\tau}\left(\ddot{\rho}_\tau - \dot{k}\rho_n - k\dot{\rho}_n\right) + \\ + \dot{\bar{n}}\left(\dot{\rho}_n + k\rho_\tau\right) + \bar{n}\left(\ddot{\rho}_n + \dot{k}\rho_\tau + k\dot{\rho}_\tau\right) = \\ = \bar{\tau}\left[\ddot{\rho}_\tau - \dot{k}\rho_n - k\left(k\rho_\tau + 2\dot{\rho}_n\right)\right] + \\ + \bar{n}\left[\ddot{\rho}_n - \dot{k}\rho_\tau + k\left(1 - k\rho_n + 2\dot{\rho}_\tau\right)\right]. \tag{9}$$

After multiplying the obtained results (9) by V_e^2 according to (8), one can derive expressions of the absolute acceleration of point B in the projections onto the unit vectors of a trihedron:

$$w_\tau = V_e^2\left[\ddot{\rho}_\tau - \dot{k}\rho_n - k\left(k\rho_\tau + 2\dot{\rho}_n\right)\right]; \\ w_n = V_e^2\left[\ddot{\rho}_n - \dot{k}\rho_\tau + k\left(1 - k\rho_n + 2\dot{\rho}_\tau\right)\right]. \tag{10}$$

The point moves in the osculating plane of the trihedron, and the trihedron moves along the curve. To find a value of the absolute acceleration of a point, it is necessary to set the speed of movement of the trihedron V_e, the parametric equations of the curve guide $x=x(s)$ and $y=y(s)$, and the law of movement of the point $\rho_\tau=\rho_\tau(s)$, $\rho_n=\rho_n(s)$. The curvature $k=k(s)$ of the curve C_e guide is found from a known formula: $k = \sqrt{\ddot{x}^2 + \ddot{y}^2}$.

More complex is the inverse problem of finding the law of the relative movement of a material point (particle), which moves under the action of the forces applied to it.

5. 2. Applying the proposed approach to solving the problem of the relative movement of cargo in a truck body

The conditions of our study stipulate that the osculating plane is the body of a truck that moves at steady speed V_e along a horizontal road with a curvilinear axis in the form of a guide curve. The role of the material point belongs to the cargo in the truck body. It is required to find the trajectory and the speed of possible slip of the cargo in the body.

The road axis, that is, the curve C_e, is set by the following parametric equations:

$$x = 2a\cdot\operatorname{arctg}\left(\frac{s}{a}\right) - s;$$

$$y = a\cdot\ln\left(\frac{a^2+s^2}{a^2}\right). \tag{11}$$

The curvature k of the curve set by the parametric equations in the arc length function s is found from a known formula:

$$k = \sqrt{\ddot{x}^2 + \ddot{y}^2}. \tag{12}$$

After substituting in (12) the second derivatives from equations (11), the curvature k expression was derived upon the differentiation of $\dot{k}$:

$$k = \frac{2a}{a^2+s^2};$$

$$\dot{k} = -\frac{4as}{\left(a^2+s^2\right)^2}. \tag{13}$$

The differential equation of absolute motion of a particle in a vector notation takes the following form: $m\overline{w} = \overline{F}$, where m is the mass of a particle, $\overline{w}$ is the vector of absolute acceleration; $\overline{F}$ is the resulting vector of forces applied to the particle. These forces are the gravity mg (g=9.81 m/s^2), the reaction N of the plane (bottom) of the body, the friction force $F=f\cdot N$ when the particle slips along the bottom of the body, where f is the friction coefficient. All forces have a strictly defined direction of action.

The gravity mg is directed downwards, that is, in the opposite direction to the unit vector of the binormal $\overline{b}$, and is balanced by the reaction N from the bottom of the truck, directed upwards. Thus, it is possible to write: $N=mg$. Therefore, the friction force has a constant value: $F=f\cdot m\cdot g$, it is directed in the opposite direction to the sliding speed V_r of the particle, which is at point B. Fig. 3 shows the curve C_e on top when the binormal $\overline{b}$ is mapped onto a point. The rectangular bottom of the truck is shaded in gray and formed at the negative values of unit vectors $\overline{\tau}$ and $\overline{n}$ of the trihedron. This location corresponds to the probable trajectory of the slip of the cargo since the centrifugal force that causes its slip is directed from the center of the curvature of the guide curve. It is required to find a unit guide vector of the action of friction force F. Projections of the relative velocity of the particle onto the unit vectors of a trihedron are found by multiplying the derivatives from the radius-vector $\overline{\rho}$ by velocity V_e due to the above reasons. Thus, the projections of the relative velocity V_r of slip and its value are written as follows:

$$V_{r\tau} = V_e\dot{\rho}_\tau;\;\; V_{rn} = V_e\dot{\rho}_n;\;\; V_r = V_e\sqrt{\dot{\rho}_\tau^2+\dot{\rho}_n^2}. \tag{14}$$

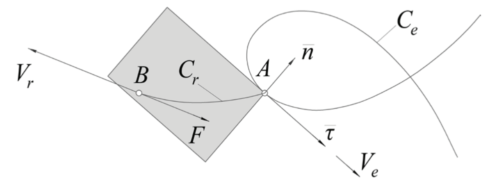

Fig. 3. Diagram showing the movement of a truck body along the curved section of the path with vectors of the portable V_e, relative V_r speeds, and the friction force F

Friction force $F=fmg$ can be decomposed into the unit vectors of a trihedron taking into consideration its direction and in accordance with the ratio of projections (14):

$$F_\tau = -fmg\frac{\dot{\rho}_\tau}{\sqrt{\dot{\rho}_\tau^2+\dot{\rho}_n^2}};\;\; F_n = -fmg\frac{\dot{\rho}_n}{\sqrt{\dot{\rho}_\tau^2+\dot{\rho}_n^2}}. \tag{15}$$

The vector equation $m\overline{w} = \overline{F}$ in the projections onto the unit vectors of a trihedron is written as:

$$mw_\tau = F_\tau;\;\; mw_n = F_n. \tag{16}$$

After substituting in equation (16) the absolute acceleration expressions (10), the friction force F (15), and after reducing by the mass m, we derive a system of two differential equations with two unknown functions $\rho_\tau=\rho_\tau(s)$ and $\rho_n=\rho_n(s)$:

$$V_e^2\left(\ddot{\rho}_\tau - \dot{k}\rho_n - k^2\rho_\tau - 2k\dot{\rho}_n\right) = -fg\cdot\frac{\dot{\rho}_\tau}{\sqrt{\dot{\rho}_\tau^2+\dot{\rho}_n^2}};$$

$$V_e^2\left(\ddot{\rho}_n + k + \dot{k}\rho_\tau - k^2\rho_n + 2k\dot{\rho}\right) = -fg\cdot\frac{\dot{\rho}_n}{\sqrt{\dot{\rho}_\tau^2+\dot{\rho}_n^2}}. \tag{17}$$

The above results make it possible to proceed to the numerical integration of the constructed system of differential equations and the graphical representation of the results obtained.

5. 3. Solving the system of differential equations of the relative movement of a particle on a plane by numerical methods

We integrated system (17) when changing the arc coordinate s from −40 m to 40 m. The following values were accepted for the variables: a=15; f=0.3; V_e=5 m/s. Fig. 4 shows the constructed charts of change in the relative and absolute speed of cargo movement. Fig. 4, *a* demonstrates that the slip of the cargo began at about $s\approx-5.5$ m and ended at $s\approx14$ m, while the maximum relative speed reached the value of $V_r\approx0.7$ m/s. A value of the arc coordinate s, at which the relative motion began, can be determined analytically. The slip of the cargo would begin when the centrifugal force exceeds $F_c = mV_e^2k$ the friction force $F=fmg$. Having equated these forces and substituting the expression $k=k(s)$ from (3), we derive the equation of the limit value of the arc coordinate s:

$$\frac{m\cdot V_e^2\cdot 2a}{a^2+s^2} = fmg,$$

hence

$$s = \sqrt{\frac{a}{fg}\left(2V_e^2 - afg\right)}. \tag{18}$$

The solution to equation (18) at the indicated constants shows that the relative movement of the cargo would begin at $s>-5.46$ m. The maximum absolute speed of the cargo movement is achieved at $s\approx 14$ m. At this moment, the slip of the cargo stops; its absolute speed begins to decrease (Fig. 4, *b*).

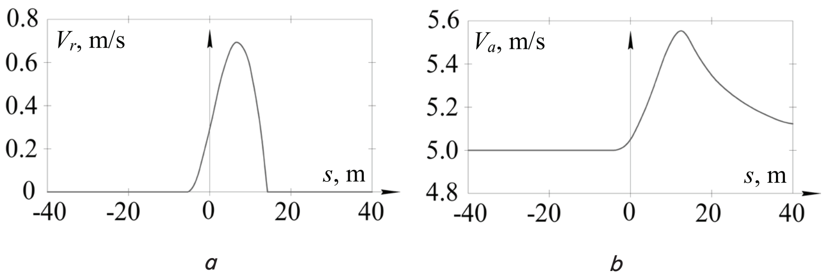

Fig. 4. Charts showing a change in the speed of cargo movement in a truck body: *a* – relative movement; *b* – absolute movement

It is also expedient to give a graphic representation of the trajectory of the cargo slip in a truck body.

5. 4. Construction of charts of the relative trajectory of the slip of cargo in a truck body

Fig. 5, *a* shows the built scaled guide curve C_e, along which a thickened line highlighted the section moving along which on the osculating plane of the trihedron (that is, the bottom of the truck body) caused the cargo to slide. The body in an enlarged scale is shown in Fig. 5, *b*; it demonstrates the built trajectory C_r of the cargo slip. Fig. 5, *b* depicts that the cargo in the body shifted about 1.5 m towards the opposite side and about 0.25 m to the side opposite to the direction of movement.

The trajectory and speed of the cargo slip largely depend on the friction coefficient and the speed of the truck.

Fig. 6 shows the constructed charts of change in the slip speed and trajectory of the cargo movement in a truck body for different values of friction coefficient *f*.

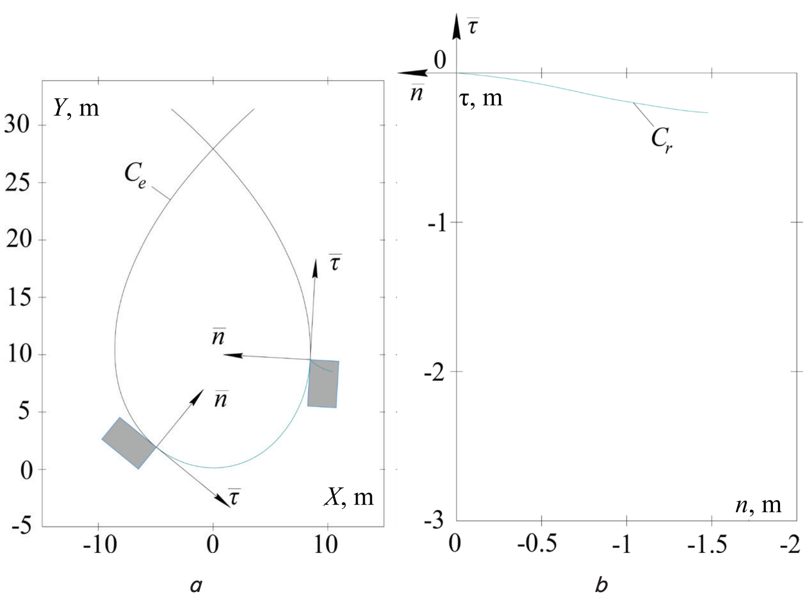

Fig. 5. Graphic illustrations showing the movement of a truck with cargo in its body shaded in gray: *a* – the trajectory C_e of the truck with its position at the beginning of the slip of the cargo and after stopping the slip; *b* – the trajectory C_r of the cargo slip

The significant impact of truck speed V_e on the trajectory of the cargo slip is explained by the fact that the speed V_e squared is part of the expression of centrifugal force $F_c = mV_e^2 k$, which causes the slip.

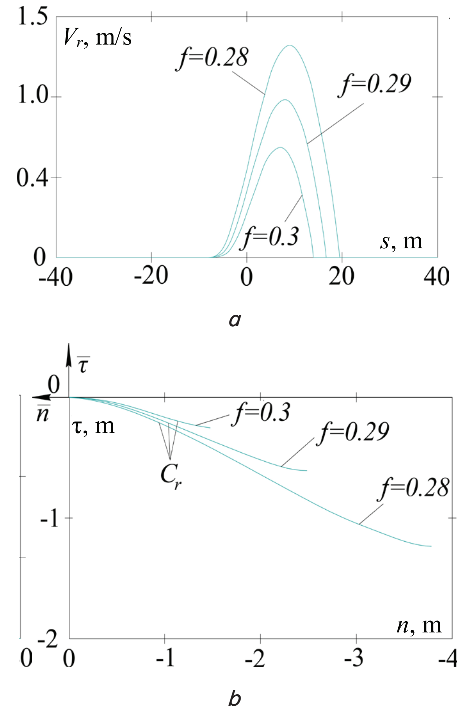

Fig. 6. Graphic illustrations showing the slip of cargo in a truck body for different values of the friction coefficient *f*: *a* – charts of changes in the relative slip speed; *b* – the trajectories of cargo slip

6. Discussion of results of studying the possibility of applying the Frenet trihedron for the complex movement of a point

The use of the accompanying trihedron of a curve as a moving coordinate system has made it possible to achieve our goal and obtain results relative to the tasks set. Solving them is due to the following factors:

– a system of differential equations (17) with the use of Frenet formulas was built;

– the guide curve of the accompanying trihedron is set by the curvilinear axis of the road along which the truck moves – this curve is set by parametric equations (11);

– the system of differential equations (17) has been solved by numerical methods, which made it possible to build the charts of change in the velocity of the relative (Fig. 4, *a*) and absolute (Fig. 4, *b*) movement of cargo in a truck body;

– the trajectories of the relative movement (slip) of cargo in a truck body for different friction coefficients (Fig. 6, *b*) were built.

Owing to the use of the Frenet trihedron and formulas, further development of the analytical notation of the complex movement of a particle on a plane is ensured.

Alternative solutions to this problem relate to the rotational motion of the plane, and advanced – to its more complex movement when a point of the plane executes not a circle but a curve of variable curvature. The evidence base of this approach is substantiated using well-known mathematical Frenet formulas from differential geometry to solve a problem of the dynamics of a material point in theoretical mechanics. The limitation of the proposed approach is the condition of using, as an independent variable, the guide curve of the length of its arc, which is possible for a limited class of curves. This study could be advanced for the spatial guide curves.

7. Conclusions

1. The shape of the spatial curve uniquely sets the movement of each face of the accompanying Frenet trihedron. If the guide curve is flat, then the pattern of movement of the trihedron depends on its curvature. The osculating plane of the trihedron. in this case, coincides with the plane of the guide curve itself.

The absolute movement of a particle consists of two movements – the portable movement of the Frenet trihedron and the relative movement of the particle in the trihedron system. To build a system of differential equations of relative particle movement, we have applied the Frenet formulas and trihedron, well known in differential geometry. The role of an independent variable, in this case, belongs to the length of the path traveled by the vertex of the trihedron when moving along the curve. We studied the particle movement in the osculating (horizontal) plane of the trihedron.

2. The osculating plane is the body of a truck that moves at a steady speed along a horizontal road with a curvilinear axis in the form of a guide curve. The role of the material point belongs to cargo in the truck body. We have built the differential equations of the absolute movement of a particle taking into consideration the forces applied to it. A system of equations has been derived that makes it possible to determine the value of the speed of the possible cargo slip in a truck body. A graphical interpretation of the trajectory of the possible cargo slip in a body has been presented.

3. We have integrated the built system of differential equations when changing the arc coordinate from −40 m to 40 m. The charts of change in the relative and absolute speed of cargo movement have been constructed. The graphical interpretation of our results demonstrated that the slip of the cargo begins at about −5.5 m and ends at 14 m. At the same time, the maximum relative speed reaches a value of 0.7 m/s. In addition, the equation of the limit value of the arc coordinate has been found analytically. Solving this equation has shown that the relative movement of cargo begins at −5.46 m, and the maximum absolute speed of cargo is reached at 14 m. The charts of changes in the speed of movement of cargo in a truck body have been built.

4. The graphic illustrations have been presented on the movement of a truck with cargo in the body, specifically: the trajectory of the truck movement with its position at the beginning of the slip of the cargo and after stopping the slip, as well as the trajectory of the cargo slip. Our analysis of graphic illustrations has revealed that the cargo in the body shifted about 1.5 m towards the opposite side and about 0.25 m to the side opposite to the direction of motion. In addition, the influence of the friction coefficient and speed of the truck on the trajectory and speed of the cargo slip has been taken into consideration.

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