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DEVELOPMENT OF A MATHEMATICAL MODEL OF VIBRATORY NON-LIFT MOVEMENT OF LIGHT SEEDS TAKING INTO ACCOUNT THE AERODYNAMIC FORCES AND MOMENTS

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The modern practice of using vibratory machines when working with fine-size light-weight seeds is faced with such an undesirable phenomenon as the impact of aerodynamic forces and moments on the kinematics of vibrational movement of particles of the seed mixture fractions.

According to the results of scientific studies devoted to the solution of this problem, only mathematical models of vibrational movement are used, where the aerodynamic factor is taken into account as taking the seeds by airflow. This is typical only for cleaning modes with the rebound of seeds from the vibrating surface. Aerodynamic forces and moments are present in them only as a force of aerodynamic resistance. The action of lateral aerodynamic forces and their moments are not taken into account. Their consideration allows to extend the range of action of the aerodynamic factor on the modes of vibration cleaning (vibroseparation) without rebound (but with sliding and rolling) which are of greater interest in terms of improving the efficiency of processing fine-size seeds.

A mathematical model of seed vibration movement taking into account the action of a complete set of aerodynamic forces (dynamic resistance forces and lateral aerodynamic forces) and moments was proposed. This makes it possible to simulate non-lifting modes of vibrational movement of seeds. A system of algebraic equations that are linear with respect to the kinematic parameters of seed movement which was obtained by translating differential equations of movement into a finite-difference form was presented. The possibility of numerical solution of equations of movement by the Euler method was shown. The results of the evaluation of the model adequacy for the processes of vibration separation of tobacco seeds and false flax were presented. As shown by the results of calculations and experiments, the developed model provides an increase in the adequacy of the simulation results by 30 % in comparison with the model where the aerodynamic factor is not taken into account

Keywords: vibratory machines, system of differential equations, aerodynamic factor, aerodynamic screen, vibrational movement, light-weight seeds

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1. Introduction

The design of vibratory machines which are commonly used for separation and cleaning fine-seed mixtures features the use of sets of work surfaces. This increases machine productivity but causes the origination of aerodynamic factor [1]. Flat channels are formed between working planes inside which a field of velocities and air pressure appears (during vibration). For seeds with distinct aerodynamic properties, this increases the effect of aerodynamic forces and moments which becomes commensurate with the action of friction forces, vibration pulses, and gravitational forces.

The aerodynamic factor significantly impairs the efficiency of vibratory machines when processing fine-seed mixtures sensitive to air movement. It is impossible to efficiently separate elite fractions and debris particles with the help of vibratory machines. This fact encourages the use of more expensive means for cleaning (separation) of such seeds. Therefore, there is a problem of improving (changing the design or operation mode) of existing vibratory machines in order to improve the efficiency of treating the seeds with distinct aerodynamic properties.

In the scientific aspect, this problem is solved on the basis of parametric studies of the processes of vibrational movement of seeds on a work surface of vibratory machines taking into account the action of aerodynamic forces and moments. The currently applied mathematical models of such seed movement take into account only action of dynamic resistance during the period of seed movement out of contact with the work surface (when the seeds rebound). The action of the aerodynamic factor is taken into account through the value of drift (transfer) of seeds by air relative to their rebound point [2].

Such modes of operation of vibratory machines remain outside the study scope when there is a continuous movement of seeds. The seeds move with slipping and rolling, without rebounding. From the point of view of efficiency of separation of fractions of seed mixes, such operating mode is more effective as the influence of physical and mechanical properties of seeds on kinematic parameters of vibrational movement is more realizable. However, for fine seeds with distinct aerodynamic properties (asymmetrical seed shape and characteristic profile promote the creation of a lifting aerodynamic force) [3], this efficiency is neutralized by the aerodynamic factor.

For non-lift modes of vibrational purification of fine-seed mixtures, the action of the aerodynamic factor manifests itself by lateral aerodynamic forces and aerodynamic moments from their action. Lateral aerodynamic forces and moments are brought about by uneven distribution of air pressure on the seed surface [3]. Because of the lateral aerodynamic forces, a partial lift of seeds above the surface occurs. The reaction support force decreases and, as a result, the pulse of vibrational movement decreases too. The action of aerodynamic moments causes a change of values of angular velocities of the seeds. All this strongly affects general kinematic parameters of movement. Therefore, there is an urgent need to construct theoretical models of vibrational non-lift movement of seeds in aerodynamic conditions. The application of such models will make it possible to reasonably adjust operating modes of vibratory machines and improve their design to raise the efficiency of treatment of seeds with distinct aerodynamic properties.

2. Literature review and problem statement

The influence of the aerodynamic resistance force on the movement of seeds in a channel with constant parameters of airflow was investigated in [1]. Analytical expressions for calculating the values of aerodynamic resistance force acting on the seeds moving in a stream of air depending on their size and form were offered. However, the seed shape was considered as a reduced sphere of a certain radius. This approach does not enable taking into account lateral aerodynamic forces and moments caused by the uneven distribution of pressure on the seed surface during the action of airflow.

The importance of taking into account the aerodynamic factor in seed processing was indicated in [2]. However, no numerical metrics have been formulated to measure these properties.

Numerical criteria for evaluating seeds as regards their aerodynamic properties were proposed in [4]. These are a coefficient of sailability (ratio of dynamic resistance force to seed weight force) and the lift velocity (vertically directed airflow velocity at which the seeds placed in the air stream are taken upwards). However, the action of lateral aerodynamic forces was not taken into account. Aeromechanical characteristics of seeds, such as their stability in the airflow (their ability to be in the airflow without rotation around their center of mass) were not assessed in any way.

Density characteristics were proposed in [5] to be used as a criterion for determining the aerodynamic properties of seeds. It was assumed that the aerodynamic factor manifests itself only by picking up and transferring the seeds by the airflow. The effect of lateral aerodynamic forces and moments of rotation around the center of mass was not taken into account.

A mathematical model of the vibrational movement of ellipsoid-shaped seeds on an inclined rough surface was proposed in [6]. However, the action of aerodynamic forces and moments was not taken into account. In addition, approximation of seeds as a body of rotation makes it impossible to take into account the aerodynamic characteristics of seeds.

The processes of interaction of seeds with work surfaces taking into account air oscillations were studied in [7] based on the analytical flat gas-dynamic models. According to the study results, regularities of distribution of airflow velocities over the height of interplanar space in the established sections were found. The influence of the aerodynamic factor on the vibrational movement of seeds through their carryover by the airflow during the seed rebound from the work surface was estimated.

A method of improving the efficiency of the process of separating sunflower seeds from debris particles by means of selecting parameters of the vibratory machines according to the criterion of minimizing the air impact was considered in [8]. The studies have shown that the selection of angles of inclination, amplitude, and frequency of oscillation of the work surfaces can reduce harmful air impacts during oilseed separation. However, this method will not give satisfactory results for light fine seeds because the parameters and aerodynamic properties of the seeds do not differ from those of the debris particles.

An attempt was made in [9] to formalize the representation of boundary conditions for gas-dynamic calculations of air in its interaction with the working bodies of seed treatment machines. However, no gas-dynamic model and methods of calculating the velocity field and air pressure were presented.

A method of calculating the aerodynamic forces and moments acting on seeds of a certain form in their specified spatial position in an airstream was offered in [3]. It takes into account the lateral aerodynamic forces and torque arising from uneven pressure distribution on the seed surface blown by the airflow.

A procedure for calculating the field of air velocities and pressures in the interplanar space of a vibratory machine unit was proposed in [10] for a three-dimensional case. Regularities of change of air dynamics in various phase positions of working bodies of the vibratory machines in various modes of their operation were obtained in [11]. The efficiency of the considered method under the condition of using modern computer technology was shown.

Thus, the above data analysis suggests that:

– the used models of vibrational movement of seeds when cleaning (sorting) seed mixtures with the help of vibratory machines do not fully take into account the harmful effects of the aerodynamic factors. Only the action of the aerodynamic resistance force in a form of carrying the seeds by air during their rebound from a work surface was considered. The non-lift modes were not taken into account in the absence of estimates of lateral aerodynamic forces and aerodynamic moments;

– elements of the calculation of the full set of aerodynamic forces and moments acting on the seeds depending on their spatial position and the phase of oscillations of the work surfaces of the vibratory machines were developed. These elements must be combined into an integrated model of the continuous vibratory movement of seeds taking into account the action of dynamic resistance forces, lateral aerodynamic forces, and aerodynamic moments.

3. The aim and objectives of the study

The study objective consisted of the construction of a model of the continuous vibrational movement of a solid body along an inclined surface taking into account the action of aerodynamic forces and moments due to the alternating air dynamics. This will make it possible to expand the scope of study of harmful effects of aerodynamic factors taking into account the modes of cleaning (sorting) fine-size seeds using the vibratory machines.

To achieve this objective, the following tasks were set:

– introduce aerodynamic forces and moments into the system of differential equations of sliding and rolling movement of a rigid body [8];

– develop a numerical algorithm of solving the system of differential equations of movement which will take into account action of aerodynamic forces and moments;

– assess the degree of adequacy of the constructed mathematical model.

4. The materials and methods used in the construction of a model of seed movement taking into account the aerodynamic factor

The following three components were used as a theoretical basis for the construction of the model of the vibrational movement of seeds:

– a procedure of calculating the aerodynamic characteristics;

– a gas-dynamic model and a numerical procedure for calculating the velocity and pressure field;

– a model of vibrational movement without taking into account the influence of the aerodynamic factor.

The first theoretical component was presented in [3] as a procedure of calculating the aerodynamic characteristics of a three-dimensional body of a certain shape depending on the angle of rotation of this body to the airflow. The aerodynamic forces and moments acting on the body were calculated by adding the elementary forces and moments calculated for the constituent elements of the body (each reduced to the corresponding elementary aerodynamic profile). Depending on the angle of attack and the geometric characteristics of such an elementary profile (chord size, thickness, and curvature coefficients), coefficients of aerodynamic forces (lifting force, reaction force) and aerodynamic moment were determined. Based on the obtained aerodynamic coefficients, elementary aerodynamic forces and moments were estimated by multiplying them by the velocity pressure acting on the body. This procedure works for any angular position of the seed relative to the airflow.

The second component was presented as a procedure of calculating the field of air velocities and pressures at the space points formed by two synchronously oscillating parallel plane surfaces. The calculation was based on solving a system of differential equations of gas dynamics of an ideal gas for a three-dimensional case. The computational scheme for solving the system of differential equations was constructed as a boundary value problem where the boundary conditions determine the design features of the packages of work surfaces including the applied aerodynamic screens [10]. The system of algebraic equations obtained for a boundary value problem in a finite-difference form was solved by the sweep method [11]. Unlike other methods for solving the systems of gas-dynamic equations (grid methods, Masso method), the run method provides stable solution convergence and does not depend on the method of dividing the integration domain. This is due to the lack of accumulation of deviations with an increasing number of nodes of the discrete scheme.

The third component was presented in [8] as a model of the movement of a rigid body sliding and rolling down a vibrating inclined rough surface. Kinematic parameters of body movement were calculated based on solving a system of differential equations of theoretical mechanics written for the conditions of equilibrium of forces and moments for a certain point in time.

To verify the adequacy of the developed mathematical model, an experimental setup (a vibratory machine with expanded capabilities as regards adjustment of the operating modes) was used. Its general view is presented in Fig. 1.

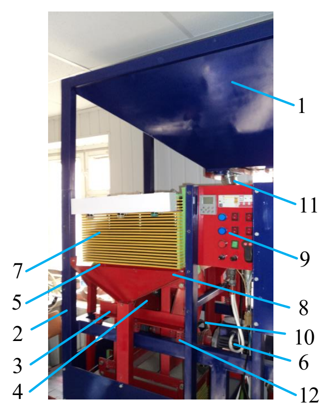

Fig. 1. General view of the experimental setup to verify the adequacy of the developed mathematical model: 1 – seed hopper; 2 – intermediate frame; 3 – vibration exciter frame; 4 – vibration exciter; 5 – frame planes; 6 – electric motor; 7 – separating planes; 8 – guides; 9 – control unit; 10 – coupling; 11 – flexible branch pipe; 12 – springs

To obtain experimental data, a calibrated seed receiver was installed on the edge of the work surface of the vibratory machine (Fig. 2).

The receiver was built as a cylindrical container divided into separate compartments. Its generatrices lay along the edge of the work surface. A certain angle of beam deviation from the Y axis (the left edge of the work surface) φ_i, $i=1, ..., N$ which characterizes the seed movement path corresponds to individual receiver containers.

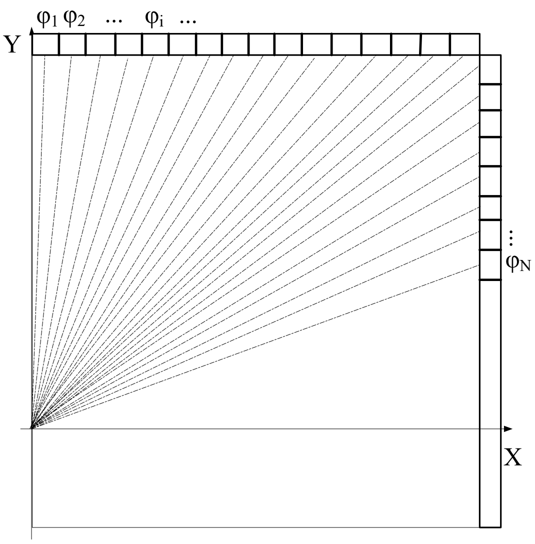

Fig. 2. Calibrated seed receiver

5. The results obtained in the study of the construction of a model of seed movement taking into account the aerodynamic factor

5. 1. Basic system of differential equations

To simulate the vibrational movement of seeds, a model of non-lift rolling and sliding movement of an ellipsoidal solid was used [8]. The movement occurs under the action of gravity, vibration (harmonic impacts) transmitted to the body from the work surface, as well as the aerodynamic forces and moments arising from the movement of air relative to the oscillating work surface. Kinematic parameters of the body movement (vector of the velocity of the center of gravity of the body and vector of its angular velocity of rotation) were determined by solving a system of equations:

$$\begin{cases} \dfrac{dL_i}{dt} = F, \\ \dfrac{dH_i}{dt} = M_i, \end{cases} \qquad (1)$$

where L_i is the amount of body movement in the adopted inertial coordinate system; H_i is the body momentum in the inertial coordinate system; F is the resultant of external forces applied to the body; M_i is the resultant moment from external forces applied to the body calculated relative to the origin of the inertial coordinate system.

A mathematical model describing the movement of a solid body without its lifting from the work surface (i.e. rolling plus sliding) on an inclined rough vibrating surface under the action of gravitational forces and the support reaction (excluding the influence of aerodynamic forces) was presented in [8].

The movement of the seed relative to the work surface was considered as a superposition of movements of its rotation relative to the point of contact **K** and the translational movement of the instantaneous center of rotation. The seed rotation is characterized by the value of angular velocity ω with which direction of the radius vector $\mathbf{R}_C$ changes in the coordinate system of the work surface. The translational displacement is characterized by the magnitude of the velocity vector $\mathbf{V}_K$ in the inertial (fixed) coordinate system. The translational movement of seeds is determined by periodic oscillations of the work surface which are transmitted to it through the point of contact.

Let us consider three coordinate systems (Fig. 3) to calculate kinematic parameters of movement of an individual seed:

– intrinsic coordinate system $CX'Y'Z'$ connected with the seed and having its origin in the center of the seed mass. Assume for convenience that the axes CX', CY', and CZ' coincide with the main axes of inertia of the seed under consideration;

– coordinate system K_0XYZ connected with the work surface and having its origin in point K_0 which is the point of seed delivering to the work surface;

– inertial (fixed) coordinate system $0X_iY_iZ_i$.

The work surface is inclined in two planes with respect to the inertial coordinate system. The inclination is given by angles α and β. The angle α characterizes the inclination of the longitudinal axis of the work surface to the horizon plane. The angle β characterizes the inclination of the transverse axis of the work surface to the horizon plane.

Assign the position of the work surface relative to the inertial coordinate system $0X_iY_iZ_i$ by the radius vector $\mathbf{r}_{vb}$ which determines the location of the point K_0. The derivative $\dot{\mathbf{r}}_{vb}$ characterizes the harmonic oscillations of the work surface under the action of the vibration exciter.

The radius vector $\mathbf{r}_{ck}$ specifies the location of the point of contact of the seed with the work surface relative to the origin of the coordinate system $0X_iY_iZ_i$ of the work surface. The derivative $\dot{\mathbf{r}}_{ck}$ characterizes the slip movement of the seed relative to the work surface.

The location of the point of contact relative to the origin of the inertial coordinate system is specified by the radius vector $\mathbf{R}_K$. The derivative of the radius vector $\mathbf{R}_K$ characterizes the movement of the seed relative to the inertial (fixed) coordinate system.

The location of the center of mass of the seed relative to the point of contact with the work surface is specified by the radius vector $\mathbf{R}_C$.

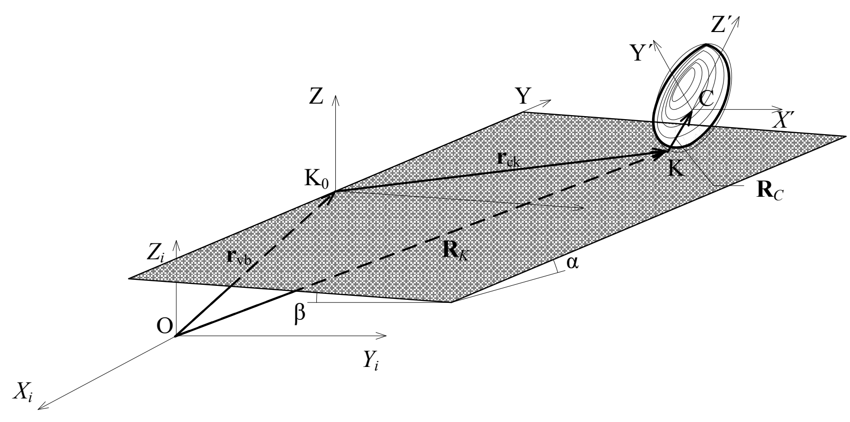

Fig. 3. Established coordinate systems for the calculation of kinematic parameters of seed movement

The system of equations (1) for the introduced coordinate systems will take the following form.

The individual seed momentum in the inertial (fixed) coordinate system is calculated using the expression:

$$\mathbf{L}_i = m\left(\dot{\mathbf{r}}_{vb} + \dot{\mathbf{r}}_{ck} + \omega \times \mathbf{R}_C\right), \tag{2}$$

where m is the seed mass.

Angular momentum of seed relative to the inertial coordinate system is as follows:

$$\mathbf{H}_i = \left(\mathbf{R}_K + \mathbf{R}_C\right) \times m\left(\dot{\mathbf{R}}_K + \omega \times \mathbf{R}_C\right) + \mathbf{H}_C, \tag{3}$$

where H_C is the angular momentum of the seed relative to its center of mass which (the momentum) is considered in the inertial coordinate system.

Taking into account (2) and (3), the system (1) will take the form:

$$m\frac{d\omega_y}{dt}R_{Cz} - m\frac{d\omega_z}{dt}R_{Cy} + m\left(\omega \cdot \mathbf{R}_C\right)\omega_x - m\omega^2 R_{Cx} + m\ddot{x}_{ck} + m\ddot{r}_{vb.x} = G_x - N \cdot f\frac{\dot{x}_{ck}}{\sqrt{\dot{x}_{ck}^2 + \dot{y}_{ck}^2}} + R_x^{aer.}, \tag{4}$$

$$m\frac{d\omega_z}{dt}R_{Cx} - m\frac{d\omega_x}{dt}R_{Cz} + m\left(\omega \cdot \mathbf{R}_C\right)\omega_y - m\omega^2 R_{Cy} + m\ddot{y}_{ck} + m\ddot{r}_{vb.y} = G_y - N \cdot f\frac{\dot{y}_{ck}}{\sqrt{\dot{x}_{ck}^2 + \dot{y}_{ck}^2}} + R_y^{aer.}, \tag{5}$$

$$mR_C^2\frac{d\omega_x}{dt} - mR_{Cx}\left(\mathbf{R}_C \cdot \frac{d\omega}{dt}\right) + m\left(\omega \cdot R_C\right)\left(R_{Cy}\omega_z - R_{Cz}\omega_y\right) + H_x - mR_{Cz}\ddot{y}_{ck} + mR_{Cy}\ddot{r}_{vb.z} - mR_{Cz}\ddot{r}_{vb.y} = R_{Cy}G_z - R_{Cz}G_y + M_{kch.x} - M_{aer.x}, \tag{6}$$

$$mR_C^2\frac{d\omega_y}{dt} - mR_{Cy}\left(\mathbf{R}_C \cdot \frac{d\omega}{dt}\right) + m\left(\omega \cdot R_C\right)\left(R_{Cz}\omega_x - R_{Cx}\omega_z\right) + H_y + mR_{Cz}\ddot{x}_{ck} + mR_{Cz}\ddot{r}_{vb.x} - mR_{Cx}\ddot{r}_{vb.z} = R_{Cz}G_x - R_{Cx}G_z + M_{kch.y} - M_{aer.y}, \tag{7}$$

$$mR_C^2\frac{d\omega_z}{dt} - mR_{Cz}\left(\mathbf{R}_C \cdot \frac{d\omega}{dt}\right) + m\left(\omega \cdot \mathbf{R}_C\right)\left(R_{Cx}\omega_y - R_{Cy}\omega_x\right) + H_z + mR_{Cx}\ddot{y}_{ck} - mR_{Cy}\ddot{x}_{ck} + mR_{Cx}\ddot{r}_{vb.y} - mR_{Cy}\ddot{r}_{vb.x} = R_{Cx}G_y - R_{Cy}G_x - M_{aer.z}, \tag{8}$$

$$N = m\frac{d\omega_x}{dt}R_{Cy} - m\frac{d\omega_y}{dt}R_{Cx} + m\left(\omega \cdot \mathbf{R}_C\right)\omega_z - m\omega^2 R_{Cz} + m\ddot{r}_{vb.z} - G_z - R_z^{aer.}, \tag{9}$$

where ω_x, ω_y, ω_z are projections of the angular velocity vector of the body relative to the instantaneous center of rotation K on the axis of the coordinate system associated with the work surface. The projections in the coordinate system of a work surface that are considered here and further; R_{Cx}, R_{Cy}, R_{Cz} are projections of the radius vector R_C; $\dot{x}_{ck}$, $\dot{y}_{ck}$ are projections of the seed sliding speed relative to the work surface; $\ddot{x}_{ck}$, $\ddot{y}_{ck}$ are projections of the seed sliding acceleration; $\ddot{r}_{vb.x}$, $\ddot{r}_{vb.y}$, $\ddot{r}_{vb.z}$ are projections of acceleration of the work surface during harmonic oscillations under the action of the vibration exciter; G_x, G_y, G_z are projections of the seed weight force; N is the magnitude of the support reaction acting on the seed; f is the coefficient of sliding friction; $M_{kch.x}$, $M_{kch.y}$ are projections of the vector of the moment of rolling resistance when seeds move on the work surface; $M_{aer.x}$, $M_{aer.y}$, $M_{aer.z}$ are projections of the vector of the aerodynamic moment of the seed which occurs when it is blown over by the airflow. The airflow is created due to the oscillations of the work surfaces of the vibratory machine; $R_x^{aer.}$, $R_y^{aer.}$, $R_z^{aer.}$ are projections of the aerodynamic force acting on the seed found in the airflow; H_x, H_y, H_z are projections of the angular momentum of the seed.

The components of equations (4) to (9) are calculated using the expressions given in [8].

To calculate the aerodynamic forces and moments acting on the seeds found in the airflow, the method of summing up the forces of resistance, lateral forces, as well as turning aerodynamic moments were calculated for the flat cross-sections of seeds [6].

The aerodynamic characteristics are calculated separately for individual projection of the air velocity vector at the location of the center of gravity of the seed. The velocity field as a function of time for the volume of air between two parallel working planes of a vibratory machine oscillating synchronously is calculated by solving the boundary value problem [10] by means of the numerical method of three-dimensional run [11].

The seeds are divided into parallel flat sections into two perpendicular planes for each projection of the air velocity vector at the point of the working space under consideration (Fig. 4).

As shown in the figure, the seeds are separated step by step using planes parallel to the Z and Y axes (vertical planes) and planes parallel to the X and Y axes (horizontal planes).

In general, total aerodynamic forces and moments for the seeds recorded in the coordinate system of the work surface can be determined using the following relationships.

The aerodynamic force acting along the Y axis is represented by the sum of the resistance force from the action of the component of air velocity along the Y axis, v_C; the lateral force from the action of the component of air velocity along the Z axis, w_C and the lateral force from the action of the component of air velocity along the X axis, u_C:

$$R_y^{aer} = \frac{\rho \cdot v_C^2}{2} \cdot \sum_{j=1}^{L_x/\Delta L} C_x\left(\alpha_j^{(zy)}\right) \cdot \Delta L \cdot b_j^{(zy)} + \frac{\rho \cdot w_C^2}{2} \cdot \sum_{j=1}^{L_x/\Delta L} C_y\left(\alpha_j^{(yz)}, \overline{f}_j^{(yz)}, \overline{x}_{f_j}^{(yz)}\right) \cdot \Delta L \cdot b_j^{(yz)} + \frac{\rho \cdot u_C^2}{2} \cdot \sum_{j=1}^{L_z/\Delta L} C_y\left(\alpha_j^{(xy)}, \overline{f}_j^{(xy)}, \overline{x}_{f_j}^{(xy)}\right) \cdot \Delta L \cdot b_j^{(xy)}. \tag{10}$$

The magnitude of the force of dynamic resistance of the seed by the action of the component of air velocity along the Y axis is calculated as follows:

$$\frac{\rho \cdot v_C^2}{2} \cdot \sum_{i=1}^{L_x/\Delta L} C_x\left(\alpha_i^{(zy)}\right) \cdot \Delta L \cdot b_i^{(zy)} = \frac{\rho \cdot v_C^2}{2} \cdot \sum_{j=1}^{L_z/\Delta L} C_x\left(\alpha_j^{(xy)}\right) \cdot \Delta L \cdot b_j^{(xy)}. \tag{11}$$

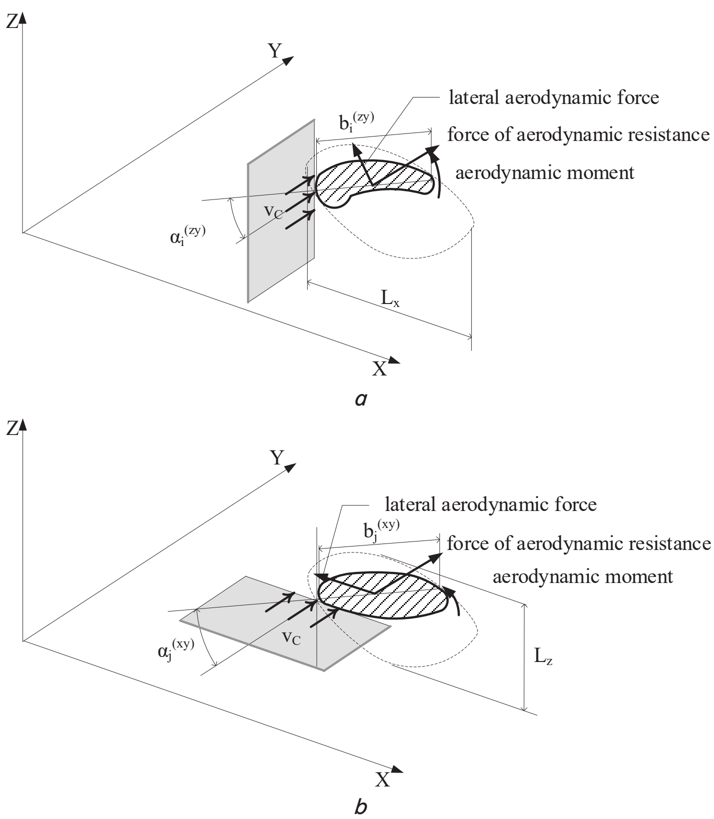

Fig. 4. Calculation of aerodynamic forces and moments for air velocity projections in the coordinate system of the work surface: *a* – velocity projection in the plane *ZOY*; *b* – velocity projection in the *XOY* plane

The aerodynamic moment that rotates the seed around the axis parallel to the *Y* axis is calculated as the following sum:

$$M_y^{aer}=\frac{\rho\cdot u_C^2}{2}\cdot\sum_{j=1}^{L_y/\Delta L}C_{m_j}^{(xz)}\cdot\Delta L\cdot b_j^{(xz)2}+ +\frac{\rho\cdot w_C^2}{2}\cdot\sum_{j=1}^{L_y/\Delta L}C_{m_j}^{(zx)}\cdot\Delta L\cdot b_j^{(zx)2}, \quad (12)$$

where

$$\frac{\rho u_C^2}{2}\sum_{j=1}^{L_y/\Delta L}C_{m_j}^{(XZ)}\Delta L\cdot b_j^{(XZ)^2}$$

is the aerodynamic moment from the action of the component of air velocity, *uC,* along the *X* axis;

$$\frac{\rho w_C^2}{2}\sum_{j=1}^{L_y/\Delta L}C_{m_j}^{(ZX)}\Delta L\cdot b_j^{(ZX)^2}$$

is the aerodynamic moment from the action of the component of air velocity, w_C, along the *Z* axis.

By analogy with (10) and (12), (13),

$$R_z^{aer}=\frac{\rho\cdot w_C^2}{2}\cdot\sum_{j=1}^{L_y/\Delta L}C_x\left(\alpha_j^{(xz)}\right)\cdot\Delta L\cdot b_j^{(xz)}+ +\frac{\rho\cdot u_C^2}{2}\cdot\sum_{j=1}^{L_y/\Delta L}C_y\left(\alpha_j^{(xz)},\bar{f}_j^{(xz)},\bar{x}_{f_j}^{(xz)}\right)\cdot\Delta L\cdot b_j^{(xz)}+ +\frac{\rho\cdot v_C^2}{2}\cdot\sum_{j=1}^{L_z/\Delta L}C_y\left(\alpha_j^{(yz)},\bar{f}_j^{(yz)},\bar{x}_{f_j}^{(yz)}\right)\cdot\Delta L\cdot b_j^{(yz)}. \quad (14)$$

$$M_x^{aer}=\frac{\rho\cdot v_C^2}{2}\cdot\sum_{j=1}^{L_x/\Delta L}C_{m_j}^{(yz)}\cdot\Delta L\cdot b_j^{(yz)2}+ +\frac{\rho\cdot w_C^2}{2}\cdot\sum_{j=1}^{L_x/\Delta L}C_{m_j}^{(zy)}\cdot\Delta L\cdot b_j^{(zy)2}. \quad (15)$$

$$M_z^{aer}=\frac{\rho\cdot v_C^2}{2}\cdot\sum_{j=1}^{L_z/\Delta L}C_{m_j}^{(yx)}\cdot\Delta L\cdot b_j^{(yx)2}+ +\frac{\rho\cdot u_C^2}{2}\cdot\sum_{j=1}^{L_z/\Delta L}C_{m_j}^{(xy)}\cdot\Delta L\cdot b_j^{(xy)2}. \quad (16)$$

Expressions (4) to (16) in their aggregate are a mathematical model of the non-lift movement of seeds with distinct aerodynamic properties on an inclined vibrating surface taking into account the action of moving air.

5. 2. Numerical algorithm for integrating the system of differential equations of movement taking into account the action of aerodynamic forces and moments

Let us use Euler's method to solve the system of differential equations (4) to (10). Determine values of the angular accelerations and the projection of slip acceleration at each step for the current time point $t=t_0+\Delta t\cdot s$ where *s* is the index of the performed calculation step. The acceleration values are found by solving a linear system of equations:

$$\begin{cases}\ddot{\theta}\cdot A_{11}+\ddot{\phi}\cdot A_{12}+\ddot{\psi}\cdot A_{13}+\ddot{x}_{ck}\cdot A_{14}+\ddot{y}_{ck}\cdot A_{15}=B_1,\\ \ddot{\theta}\cdot A_{21}+\ddot{\phi}\cdot A_{22}+\ddot{\psi}\cdot A_{23}+\ddot{x}_{ck}\cdot A_{24}+\ddot{y}_{ck}\cdot A_{25}=B_2,\\ \ddot{\theta}\cdot A_{31}+\ddot{\phi}\cdot A_{32}+\ddot{\psi}\cdot A_{33}+\ddot{x}_{ck}\cdot A_{34}+\ddot{y}_{ck}\cdot A_{35}=B_3,\\ \ddot{\theta}\cdot A_{41}+\ddot{\phi}\cdot A_{42}+\ddot{\psi}\cdot A_{43}+\ddot{x}_{ck}\cdot A_{44}+\ddot{y}_{ck}\cdot A_{45}=B_4,\\ \ddot{\theta}\cdot A_{51}+\ddot{\phi}\cdot A_{52}+\ddot{\psi}\cdot A_{53}+\ddot{x}_{ck}\cdot A_{54}+\ddot{y}_{ck}\cdot A_{55}=B_5,\end{cases} \quad (17)$$

where A_{ij} and B_i, $i=1, 2,..., 5$, $j=1, 2,..., 5$ are the constants for each considered calculation step, the coefficients which are found either from initial conditions of seed movement or from the results of calculations of the previous step. That is, when solving the system of equations for the points of time $t=t_0+\Delta t$, values of the coefficients are taken as follows: $A_{ij}=A_{ij}(t-\Delta t)=(A_{ij})_{s-1}$, $B_i=B_i(t-\Delta t)=(B_i)_{s-1}$.

For the obtained values of the second derivatives (accelerations) of angular and linear coordinates, velocities are calculated first and then kinematic parameters themselves are found using Euler's relationships.

Location of the center of the seed mass relative to the work surface is determined by step-by-step integration of the path gone up to the point of time $t=t_0+\Delta t\cdot s$:

$$x_s=\sum_{k=1}^{s}\left(x_{ck}\right)_k+\sum_{k=1}^{s}\left[\left(x_K\right)_k-\left(x_K\right)_{k-1}\right]+\left(R_{Cx}\right)_s, \quad (18)$$

$$y_s=\sum_{k=1}^{s}\left(y_{ck}\right)_k+\sum_{k=1}^{s}\left[\left(y_K\right)_k-\left(y_K\right)_{k-1}\right]+\left(R_{Cy}\right)_s, \quad (19)$$

where $(x_{ck})_k$, $(y_{ck})_k$ are projections of the vector of the sliding path traveled by the seeds at the *k*-th calculation step, $k=1, 2,..., S$; $(x_k)_{k-1}$, $(x_k)_k$, $(y_k)_{k-1}$, $(y_k)_k$ are the coordinates of the point of the seed contact with the work surface in the coordinate system of the work surface when performing *k*–1-th and k-th calculation steps, respectively; $(R_{Cx})_s$, $(R_{Cy})_s$ are projections of the radius vector connecting the point of contact with the center of mass of the seed in the coordinate system of the work surface for the S-th calculation step.

5. 3. Assessment of the degree of adequacy of the constructed mathematical model

To verify the adequacy of the mathematical model of vibration movement taking into account the aerodynamic factor, seed movement was modeled taking and without taking into account the influence of aerodynamic forces and moments. At the same time, estimates of the angles of inclination of the calculated paths of seed movement relative to the work surface were determined without taking into account φ_1^{rzr}, and taking into account φ_2^{rzr}, the action of aerodynamic forces and moments (Fig. 5).

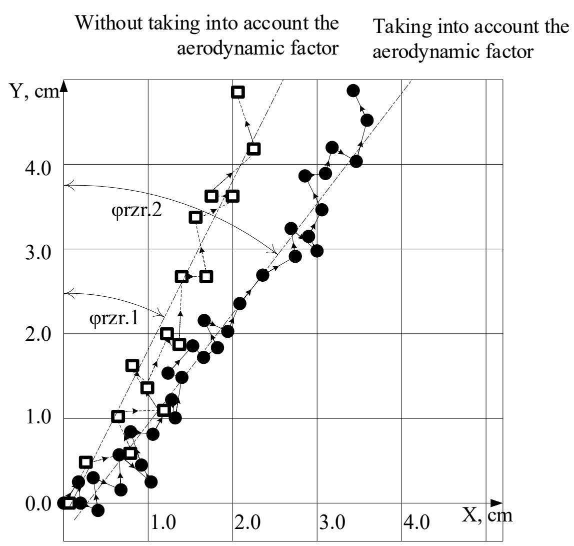

Fig. 5. The results of modeling the movement of seeds taking and without taking into account the aerodynamic forces and moments

The calculated angles of deviation of the seed path from the Y axis, φ_1^{rzr} and φ_2^{rzr} were determined according to the simulation results without taking into account and taking into account action of aerodynamic forces and moments, respectively. Certain initial conditions such as the initial kinematic parameters, geometric dimensions, and mass of the modeled seeds were set at the model input. The seed movement on the work surface was numerically simulated for the set input data. Coordinates of the location of the center of seed mass relative to the work surface were memorized for each step of the model: $\{x_s; y_s\}$, $s=1,..., S$ where S is the number of modeling steps until the seeds reach the set edge of the work surface.

The coefficients of the straight line that sets the general direction of seed movement were calculated on the formed array of points of the modeled seed path by the method of least squares. The line equation takes the following form:

$$x = a_1 y + a_0, \quad (20)$$

where

$$a_1 = \frac{\sum_{s=1}^{S} y_s \cdot x_s}{\sum_{s=1}^{S} y_s^2}, \quad (21)$$

$$a_0 = \frac{\sum_{s=1}^{S} x_s}{S}. \quad (22)$$

The value of the calculated angle of deviation of the seed path from the Y axis (from the left edge of the work surface) was determined using the following expression:

$$\phi^{rzr} = \mathrm{arctg}(a_1)\cdot 57, [\text{deg.}]. \quad (23)$$

The obtained results of the numerical experiment were compared with the field test data. The comparison was based on an assessment of the adequacy degree:

$$K_i = \frac{\left|\phi_i^{rzr.} - \phi^{exper.}\right|}{\phi^{exper.}} \cdot 100\,\%, \quad (24)$$

where φ_i^{rzr}, $i=1, 2$; $\varphi^{exper.}$ are the angles of inclination of the seed path determined by numerical (without taking into account, $i=1$, and taking into account, $i=2$, aerodynamic forces and moments) and the full-scale experiments, respectively.

During the experiments, a control sample consisting of M seeds was fed to the feed hopper and the vibratory machine was started. Seeds of the control sample were fed from the feeder to the controlled work surface. Vibrational movement of seeds on the work surface was directed to various paths. After descending from the work surface, the seeds fell into corresponding compartments of the calibrated receiver (Fig. 2). After freeing the work surface from all seeds of the control sample, the vibratory machine was turned off and the seeds were counted in the compartments of the calibrated receiver.

The experimental angle between the beam of the average statistical seed path and the left edge of the work surface was calculated as a weighted average sum taking the form:

$$\phi^{exper.} = \sum_{i=1}^{N} \frac{m_i}{M} \phi_i, \quad (25)$$

where m_i is the number of seeds from the control sample that got into the i-th compartment of the calibrated receiver; φ_i is the value of the angle of the beam deviation which corresponds to the i-th compartment of the receiver.

The conclusion on the adequacy of the mathematical model of seed vibrational movement taking into account the aerodynamic forces and moments was drawn based on the established relative deviation of the numerical modeling results from the experimental data, K_2.

Conclusion about the extent to which the model which takes into account action of aerodynamic forces and moments was more adequate than that for the model where these factors were not taken into account was drawn based on the correlation of relative deviations K_1 and K_2.

The results of estimating the adequacy of the mathematical model for different seed crops are given in Table 1.

As can be seen from the results for the seeds with distinct aerodynamic properties (rustic tobacco, common tobacco) given in Table 1, there is a significant increase in the level of adequacy of the mathematical model taking into account the aerodynamics. For these crops, a discrepancy between the calculated and experimental values of the angle of trace deviation decreases by approximately 30 % ($K_1/K_2 \approx 0.7$). For crops with less distinct aerodynamic properties (false flax seeds), taking into account the aerodynamics does not provide such a significant increase in the model accuracy: accuracy increases by 20 % ($K_1/K_2 \approx 0.8$).

Table 1

Estimation of the adequacy of the mathematical model

Crop name (the model application mode)	Mode of vibrating separation		Angle of trace inclination		Index of model adequacy
	vibration amplitude, mm	vibration frequency, 1/s	calculation, deg.	experiment, deg.	
1. Rustic tobacco (without taking into account the aerodynamics)	1.5	2200	21	34	38
2. Rustic tobacco (taking into account the aerodynamics)	1.5	2200	41		20
3. Tobacco (without taking into account the aerodynamics)	1.5	2200	21	29	28
4. Tobacco (taking into account the aerodynamics)	1.5	2200	32		10
5. False flax (without taking into account the aerodynamics)	1.5	2200	20	24	17
6. False flax (taking into account the aerodynamics)	1.5	2200	27		13

6. Discussion of the results obtained in the study of constructing a model of the vibrational non-lift movement of seeds taking into account the aerodynamic factor

Fig. 5 shows the results of modeling the movement of seeds along the work surface of the vibratory machine. The results of modeling the movement of seeds which are affected by aerodynamic forces and moments are presented using marks in the form of black circles. The movement of seeds with similar physical and mechanical properties but with zero aerodynamic forces and moments is shown with the help of marks in the form of white squares. As can be seen from the above results, the influence of air makes it difficult to move the seeds relative to the work surface under the action of vibration pulses. In the first case (the path of movement formed by the marks in the form of black circles), the seed path on the work surface up to the moment of its descending consists of a significantly larger number of steps-movements of the modeled seed. Each such step obtained under the influence of the momentum from the oscillating rough work surface has a shorter length. Also, as a result of the influence of moving air, there is a greater deviation of the middle axis of the common path of the seed on the work surface down the slope. There is a less acute angle of the middle axis of movement of the seed relative to the Y axis of the coordinate system of the work surface. The difference in angles of the axes of movement of the seeds in relation to the Y axis without taking into account and taking into account the aerodynamic forces and moments is 15–25 %.

Also, it should be noted that the mathematical model taking into account the aerodynamics gives the angle of deviation of the seed trace overrated relative to the experimental data (the beam of movement deviates from the Y axis). The mathematical model without taking into account the aerodynamics underrates this characteristic (the beam of movement is pressed toward the Y axis).

However, taking into account the aerodynamic forces and moments, in general, brings the calculated kinematic parameters closer to real data.

Thus, the mathematical model of seed vibration movement without taking into account the aerodynamics [8] for small size seeds with distinct aerodynamic properties gives the calculated movement path which coincides with the direction of vibration.

The proposed model of movement for seeds of this species gives a result when the average statistical path is quite noticeably (up to 30 %) deviating to a side because of weakening of vibration under action of aerodynamic factors.

The movement path of similar seeds obtained experimentally lies inside the sector formed by rays of the average statistical movement paths of the seeds determined without taking into account and taking into account the aerodynamics. In this case, the path determined taking into account the aerodynamics deviates from the experimental path by a smaller angle.

A mathematical model was constructed that takes into account the action of aerodynamic forces and moments on seed movement. The aerodynamic component depends on the design features and mode of operation of the vibratory machine. That is, a toolkit has been created for conducting parametric studies in the improvement of vibratory machines and adjusting their modes of operation for the treatment of fine-size seeds.

The constructed mathematical model makes it possible to take more fully into account the harmful impact of the aerodynamic factor during cleaning (sorting) of fine-seed mixtures for the whole range of operating modes of the vibratory machine. This is quite possible due to the introduction of lateral aerodynamic forces and moments into the vibration model in addition to the force of dynamic resistance. This makes it possible to extend the action of the aerodynamic factor on the non-lifting operating modes of vibratory machines. Such modes are more relevant in the implementation of vibration treatment of fine seeds.

The study, the results of which are presented in this paper, was performed for constantly set boundary conditions for the air area between the work surfaces (without the use of aerodynamic screens) and for one operating mode (amplitude, frequency, and direction of oscillations) of the vibratory machine. The study data volume was too bulky for the format of a scientific article to consider all these factors. Meanwhile, these factors also have a significant impact on the integrated kinematic parameters of seed movement. A full-scale study should be conducted to establish rational design parameters and modes of operation of vibratory machines in order to improve the efficiency of the processes of cleaning (sorting) the fine-size seeds.

In the future, the authors will conduct a full-factor parametric study of the seed cleaning processes with distinct aerodynamic properties on the basis of the constructed mathematical model. This will result in appropriate design characteristics of aerodynamic screens for work surfaces of vibratory machines.

7. Conclusions

1. A system of differential equations of non-lifting vibrational seed movement was obtained. It was supplemented by a complete set of aerodynamic forces and moments. In contrast to the existing models of vibrational movement where

the aerodynamic factor is given only in a form of a dynamic resistance force, the action of lateral aerodynamic forces was additionally taken into account. This makes it possible to extend the modeled range of operation of vibratory machines taking into account the aerodynamic factor. Due to the developed system of differential equations, it is possible to calculate kinematic parameters of movement of fine-size seeds under non-lift modes of vibrational movement (with sliding and rolling). In existing models, aerodynamics is taken into account only for vibration modes with the seed rebound which are less relevant for cleaning (sorting) the fine-size seeds.

2. It was shown that the system of differential equations of vibrational movement taking into account the action of aerodynamic forces and moments can be solved through numerical integration using the Euler method. The algebraic equations obtained by converting the differential equations of movement into a finite difference form are linear. This fact as well as the relatively small dimension of the equation system does not imply any difficulties in numerical solving the equations of vibrational movement of seeds.

3. According to the results of numerical and field experiments, taking into account the aerodynamic factor improves the adequacy of the mathematical model. The value of the relative difference between the calculated and experimental values of the angle of seed trace during vibrational movement on the work surface was proposed as a quantitative indicator of the level of adequacy of the used mathematical model. It was found that for rustic tobacco seeds, the model without taking into account the aerodynamic factor gives a 38 % deviation from the experiment. Taking into account the aerodynamics gives a 20 % deviation. The deviation amounts to 28 and 10 % for tobacco seeds and 17 and 13 % for false flax seeds, respectively. On average, the degree of adequacy of the mathematical model taking into account the aerodynamic factor increases by 30 % compared with the model where aerodynamics is not taken into account.

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