

The processes to form the compositions of loose materials in centrifugal mixers of continuous action have been considered. Based on the method of discrete elements, a mathematical model of the movement of particles in the rotor of the centrifugal mixer was built, taking into consideration their geometric and physical-mechanical parameters. To assess the extent of influence of these parameters on the nature of particle movement, a well-known mathematical model in the form of a system of differential equations was used, which was built on the basis of classical laws of mechanics. The process of mixing particles of two loose materials under different initial conditions of movement was modeled. The trajectories of individual particles along the bottom and side wall of the rotor were calculated.

The results of the research reported here have established that the model built on the basis of the discrete element method makes it possible to improve the accuracy of determining the parameters of the movement of loose materials in the mixing zone. Calculations that involved this method show that the length of the particle trajectory is 2.9, and the movement time is 9 times greater than those calculated by the system of differential equations. The built and known mathematical models demonstrated the same nature of the distribution of components in the mixer. The value of the Pearson correlation coefficient between the calculated values of the coefficients of variation is 0.758. The best homogeneity is achieved by separating the flows of the mixture components and reducing the distance between their centers.

The experimental study was carried out using a centrifugal mixer of continuous action with a conical rotor. Particle trajectories were constructed; it was established that the shape of the trajectory built by a discrete element method is closer to the experimental one.

The results reported in this paper make it possible to predict the impact of the structural and technological parameters of the mixers of continuous action on the uniformity of the mixture

Keywords: discrete element method, centrifugal mixer, loose material, continuous mixing, mixture homogeneity

DETERMINING THE LOOSE MEDIUM MOVEMENT PARAMETERS IN A CENTRIFUGAL CONTINUOUS MIXER USING A DISCRETE ELEMENT METHOD

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1. Introduction

In the light, chemical, and other industries, the preparation of compositions of loose materials is carried out with the help of mixers of various types and designs [1–3]. The requirements for such equipment and the principle of its operation depend on the physical-mechanical properties of the starting materials, their particle size distribution, as well as the predefined quality parameters of the finished composition [4, 5]. The number of mixed components is determined by the formulation of the mixture. The particle dimensions of the starting materials vary widely depending on the form used to supply the material (powders, granules), which makes it almost impossible to design universal mixing equipment [6, 7]. At the same time, at present, there is a tendency to increase the use of granules [3, 8, 9] when forming the mixtures of polymeric materials. That is associated with the increased offer of various commercially available super

concentrates of dyes, which are supplied precisely in the form of granules. Compared to powdered dyes in the form of powders, those ones avoid problems related to cleaning the equipment when changing the color of the dye, air pollution, pouring powder into the lower layers of the mixture, and others. In addition, in the manufacture of a series of articles, secondary raw materials are often used, which are also supplied in the form of granules. Thus, in many cases, the components of polymer compositions are particles of spherical, ellipsoidal, cylindrical shapes with a diameter in the range of 5 to 7 mm [1, 6]. The percentage of components varies from ten to tens of percent but, in any case, the mixing equipment should provide the necessary value of the basic parameter of the mixture quality – its homogeneity [6]. For the preparation of compositions, periodic and continuous mixers are used, which employ certain physical effects to mix components. A special role belongs to centrifugal mixers of continuous action (CMCA) that ensure the required

uniformity of the mixture, relatively low impact of working bodies on particles, as well as high efficiency [2]. At the same time, it is technically difficult to ensure control over the homogeneity of the loose medium in continuous mixers and, accordingly, to manage the process of preparation of the composition. Constructing mathematical models of loose media movement in the working volume of mixers of this type could make it possible to estimate the nature of particle movement. And determine the effect of particle size distribution, the nature of loading, the design of the mixer, as well as the speed of movement of working bodies on the trajectory of movement. This would provide for the conditions to design mixers with parameters close to optimal, which predetermines the relevance of such research.

2. Literature review and problem statement

The authors of paper [10] reported the results of studying the movement of polydisperse particles of loose materials in drums. Mathematical modeling was carried out using the method of Monte Carlo and a discrete element method (DEM). They established the interrelations between the technological parameters of the equipment and the uniformity of the mixture. It should be noted that the cited paper considers a mixing mode under which interactions between particles exert a decisive effect on the movement of the entire mass of the material. That complicates the application of the proposed approach to studying the operation of continuous mixers. Work [11] examines the nature of the movement of individual particles of the loose environment, the trajectory of movement, and the time they spend in the working area of a centrifugal mixer of continuous action. The results make it possible to predict the nature of movement and the shape of the trajectory of particles inside the mixer rotor. At the same time, the built mathematical models do not take into consideration the possible interactions of particles with each other.

The results of studying the process of mixing polydisperse materials in drum and conical mixers are reported in [12]. The author establishes the dependence of mixing quality on the technological and structural parameters of mixers and particle sizes. The results of the research were confirmed by experimental verification. At the same time, the proposed models can only be used for periodic mixing equipment.

Work [13] addresses the processes of continuous preparation of mixtures for the pharmaceutical industry. The results of studying the effect of the pulsations of the input flows of the components and the smoothing ability of the mixer on the quality of the mixture are presented. The author analyzed the nature of particle movement inside the equipment and determined the impact of structural and technological parameters on the quality of the mixture. It should be noted that the homogeneity is determined at the outlet of the mixer in the output nozzle area. That approach does not make it possible to obtain information about the impact of working bodies on the effectiveness of mixing and determine their impact on the homogeneity of the mixture in general.

The authors of work [14] proposed a mathematical model of the process of mixing dispersed materials, which was constructed on the basis of correlation analysis methods. The movement of loose materials in centrifugal mixers of continuous action with a conical rotor, which implies the presence of advanced particle flows, has been investigated. The authors established a relationship between the vari-

ances of input and output signals, due to which they were able to predict the homogeneity of the flow based on known recirculation coefficients. The use of that approach in practice is complicated by the need to build models linking the structural and technological parameters of the equipment to recirculation coefficients.

Models of mixing loose materials for the pharmaceutical industry in mixers of continuous action of two types are proposed in paper [15]. The research was carried out using a discrete element method and the systems of differential equations, built on the basis of classical laws of mechanics. The cited paper considers mixing modes under which there are practically no sparse flows of material, which makes it difficult to use the obtained results for centrifugal mixers.

Work [16] analyzed the process of mixing particles of loose materials inside the rotating drums. The mathematical models, built on the basis of a discrete element method, made it possible to determine the effect of particle parameters (friction coefficients) on the trajectory of their movement. At the same time, the mutual impact of particles is much higher than in centrifugal mixers, which does not make it possible to apply the results obtained for this type of equipment.

The mathematical models of the micromechanics of movement of loose materials, built on the basis of DEM, are considered in work [17]. Using them, the authors managed to investigate vortex evolution during the movement of loose materials and its impact on movement trajectories. The cited work mainly investigates the dense streams of materials with a high level of interaction of particles with each other.

The process of mixing three-phase combined systems in centrifugal mixers of continuous action with a conical rotor is explored in [18]. The influence of liquid on the homogeneity of the mixture of loose materials was determined. The proposed mathematical model is represented in the form of differential equations and does not take into consideration the trajectories of individual particles.

The effectiveness of using a discrete element method to study the movement of particles in bunkers and plate feeders of continuous action is shown in work [19]. It was demonstrated that the application of DEM made it possible to predict the presence and magnitude of pulsations in the streams of loose materials. The results were confirmed by the experimental study. At the same time, it is almost impossible to apply the obtained results for the analysis of particle movement in centrifugal mixers. This relates to the fact that in the system "bunker–plate feeder" particles move in a continuous flow. The number and intensity of interactions "particle–particle" are, in this case, much greater than in the mixer rotor.

An analytical study of particle movement in the rotor of the centrifugal mixer is reported in paper [20]. Based on the laws of classical mechanics, a mathematical model of particle movement in the rotor was built, which is represented in the form of a system of differential equations. The effect of the initial position of particles on their distribution inside the rotor and on the homogeneity of the mixture was determined. It should be noted that the proposed model is idealized. Particles are considered as material points, the impact of possible impact interactions between particles and working bodies is not taken into consideration.

Our review of the above studies shows that the movement of loose materials is examined using the mathematical models built on the basis of the theory of motion of solid media, rheological models, and a discrete element method. In

this case, it is the DEM that makes it possible to analyze the interaction of individual particles with each other and with working bodies of equipment, which is important for studying the movement of material in centrifugal mixers of continuous action. Since none method fully reflects the physical processes that occur during the mixing of loose materials, it is advisable to compare the results obtained from different mathematical models and check their correspondence to the actual trajectories of particles. In addition, an important task when investigating continuous mixers is to determine the effectiveness of the influence of working bodies on the mixing process, that is, it is necessary to investigate their effect on a change in the uniformity of the mixture. For CMCA, that means the construction of mathematical models that make it possible to determine the distribution of components at the rotor outlet, depending on the predefined initial conditions of particle movement, the structural and technological parameters of the equipment.

3. The aim and objectives of the study

The purpose of this work is to determine the parameters of the movement of particles of the loose medium in the rotor of the centrifugal mixer, the effect of their initial position on the shape of trajectories, and the mutual location of particles at the rotor outlet.

To accomplish the aim, the following tasks have been set:

- to determine the influence of taking into consideration the geometrical and physical-mechanical parameters of particles in the mathematical models of their movement inside a CMCA rotor on the shape and length of trajectories, the time the particles spend in the mixer;
- to determine the effect of the initial positions of particles on their distribution inside the rotor using the built mathematical model;
- to conduct an experimental study of particle movement, estimate the accuracy of the constructed and known mathematical models.

4. Materials and methods to study particle movement in a centrifugal mixer

This paper examines mathematical models constructed on the basis of classical laws of mechanics [20] and a discrete element method. The model in [20] was a system of differential equations. It was solved by the Runge-Kutta method of the 4th order using software written in the Python programming language (USA). To determine the influence of geometric dimensions and the physical-mechanical parameters of particles a model was built based on DEM that consisted of two parts. The first was a model of a conical rotor, constructed using the SolidWorks system (France). The second represented the models of particles of loose material and their interaction with each other and with a rotor, built in the EDEM 2017 system (Great Britain). The calculation of the normal component of the forces of interaction between particles was carried out in accordance with the contact theory by Hertz (Hertz, 1882). The tangential components were calculated on the basis of work by Midlin-Deresiewicz [21]. The damping components of the constituent forces of interaction of particles were calculated on the basis of elasticity coefficients [22, 23]. The tangential friction force was deter-

mined by the law of Coulomb. Rolling friction force was determined on the basis of the rotational moment of particles.

The experimental study was carried out at a research bench. It included a continuous mixer, rotor speed regulator, tachometer. During analytical research, the rotation speed of the rotor and its geometric dimensions were equal to similar parameters of the research bench.

Mathematical processing of results, the construction of charts and diagrams involved the software written in the Python programming language.

5. Results of studying the movement of a loose medium in a centrifugal mixer of continuous action

5.1. Analytical study of the dynamics of particle movement inside the rotor of a centrifugal mixer

Loose material is fed into the mixer by dosing devices [19, 24, 25]. Typically, it comes in the form of streams of particles whose number is equal to the number of components in the mixture. The vectors of particle speeds in the streams almost coincide. Since the task of the mixer is to prepare a homogeneous composition, the particles must be distributed as evenly as possible inside its volume. This task in mixing equipment is solved with the help of working bodies that change the vectors of particle movement by interacting with their individual groups. The main working body of centrifugal mixers is the rotor that rotates at the predefined angular velocity. The streams of the mixture components fall to the bottom of the rotor, and the particles are in the field of action of centrifugal forces. Since the initial magnitude of these forces depends on the distance to the axis of rotation, then the minimal differences in the position of particles in the input flows lead to a significant difference in the initial conditions of their movement inside the rotor. In the ideal case, the particles are distributed evenly in the volume of the rotor. The mixing process takes place in thin layers of particles moving along the inner surface of the rotor when intersecting flows with different directions of particle movement and interaction with the structural elements of the mixer. Intensification of the mixing process is provided with the help of additional working bodies: turbulizers, blades, additional rotors, etc. Despite the large variety of designs and modes of operation of such devices, their main purpose is to expand the range of particle speeds and directions, that is, maximum chaotization. As a result, the probability of local particle groupings is reduced, which increases the homogeneity of the mixture. Fig. 1 shows the structure of a continuous centrifugal mixer with conical rotor 1, which rotates around the vertical axis Z passing through its center. Particles of two components from flow molders 2, 3 enter the inlet nozzle of mixer 4 and, under the action of gravity, fall to the bottom of the rotor. Further, under the action of centrifugal forces, they follow spiral trajectories to ascend along the lateral surface of the rotor. After that, they bounce off the upper part of case 5 and fall into the zone between rotor 1 and the lower part of case 6. The finished mixture is unloaded through outlet nozzle 7 with the help of blade 8 rotating around the Z axis.

During the operation of the mixer, the centrifugal forces of inertia, friction force, gravity, Coriolis, and aerodynamic forces act on the mixture particles. The movement of components at high speeds also leads to the destruction of particle conglomerates during their interaction with the working bodies of the equipment.

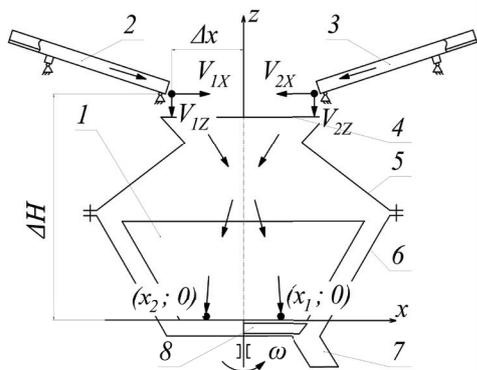


Fig. 1. Schematic of a centrifugal mixer of continuous action

The initial coordinates of the particle movement at the bottom of the rotor are determined by their distance from the edge of the molders to the axis of rotation of the rotor Δx and the speeds (V_1, V_2) of particles, as well as the distance from the bottom of the rotor to the molders ΔH .

Earlier study [20] derived a mathematical model of particle movement inside the conical rotor, based on the laws of classical mechanics [22, 23]. When building the specified model [20], a series of simplifications were adopted. In particular, particles were considered in the form of material points, their elastic properties were not taken into consideration. Within the framework of the analytical study reported in this work, a mathematical model of particle movement has been constructed, based on the method of discrete elements [26]. It takes into consideration the geometrical parameters of particles, the value of their density, Poisson coefficients, and shear modules (Table 1). The resulting model helped calculate the trajectories of particles in the conical rotor of the centrifugal mixer of continuous action. Particle trajectories were compared with the results reported in work [20]. We estimated the accuracy of calculations on the basis of comparison with the results of the experimental study.

The laws of mechanics, which are used to determine the coordinates and speeds of particles, characteristics of their interaction with other particles and equipment elements, underlie DEM. Loose material is considered as a set of discrete particles that may have different properties and shapes of the surface. Since the processes of mixing granules of polymeric materials are investigated in this paper, the particles are modeled as spheres with a constant radius.

Table 1

Particle parameters

Parameter name	Parameter value
Shape	Sphere
Radius, m	0.001
Density, kg/m ³	2,300
Friction factor	0.3
Shear module, Pa	5.692·10 ⁷
Poisson coefficient	0.3

The input parameters for the modeling process involving DEM are the position ($x_i; y_i; z_i$), the initial linear ($v_{xi}; v_{yi}; v_{zi}$) and angular ($\omega_{xi}; \omega_{yi}; \omega_{zi}$) velocities of all particles. In addition, we set the position and parameters of movement

of working bodies of the equipment interacting with the material. The modeling process is iterative. Schematically, the steps for calculating DEM for the i -th and j -th particles are shown in Fig. 2. The first two steps demonstrate the interaction of particles with each other, and the third – with the surface of the equipment.

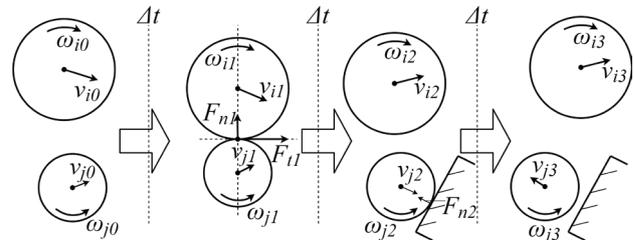


Fig. 2. Scheme for calculating the movement of particles by a discrete element method

At each step, the particle position is calculated, and the possible interactions of particles with the elements of the equipment and with each other are checked. In the presence of such interactions, the values of tangential and normal forces are calculated, and new vectors of particle speeds are calculated. On their basis, the position of the particles is calculated at the next step of modeling (after time Δt). Next, the calculation cycle is repeated.

Before starting the calculation, the Δt modeling step is determined. There are several criteria by which the value of Δt is determined. This paper has employed a Rayleigh criterion [21]:

$$\Delta t = \frac{\pi R}{0.163\nu + 0.87766} \sqrt{\frac{\rho}{G}}, \tag{1}$$

where R, ρ, G, ν are the radius, density, shear module, and Poisson coefficient of the particle, respectively.

The calculation of normal and tangential forces of interaction between particles and their moments of inertia was carried out on the basis of well-known laws of mechanics; it was discussed in detail in work [19].

Fig. 3 shows the simulated movement of a group of particles in the rotor.

The structural and technological parameters of the conical rotor are given in Table 2. The choice of the rotor of the conical form is carried out on the basis of the results reported in [1], which proved that such equipment ensures a better mixing effect.

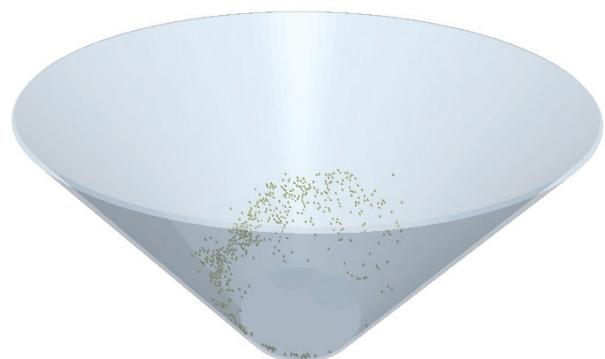


Fig. 3. Simulated movement of particles in a conical rotor orbiting a vertical axis

Table 2
Parameters of the mixer's conical rotor

Parameter title	Value
Base radius, m	0.051
Height, m	0.18
Funnel angle half, degree	30
Outer radius, m	0.155
Rotation angular velocity, rad/s	120

5. 2. Studying the trajectories of particles of loose materials inside the rotor

During the operation of the mixer, the components enter the rotor and fall to its bottom. Then, under the action of centrifugal force, they rise along the side wall of the rotor. As shown in work [19], the velocity and coordinates of the particles at the inlet to the mixer are random variables. At the same time, the location and structural parameters of flow molders ensure that particles get inside the mixer. This means that the components of the mixture enter the mixer by streams in which the particles have almost the same speed vectors. The number of such streams is at least equal to the number of components of the mixture. In the case of using a turbulizer that divides streams into constituent parts, it is a multiple of the number of input components and depends on the speed of its rotation. To determine the effect of additional considered particle parameters on their trajectories and distribution, compared to the results reported in [20], similar assumptions were made regarding the characteristics of input streams.

In particular, the initial positions of particle flows, their number, and the number of particles in them were set in the same way as in work [20], namely:

1) Experiment 1. The flow of the main component (MC) (400 particles) was sent to the first quadrant of the rotor base, the flow of the key component (KC) (100 particles) – into the second quadrant.

2) Experiment 2. The flow of MC (400 particles) came into the first quadrant, the KC stream (100 particles) – into the third quadrant.

3) Experiment 3. MC came in two streams, which consisted of 200 particles each and fell into the first and second quadrants, KC arrived in two streams of 50 particles, which fell into the third and fourth quadrants.

4) Experiment 4. MC came in two streams, which consisted of 200 particles each and fell into the first and third quadrants, KC came in two streams of 50 particles that fell into the second and fourth quadrants.

The results of modeling the trajectories of particle movement according to the developed mathematical model involving DEM are shown in Fig. 4.

We compared our results with the results reported in [20] according to certain values of the parameters of the trajectories of particles, which are given in Table 3. For each parameter, we calculated the average value, variance, and root mean square deviation.

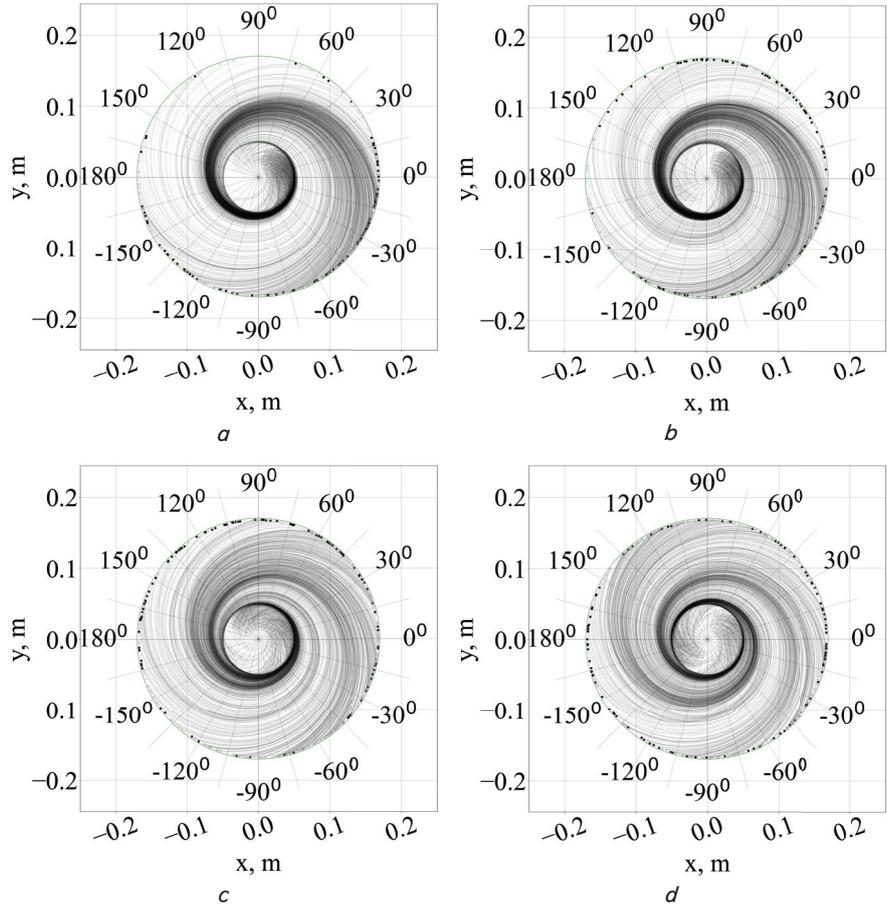


Fig. 4. Trajectories of particles of loose material: *a* – experiment 1; *b* – experiment 2; *c* – experiment 3; *d* – experiment 4

Table 3

Parameters of particle trajectories in the conical rotor

Parameter		DEM	System of DE's
The length of the trajectory along the bottom of the rotor, m	Average value, m	0.110	0.031
	Variance	1.38E-03	5.04E-04
	RMS deviation	2.97E-02	1.75E-02
The length of the trajectory along the side wall, m	Average value, m	0.675	0.239
	Variance	6.71E-03	3.15E-05
	RMS deviation	6.61E-02	4.73E-03
Particle movement time along the bottom, s	Average value, m	0.293	0.029
	Variance	3.40E-03	2.65E-04
	RMS deviation	4.85E-02	1.25E-02
Particle movement time along the side wall, s	Average value, m	0.373	0.045
	Variance	2.26E-03	5.66E-06
	RMS deviation	3.64E-02	1.54E-03

The values obtained using DEM significantly exceed the values of the corresponding parameters, which are calculated from the system of differential equations. This makes it possible to quantify the impact of the parameters included in the model involving DEM.

5. 3. Determining the effect of the initial positions of particles on their distribution inside the rotor

We estimated particle distribution unevenness by the method proposed in work [20]. The rotor is divided into segments in increments of 15° (Fig. 4). In each of these segments, we determined the number of particles of the main

and key components at the time of their exit from the rotor. Histograms showing the number of MC and KC particles in each sector are shown in Fig. 5.

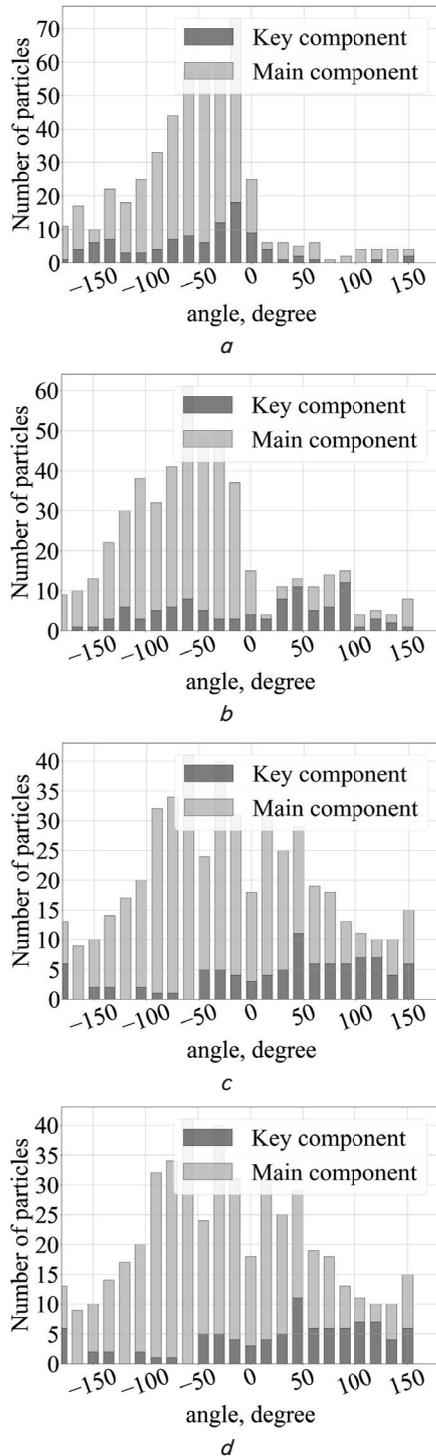


Fig. 5. Histograms showing particle distribution at the exit from the conical rotor: *a* – experiment 1; *b* – experiment 2; *c* – experiment 3; *d* – experiment 4

We numerically estimated particle distribution using the variance coefficient V_c [20]. The results of calculations for all four experiments are given in Table 4.

We numerically estimated the relationship between the values of variance coefficients based on DEM and those reported in work [20] by using the Pearson correlation coefficient

$$r = \frac{\sum(V_{CiDE} - \overline{V_{CiDE}})(V_{CiDEM} - \overline{V_{CiDEM}})}{\sqrt{\sum(V_{CiDE} - \overline{V_{CiDE}})^2 \sum(V_{CiDEM} - \overline{V_{CiDEM}})^2}}, \quad (2)$$

where $\overline{V_{CiDE}}$, $\overline{V_{CiDEM}}$ are the average values of variance coefficients calculated from a system of differential equations [20] and by a discrete element method, respectively; V_{CiDE} , V_{CiDEM} are the values of variance coefficients calculated from a system of differential equations [20] and by a discrete element method, respectively.

Table 4

Dependence of a variance coefficient value on the initial position of the flows of mixture components in the rotor

Experiment No.	V_c calculated by DEM
Experiment 1	92.7 %
Experiment 2	144.1 %
Experiment 3	108.1 %
Experiment 4	78.9 %

The resulting value is $r=0.758$, which confirms the existence of a relationship between the variance coefficients. Accordingly, the nature of the particle movement, calculated according to the mathematical model proposed in this work, coincides with the results of calculations reported in [20].

The difference between the calculated values of the variance coefficients is much less than the difference between the parameters of particle trajectories (Table 2). This indicates that the determining effect on the distribution of components is exerted by the nature of their movement (the spiral shape of the trajectory), rather than the parameters of interaction of particles with each other and with the elements of equipment.

5. 4. Experimental study of particle movement inside the rotor

In order to verify the obtained results, we studied the movement of particles inside the conical rotor. For our research, a particle was covered with paint and sent to the bottom of the rotating rotor at a predefined speed. Its inner surface was covered with a paper insert, on which the imprint of the paint remained during the movement of the particle. The centrifugal-driven particle moved on the base of the rotor, its side wall, and flew out of the mixer through the outlet nozzle. After the particle exit, the mixer stopped, and a paper insert was replaced in the rotor. Next, the rotor speed increased again to the predefined value, after which the next particle was sent to the mixer. The experiment was repeated for 20 particles. As a result, the fingerprints of 20 trajectories were obtained. For each particle transition point from the bottom of the rotor to its side wall, the coordinates $(R_0; 0; 0)$ were set. Thus, all particle trajectories were reduced to one point. Next, the side surface was divided into sectors in increments of 10° . The coordinates of the intersection points of all particle trajectories with the edges of sectors were determined. After that, the average values of these coordinates for each sector were calculated. These calculations were carried out only for a solid part of the trajectories. As a result of vibrations that occurred during the research bench operation, “gaps” and

“jumps” appeared in the trajectories. Most of these “gaps” were observed at the end of the trajectory. Therefore, the processing of experimental data took into consideration part of the trajectory from the base of the rotor to the level $z=0.0463$ m. The imprints of trajectories of all particles without “gaps” were obtained for this section. As a result, the “average” trajectory of the particle along the side wall of the rotor was calculated.

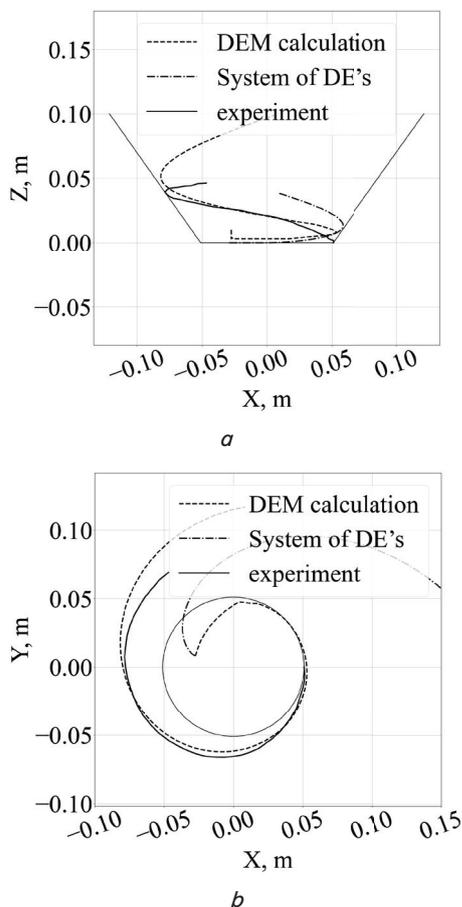


Fig. 6. Trajectories of the particle movement, determined on the basis of DEM, the mathematical model from work [20], and experimentally: *a* – side view; *b* – top view

Our results are shown in Fig. 6.

The shape of the trajectory determined by DEM is much closer to the experimental one. To quantify the difference between the trajectories obtained by different methods, the lengths of all three trajectories for the area from the beginning of the lateral wall of the rotor to the level $z=0.0463$ m were calculated. The results are:

- the length of the trajectory determined experimental-ly: $L_{exp}=0.288$ m;
- the length of the trajectory calculated according to the mathematical model from work [20]: $L_{DE}=0.021$ m (13.7 times less than the length of the experimental one);
- the length of the trajectory calculated by a discrete element method: $L_{MDE}=0.238$ m (1.2 times less than the length of the experimental one).

These results confirm that taking into consideration the geometrical and physical-mechanical parameters of particles makes it possible to increase the accuracy of calculating the parameters of their movement.

6. Discussion of results of studying the loose environment movement in the centrifugal mixer of continuous action

All experiments carried out in this work (Fig. 4–6) confirm that the trajectory of particles in the rotor takes the shape of a spiral. At the same time, when calculating based on DEM, this spiral has a greater number of turns and, accordingly, a longer trajectory. This is demonstrated in Fig. 4 by the thicker positioning of the turns compared to similar figures in [20]. The mathematical model proposed in this work showed the patterns of particle movement when moving from the base to the side wall of the rotor. In this zone, the shape of the particle trajectory becomes circular for a while and later changes again to a spiral one. As a result, particles move along the side wall at lower initial speeds, which leads to an increase in the length of the trajectory. This effect can be explained by hitting the particle on the lateral surface of the cone, which reduces the component of the velocity that is directed along the radius. The mathematical model reported in work [20] does not take into consideration this effect. The presence of circular trajectories in the transition zone from the base to the lateral wall of the rotor indicates an increase in the number of interactions between particles. These interactions affect the shape of trajectories and are taken into consideration when DEM is applied.

The trajectory parameters that are determined according to the model given in our paper and the model proposed in work [20] are also significantly different. Thus, the length of the trajectory differs by 2.9 times, and the movement time – 9 times. At the same time, the calculations according to the model from [20] produce lower values of movement time and trajectory length. This result indicates a significant influence of factors taken into consideration in the model constructed on the basis of DEM, namely: the geometrical parameters of particles, the value of their density, Poisson coefficients, shear modules, and interaction between particles. Thus, DEM makes it possible to determine with greater accuracy the velocity of particles, the time of their movement, and the supply of material in the rotor.

From the point of view of homogeneity assessment, the influence of rotor parameters on the location of particles at its output is fundamentally important.

Histograms of particle distribution that are shown in Fig. 5 indicate that the particles are not evenly distributed along the outer edge of the rotor. In the case of an ideal distribution, the number of particles in all sectors should be the same, but in practice, the particles are arranged unevenly. Analysis of homogeneity estimation (Table 4) showed that the lowest values of variance coefficients were obtained under the conditions of dividing MC and KC into two streams and their arrival into opposite quadrants of the mixer rotor. This result is confirmed by both mathematical models, which is clearly seen in Fig. 4, *d* and Fig. 5, *d*. Thus, it is possible to ensure an increase in the homogeneity of the mixture by coalescing the centers of input flows of its components. In this case, the results of calculations according to the mathematical model based on DEM almost coincide with the results reported in work [20].

The experimental study has confirmed that particles move along spiral trajectories. At the same time, the results obtained with the help of DEM are closer to experimental ones. This is due to the greater number of particle parameters that are taken into consideration when the model was

built, and taking into consideration the elastic interaction between the particles and the side wall of the rotor.

In general, the DEM-based models provide for greater accuracy but, compared to models based on the classical systems of differential equations [20], their construction requires more information about the properties of objects and the parameters of their interaction.

It should be noted that both models do not take into consideration a series of factors that affect the movement of particles in actual equipment. In particular, the following ones are not taken into consideration: vibrations of the rotor, effect of the shape of particles, possible changes in friction coefficients due to external conditions (for example, humidity), the presence of electrostatic forces. These parameters can be included in the model based on DEM. However, determining the actual values of these parameters and the ranges of their changes under real conditions requires further research.

7. Conclusions

1. A mathematical model based on DEM was constructed, which takes into consideration the geometric and phys-

ical-mechanical parameters of particles. It was determined that in the calculations involving DEM the length of the particle trajectory is 2.9, and the movement time is 9 times greater than those derived from a system of differential equations. The presence of circular trajectories in the transition zone from the base to the side wall of the rotor was detected, which confirms the effectiveness of the use of the proposed model.

2. It has been established that the particle distribution models defined according to the developed and known mathematical models are of the same nature (the correlation coefficient is 0.758). The difference between the variance coefficients, corresponding to the initial positions of MC and KC considered in this study, is 42.9 %. This result confirms the effectiveness of the use of turbulizers in CMCA.

3. Based on the experimental study, the particle movement was studied and the accuracy of the constructed and known mathematical models was tested. It was established that the length of trajectories, calculated according to DEM, is 1.2 times less than the experimental one. At the same time, the length of trajectories, calculated according to known mathematical models from [20], is 13.7 times less than the experimental one.

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