------

-0

D-

UDC 621.45.026.2

DOI: 10.15587/1729-4061.2021.233850

FOR SELECTING A

**ENGINE CONTROL** 

**CRUISING MODE AND** 

A RAMJET AIRCRAFT

DEVELOPMENT

**OF A METHOD** 

**PROGRAM OF** 

PhD, Associate Professor\*

Mykhailo Shevchenko

E-mail: mikleshevchenko@gmail.com

National Aerospace University

«Kharkiv Aviation Institute»

\*Department of Aviation Engines Theory

Chkalova str., 17, Kharkiv, Ukraine, 61070

Oleh Kislov

Corresponding author

Postgraduate Student\*

#### CONTROL PROCESSES

For supersonic cruising, combined power plants can be used, in which a gas turbine engine reaches the cruising mode, and a ramjet is used for cruising. Supersonic transoceanic flights are characterized by a long cruising segment, which is decisive in terms of required fuel mass. Therefore, the selection of cruising and engine operation parameters is an important task. As a rule, when selecting the cruising mode, the range parameter is used, which depends on the flight and engine operation modes. To take into account the influence of the ramjet operating mode on the range parameter, dimensionless relationships of engine parameters with control factors were obtained. Using the obtained relationships together with the equations of aircraft motion in steady horizontal flight, it is shown that the values of the engine control factors and the range parameter do not change at the altitudes of 11...20 km. This made it possible to conclude that the range parameter can be increased only by selecting the cruising and engine parameters that provide the minimum specific fuel consumption. The variable cruising parameters are speed and initial altitude. A method for selecting the cruising and ramjet operation parameters was developed, based on the analysis of the relationship between the range parameter and the flight speed and initial altitude at the most advantageous values of the engine control factors. The obtained relationships allow selecting the cruising parameters and the engine operating mode, taking into account the restrictions. It is shown that the specific fuel consumption decreases by 0...30 %, depending on the engine operating mode, when the control program is optimized

Keywords: cruising, range parameter, ramjet, engine control program -0

**D-**

Received date 30.04.2021 Accepted date 07.05.2021 Published date 30.06.2021 How to Cite: Kislov, O., Shevchenko, M. (2021). Development of a method for selecting a cruising mode and engine control program of a ramjet aircraft. Eastern-European Journal of Enterprise Technologies, 3 (3 (111)), 6-14. doi: https://doi.org/ 10.15587/1729-4061.2021.233850

### 1. Introduction

In [1, 2], projects of transoceanic supersonic cruising aircrafts are considered. It is known that for low supersonic cruising speeds, turbofan engines are preferable, and for high supersonic speeds - ramjets [3]. But ramjets have no starting thrust and low efficiency at subsonic speeds, which necessitates the use of combined power plants (PP) [4-6]. In this case, takeoff, climb and achieving supersonic cruising speed are carried out using a gas turbine engine and cruising - a ramjet.

Supersonic passenger transport is characterized by a flight profile with a pronounced cruising segment [1, 2]. Therefore, for a preliminary assessment of the fuel mass, only the cruising segment can be used, and the remaining flight segments can be taken into account by the empirical coefficients [7]. Since the cruising segment is decisive, aircraft flight and engine operation modes must be selected from the condition of a minimum required relative fuel mass.

The required relative fuel mass  $\bar{m}_{f}$  and cruising segment length *L* are related by the Breguet range equation:

$$L = \frac{3.6 \cdot K \cdot V}{g \cdot c_R} \ln \frac{1}{1 - \overline{m}_f} = \frac{3.6 \cdot a}{g} \cdot \frac{K \cdot M_a}{c_R} \ln \frac{1}{1 - \overline{m}_f},$$
 (1)

where K – lift-to-drag ratio of the aircraft, depending on the flight Mach number  $M_a$  and the angle of attack of the aircraft; a, m/s – local speed of sound;  $c_R$ , kg/N·h – specific fuel consumption (depending on flight conditions and engine control program).

It follows from (1) that for a given *L*, the minimum fuel consumption is provided at the maximum of the complex  $P = KV/c_{\rm p}$ , which is sometimes called the range parameter. It can be seen that *P* depends on both the aircraft flight mode and the engine operating mode. In any steady horizontal flight mode, the thrust of a ramjet PP can be implemented by a variety of combinations of control factors (CF), i.e. with different ramjet control programs (CP). CP should be selected from the condition of minimum  $c_R$  to increase P. Clearly, the *P* value will be different in different steady flight conditions. A mode that provides the maximum value of Pis selected as a cruising one.

With regard to the development of transoceanic supersonic aircrafts [1, 2], including ramjet as the PP [4-6], the problem of developing a method for selecting the cruising mode and ramjet PP control program providing the maximum P value is relevant.

## 2. Literature review and problem statement

The general approach to selecting flight parameters is based on the application of aircraft motion equations with

complex consideration of aerodynamic and mass characteristics, as well as PP characteristics. Moreover, it is emphasized that the control program of the PP has to be optimized for each flight mode – altitude and speed [8]. Optimization of the PP parameters is carried out directly in the process of flight path calculation using its mathematical model without using dimensionless universal relationships between engine parameters and CF values. This optimization method requires significant computational costs. In [9], a similar approach is used to determine the most favorable flight conditions in the presence of restrictions.

When selecting the cruising mode, the range parameter is widely used. In [10], the parameter P was used to optimize the airframe design. In [11], P was used to determine the cruising flight range under given flight conditions (H and  $M_a$ ) and fuel mass of a maneuverable aircraft in the preliminary design under uncertainty. In [1], the parameter P was used to select the supersonic cruising speed. However, in these works, the influence of the PP parameters on the parameter P was taken into account without considering the engine CP.

The required fuel mass for a ramjet aircraft was estimated by the same approach based on P [5]. At the same time, the influence of the engine CP on P is also not taken into account explicitly.

The general approach to selecting the aircraft cruising mode using P, taking into account the engine CP, is described in [12]. This approach assumes the maximization of P when solving differential aircraft motion equations. Maximizing P requires minimizing  $c_R$  at each point of the P domain. To minimize  $c_R$ , direct modeling of the PP without universal dimensionless relationships between the PP parameters and CF is used. This complicates the decomposition of the complex problem of maximizing P into simpler problems to select the cruising parameters and the PP operating mode. In addition, direct modeling of the PP requires significant computational resources.

Thus, there are approaches to selecting flight parameters based on maximizing *P* that imply the optimization of the PP parameters using its mathematical model without universal dimensionless relationships between the PP parameters and CF, which complicates the decomposition of the problem and requires significant computational resources. Therefore, it is necessary to develop a method for selecting a supersonic cruising mode of a ramjet aircraft using universal dimensionless relationships between the PP parameters and CF.

This method will make it possible not only to substantiate the cruising parameters and the PP operating mode, but also to decompose the problem into two parts. The first part is obtaining universal dimensionless relationships between the PP parameters and the CF values. The second part is determining P with the most advantageous CF combinations in terms of cruise thrust obtained using universal relationships.

#### 3. The aim and objectives of the study

The aim of the study is to develop a method for selecting a cruising mode based on the consideration of all steady flight modes with a ramjet CP that provides the minimum fuel consumption at the altitudes H=11...20 km. The resulting ramjet control program will make it possible to implement not only cruising, but also any steady flight modes, providing the minimum required fuel mass.

To achieve the aim, the following objectives were set:

 using the similarity theory, to obtain the criterion relationships between the ramjet parameters and the control factors; - to justify the constancy of *P* in steady flight at the altitudes H=11...20 km at constant CF values;

 to develop a method for selecting the most advantageous cruising and ramjet operation parameters, which provide the maximum range parameter;

- to evaluate the reliability of the developed method;

– using the developed method, to substantiate the most advantageous cruising parameters in terms of fuel mass and obtain a ramjet CP for different conditions of steady horizontal flight.

#### 4. Materials and methods of the study

The study uses theoretical methods and mathematical modeling of gas-dynamic processes in ramjets.

The main hypothesis of the study is that in the altitude range H=11...20 km, the condition for the similarity of ramjet operation modes is the invariability of the ramjet CF values (fuel air ratio  $q_f$  and relative nozzle-throat area  $\overline{F}$ ). This makes it possible to represent the dimensionless characteristics of the ramjet in a universal form, regardless of the height, for each  $M_{q}$ .

The second hypothesis of the study is the statement that the range parameter does not change in the altitude range H=11...20 km during steady flight for  $M_a=$  const and at constant CF values. This makes it possible to avoid splitting of the cruising path into segments, which simplifies the calculation and analysis of the obtained results.

The initial data were the flight profile, aerodynamic and mass characteristics of a supersonic transoceanic passenger aircraft [1].

The study was carried out in the following sequence. Initially, functional relationships between the dimensionless ramjet parameters and the dimensionless CF were obtained using the similarity theory. The form of these relationships was obtained by calculation [6].

Then, the required values of the angle of attack and the thrust parameter (the ratio of thrust to atmospheric pressure  $R/p_a$ ) using aircraft motion equations were determined for different  $M_a$  numbers of the steady flight. For the known value of  $R/p_a$  using the dimensionless ramjet characteristics, the CF providing the minimum  $c_R$  were determined. The minimum value of P for a given  $M_a$ . The relationships  $P(M_a)$ ,  $q_f(M_a)$  and  $\overline{F}(M_a)$  were plotted based on the obtained results. The relationship  $P(M_a)$  allows finding the most advantageous steady flight speed of the aircraft that provides the minimum required fuel mass. The relationships  $q_f(M_a)$  and  $\overline{F}(M_a)$  are the ramjet CP depending on the  $M_a$  number at a given initial altitude of the steady flight segment  $H_0$ .

Since the solution of the motion equations depends on  $H_0$ , in order to take into account its influence, using the method described above, the relationships  $P(M_a)$  for different  $H_0$  were obtained. This makes it possible to obtain the ramjet CP in the form of  $q_f(M_a, H_0)$  and  $\overline{F}(M_a, H_0)$ .

#### 5. Results of the study of supersonic cruising parameters

# 5.1. Criteria relationships between ramjet parameters and control factors

In such modes, all dimensionless flow parameters are the same. Moreover, both the similarity of the entire power plant and the similarity of the operating modes of its elements are provided.

The ramjet power plant consists of the inlet diffuser, nozzle and combustion chamber (Fig. 1).



Fig. 1. Ramjet scheme: a - freestream section in front of the engine; in - combustion chamber inlet section; b - combustion chamber outlet section; nt - nozzle-throat area; n - nozzle exit area

The similarity conditions of each PP element are known [13].

For the inlet diffuser, this is the identity of the  $M_a$  numbers of the freestream (section a-a) and at the outlet of the inlet diffuser  $M_{in}$  (section in-in) with the geometric similarity of the flow path, i. e.:

$$\begin{cases}
M_a = idem, \\
M_{in} = idem.
\end{cases}$$
(1)

Instead of the  $M_{in}$  number, it is more convenient to use the gas dynamic flow function  $q(\lambda_{in})$  unequivocally associated with it.

For the combustion chamber with geometric similarity, the following conditions are usually taken as the flow similarity conditions:

$$\begin{cases} \sigma_b = idem, \\ \eta_b = idem, \end{cases}$$
(2)

where  $\sigma_b = p_b^* / p_m^*$  – total pressure recovery factor in the combustion chamber, and  $\eta_b$  – fuel combustion efficiency.

For the nozzle, while ensuring the geometric similarity of streamlined bodies, the similarity conditions for the modes are:

$$\begin{cases} \pi_d = idem, \\ M_a = idem, \end{cases}$$
(3)

where  $\pi_d = (1/\pi(M_a)) \cdot \sigma_{in} \cdot \sigma_b$  – available nozzle pressure ratio. Here,  $\pi(M_a)$  – gas dynamic pressure function, and  $\sigma_{in}$  – total pressure recovery factor in the inlet diffuser. The gas dynamic pressure function  $\pi(M_a)$  is equal to:

$$\pi \left( M_{a} \right) = \left( 1 + \frac{k-1}{2} M_{a}^{2} \right)^{\frac{k}{1-k}}, \tag{4}$$

where k – specific heat ratio.

To provide the similarity of the external flow around the nozzle, it is necessary to ensure the condition  $M_a = idem$ .

If the PP is geometrically similar, the conditions for the similarity between the ramjet PP operating modes are:

$$\begin{cases} M_a = idem, \\ \theta^* = idem, \end{cases}$$
(5)

where  $\theta^* = T^*_{nt}/T^*_a$  – ratio of the total chamber outlet temperature to the total freestream temperature.

Since when conditions (5) are met, the operation modes of all elements of the ramjet control system are similar (1)-(3).

Indeed, from the conditions for the airflow balance in the nozzle throat and at the outlet of the inlet diffuser, it follows that  $q(\lambda_{in})$  is determined by the value  $\theta^*$  at a supercritical nozzle pressure ratio:

$$q(\lambda_{in}) = \frac{m_b}{m_a} \cdot \overline{F} \cdot \frac{1}{\sqrt{\theta^*}} \cdot \frac{q(\lambda_{in})}{1+q_f} \approx \text{const} \cdot \frac{1}{\sqrt{\theta^*}}, \tag{6}$$

where  $m_a$  and  $m_b$  – airflow factors in sections a–a and b–b (Fig. 1);  $\overline{F}$  – ratio of the nozzle-throat area  $F_{nt}$  to the nozzle exit area  $F_{in}$ .

Therefore, if conditions (5) are met, conditions (1) are also provided, i.e. the similarity of the inlet diffuser operating modes.

Conditions (2) are provided in a wide range of PP operation modes, so it can be considered that the similarity of the operation modes of the combustion chamber is provided in all modes. Conditions (3) are also provided when conditions (5) are met, since the values of  $\sigma_{in}$  and  $\sigma_b$  are unchanged for similar modes.

The value  $\theta^*$  can be derived from the fuel-air ratio  $q_f$  [14]:

$$\boldsymbol{\theta}^* = \frac{\left(T_a^* + H_u \cdot q_f \cdot \boldsymbol{\eta}_b\right)}{T_a^*} = 1 + \frac{H_u \cdot q_f \cdot \boldsymbol{\eta}_b}{\left(\frac{T_a}{1 + \frac{k - 1}{2}M_a^2}\right)} = 1 + \text{const} \cdot \frac{q_f}{T_a}, (7)$$

where  $H_u$ , kJ/kg – specific lower fuel calorific value.

Then conditions (5) can be rewritten as:

1 .

$$\begin{cases} M_a = idem, \\ \frac{q_f}{T_a} = idem. \end{cases}$$
(8)

In a particular case, for  $T_a=idem$ , conditions (8) have the form:

$$\begin{cases} M_a = idem, \\ q_f = idem. \end{cases}$$
(9)

Conditions (9) are convenient for use in the range of flight altitudes H=11...20 km, since the standard atmospheric temperature for these altitudes is constant ( $T_a=216.5$  K).

For similar modes of the ramjet PP, the identity of the PP similarity parameters is ensured:

$$\begin{cases} \frac{R_{s}}{\sqrt{T_{a}^{*}}} = idem, \\ \frac{c_{R}}{\sqrt{T_{a}^{*}}} = idem, \\ \frac{R}{p_{a}} = idem, \end{cases}$$
(10)

where  $R_s$ ,  $N \cdot s/kg$  – specific thrust. Insofar as:

$$\frac{R_s}{\sqrt{T_a^*}} = \frac{c_n - V}{\sqrt{T_a^*}} = \frac{c_n}{\sqrt{T_{nt}^*}} \cdot \sqrt{\frac{T_{nt}^*}{T_a^*}} - \frac{V}{\sqrt{T_a^*}} =$$
$$= \operatorname{const} \cdot \lambda_n \sqrt{\frac{T_{nt}^*}{T_a^*}} - \operatorname{const} \cdot \lambda_a =$$
$$= \operatorname{const} \cdot \lambda_n \cdot \sqrt{\Theta^*} - \operatorname{const} \cdot M_a = idem,$$

and for the similar modes, the values of the reduced velocity  $\lambda$  and  $\theta^*$  are the same. Here  $c_n$ , m/s – exhaust velocity in section n–n, V, m/s – flight speed.

$$\begin{aligned} \frac{c_{R}}{\sqrt{T_{a}^{*}}} &= \frac{3,600 \cdot q_{f}}{R_{s} \cdot \sqrt{T_{a}^{*}}} = \frac{3,600 \cdot C \cdot (T_{mt}^{*} - T_{a}^{*})}{H_{u} \cdot \eta_{b} \cdot R_{s} \cdot \sqrt{T_{a}^{*}}} = \\ &= \frac{3,600 \cdot C}{H_{u} \cdot \eta_{b}} \cdot \left(\frac{T_{mt}^{*}}{T_{a}^{*}} - 1\right) \cdot \frac{1}{\frac{R_{s}}{T_{a}^{*}} \cdot \sqrt{T_{a}^{*}}} = \\ &= \frac{3,600C}{H_{u} \cdot \eta_{b}} \left(\theta^{*} - 1\right) \cdot \frac{1}{\frac{R_{s}}{\sqrt{T_{a}^{*}}}} = idem \end{aligned}$$

and

$$\frac{R}{p_a} = \frac{R_s \cdot \dot{m}}{p_a} = \left(\frac{R_s}{\sqrt{T_a^*}}\right) \cdot \frac{\dot{m} \cdot \sqrt{T_a^*}}{p_a^* \cdot \pi(M_a)} = idem,$$

insofar as:

$$\frac{R_s}{\sqrt{T_a^*}} = iden$$

for similar modes, and

$$\frac{\dot{m}\cdot\sqrt{T_a^*}}{p_a^*\cdot\pi(M_a)} = \frac{m_a F_{in}\cdot q(\lambda_{in})\cdot\sigma_{in}}{\pi(M_a)} = \operatorname{const}\frac{q(\lambda_{in})}{\pi(M_a)}$$

for similar modes is also invariable due to the invariability of  $\lambda_{in}$  and  $M_a$ . Here  $\dot{m}$ , kg/s – mass airflow.

When  $\overline{F}$  changes, the following values are changed too:

$$\frac{R_s}{\sqrt{T_a^*}}, \ \frac{c_R}{\sqrt{T_a^*}}, \ \frac{R}{p_a}.$$

Thus, the ramjet characteristics at  $M_a$  = const can be written as relationships:

$$\frac{R}{p_a} = f\left(\frac{q_f}{T_a}, \frac{F_{nt}}{F_{in}}\right),\tag{11}$$

$$\frac{c_R}{\sqrt{T_a^*}} = f\left(\frac{q_f}{T_a}, \frac{F_{nt}}{F_{in}}\right).$$
(12)

Relationships (11) can be made more universal by the transition from thrust to thrust divided by  $F_{in}$ ,

$$\overline{R} = \frac{R}{p_a \cdot F_{in}} = f\left(\frac{q_f}{T_a}, \frac{F_{nt}}{F_{in}}\right).$$
(13)

The standard atmospheric temperature  $T_a$  is constant at the flight altitudes H=11...20 km, therefore, instead of relationships (12), (13), it is convenient to use:

$$\overline{R} = f\left(q_f, \overline{F}\right),\tag{14}$$

$$c_R = f\left(q_f, \overline{F}\right). \tag{15}$$

Relationships (14), (15) relate the main ramjet parameters with the CF values and they are convenient to use as ramjet characteristics in the steady flight of an aircraft in order to determine the most advantageous combinations of CF values.

## 5. 2. Justification of the invariability of the range parameter in steady flight

Fuel consumption on an elementary segment of the path depends on the range parameter P. The mass of the aircraft in steady flight decreases, but the resultant of the forces acting on it is equal to zero. The flight parameters are changed in this case. It is important to determine how this affects the value of P in order to ensure its maximum throughout the entire segment of the steady flight.

At the altitudes H=11...20 km, the atmospheric temperature and the speed of sound are constant, so it is convenient to use the following complex as the range parameter:

$$P_0 = \frac{K \cdot M_a}{c_R}.$$

٢

The aircraft motion under steady flight conditions is described by the system of equations:

$$\begin{cases} R \cdot \cos(\alpha + \varphi) - c_x \cdot 0.7 \cdot M_a^2 \cdot S_w = 0, \\ c_y^{\alpha} \cdot (\alpha - \alpha_0) \cdot 0.7 \cdot p_a \cdot M_a^2 \cdot S_w - \\ -m_A \cdot g + R \cdot \sin(\alpha + \varphi) = 0, \\ c_y^{\alpha} = f_1(M_a), \\ c_x = f_2(c_y^{\alpha}, \alpha, M_a), \end{cases}$$
(16)

where  $\alpha$  – angle of attack;  $\varphi$  – angle of the geometric axis of the PP, relative to the aircraft water line;  $c_x$  – drag coefficient;  $S_w$ , m<sup>2</sup> – wing area;  $c_y^{\alpha} = dc_y/d\alpha$  – lift slope;  $\alpha_0$  – zero-lift angle;  $m_A$ , kg – aircraft mass at the beginning of the steady flight segment; g, m/s<sup>2</sup> – gravitational acceleration.

This system can be represented as a system of four equations with six variables  $(\alpha, M_a, R/p_a, m_Ag/p_a, c_y^{\alpha}, c_x)$ :

$$\begin{cases} \frac{R}{p_a} \cdot \cos(\alpha + \varphi) - c_x \cdot 0.7 \cdot M_a^2 \cdot S_w = 0, \\ c_y^{\alpha} \cdot (\alpha - \alpha_0) \cdot 0.7 \cdot M_a^2 \cdot S_w - \\ -\frac{m_A \cdot g}{p_a} + \frac{R}{p_a} \cdot \sin(\alpha + \varphi) = 0, \\ c_y^{\alpha} = f_1(M_a), \\ c_x = f_2(c_y^{\alpha}, \alpha, M_a). \end{cases}$$
(17)

The initial aircraft mass, aerodynamic characteristics  $c_y^a = f_1(M_a)$  and  $c_x = f_2(c_y^a, \alpha, M_a)$ , as well as  $H_0$  and  $M_a$  number are considered to be known. Under these conditions,  $c_y^a$ ,  $m_Ag/p_a$  become known, and system (17) turns into a system of three equations with three unknowns. From this system of equations,  $R/p_a$ ,  $\alpha$ ,  $c_x$  are uniquely determined. Based on the obtained values of the aerodynamic parameters of the aircraft, K is determined, and  $R/p_a$  is the required value of the thrust parameter in the elementary steady flight segment used to determine the required ramjet operating mode. Relationships (14), (15) are used to determine it, which allow finding the CF values that ensure the minimum  $c_R$  according to the known value of  $R/p_a$ .

From system (17), it follows that during the flight on an elementary section of the path with the given  $M_a$ =const and  $\alpha$ =const,  $R/p_a$  and  $m_Ag/p_a$  also remain unchanged. Since the aircraft mass decreases, at  $m_Ag/p_a$ =const the atmospheric pressure  $p_a$  also has to decrease in proportion to the aircraft mass, which corresponds to a continuous increase in the cruising altitude.

An arbitrary change in  $\alpha$  on the following elementary segment of the path in order to find the largest value of *P* in steady flight is impossible without violating the conditions of steady horizontal flight – the equality of the resultant forces to zero.

Indeed, the solution of the system of equations (17) changes with a change in  $\alpha$ , and hence the value of  $m_A g/p_a$  is changed too. Since  $m_A$  remains the same as at the end of the previous elementary segment of the path, the change in  $m_A g/p_a$  must occur via the change in  $p_a$ , and hence *H*. This is impossible without violating the conditions of steady horizontal flight.

Thus, the change in P in the steady horizontal flight is impossible, so the selection of cruising parameters should be performed by varying  $H_0$  and  $M_a$  at the starting point of cruising.

### 5. 3. Method of selecting the most advantageous cruising and ramjet operation parameters

As shown above, P does not change in steady flight, so the selection of flight parameters at the beginning of the cruising segment has particular importance. Flight parameters and engine operating mode should be selected so that to ensure the maximum P.

For this, it is necessary to vary  $H_0$  and  $M_a$  at the starting point of cruising.

The required value of  $R/p_a$  and K are determined solving the system of equations (17) for each combination of  $H_0$  and  $M_a$  with a known aircraft mass.

The criterion relationships between the ramjet parameters and CF are constructed in the form of (14), (15) for each  $M_a$  number according to the method [6].

These relationships are used to determine the combination of CF ( $q_f$  and  $\overline{F}$ ) that provides the minimum  $c_R$  at the required value of  $R/p_a$ .

 $P_0$  is calculated, and the relationships  $P_0(M_a, H_0)$ ,  $q_f(M_a, H_0)$  and  $\overline{F}(M_a, H_0)$  are plotted. These relationships are used to determine the most advantageous  $M_a$ ,  $H_0$ ,  $q_f$ ,  $\overline{F}$ , which provide the maximum  $P_0$  and the minimum fuel mass for cruising.

The relationships  $q_f(M_a, H_0)$  and  $\overline{F}(M_a, H_0)$  determine the required CF values to ensure the minimum fuel mass for a given flight mode, which differ from the most advantageous values of  $M_a$ ,  $H_0$  of the steady flight.

The relationships  $q_f(M_a, H_0)$  and  $\overline{F}(M_a, H_0)$  represent the ramjet control program, which ensures the minimum fuel consumption in the steady flight.

### 5. 4. Evaluation of the reliability of the developed method

Since the method is based on solving the aircraft motion equations and using criterion relationships between the ramjet parameters and CF, evaluation of the reliability of both the solution of the system of equations and the model for calculating the ramjet parameters is required.

The reliability of the solution of the motion equations was evaluated by comparing the results of calculating the lift-to-drag ratio of the aircraft in cruising mode, which depends on the  $M_a$  number and the angle of attack of the aircraft, with the data of [1]. The comparison of these results is shown in Fig. 2. The difference in the results was less than 0.5 %.

The reliability of the model for calculating the ramjet characteristics was evaluated by comparing the speed calculation results with the data of [15]. The comparison of these relationships is shown in Fig. 3.





----- calculated values

The thrust coefficient  $k_R$  and specific impulse I are calculated as:

$$k_{R} = \frac{R}{\frac{k}{2} \cdot p_{a} \cdot M_{a}^{2} \cdot F_{n}}$$
$$I = \frac{3,600}{g \cdot c_{R}}.$$

Fig. 3 shows that the calculation error is less than 3 %. The source of the error is some methodological differences in the method of accounting for total pressure losses.

## 5. 5. Substantiation of cruising parameters and ramjet control program

Solutions of the system of equations (17) were obtained using the developed method for different cruising conditions. The relationships between  $\overline{R}$  and lift-to-drag ratio  $K = c_y^{\alpha} (\alpha - \alpha_0) / c_x$  and  $M_a$  and  $H_0$  are shown in Fig. 4.

The dimensionless characteristics were calculated in the form of relationships (14), (15) for different  $M_a$  numbers. These relationships have the form shown in Fig. 5.

The CF combinations providing the minimum  $c_R$  are determined by the obtained relationships  $\overline{R}(M_a, H_0)$  using the dimensionless characteristics of the ramjet. The relationships  $q_f(H_0, M_a)$  and  $\overline{F}(M_a, H_0)$  are shown in Fig. 6. These relationships represent the ramjet CP, depending on cruising conditions, providing a minimum fuel mass.

The  $c_R$  relationships of this control program are shown in Fig. 7.

The relationship  $P_0(M_a, H_0)$  was obtained using the relationships  $K(M_a, H_0)$  and  $c_R(M_a, H_0)$  and it is shown in Fig. 8.



Fig. 4. Relationships between the thrust parameter and lift-to-drag ratio of the aircraft and the  $M_a$  number for different altitudes:  $\bullet - H_0 = 11$  km;  $\blacksquare - H_0 = 15$  km;  $\blacktriangle - H_0 = 17$  km;  $- \overline{R}$ ;  $- \overline{R}$ ;  $- \overline{K}$ 



Fig. 5. Relationships between the thrust parameter  $\overline{R}$  and specific fuel consumption and the area ratio  $\overline{F}$ , at  $M_a = 2.58$  for different  $q_f$  values for altitudes H = 11...20 km:  $\bullet - q_f = 0.0075$ ;  $\blacksquare - q_f = 0.01$ ;  $\blacktriangle - q_f = 0.0125$ ;  $\blacksquare - \overline{R}$ ;  $\blacksquare - c_R$ 



Fig. 6. Relationships between the fuel-air ratio and the area ratio and the  $M_{\sigma}$  number for altitudes: •  $-H_0 = 11 \text{ km}; = -H_0 = 15 \text{ km}; A - H_0 = 17 \text{ km}; - q_f; - f$ 

Fig. 8 shows that  $P_0$  increases with increasing  $H_0$ . There is the most advantageous  $M_a$  number, providing the maximum  $P_0$  for each  $H_0$ . The value of the most advantageous  $M_a$  also increases with increasing  $H_0$ .

The influence of ramjet control on  $c_R$ , and hence on parameter  $P_0$ , was estimated by comparing  $c_R$  for optimal values of the ramjet CF with  $c_R$  for a ramjet with an invariable-geometry nozzle (Fig. 9).

Fig. 9 shows the thrust value  $\overline{R}$  and the relative decrease in specific fuel consumption  $\delta c_R$  in percentage, calculated by the equations:

$$\overline{R} = \frac{\overline{R}_i}{\overline{R}_c},$$

$$\delta c_R = \left(1 - \frac{c_{Rvar}}{c_{Rinv}}\right) \cdot 100\%,$$

where  $\overline{R}_c$  – value of the thrust parameter in the design mode;  $\overline{R}_i$  – value of the thrust parameter in the off-design mode;  $c_{Rvar}$  – specific fuel consumption of a ramjet with a variable-geometry nozzle;  $c_{Rinv}$  – specific fuel consumption of a ramjet with an invariable-geometry nozzle.





of a ramjet with variable and invariable-geometry nozzles and the engine thrust parameter for  $M_a=2$ : ----- ramjet with an invariable-geometry nozzle; ------ ramjet with a variable-geometry nozzle; ------ relative decrease in  $\delta c_R$  due to control

Fig. 9 shows that the possible effect of optimal control is from 0 to 30 % reduction in specific fuel consumption compared to a ramjet with an invariable-geometry nozzle.

## 6. Discussion of the results of studying the supersonic cruising parameters

The solutions of the system of equations (17) depending on  $M_a$  and  $H_0$  (Fig. 4) show that an increase in  $M_a$  leads to a decrease in K, and an increase in  $\overline{R}$ , and  $H_0$  leads to an increase in K and  $\overline{R}$ . The increase in the required thrust with increasing  $M_a$  is explained by the increase in aerodynamic drag. The decrease in Kis caused by a decrease in the angle of attack and displacement of its value from  $\alpha_{ad}$  at which  $K_{\max}$  is reached. On the contrary, the increase in K with increasing  $H_0$  is explained by the increase in  $\alpha$ and approaching of its value to  $\alpha_{ad}$ . The increase in  $\overline{R}$  with increasing  $H_0$ , despite the decrease in R, is explained by the stronger effect of a decrease in  $p_a$ .

The obtained universal relationships of the ramjet parameters  $\overline{R}$  and  $c_R$  with the CF values  $q_f$  and  $\overline{F}$  for the given  $M_a$  number, independent of the altitudes and frontal dimensions of the ramjet, are shown in Fig. 5.

It is characteristic that  $\overline{R}$  increases with increasing  $q_f$  due to the increase in  $T_m^*$  for each given value of  $\overline{F}$ . The change in  $\overline{R}$  is non-monotonic with the increase in  $\overline{F}$  for the given value of  $q_f$ .

The relationship between  $c_R$  and  $\overline{F}$  at  $q_f$ =const has the minimum. An increase in  $q_f$  leads to an increase in  $c_R$  at small values of  $\overline{F}$  and to a decrease in  $c_R$  at large values of  $\overline{F}$ .

There are two CF combinations that provide the same value of  $\overline{R}$  for a given value of  $q_f$ . The solution for a small value of  $\overline{F}$  refers to the case of small  $\dot{m}$  and high  $T_{nt}^*$ . The value of  $\dot{m}$  is larger and  $T_{nt}^*$  is lower for a larger value of  $\overline{F}$ . In this case,  $c_R$  has a greater value for larger  $\overline{F}$ . This is due to the fact that:

$$c_{R} = \frac{3,600\dot{m}_{f}}{R} = \frac{3,600 \cdot q_{j}}{R/\dot{m}}$$

and for the given  $q_f$  and  $\overline{R}$ , the value of  $c_R$  will be greater for a larger value of  $\dot{m}$ .

Fig. 6 shows the relationships between the CF and the cruising conditions ( $M_a$  and  $H_0$ ), providing a minimum  $c_R$ . In this case, with increasing  $H_0$ , it is required to increase  $q_f$  and  $\overline{F}$  due to an increase in the required value of  $\overline{R}$ . The obtained  $c_R$  values are shown in Fig. 7. The change in  $c_R$  is mainly determined by the change in  $q_f$ , so their relationships are similar.

Fig. 8 shows the obtained relationships  $P_0(M_a, H_0)$ . The increase in  $P_0$  with increasing  $H_0$  is explained by the increase in K, and the maximum  $P_0$  with  $H_0$ =const and change in  $M_a$  is explained by the presence of the function minimum  $c_R=f(M_a)$ .

The resulting ramjet CP provides the minimum  $c_R$  at all steady flight conditions of the aircraft. The joint consideration of the aircraft motion equations and ramjet CP made it possible to determine the most advantageous cruising conditions in terms of the required fuel mass.

The advantages of this study over the known ones are:

 decomposition of the problem of selecting cruising and PP operation parameters;

- consideration of a supersonic steady flight of a ramjet aircraft, taking into account the engine CP;

- ramjet CP, providing the minimum  $c_R$  in steady flight conditions was obtained;

 regularities that allow developing recommendations for selecting the cruising parameters of a ramjet aircraft were determined.

When selecting the cruising mode of the aircraft, it is necessary to take into account that at H>20 km, the temperature  $T_a$  increases with altitude and this adversely affects the engine parameters.

In addition, a higher  $H_0$  increases the relative proportion of the climb segment causing an increase in the required fuel mass in this segment. Therefore, the final selection of cruising parameters should be made taking into account the fuel consumption in the climb segment.

The disadvantage of the developed method is the limited scope of application only by steady flight modes. Therefore, further development of the method is advisable in order to extend it to transient flight modes.

#### 7. Conclusions

1. The universal relationships  $\overline{R} = f(q_f, \overline{F}, M_a)$  and  $c_R = f(q_f, \overline{F}, M_a)$  are substantiated using the similarity theory. Their feature is the independence of the flight altitude (at H=11...20 km) and geometric dimensions of the ramjet. These relationships allow selecting CF combinations that provide the minimum required fuel mass in the steady horizontal flight.

2. It is shown that the range parameter in the altitude range H=11...20 km in steady flight with  $M_a=$  const and at constant values of the ramjet CF does not change. Due to this, it was concluded that it is necessary to select the cruising conditions by varying the values of  $H_0$  and  $M_a$  number at the beginning of the steady flight segment.

3. The method of determining the most advantageous cruising and ramjet operation parameters in terms of the minimum required fuel mass was developed. Its feature is the use of equations of aircraft motion in steady horizontal flight and universal relationships between the ramjet parameters and CF. This method made it possible to determine the most efficient ramjet operation mode for all steady horizontal flight modes. This allows selecting the cruising parameters providing the minimum required fuel mass.

4. Verification of the method was carried out by comparing the calculation results with the published calculation results of other authors. The results of modeling the cruising mode made it possible to determine the aircraft parameters, which differ from the available results by less than 0.5 %. The reliability of the model for calculating the ramjet characteristics was evaluated by comparing the speed calculation results with the published data. The difference between the modeling results is less than 3 %.

5. The CF change laws, which ensure the minimum specific fuel consumption for all steady flight modes were obtained. Their features are monotonic changes in altitude and non-monotonic changes in flight speed. In this case, the relationship between the fuel-air ratio and the flight speed has a minimum, and the nozzle-throat area decreases with increasing flight speed. These laws make it possible to take into account the effect of the ramjet when maximizing the range parameter. The change patterns of the range parameter depending on the speed and initial altitude of the steady horizontal flight were determined. Their features are an increase in the range parameter with increasing initial altitude of the steady flight in the range H=11...20 km, as well as the presence of the most advantageous speed for a given initial flight altitude, at which the range parameter has a maximum. Moreover, the most advantageous speed increases with increasing initial altitude. These patterns make it possible to select the cruising parameters of the ramjet aircraft.

## References

- Morgenstern, J. et. al. (2015). Advanced Concept Studies for Supersonic Commercial Transports Entering Service in the 2018–2020 Period Phase 2. NASA Report. Available at: https://ntrs.nasa.gov/citations/20150015837
- Sun, Y., Smith, H. (2017). Review and prospect of supersonic business jet design. Progress in Aerospace Sciences, 90, 12–38. doi: https://doi.org/10.1016/j.paerosci.2016.12.003
- 3. Walsh, P. P., Fletcher, P. (2004). Gas Turbine Performance. Blackwell Science Ltd. doi: https://doi.org/10.1002/9780470774533
- Zheng, J., Tang, H., Chen, M., Yin, F.-J. (2018). Equilibrium running principle analysis on an adaptive cycle engine. Applied Thermal Engineering, 132, 393–409. doi: https://doi.org/10.1016/j.applthermaleng.2017.12.102

- Chen, M., Jia, Z., Tang, H., Xiao, Y., Yang, Y., Yin, F. (2019). Research on Simulation and Performance Optimization of Mach 4 Civil Aircraft Propulsion Concept. International Journal of Aerospace Engineering, 2019, 1–19. doi: https://doi.org/10.1155/ 2019/2918646
- Kislov, O. V., Shevchenko, M. A. (2020). Calculation and regulation features of duct-burning turbofan engine at ramjet modes. Aerospace technic and technology, 6, 15–23. doi: https://doi.org/10.32620/aktt.2020.6.02
- 7. Eger, S. M., Mishin, V. F., Liseytsev, N. K. (1983). Proektirovanie samoletov. Moscow: Mashinostroenie, 616.
- Hendricks, E. S., Falck, R. D., Gray, J. S. (2017). Simultaneous Propulsion System and Trajectory Optimization. 18th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference. doi: https://doi.org/10.2514/6.2017-4435
- 9. Jasa, J. P., Brelje, B. J., Gray, J. S., Mader, C. A., Martins, J. R. R. A. (2020). Large-Scale Path-Dependent Optimization of Supersonic Aircraft. Aerospace, 7 (10), 152. doi: https://doi.org/10.3390/aerospace7100152
- 10. Grebenikov, A. G., Zhuravel', S. V., Bochko, A. Yu. (2014). Project of medium-haul passenger aircraft KhAI-150. Otkrytye informatsionnye i komp'yuternye integrirovannye tekhnologii, 65, 5–22.
- Veresnikov, G. S., Pankova, L. A., Pronina, V. A., Ogorodnicov, O. V., Ikryanov, I. I. (2017). Determining maneuverable aircraft parameters in preliminary design under conditions of uncertainty. Procedia Computer Science, 112, 1123–1130. doi: https://doi.org/ 10.1016/j.procs.2017.08.143
- 12. Yugov, O. K., Selivanov, O. D. (1989). Osnovy integratsii samoleta i dvigatelya. Moscow: Mashinostroenie, 304.
- 13. Nechaev, Yu. N., Fedorov, R. M., Kotovskiy, V. N., Polev, A. S. (2006). Teoriya aviatsionnyh dvigateley. Moscow: VVIA im. prof. N. E. Zhukovskogo, 448.
- Kislov, O., Ambrozhevich, M., Shevchenko, M. (2021). Development of a method to improve the calculation accuracy of specific fuel consumption for performance modeling of air-breathing engines. Eastern-European Journal of Enterprise Technologies, 2 (8 (110)), 23–30. doi: https://doi.org/10.15587/1729-4061.2021.229515
- 15. Bondaryuk, M. N., Il'yashenko, S. M. (1958). Ramjet engines. Moscow: Gosudarstvennoye Izdatel'stvo Oboronnoy Promyshlennosti, 452. Available at: https://apps.dtic.mil/sti/pdfs/AD0607169.pdf