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In modern conditions, due to the vastness of the territory of Kazakhstan, with a certain probability, natural disasters such as earthquakes, floods, avalanches, as well as accidents, destruction of buildings, epidemics, release of chemical toxic substances at industrial enterprises, fires in educational and medical institutions are possible, which justifies the relevance of modern methods and technologies for solving the problem of evacuation.

The peculiarity of this work lies in the formation of an integrated approach for organizing the evacuation process both in peacetime as training for the event of an emergency situation (emergency), and in the event of the emergency itself. A conceptual diagram of an evacuation system is proposed that uses heterogeneous sources for receiving and transmitting information about the onset of an emergency. The input and output sources for receiving and transmitting information about the number of people in the building are determined. The main purpose of the system is to form an operational real-time evacuation plan.

This work is the result of a phased implementation of an integrated evacuation system, which consists in building a mathematical model and a method for solving the problem of maximum flow in the network. A mathematical model has been developed for the optimal flow distribution along the Grindshiels network with the analysis of the flow formation and the characteristics of people's motion in enclosed spaces. A game-theoretic approach and mathematical methods of the theory of hydraulic networks for finding an equilibrium state in flow-distribution networks have been developed. An algorithm for solving the evacuation problem using the graph approach is proposed.

The results of this paper make it possible to systematically organize training evacuations, prepare resources, train the personnel responsible for evacuation in order to quickly respond in an emergency and carry out the evacuation process in order to avoid major consequences

Keywords: *maximum flow, optimal plan, Grindshiels network, Nash equilibrium, evacuation planning*

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DEVELOPMENT OF A SYSTEMATIC APPROACH AND MATHEMATICAL SUPPORT FOR THE EVACUATION PROCESS

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1. Introduction

The relevance of evacuation from a building as a way to protect the population in peacetime has increased in recent years. The practice of modern life suggests that the popula-

tion is increasingly exposed to dangers as a result of natural disasters, accidents and catastrophes in industry, fire in buildings, terrorist attacks, and this need may also be caused by poor-quality construction of administrative and residential premises. Especially prompt and successful evacuation is

of great importance in the earthquake-prone regions of Central Asia and the Caucasus. The cost of the issue of prompt evacuation is the safety of people's lives and material and technical values.

Scientists from many countries consider life safety a new science at the first stage of development – the systematization of knowledge. Only recently, attempts to explain and predict have appeared and formed, so various issues of the theory and practice of life safety were scientists [1].

The theoretical foundations of the mathematical modeling of the movement of people flows inside buildings and assessment of evacuation plans of buildings were developed [2]. In [3], a graphic-analytical method for calculating the total evacuation time was developed, in which empirical dependences of the speed of movement of people on the density of the human flow were revealed.

At the present stage, the most effective tool for research and optimization of the evacuation process are computer evacuation models. By the present time, a large number of similar computer models have been created [4].

The work [5] contributed to the development of computer evacuation simulation models (CESM).

Modern CESM allow to some extent simulate the dynamics of changes in the parameters of the human flow during evacuation from a building, estimate the total duration of evacuation and solve the problem of choosing evacuation routes. However, the overwhelming majority of modern CESM do not fully take into account the possibility of flow stratification according to velocities [6]. In addition, they practically do not take into account the specifics and architectural features of educational institutions and enterprises. As you know, the main feature of streaming in buildings of educational institutions is the non-stationarity of people distribution in the internal premises of the building, associated with the schedule of classes [7]. This leads to the dependence of evacuation plans on the time of day, and also requires an assessment of the training schedule in terms of organizing the smooth movement of people during evacuation. The solution of these problems for buildings of educational institutions is complicated by the presence of time intervals when people move from one room to another, for example, during breaks between classes [8].

Thus, the development of new models and methods for evacuating people in educational institutions under conditions of non-stationarity and variability of people distribution in the premises of the building at different points in time, allowing to evaluate the curriculum in terms of the ease of evacuation, is an urgent task [9].

Undoubtedly, one of the important components of the successful implementation of evacuation measures is the warning system. The ROXTON-8000 warning system is a wide range of devices based on modern digital (micro-processor) technologies and components that function both independently and under software control [10]. And also for notification, the development and use of mobile applications for mobile phones, monitors in classrooms and other means of notification are provided.

The significance of the proposed system for solving evacuation problems globally is also very important, as the countries with developed economies, due to political differences, terrorism and conflict situations often face the evacuation tasks. The densely populated urban areas, natural disasters and industrial accidents also justify the need to solve evacuation problems effectively. And in that case, the methods and tech-

nologies of evacuation, proposed by the executors are significant, since upon the literature review of evacuation problems we have not found similar complex solutions.

Development of modern means of security in places of mass congestion requires the development of new methods, in particular, methods for modeling and optimizing evacuation processes in the event of emergencies based on mathematical models and modern technical means. Training, organizational capabilities of the people responsible for evacuation, as well as the ability to manage equipment, are also very important. The training and being tested work regimes of the proposed work provide preparedness for the evacuation of both the organizers and the people in the building.

In the case of market relations, the standards for material and technical supply lose their significance, each subject of the economy independently assesses the situation and makes a decision.

The results of the work are socially and economically significant, since the price of successful implementation of the goals and objectives of the work in emergency situations is the sum of saved lives and material values.

The result of the work is aimed at ensuring the safety of people or reducing the impact of the consequences of emergencies on the most important resources – human. The use of mathematical methods and information technologies significantly increases the efficiency of evacuation systems, therefore, the development of new integrated and infocommunication approaches to the evacuation problem is currently relevant. In scientific terms, mathematical models of multicriteria optimization are developed based on Nash conditions, models for optimal coverage of building areas, technical means for information transfer, technology for processing large data in the flow distribution problem, and the training regimes for evacuation processes are proposed. Successful implementation of these solutions ensures effective evacuation, which, surely, entails a social and economic effect.

The practical importance of the work is in developing models and methods for solving the evacuation problem, simulating the information model of the evacuation system, taking into account the minute-to-minute recording of the number of people in the audience using touch sensors and obtaining an hourly optimal operational evacuation plan. The use of this system will cut evacuation time and reduce people congestion. The results and main points of the work can be used for educational purposes, as training upon emergencies.

Evacuation is one of population protection means. It is taking out or withdrawal of people from hazardous areas. It can take place both in peacetime and wartime. Evacuation as a means of population protection has long been used.

The actuality of evacuation as the means of population protection in wartime and peacetime during recent years increased rather than decreased. Contemporary life experience says that the population increasingly runs into danger as a result of natural calamities, accidents and disasters in industry and transport. Think for instance of natural calamities, earthquakes, floods, snow slides, mud streams and earth falls, wild scale forest fires. In such cases, evacuation is usually unavoidable. Evacuation measures are taken at accidents at atomic power stations, emissions and flood of hazardous chemicals and biologically damaging substances, at vast fires at petrochemical plants and oil refineries.

In the posted task, we consider people evacuation from the educational institution in an emergency situation. The main peculiarity of educational institution's buildings is the insta-

bility of people distribution in internal premises due to the lectures timetable. This requires assessing the lessons schedule with regard to organizing unobstructed movement of people. As announced earlier, the topic herein is and will be acute as, unfortunately, emergencies happen increasingly frequently.

2. Literature review and problem statement

For the evacuation planning process, a variety of methods and algorithms have been presented. In [11], evacuation planning was considered directly on dynamic issues related to time-varying and volume-dependent.

The paper [12] distinguishes between macroscopic and microscopic models of evacuation, which are able to record the movement of evacuees in time.

The paper [13] considers a mathematical model describing the motion of dynamic flows in a directed graph. The model parameters include the undirected graph as the building model, the initial flow values, the flow sources, and their receivers.

The paper [14] presents a new mathematical model of rescue-evacuation and developed a method for a quick solution for emergency response in real time for various population groups and various means of evacuation, based on the iterative use of a modification of the planning algorithm.

The paper [15] describes the maximum flow in a bipartite dynamic network. The main idea behind these improvements is the rule of pushing out two arcs in the case of maximal algorithms.

The paper [16] proposed and analyzed an algorithm for Dynamic Real-Time Bandwidth Sharing Routing (DRTCCR). This algorithm allows investigating the capacity constraints of the evacuation network in real time by modeling capacity based on time series to improve current solutions to the emergency route planning (ERP) problem.

The paper [17] presents a highly polynomial time algorithm for calculating an approximate solution to the fastest partial contraflow problem on two terminal networks, which is justified by numerical calculations that consider the Kathmandu road network as an evacuation network.

The work [18] describes the evacuation decision model proposed in this paper consisting of three parts: a model for predicting the distribution of pedestrians, a model for calculating pedestrian flow, a situation on the way, and a model for correcting feedback.

The practice of modern life shows that the population is increasingly exposed to dangers as a result of natural disasters, accidents and catastrophes in industry and nuclear power plants, earthquakes, floods, avalanches, mudflows, landslides, mass forest fires, spills of chemically hazardous substances and biologically harmful substances, large fires at petrochemical plants and refineries. In all those cases, it is almost necessary to resort to evacuation. The task of constructing effective evacuation measures is of paramount significance, since people's lives and the preservation of material values depend on that [2, 19].

Evacuation models are designed primarily for the operational conduct of evacuation processes, aimed at saving people's lives and material assets of the enterprise, forecasting and timely determining the time of people evacuation. Very often, such models make it possible to determine possible areas of congestion during evacuation [5]. Many existing models include features such as visualizing the people flow, modeling human behavior, determining the best evacuation

routes, etc. The use of mathematical methods and information technologies significantly increases the efficiency of evacuation systems, therefore, the development of new integrated and intelligent infocommunication approaches using sensors for receiving and transmitting information to solve the evacuation problem is currently very relevant.

At the present stage, evacuation computer models are the most effective tools for investigating and optimizing the evacuation process. By now, a large number of such computer models have been created [1].

Thus, the scientific and technical significance of the work consists in constructing an optimal plan for real-time evacuation from different buildings based on the development of mathematical and information optimization models with the condition of fulfilling Nash equilibrium and software and hardware for the implementation of flow distribution for various types of buildings [20].

Based on the above sources, the works practically do not take into account the specifics and architectural features of educational institutions and enterprises. The main feature of the flow formation in the buildings of educational institutions is the non-stationarity of people distribution in the building.

This work is the result of a phased implementation of an integrated evacuation system, which consists in building a mathematical model and a method for solving the problem of maximum flow in the network. The optimal solution to the problem of maximum network flow is implemented using a game-theoretic approach.

3. The aim and objectives of the study

The aim of this study is to develop a mathematical and informational model of multicriteria flow distribution in networks for selected types of buildings and academic buildings. The practical significance of the project is to build an optimal plan for real-time evacuation from different buildings based on the development of mathematical and information optimization models with the condition of Nash equilibrium and software and hardware for implementing flow distribution for different types of buildings.

To achieve the aim, the following objectives were set:

- to build mathematical and information models of the Grindshields distribution network in enclosed spaces;
- to develop a mathematical method for finding equilibrium in flow distribution networks;
- to solve the problems of maximum network flow;
- to develop an algorithm for finding the equilibrium state according to Nash.

4. Materials and methods

We offer a conceptual diagram of the evacuation task based on heterogeneous systems of reception and transmission of information (Fig. 1). The development of a computer model of the information system will be carried out in accordance with the proposed conceptual schemes shown in Fig. 1.

In terms of mathematical modeling, effective coverage areas for the selected types of buildings will be investigated (universities, schools, industrial premises, office buildings, business centers, etc.). As a result of the study on finding the optimal coverage, sensors for receiving information will be installed, information flows will be determined,

a technology for processing information from sensors will be proposed, warning methods for evacuation processes will be proposed, both in the training mode and in cases of emergency requiring evacuation.

The use of mathematical methods and information technologies significantly increases the efficiency of evacuation systems, therefore, the development

A ten-story educational institution is presented. Suppose that people need to be evacuated from a building due to an emergency. Since the alarm is announced during the class, all classrooms will accordingly be occupied. Each classroom has a certain number of students. There are 25 to 35 classrooms on each floor. There are 2 stairways between the floors. There are 2 exits in the building, two of them are the main exit, two are the emergency exit.

Based on the timetable in educational institutions, we can say that there are different numbers of people in different rooms. Since the number of people in the classroom varies, the evacuation plan must be appropriate.

It is very important for people in emergency situations (fire, earthquake, etc.) to quickly leave the building.

The objective of the project is to develop the most effective evacuation plan, which will be based on the following data.

Model (object) – Kazakh National Research Technical University named after K. I. Satbayev. The model of the university interior in the AutoCAD program is shown in Fig. 2.

Suppose that an emergency has happened in the educational institution bringing the necessity to evacuate people. There are 24 classrooms with 30 students in each, and 10 stair wells and 2 exits (Fig. 3). It is necessary to calculate the time, speed and direction of students' evacuation from the educational institution. Let us specify a graph $G = \langle E, V, H \rangle$, in which the direction of every arc $v \in V$ identifies the direction of flow motion, the flowing capacity of each arc equals to dv . Auditoriums are in the E vertices multiple. Two vertexes «start» and «end» are identified in the E vertexes multiple. Vertex 0 is the flow source, 35 flowing. For i from E there are 2 numbers: amount of people sitting there and amount of people rushing out of there per unit time. Arcs are corridors and stair wells between the nodes.

The main goal of the project is to create the most optimal plan for emergency evacuation in educational institutions ac-

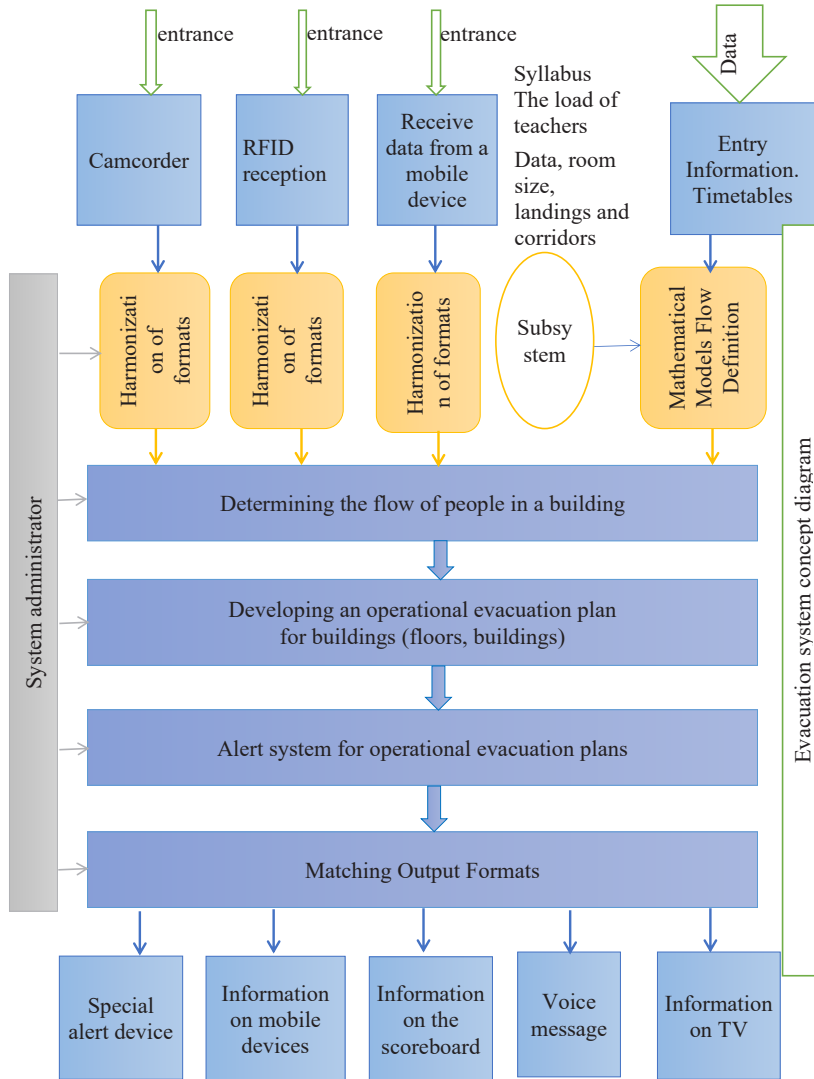


Fig. 1. Conceptual diagram of the evacuation system

of new integrated and infocommunication approaches to the evacuation problem is currently relevant. In scientific terms, mathematical models of multicriteria optimization are developed based on Nash conditions, models for optimal coverage of building areas, technical means for receiving information, technology for processing large data in the flow distribution problem, and the training regimes for evacuation processes are proposed. Successful implementation of these solutions ensures effective evacuation, which, surely, entails a social and economic effect.

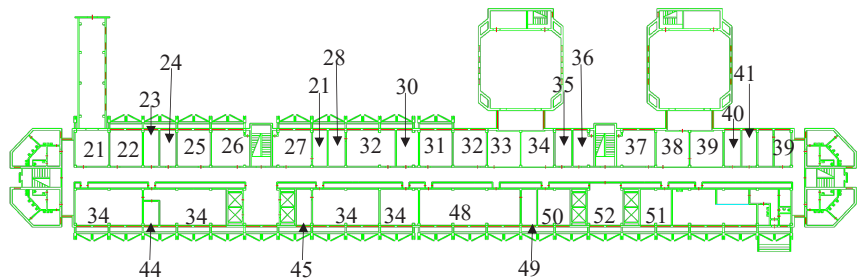


Fig. 2. Plan of the Kazakh National Research Technical University named after K. I. Satbayev building (main educational building, 2nd floor)

ording to the schedule at a certain time.

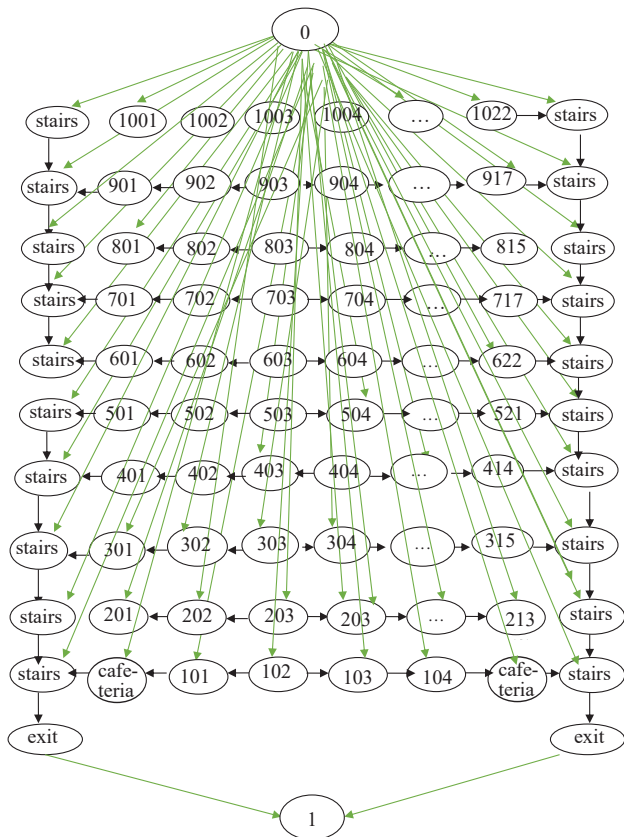


Fig. 3. Main educational building of the Kazakh National Research Technical University named after K. I. Satbayev in the form of a graph

The evacuation should be based on the number of people in the auditors, which differs depending on time and schedule, in order to avoid any crowding during the evacuation process. This approach provides a quick evacuation without any crush and density, which in the future will help save a lot of time and reduce the risk to human life.

5. Results of mathematical and informational representation on the example of evacuation from the educational building

5.1. Construction of mathematical and information models of Grindshiels people distribution network in closed rooms

It is obvious that in order the movement along total length was minimal, it is necessary that the motion speed of every participant on the arc was maximum. However, other participants involuntarily affect the speed of a certain participant. They as well strive to speed maximization, selecting own motion parameters. Flow increase leads to motion speed decrease of a considered driver, which results in time increase.

Let us consider motion only along one arc, and omit all indices concerning the arcs. Let us introduce the following designations: L – length of the circuit section, T – time of movement along the section, x – flow having passed through a road section per unit time, ρ – flow density, s – number of lanes in the corridor, w – flow speed, λ – average length of the corridor.

According to determination, the density is $\rho=1/\lambda$. Let w – speed of a student, w_{max} – maximum speed. The time spent by a person to travel a section of length λ equals $\tau=\lambda/v$.

The number of students per unit time will be equal to $\kappa=1/\tau$. Therefore,

$$x = \kappa s = \frac{1}{\tau} s = \frac{v}{\lambda} = w \rho s.$$

We assume that the flow rate and density are linearly related, and we obtain the Grindshiels formula:

$$w/w_{max} + \rho/\rho_{max} = 1.$$

From the Grindshiels formula, we obtain the following equation:

$$w = w_{max} (1 - \rho/\rho_{max})$$

or

$$\rho = \rho_{max} (1 - w/w_{max}).$$

Substituting it into the flow, we obtain:

$$x = s w \rho_{max} (1 - w/w_{max}).$$

The obtained function is a parabola with downward-directed branches, the maximum is reached at:

$$w = w_{max} / 2$$

and accordingly

$$x_{max} = s (w_{max} \rho_{max}) / 4.$$

Thus, we obtained the value of maximum flow, which can be passed through the road.

Let us substitute instead of ρ its expression, we'll obtain:

$$w^2 - w_{max} w + (w_{max} / (s \rho_{max})) x = 0.$$

Using the Vieta's formula, we obtain:

$$w = w_{max} (1 + \sqrt{1 - x/x_{max}}) / 2.$$

with account that every participant strives to maximize own speed. Herefrom, we find that motion time along the circuit section is expressed by the following dependence:

$$\tau(x) = 2 \tau_{min} / (1 + \sqrt{1 - x/x_{max}}),$$

where τ_{min} – minimal motion time along the section in case when the flow along it equals zero. For descriptive reasons of that function (Fig. 4), we provide the graph of the function:

$$\tau(x) = 1 / (1 + \sqrt{1 - x}).$$

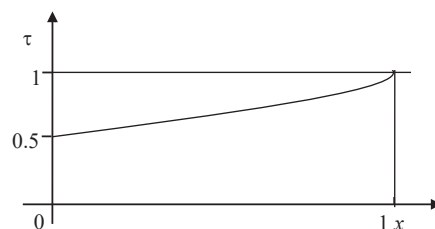


Fig. 4. Graph of the function for time calculation

The mathematical methods and models of evacuation were analyzed. Research and analysis of the subject area and mathematical formulation of the evacuation problem were carried out. Models of the Grindshiels network for the distribution of people in enclosed spaces have been built, taking into account the architectural features of the building.

5. 2. Developing the mathematical methods for searching the equilibrium in flow distribution networks

As every arc has a limited flowing capacity, the check of existing permissible flows along with their search can be fulfilled by means of the task on maximum flow and solving it with the Ford-Fulkerson algorithm [21].

In the task on maximum flow, the flow is passed from one initial vertex to one final. All arcs have prescribed flowing capacity. To arrange that type of the task, let us add two dummy vertexes *ii* and *kk*. Let us connect *ii* with the stream source *i₀*. Its flowing capacity equals to *q_i⁰(i₀)*. The flowing *q_i(i) > 0* is connected with arcs to the vertex *kk*. The capacity of those arcs is *q_i(i)* accordingly. We obtain the task on standard maximum flow and apply any known algorithm to solve it. If it turned out that maximum flow is less than *q_i⁰(i₀)* then the initial task of one layer and accordingly the whole task has no solution. In that case, the minimal cut is beyond additional arcs [22].

If it turned out that the maximum flow equals to *q_i⁰(i₀)*, we obtain permissible flow, which is transferred to the state of equilibrium by invariant transformations.

Let us describe people’s flow movement along a corridor and staircase by means of Grindshiels formula. Let us introduce the following designations: *L* – network section length, *T* – time of moving along the section, *x* – flow having passed the road section per unit time, *P* – flow density, *S* – number of lanes, *W* – flow speed, *λ* – average corridor length.

As defined, the density is $\rho = 1/\lambda$. Assume *W* – student’s speed, *W_{max}* – maximum speed. The time, which gets a man to travel a route section of length *λ* equals to $\tau = \lambda/v$. The number of students per unit time will be equal to $\kappa = 1/\tau$. Therefore, $x = \kappa s = (1/T)S = w/\lambda = wps$. We consider that the flow speed and density are interconnected due to linear dependence $w/w_{max} + p/p_{max} = 1$ (Grindshiels formula) [23].

Therefrom, $w = w_{max}(1 - p/p_{max})$, or $p = p_{max}(1 - w/w_{max})$. Let us insert it, and obtain $x = swp_{max}(1 - w/w_{max})$. The obtained function is a parabola with downward-directed branches, the maximum is achieved at $w = w_{max}/2$, and accordingly $x_{max} = s(w_{max}p_{max})/4$.

Thus, we obtained the magnitude of maximum flow, which can be passed through.

Let us insert instead of ρ the expression and obtain the following formula:

$$w^2 - w_{max,x}w + \left(\frac{w_{max}}{sp_{max}}\right)x = 0.$$

According to the Vieta’s formula, we obtain:

$$w = w_{max,x} \left(1 + \sqrt{1 - \frac{x}{x_{max}}} \right) / 2,$$

taking into account that every member strives to maximize own speed. From here, we find that the time of travelling along the network section is expressed by the following dependence:

$$T(x) = 2T_{min} / \left(1 + \sqrt{1 - \frac{x}{x_{max}}} \right),$$

where *T_{min}* – minimal travelling time along the section in case the flow along it equals to zero. Let us consider evacuation movement route. Based on investigation data, the width can be accepted as 0.6 m, with supposition of its small reduction for the roads with the width in several flows. Apart from that, in view of necessity, irrespective of the road width, in case of possibility of occasional opposing traffic or overdrive at traffic delay, the path with the width of one flow should be accepted with some width reserve. Considering this and the necessity of flows number at existing and adaptable evacuation routes, we can give Table 1 for defining flows number per width both of horizontal route and of staircases.

Table 1

Determination of flows number

Number of elementary flows	Width of evacuation route		
	Normal	Minimal	Maximum
1	0.9	0.9	1.2
2	1.2	1.2	1.7
3	1.8	1.7	2.3
4	2.4	2.3	3

In practice, mass movement speed fluctuates from 5 to 75 m per minute. At sustained motion, density cannot reach physically maximum amount, therefore, it is rational to accept the length of the route as calculation basis. So, specified speed values are defined for horizontal path as 16 meters per minute, for descent down staircase as 10 meters and for ascent 20 % less, that is as 8 meters per minute [24].

The flowing capacity of the elementary flow per minute is defined as a fraction of speed division by flow density. Total capability is defined by multiplying the obtained value by the flows number at route width and by the number per minute, making up evacuation duration. It is evident hereof that such product, depending on total evacuation motion factors, cannot be a constant value, as it is recommended by existing norms, but it is, to a significant extent, a variable value, depending on local conditions and increasing proportionally to an increase in permissible evacuation duration. Time allowance directly influences permissible route length.

For the first stage, the route length characterizes ultimate moving away from the exits and has importance mainly for large buildings. For the sum of the first and second stages, the norms herein determine the layout of separate floors in a ratio of the number and location of exits to outside or to the staircases. For the sum of three stages, the same norms influence the layout in the whole limiting number of floors and prescribe premises grouping per floors in such a way that the first and second stages could decrease in proportion to the increase of the third one [25].

5. 3. Solving the problem of the maximum flow in the network

In many network problems, it is meaningful to consider the arcs as certain communication having definite flowing capacity. In this case, as a rule, the task of some flow maximization, directed from the selected vertex (source) to some other vertex (outflow) is considered. This type of task is called the problem of maximum flow.

Let us assume that there is an oriented graph $G = \langle E, V, H \rangle$, in which the direction of every arc $v \in V$ denotes the flow motion direction, the flowing capacity of each arc equals to *dv*.

At vertexes of the multiple E , two vertexes are distinguished: start and end.

Vertex h is the source of the flow, k is the outflow. It requires maximum flow, which can pass from vertex h to k .

Let us denote as x_v flow level passing along the arc v . It is obvious that:

$$0 \leq x_v \leq d_v, v \in V. \tag{1}$$

In every vertex $i \in E \setminus \{h, k\}$, the incoming flow level equals to the outgoing flow level. That is, the following congruence is true:

$$\sum_{v \in V_i^+} x_v = \sum_{v \in V_i^-} x_v \tag{2}$$

or

$$\sum_{v \in V_i^+} x_v - \sum_{v \in V_i^-} x_v = 0. \tag{3}$$

Accordingly, for vertexes h and k , the following is true:

$$\sum_{v \in V_h^+} x_v - \sum_{v \in V_h^-} x_v = -Q, \tag{4}$$

$$\sum_{v \in V_k^+} x_v - \sum_{v \in V_k^-} x_v = Q. \tag{5}$$

The magnitude Q is the value of the flow, outgoing from vertex h and incoming into vertex k .

Problem. Define:

$$Q \rightarrow \max, \tag{6}$$

at delimitations (1)–(5).

The values $(Q, x_v, v \in V)$ satisfying delimitations (1)–(5) will be named as flow in the network, and if they maximize the magnitude Q , then as maximum flow. It is easy to see that the values $Q=0, x_v=0, v \in V$ are the flow in the network. Problem (1)–(5) is the task of linear programming and can be solved applying the simplex algorithm.

Let us break the multiple of vertex E into two nonintersecting parts E_1 and E_2 in such a way that $h \in E_1, k \in E_2$. Crosscut $R(E_1, E_2)$, separating h and k we will name such multiple $R(E_1, E_2) \subset V$ that for every arc $v \in R(E_1, E_2)$ or $h_1(v) \in E_1$ and $h_2(v) \in E_2$, or $h_1(v) \in E_2$ and $h_2(v) \in E_1$.

There is the multiple $E_1 = \{1, 4, 7\}$ in Fig. 5, these vertexes have dark filling. $E_2 = \{2, 3, 5, 6, 8, 9\}$. Crosscut $R(E_1, E_2)$ represents arcs, which dotted line went through.

Let us break the multiple $R(E_1, E_2)$ into two parts as follows:

$$R^+(E_1, E_2) = \{v \in R(E_1, E_2) \mid h_1(v) \in E_1 \text{ and } h_2(v) \in E_2\},$$

$$R^-(E_1, E_2) = \{v \in R(E_1, E_2) \mid h_2(v) \in E_1 \text{ and } h_1(v) \in E_2\}.$$

Elements of the multiple $R^+(E_1, E_2)$ we will name straight arcs, they lead from the multiple E_1 to E_2 . Elements of the multiple $R^-(E_1, E_2)$ are backward arcs, they lead from the multiple E_2 to E_1 . The flow through the crosscut we will name the value:

$$X(E_1, E_2) = \sum_{v \in R^+(E_1, E_2)} x_v - \sum_{v \in R^-(E_1, E_2)} x_v.$$

Crosscut flowing capacity we will name the value:

$$D(E_1, E_2) = \sum_{v \in R^+(E_1, E_2)} d_v.$$

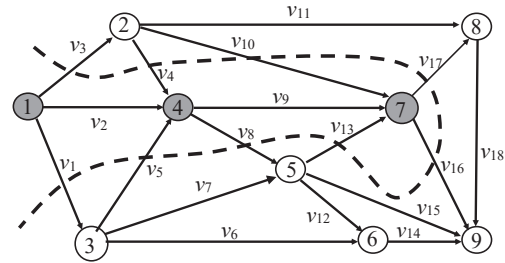


Fig. 5. Search for crosscut

It is obvious that $0 \leq X(E_1, E_2) \leq D(E_1, E_2)$. The following theorem is true.

Theorem 1. On maximum flow and minimal crosscut.

In any network, the magnitude of maximum flow Q from the source h to the overflow k equals the minimal flowing capacity $D(E_1, E_2)$ amongst all crosscuts $R(E_1, E_2)$, separating vertexes h and k .

Crosscut $R(\bar{E}_1, \bar{E}_2)$, with $Q = D(\bar{E}_1, \bar{E}_2)$ we will name constraining. At constraining crosscut, the following is true:

$$x_v = \begin{cases} d_v, & \text{if } v \in R^+(\bar{E}_1, \bar{E}_2), \\ 0, & \text{if } v \in R^-(\bar{E}_1, \bar{E}_2). \end{cases}$$

Let us assume that $(Q, x_v, v \in V)$ is a flow in the network, and succession $h=i_0, v_1, i_1, v_2, i_2, \dots, v_k, i_k=k$ is a circuit connecting vertexes h and k . Define on that circuit motion direction from vertex h to k . The arc v_j from that circuit is called straight, if its direction coincides with the motion direction from h to k , and backward, if not. The circuit will be called flow increasing circuit, if for straight arcs of the circuit v $(d_v - x_v) > 0$ and for backward $x_v > 0$. Through the circuit thereof, it is possible to pass additional flow q from h to k with the value $q = \min(q_1, q_2)$, where $q_1 = \min(d_v - x_v)$, minimum is taken from all straight arcs of the circuit, $q_2 = \min(x_v)$, minimum is taken from all backward arcs of the circuit.

Theorem 2. The flow $(Q, x_v, v \in V)$ is maximum if and only if there is no way to increase the flow.

The proposed algorithm for solving the problem of maximum flow in the network is based on searching an increasing flow in the circuit from h to k . The search, in its turn, is based on the process of vertexes marks disposition similar to the Dijkstra's algorithm.

Let us add mark $P_i = [g_i, v_i, \theta]$ to every vertex i , where g_i – value of additional flow entered the vertex i , v_i – arc through which the flow entered, θ – sign «+», if the flow entered along the arc v_i , directed to i (along the straight arc); θ – sign «-», if the flow entered along the arc v_i , directed from i (along the backward arc).

Let us say that vertex i :

- is not labeled, if the additional flow does not reach it, the label will have the form $P_i = [0, -, \theta]$;
- is labeled, but not viewed, if the flow has reached it, but has not been allowed to go further, the label will have the form $P_i = [g_i, v_i, \theta]$, where $g_i > 0$;
- labeled and viewed, if the flow reached it and is allowed to go further, the label will have the form $P_i = [g_i, v_i, \theta]$.

Let us consider the solution algorithm:

0. For all $v \in V$ assume that $x_v = 0$, assume that $Q = 0$.

1. All vertexes are unlabeled. Vertex h is labeled, but not viewed with the label $P_h = [\infty, -, -]$. It means that the unlimited volume flow enters that vertex.

2. Search labeled but not viewed vertex. If it is not available, then the found flow $Q, x_v, v \in V$ is maximum and the

algorithm completes its function. If such vertex is found, i – is its number, then pass on to 3.

3. View vertex i :

– for all $v \in V_i^-$ assume $j=h_2(v)$. If vertex j is unlabeled and $(dv-xv)>0$, then mark it with the label $P_j=[q, v, +]$, where $q=\min(q_i, (dv-xv))$, if $j=k$, then pass on to point 4;

– for all $v \in V_i^+$ assume $j=h_1(v)$. If vertex j is unlabeled and $xv>0$, then mark it with the label $P_j=[q, v, -]$, where $q=\min(q_i, xv)$, if $j=k$ then pass on to point 4;

– label vertex i as viewed and pass on to point 2.

4. Pass additional flow. Let us assume that $j=k, q=gk$ and $v=vj$:

– if $\theta=\llcorner\lrcorner$, then it is necessary to fulfill: Let us assume that $xv=xv+q, i=h_1(v)$, if $i=h$, then pass on to point 1, otherwise put $j=i$ and pass on to $v=vj$;

– if $\theta=\lrcorner\lrcorner$, then it is necessary to fulfill: Let us assume that $xv=xv-q, i=h_2(v)$, if $i=h$, then pass on to point 1, otherwise put $j=i$ and pass on to $v=vj$.

Because of the algorithm execution, the flow $(Q, xv, v \in V)$ will be obtained. To search the crosscut with minimal flowing capacity, part of vertexes should be labeled and viewed at the final stage of algorithm operation in point 2, we include these vertexes into the multiple $\bar{E}_1, \bar{E}_2 = \bar{E} \setminus \bar{E}_1$. Cross cut $R(\bar{E}_1, \bar{E}_2)$ will be the sought for [26].

The method of potentials for solving the problem thereof supposes that at the initial graph, in some way, the initial radix tree $G'=\langle E, V', H \rangle$ is defined (let's name its arcs as basic), along which transportation is performed in such a way that:

$$x_v = \begin{cases} \geq 0, & \text{if } v \in V', \\ = 0, & \text{if } v \in V \setminus V'. \end{cases}$$

We can use the following algorithm for defining the values $x_v \in V'$.

0 – step. $E':=E$.

K – step. If $E' \neq \emptyset$, find the vertex for which $(|V_i^{'+}| + |V_i^{-}| = 1)$, that is, the vertex i is final at $G'=\langle E', V', H \rangle$.

If $|V_i^{'+}| = 1$, then for the arc $v \in V_i^{'+}$ perform:

$$x_v := b_i, b_{h_1(v)} := b_{h_1(v)} + x_v.$$

If $|V_i^{-}| = 1$, then for the arc $v \in V_i^{-}$ perform:

$$x_v := -b_i, b_{h_2(v)} := b_{h_2(v)} - x_v.$$

Move to the next step. We can obtain an infeasible solution as a result of algorithm operation, that is, for some $v \in V, x_v < 0$. In that case, the initial radix tree shall be changed. However, it does not guarantee that we will obtain feasible solution for it. Further, the algorithm will be described that allows either defining basic solution or demonstrating that it does not exist.

Let us consider that the initial radix tree G' is found and for $v \in V, x_v > 0$ is defined. In order to determine whether the obtained solution is optimal, we will make use of optimality criterion.

Let $x_v, v \in V$ is such a solution of the problem that for $v \in V \setminus V', x_v = 0$ and for $v \in V', x_v \geq 0$. The solution herein is optimal, then and only then, when there exist numbers $u_i, i \in E$, called potentials, such that:

$$u_{h_2(v)} = u_{h_1(v)} + c_v, v \in V, \tag{7}$$

$$u_{h_2(v)} \leq u_{h_1(v)} + c_v, v \in V \setminus V'. \tag{8}$$

For potentials calculation, the following algorithm is applied.
0-step. For certain (only one) vertex $i \in E$, we assume $u_i := 0$.
 k -step. Find the arc $v \in V'$, for which the potential is known, of only one of its vertexes. If there is no such arc, it is the end of operation, otherwise, using the dependence $u_{h_2(v)} = u_{h_1(v)} + c_v$, we define the potential in the vertex, in which it is unknown, and pass on to step $(k+1)$.

Let us consider the following algorithm.

1. Amongst all arcs $v \in V \setminus V'$, we search the arc v_0 such that $u_{h_2(v_0)} > u_{h_1(v_0)} + c_{v_0}$.
2. If there is no such arc, then the initial problem is solved, otherwise, it is needed to accomplish the algorithm of transfer to a new radix tree.

$V' := V' \cup \{v_0\}$, where v_0 is the arc, found in the previous algorithm. Now the graph $G'=\langle E, V', H \rangle$ contains exactly one cycle $G''=\langle E'', V'', H \rangle$, at that, $v_0 \in V''$. At the subgraph G'' , we define girdle direction coinciding with the direction of the arc v_0 , and pass along the subgraph G'' an additional flow in girdle direction of value θ . Every arc $v \in V''$ is ascribed a symbol $\llcorner+\theta\lrcorner$, if the direction of the arc v coincides with girdle direction, and a symbol $\lrcorner-\theta\lrcorner$, if not. Assume that $\theta = \min x_v$ amongst $v \in V''$, which are ascribed a symbol $\llcorner-\theta\lrcorner$.

Assume that $x_{v_0} := \theta$, for all $v \in V'' \setminus \{v_0\}$:

$$x_v := x_v + \theta, \text{ if the arc } v \text{ is ascribed a sign } +\theta,$$

$$x_v := x_v - \theta, \text{ if the arc } v \text{ is ascribed a sign } -\theta.$$

From the multiple $V' \setminus \{v_0\}$, we exclude the arc, for which $x_v = 0$. If there are several such arcs (degenerate case), we remove only one in such a way that graph connectivity $G'=\langle E, V', H \rangle$ was not broken.

For the obtained solution, we over again calculate potentials with the second algorithm and study it, concerning optimality with the third algorithm, while the optimal solution is found.

Let us consider the algorithm of initial radix tree search. Upon initial radix tree search, the described above method for the next «toy» problem is applied.

Construct the graph $G=\langle E, V, H \rangle$, in which

$$E = E \cup \{i_0\},$$

where i_0 – additional value,

$$V = V \cup V_1 \cup V_2,$$

where V_1 – multiple of additional arcs, directed from vertexes – points of production to additional vertex i_0 , and V_2 – multiple of additional arcs, directed from i_0 intermediate vertexes and vertexes-points of consumption:

- for $v \in V$ assume $c_v = 0$;
- for $v \in V_1$ assume $c_v = 1$;
- for $v \in V_2$ assume $c_v = 1$.

Assume $b_{i_0} = 0$. The initial radix tree is the subgraph $G=\langle E, V_1 \cup V_2, H \rangle$. If as a result of solving this problem, it turned out that the optimal value of a functional is strictly greater than zero, then the initial problem has no solution, otherwise, the radix tree of the initial problem will be obtained [26, 27].

Let us assume that some permissible flow has been already found. Let us ask two questions: how, having permissible flow, to define whether it is optimal and how to obtain permissible flow greater by value if the permissible flow thereof is not optimal.

For that purpose, it is necessary to identify what of given below properties owns every arc in the circuit. For the first, the flow along the arc (i, j) is less than the flowing capacity of the arc, (i, j) , which naturally means that the flow along the arc can be increased. Let us denote the multiple of such arcs in the circuit as i . For the second, the flow along the arc (i, j) is positive, which means that it can be reduced. Let us denote the multiple of such arcs as R . Let us describe the procedure of the Ford and Fulkerson method for labels disposition to construct the greater flow.

Step 1. Assign label to the source (vertex 1).

Step 2. Assign other labels to the vertexes and arcs proceeding from the next rules. If vertex x has a label, and vertex y has no mark and the arc $(x, y) \in I$, then label the arc (x, y) and vertex y . In this case, the arc (x, y) is the straight direction arc. If vertex x has a label, and vertex y is unmarked and the arc $(y, x) \in R$, then label the arc (y, x) and vertex y . In this case, the arc (y, x) is a backward direction one.

Step 3. Continue the procedure of labels disposition until the outflow is labeled, or there are no unlabeled arcs left.

If in case of the given procedure implementation, the outflow turned out to be labeled, we can say that there exists a sequence of labeled arcs (name it C) from the source to outflow. Changing arcs flows entering C , we can construct the flow of greater value compared to the initial one. In order to be sure, let us consider two cases: succession C contains only arcs of straight direction and succession C contains both straight and backward direction arcs.

In every case, we can say how to obtain the flow of greater value compared to the given one.

Let us consider case 1. Let $i(x, y)$ – a maximum value, the flow along the arc can be increased without violation of delimitation on flowing capacity. Assume that:

$$k = \min_{(x,y) \in C} i(x,y).$$

Then $k > 0$. In order to modify the flow upwards, let us increase the values of flows on all arcs from C per value k . In this case, not a single delimitation of flowing capacity will be violated. It is easy to note, the flow preservation conditions for all vertexes will be satisfied. It follows that a new flow, on the one hand, is permissible, and, on the other hand, it has a value for k greater than the initial one.

Let us consider case 2. In this case, succession C contains both straight direction and backwards direction arcs. Let $r(x, y)$ – maximum value, the flow can be decreased along the arc (x, y) . Assume that:

$$k_1 = \min_{(x,y) \in C \cap R} r(x,y),$$

$$k_2 = \min_{(x,y) \in C \cap I} i(x,y).$$

Both values k_1 and k_2 , and, accordingly, $\min(k_1, k_2) > 0$. In order to modify the flow upwards, let us increase the flows values along all straight direction arcs from C for the value $\min(k_1, k_2)$, and at all backward directions arcs from C decrease for the same value $\min(k_1, k_2)$. In this case, not a single delimitation of flowing capacity will be violated. It is easy to note, the flow preservation conditions for all vertexes will be satisfied as well. Accordingly, a new flow, on the one hand, is permissible, on the other hand, it has a value less for $\min(k_1, k_2)$ compared to the initial one.

If outflow cannot be labeled, it means that the flow is maximum. To ground this consideration, let us study the crosscut notion.

Let us select any multiple V , containing an outflow, but without the source. Then the multiple of arcs (x, y) , for which x does not belong to V , and $y \in V$ is called a circuit crosscut. In other words, crosscut is a multiple of arcs, excluding which out of the circuit we would separate the source from the outflow. Crosscut value is the sum of flowing capacities of the arcs entering the crosscut. Crosscut is a multiple of arcs removal, which leads to the impossibility to pass from the source to the outflow along the remained arcs. There are several crosscuts in the circuit. Lemma 1 and lemma 2 establish connection between crosscuts and maximum flow. Lemma 1 concludes that the value of any permissible flow from the source to the overflow is not greater than the value of any crosscut. Let us consider any crosscut, defined by the multiple V . Assume W – all other circuit vertexes, not included into the multiple V . Let x_{ij} – value of the flow for the arc (i, j) , and z – overall value of the flow from the source to outflow. If to summarize conditions of flow preservation for all vertexes from the multiple W , the values of flows for arcs (i, j) , for which vertex i and vertex j belong to the multiple W , will reduce, then in the result remains:

$$\sum_{\substack{i \in W \\ j \in V}} x_{ij} - \sum_{\substack{i \in W \\ i \in V}} x_{ij} = z.$$

Taking into account that the first sum from the given ratio is not bigger than the crosscut value, it can be concluded that Lemma 1 is true [12].

Lemma 2 lies in the fact that if the outflow cannot be labeled, then the value of some crosscut equals to the flow value. Let V be the multiple of unmarked vertexes, and W be the multiple of labeled vertexes. Let us consider arcs (i, j) , for which $i \in W, j \in V$, then for them $x_{ij} = c_{ij}$ is true. It follows because in the contrary case we could mark vertex j from the multiple V (as the arc (i, j) is the straight direction arc), which would contradict to determination of the multiple V .

Let us consider arcs (i, j) , for which $i \in V, j \in W$, then for them $x_{ij} = 0$ is true. It follows because in the contrary case we could label vertex i from the multiple V (as the arc (i, j) is the backward direction arc), which would contradict to determination of the multiple V . Thus, it is seen from the ratio that the crosscut value equals to the flow value.

5. 4. Development of algorithms for finding the Nash equilibrium state

Nash equilibrium is the situation, upon which none of the players can increase own bending of the game, changing, on a unilateral basis, own decision. In other ways, it is the situation in which the strategy of both players is the best response to the opponent's actions.

Rational approach to finding the game solution supposes that any player i forms an opinion on the other players' actions and selects as S_i own best answer. Situation S_N in the game is called Nash equilibrium, if for any player i and for his any strategy $s_i \in S_i$ there is fulfilled in equation $U_i(S_n) \geq U_i(S_i, S_i^*)$. Put it otherwise, S_i^* is the best response for every player i . The given situation is such that it is not beneficial for anybody to deviate from it. If others confine themselves to it.

Nash equilibrium is the main concept for solving in no cooperative case. Notion of equilibrium connects two hypotheses on players' behavior. The first – if the situation S_N is unbalanced, it cannot be considered as stable state. That is, if a player sees that deviation from S_i will bring the

bigger bending game, then he/she, most likely, will deviate. It matches the rationality hypothesis. However, the player surely understands that his deviation can arouse an unpredictable chain of responses from other players, the final consequences of which are difficult to overestimate. Such deviation is justified only in case if there is confidence that other players keep unchanged their strategies.

The second hypothesis – if every player sees that deviations from S_i^* bring no improvement, he will maintain that strategy. Equilibrium bending of the game cannot be less than guaranteed level α_i .

Lemma 1 lies in the fact that if S_i^* – Nash equilibrium, then $U_i(S_N^*) \geq \alpha_i$ for any player – i .

Lemma 2 supposes that for every player, subtotals are prescribed $S_i' \subset S_i$. Suppose that S_N^* – equilibrium in the game $(N, (S_i), (U_i))$, and $S_i^* \in S_i'$ for any i . Then S_N^* is equilibrium in the game. If G' is a game, obtained after iterated elimination of strongly dominated strategies, then $NE(G) \subset NE(G')$. We can show that any equilibrium in the game G' is equilibrium in the initial game G , that is, we can record $NE(G) \subset NE(G')$. The given congruence explains the sense of elimination of heavily dominated strategies. If after sequential exclusion there is one profile remained, it is in equilibrium in the initial game, but if several profiles remained, it is necessary to find the balanced one among them.

Nash theorem. Let us assume that in the game $(N, (S_i), (U_i))$, all multiples S_i are convex, and functions of bending of the game U_i are persistent and hill-shaped per variable, then there exists at least one Nash equilibrium.

6. Discussion of experimental results of basic approaches to modeling the movement of people inside buildings

The most commonly used techniques for modeling the movement of people are the representation of people movement as fluid flow (hydroanalogy), cellular automata, and network models.

The first approach is the most common. In it, the movement of people flows along the corridors is represented as the flow of liquid through pipes. This approach assumes that the human flow consists of the same elements having the same characteristics. However, unlike liquid particles, people in a flow have different individual behavior. The individual behavior of each person in the flow has a strong influence on the behavior of the entire group of people as a whole.

The second approach involves the use of cellular automata, which are simplified discrete models. In this kind of models, it is assumed that the movement of people flow has two components: directional and chaotic. But in this form, the model does not reflect the fact that people in the flow move at different speeds.

The third approach involves the representation of the building's constituent parts in the form of nodes connected by arcs. Each arc is associated with some intersection time. The movement of people is calculated from node to node. With this approach, it is possible to simulate people movement at different speeds. Thus, network models take into account the individual characteristics of people.

The main methods for calculating the parameters of the human flow are:

- methodology for calculating the total evacuation time and flow parameters using the formulas state standard 12.1.004-9;

- calculation of the total evacuation time and flow parameters using the graphic-analytical method;

- calculation of the flow parameters taking into account the influence of the flow density and emotional state of people on the speed of their movement.

Modeling using this technique shows that the density in front of the flow increases gradually, which corresponds to the data obtained in [4]. In accordance with [4], flows with high densities are formed gradually, the time to reach the density from the moment Edmach (the flux density $\xi >$ at which the flux intensity reaches its maximum) to D (the maximum flux density $\xi >$) is 5–7 s. This fact is not reflected by the state standard 12.1.004-91 methodology and the graphic-analytical method. As a result, these two methods inadequately reflect the dynamics of formation and the time of existence of people crowds with a high density, and therefore, the total evacuation time is calculated incorrectly.

The proposed algorithm for solving the problem of the maximum flow in the network is based on the search for a chain of increasing flow from h to k . This search, in turn, is based on a vertex marking process similar to the Dijkstra's algorithm.

In the pilot version, in the simulation and training mode, the time of evacuation from the building of the Kazakh National University was calculated, taking into account the architectural features of the two floors of the main educational building. The results of the calculation showed that the evacuation based on the formed optimal plan showed its effectiveness. The total evacuation time based on the training schedule was reduced by 20 %, which is an advantage of the proposed approach. When other educational buildings with different locations of classrooms and other premises are registered, evacuation based on an integrated approach using mobile means of signal transmission becomes a complex multifactorial task. The calculated dependences of the intensity and speed of human traffic on the density of traffic on different roads are shown in Fig. 6.

Thus, the scientific and technical significance of the work consists in constructing an optimal plan of real-time evacuation from different buildings based on the development of mathematical and informational optimization models with the condition of fulfilling the Nash equilibrium and software and hardware for realizing flow distribution for various types of buildings [20].

By conducting a comparative analysis of mathematical evacuation models, the main requirements for modern simulation evacuation models were determined: the choice of evacuation route; modeling people's behavior; features of evacuation accounting when modeling person's characteristics (age, gender, degree of study, degree of familiarity with the building project) were considered.

The application of mathematical models in transport problems to the graph of the studied building with modeling of all classrooms, corridors and door cavities of the building, passageways in the form of graphs, in the form of a special tree is considered. General characteristics of the main parameters of human flow (state standard 12.1.004-91) are given in Table 2.

The calculated dependences of the intensity and speed of human traffic on the density of traffic on different roads are shown in Fig. 6.

In the mathematical representation of the evacuation problem, the apparatus of game theory based on Nash equilibrium is used for optimal flow distribution and redistribution. When taking into account flows, compliance with the first and second Kirchhoff rules is taken into account.

Table 2

Description of the main parameters of human flow

Basic flow parameters	Parameter values					
	Road type, k	$V_{0,k}$, m/min	$s(V_{0,k})$, m/min	$D_{0,k}$, person/m ²	ak	m
Number of people in the flow N ; – D (person/m ²) density; – V (m/min) speed; – PV flow rate; – $D_{max}=0.92$ m ² value of the maximum density of human flow; – $D_n=1.15$ m ² . The value of the flow density increases in the case of narrowing of the given roads	In the building horizontal	100	5	0.51	0.295	1
	Horizontal outside the building	100	5	0.70	0.407	1
	Door cavity	100	5	0.65	0.295	1.25–0.05
	Down the stairs	100(80)	5	0.89	0.400	1
	Up the stairs	60	2.5	0.67	0.305	1

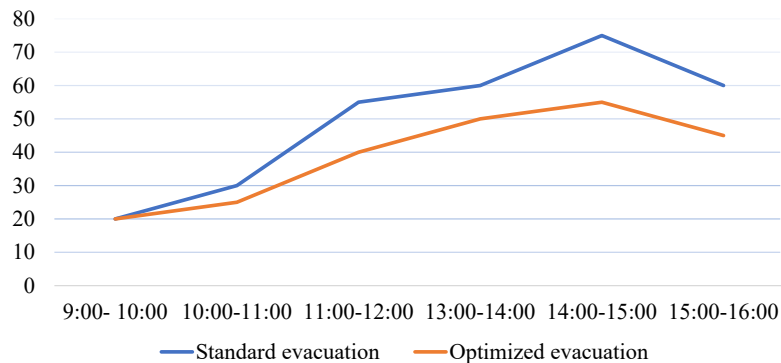


Fig. 6. Time spent for standard and extended evacuation

In the process of solving the problem, the formation of the maximum flow in the network is described, and the algorithm for its solution (distribution) is considered.

7. Conclusions

1. A mathematical and informational model of the Grindshields distribution network in closed rooms was built with the data of the plan of the Kazakh National Research Technical University named after K. I. Satbayev. A mathematical model of the Grindshields people distribution network in enclosed spaces has been built, taking into account the architectural features of the building.

2. A mathematical method for finding equilibrium in flow distribution networks using the algorithm for finding an equilibrium state was developed. We are looking for an arc for which (a small enough number), if such an arc is not found, then we stop linking the layer, for this arc we solve the problem, and proceed to the algorithm again. For multilayer systems, the search for an arc must be carried out in all layers and, accordingly, within a layer. A mathematical method for finding equilibrium in distribution networks based on the game-theoretic approach and the theory of hydraulic networks was developed.

3. The problem of the maximum flow in the network has been solved, if it turns out that the maximum flow is equal, then we get the admissible flow, which we transfer to the equilibrium state by invariant transformations.

4. An algorithm for finding the equilibrium state according to Nash was developed, if it is a game, obtained after iterated elimination of strongly dominated strategies, then we can show that any equilibrium in the game is equilibrium in the initial game, that is, we can record. The given congruence explains the sense of elimination of heavily dominated strategies. If after sequential exclusion there is one profile remained, it is in equilibrium in the initial game, but if several profiles remained, then it is necessary to find the balanced one among them.

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