

*This study was conducted to determine the impact of the HACCP management system on the safety of final products in baked goods production. The object of the study is two critical control points of the production of bakery products, which in turn affect the indicator of microbiological quality and safety of finished products. Using a mathematical model, the critical limits of critical control points (CCP) were determined – in the baking and cooling processes of baked goods. For the two CCPs, measures for continuous monitoring have been defined and critical limits have been developed. For CCT 1, a theoretical calculation using a mathematical model is given: the critical limit of the baking process (95.2–99.1 °C in the center of the crumb), by controlling this process, the company controls the biohazard factor, for CCP 2 the critical limit of cooling the finished product to a temperature of 22.84 °C in the center of the crumb for safe packing. The presentation of the results of the development of critical control limits and critical points of the production of bakery products using the construction of mathematical modeling is the theoretical beginning of further research.*

*When compiling the model, the physical parameters of a specific dough with a certain geometric shape were used, from which the bakery product is baked. The use of a mathematical model allows predetermining the baking parameters for the dough of any initial concentration.*

*The results can be used in the production of baked goods to better ensure the quality and safety of the final product*

*Keywords: HACCP, critical control points, bakery products, food safety and quality*

# DEVELOPMENT OF A METHODOLOGY FOR DETERMINING THE CRITICAL LIMITS OF THE CRITICAL CONTROL POINTS OF THE PRODUCTION OF BAKERY PRODUCTS IN THE REPUBLIC OF KAZAKHSTAN

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## 1. Introduction

The strategic vision of ensuring food security in the Republic of Kazakhstan is to ensure the security of the economy of Kazakhstan, including the agro-industrial complex, in which the state can provide physical and economic access of the entire population to high-quality and safe food products sufficient to meet the physiological norms of consumption and demographic growth [1].

Effective and efficient legally defined food control systems are essential to protect the health of consumers. Be-

sides, they are essential to creating an environment in which countries can ensure the safety and quality of food products entering the international market and verify that imported food products meet national requirements.

Undoubtedly, one of the motivations for the introduction of HACCP systems for food enterprises is to increase export potential. The European market is, for many reasons, extremely attractive for food producers from many countries of the world, and therefore knowledge of the EU food legislation is an important factor for them to successfully enter this market [2].

Food safety is considered in European legislation within the framework of an integrated approach based on the principle of “from the field (stall) to the table” as a single continuous chain that begins with the production of animal feed, and includes (but is not limited to) the production of primary products, processing, packaging, transportation, and marketing, and ends with the consumption of the food product by the end consumer [3].

Food producers, regardless of the stage of the food chain in which they operate, are fully responsible for the quality and safety of food products produced and delivered by them to the consumer market [4].

If any changes are made to the product, to the process, or one of the production stages, the food industry enterprises should review the HACCP system and make the necessary changes to it. All businesses in the food chain must ensure that food is traceable on a “step back, step forward” basis.

Some enterprises, when implementing the HACCP system, introduce the theoretical part of this system, forgetting the scientific approach. The latest version of the ISO 22 000:2018 standard encourages enterprises to take a scientific approach. For example, the definition of a critical limit is given as follows: a measurable value that allows you to separate acceptability from unacceptability, with the following notes: critical limits are set to determine whether a critical control point remains under control (in the acceptable range of values). If the critical limit is exceeded or does not meet the norm, the products that were affected as a result should be considered potentially unsafe [5].

The microbiological quality of a product or its shelf life is determined mainly by organisms, the presence of which (or their metabolites) in certain quantities makes it possible to assess the current quality of the product or, more preferably, to assess its shelf life. For an organism to be suitable for use as such an indicator, it must meet the following requirements as much as possible:

- 1) it must be present in definable quantities in all products, the quality is checked with its help;
- 2) its growth and quantity should directly negatively correlate with the quality of the product;
- 3) it should be easily distinguishable from other organisms and should be easily identified and counted using the simplest methods possible;
- 4) counting its amount should take as little time as possible – the best option is no more than one day;
- 5) its growth should not be influenced by other types of microbiota of the product [6].

The microorganisms of the genus *Bacillus* were determined as an indicator organism of the quality and safety of bakery products. The causative agents of potato disease are widespread in nature (in soil, plants, water, air) and often contaminate grain and flour. A characteristic feature of the causative agents of potato disease is their ability to form spores – dormant forms, which have an extremely high resistance to the effects of unfavorable environmental factors. If the vegetative cells of spore-forming bacteria of the genus *Bacillus* cannot withstand heating to 80 °C, then the spores die at 100 °C only after 6 hours, at 113 °C after 45 minutes, at 125 °C after 10 minutes, boiling and drying are easily tolerated.

For all CCPs, critical limits can be set for one or more parameters. Typically, these parameters are: time, temperature, pH, humidity, water activity, acidity, etc. In many literary sources for the production of bakery products,

generalized critical limits for CCP “baking (danger – development of *Bacillus subtilis*)” are given – the temperature inside the product is not lower than 95 °C. Of course, the controller can easily measure the temperature in the center of the crumb through measuring equipment (thermocouples, etc.), but this leaves a number of questions such as: does this temperature depend on the geometric shape of the bakery product, is it possible to raise the temperature above 95 °C, etc. After all, it is known that the higher the given temperature, the less the influence of the dangerous risk of the development of *Bacillus subtilis*. That is, there is not enough theoretical official data on the parameters, which would make it possible to push off for further research and other works.

Food production requires the use of mathematical equations and statistics to predict the behavior of intermediate production processes over time. The science of mathematical modeling of controlled release has gained a tremendous advancement in drug delivery in recent years. Many of these modeling methods could be transferred to food [7]. Developing a system for improving control requires an understanding of the underlying physical and chemical processes, mechanisms, and principles of finished products.

In the study of food quality, chemical, physical and microbiological control methods are widely used. The study of the physical changes that occur in baked products during production (processing) makes it possible to recognize those that are most important in relation to safety, integrity and overall quality.

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## 2. Literature review and problem statement

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The scientific work [8] presents the results of studies of the impact of personnel training on the food safety management system. The training process is very important and affects the critical control point as the hygiene of personnel in food production.

The paper [9] provides the results of research on the parameters of food storage in the refrigerator and the critical limits of temperature and time. But these critical limits apply to finished products.

The work [10] provides the results of studies on the integration of international standards such as ISO 9001 and the HACCP system. But in this work, the greatest emphasis is placed, firstly, on the integration method and on the method for determining critical control points, rather than critical limits.

The research work [11] provides the results of studies on the kinetics of deterioration in food quality. The work provides in-depth research on shelf life, gives different assessments of the approaches to the shelf life of finished products.

Based on the nature of food products as physically, chemically and biologically active systems, the quality of food products is a dynamical state, continuously changing to ever lower levels [12]. Generally, the overall quality of food is better described by organoleptic assessment. Consequently, for each individual product there is a certain finite time from the moment of its production, during which, under given storage conditions, it retains the required level of organoleptic properties and safety [13]. But organoleptic information is not objective enough to take managerial measures, as well as in controversial cases. All this demonstrates the need to

search for alternative possibilities for a cleaner assessment of production processes before receiving the finished product [14]. This study examines the heat transfer in bakery products during heating and cooling due to thermal conductivity in the absence of internal heat sources when the temperature of the system changes not only from point to point but also over time. Such processes of thermal conductivity, when the temperature field in the body changes not only in space but also in time, are called non-stationary [14, 15].

Among the practical problems of non-stationary thermal conductivity, two groups of processes are of the greatest importance:

- a) the body tends to thermal equilibrium;
- b) the body temperature undergoes periodic changes [16].

The first group includes the processes of heating or cooling bodies placed in an environment with a given thermal state, for example, heating the molded dough in the oven, cooling the finished bakery products, etc.

Fig. 1 shows the nature of the curves obtained by heating a homogeneous solid in a medium with a constant temperature  $t_j$ . As it heats up, the temperature at each point asymptotically approaches the temperature of the heating medium. The temperature of the points lying near the surface of the body changes most rapidly. As the heating time increases, this difference will decrease and theoretically, after a sufficiently long period, it will be zero [17].

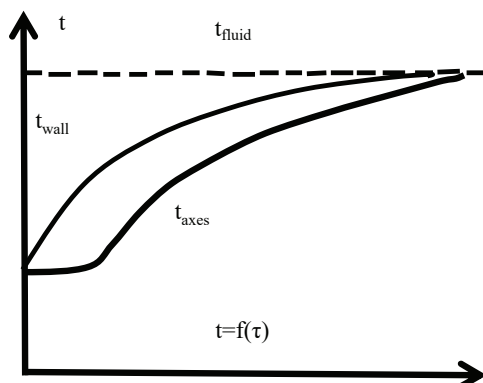


Fig. 1. Nature of changes in the temperature of the bakery product over time  $\tau$  [17]

In this chapter, we consider only a few of the most important problems related to the processes in which the body tends to thermal equilibrium. The purpose of this review is to show the general physical features of such processes, to get acquainted with the method of solving the problem of non-stationary thermal conductivity, and to obtain mathematical relations for practical calculations. Critical limit data is necessary in order to understand whether a CCP is under control or not.

### 3. The aim and objectives of the study

The aim of the study is to develop parameters for controlling the heat treatment of bakery production processes for the shelf life of high quality, as well as for the practical shelf life.

To achieve the aim, the following objectives were set:

- on the basis of the calculation methodology for controlling the heat treatment of bakery production processes,

to develop a critical limit for the baking process (CCT No. 1) using a mathematical model in a non-stationary mode;

- to carry out intermediate production control of heat treatment of bakery production processes, calculate the cooling process (CCT No. 2) of bakery products in a non-stationary mode to determine the critical limit.

The work shows that the temperature in the center of the crumb (during baking, during cooling) of bakery products, which is used to control the CCT, is different, not generalized. It is also necessary to preliminarily make a theoretical calculation of the thermal conductivity of bakery products, which depends on the shape and composition of bakery products. The implementation of the obtained data will be used in monitoring according to the Practical Plan for HACCP Control and Critical Points for the production of bread with a specified geometric shape.

## 4. Materials and methods of analysis of the development of critical limits for heat treatment processes in the production of bakery products

### 4.1. Research materials

In the study, we consider (Fig. 2) two critical control points of the production of bakery products, which in turn affect the indicator of microbiological quality and safety of finished products.

After we have determined the critical control points, then we determine the critical limits for each CCP. The critical limit is a criterion that separates the permissible and unacceptable values of the controlled value about the above indicator organisms of the quality and safety of bakery products [18]. In this method, the maximum possible temperatures for the baking process (maximum high) and the cooling process (maximum low under normal conditions) were determined using mathematical data processing. This technique is being developed for high-quality shelf life (HQShL) as well as for practical shelf life (PShL) [10].

### 4.2. Development of a methodology for calculating critical limits for heat treatment processes in the production of bakery products

#### 4.2.1. Analytical description of the process

The analytical description of the heat conduction process includes a differential equation and unambiguity conditions [19].

The differential equation of thermal conductivity when heating or cooling

$$\frac{\partial t}{\partial \tau} = a \left( \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} \right). \quad (1)$$

The unambiguity conditions are set in the form:

- physical parameters  $\lambda c, \rho$ ;
- the shape and geometric dimensions of BP  $l_0, l_1, l_2 \dots l_n$
- body temperature at the initial time  $\tau=0, t=t_0=f(x, y, z)$ .

The boundary conditions can be defined as boundary conditions of the third kind (2):

$$\left( \frac{\partial t}{\partial n} \right)_{n=0} = -\frac{\alpha}{\lambda} (t_{n=0} - t_f). \quad (2)$$

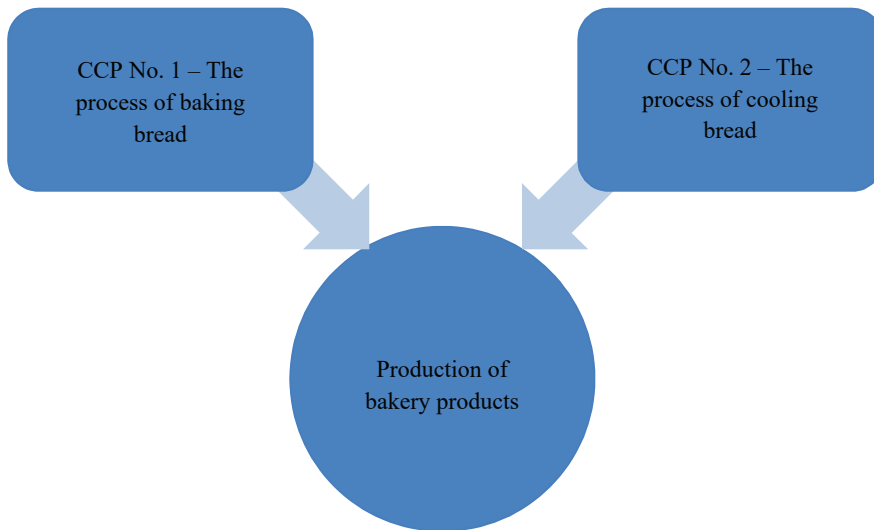


Fig. 2. Control and critical points for the production of bakery products

The differential equation of thermal conductivity (1) together with the conditions of unambiguity (2) gives a complete mathematical formulation of the problem under consideration. The solution to it can be found in the search function:

$$t = f(x, y, z, \tau, \alpha, a, t_0, t_f, l_0, l_1, \dots, l_n). \tag{3}$$

Let us consider in detail the solution of the problem of cooling a flat homogeneous wall and obtain for this case a specific form of function (3). Having studied the method of solving the problem for a flat body, we can understand the principle of solving problems for bodies of other geometric configurations.

**4. 2. 2. Cooling (heating) of an unlimited flat body**

*Problem statement.* If the thickness of a flat body is small compared to the length and width, then such a body is usually considered unlimited.

Under the given boundary conditions, the heat transfer coefficient is the same for all points of the body surface. The temperature changes only in one direction  $x$ , the temperature does not change in the other two directions  $\partial t / \partial y = \partial t / \partial z = 0$ , therefore, in space, the problem is one –  $t(x, 0) = f(x)$  dimensional, the initial temperature distribution is given by some function  $t(x, 0) = f(x)$ . Cooling occurs in a constant temperature environment  $t_f = \text{const}$ . On both surfaces, heat removal is carried out at a constant heat transfer coefficient in time. We will count the temperature of a flat body for any moment of time from the ambient temperature, i.e., from the temperature of the environment  $t - t_f = \Theta$ . Since the problem in space is one-dimensional, the differential (1) takes the form (4):

$$\frac{\partial \Theta}{\partial \tau} = a \left( \frac{\partial^2 \Theta}{\partial x^2} \right). \tag{4}$$

Initial conditions: where  $\tau = 0$ ,

$$\Theta = \Theta_0 = f(x) - t_f = F(x). \tag{5}$$

Under the given cooling conditions, the problem becomes symmetric and the origin is conveniently placed on the axis of a flat body, as shown in Fig. 3 [20].

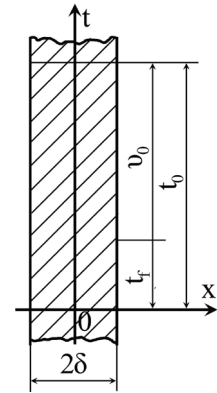


Fig. 3. Flat unbounded body: when  $\tau=0, \tau=0$  specified  $t_0=\text{const}$  and  $\Theta_0=\text{const}$

In this case, the boundary conditions on the axis and on the surface of the plate are written as follows (6):

a) on the axis of a flat body at

$$x = 0, \left( \frac{\partial \Theta}{\partial x} \right)_{x=0} = 0;$$

b) on the surface of the body when

$$x = \delta, \left( \frac{\partial \Theta}{\partial x} \right)_{x=\delta} = -\frac{\delta}{\lambda} \Theta_{x=\delta}. \tag{6}$$

The differential (4) together with the initial (5) and boundary (6) conditions uniquely forms the problem. The solution of the differential equation (4) with the initial and boundary conditions taken into account gives the desired temperature distribution in a flat plate.

The solution of the differential equation (4) is sought in the form of a product of two functions, of which one is a function only  $\tau$ , and the other is only  $x$  (the method of separation of variables):

$$\Theta = \Theta(\tau, x) = \phi(\tau) \psi(x). \tag{7}$$

After substituting the last expression in the differential equation (4), we get:

$$\frac{\partial \phi(\tau)}{\partial \tau} \psi(x) = a \frac{\partial^2 \psi(x)}{\partial x^2} \phi(\tau) \tag{8}$$

or

$$\phi'(\tau) \psi(x) = a \psi''(x) \phi \tau. \tag{9}$$

In this equation, the variables are easily separated, and it can be written as follows:

$$\frac{\phi'(\tau)}{\phi(\tau)} = a \frac{\psi''(x)}{\psi(x)}. \tag{10}$$

The left-hand side of equation (10) is a function of only  $\tau$ , and the right one is a function only  $x$ .

If you fix the argument  $x$  and change only  $\tau$ , then, for any of its values, the left side of equation (10) is equal to the constant

value in the right side, i.e.  $\phi'(\tau)/\phi(\tau) = \text{const}$ . Similarly, when  $x$  is fixed and changed, the right-hand side of equation (10) for any value of  $x$  must be equal to the constant left-hand side, which depends only on  $\tau$ , i. e.  $\psi''(x)/\psi(x) = \text{const}$ .

Since equality (10) must hold for any values of  $x$  and  $\tau$ : then both of its parts must be equal to the same constant value. Denote the latter by  $\varepsilon$  and rewrite the relation (10):

$$\frac{1}{a} \frac{\phi'(\tau)}{\phi(\tau)} = \frac{\psi'(x)}{\psi(x)} = \text{const} = \varepsilon. \tag{11}$$

Note that we get the nontrivial solution for the function  $\psi(x)$  not for all values of  $\varepsilon$ , but when  $\varepsilon < 0$ . As  $\varepsilon$  long as an arbitrary constant in numerical value, then we assume  $\varepsilon = -k^2$ . Substituting this value for  $\varepsilon$ , we get:

$$\frac{1}{a} \frac{\phi'(\tau)}{\phi(\tau)} = \frac{\psi'(x)}{\psi(x)} = -k^2. \tag{12}$$

Where from:

$$\phi'(\tau) + ak^2\phi(\tau) = 0, \tag{13}$$

$$\psi''(x) + k^2\psi(x) = 0. \tag{14}$$

The constant  $k$  is determined from the boundary conditions, and the minus sign is chosen from physical considerations. For thermal processes tending to thermal equilibrium, the sign can only be a minus. As a result, we have obtained a system of ordinary differential equations (13), (14), which are easily integrated. Equation (13) is satisfied by the function  $\phi(\tau) = C_1 e^{-ak^2\tau}$ .

Equation (14) is satisfied by a function of the form:

$$\psi(x) = C_2 \sin(kx) + C_3 \cos(kx). \tag{15}$$

Substituting the resulting expressions for  $\phi(\tau)$  and  $\psi(x)$  in equation (7), we obtain a partial solution:

$$\theta = [C_2 \sin(kx) + C_3 \cos(kx)] C_1 e^{-ak^2\tau}. \tag{16}$$

Expression (16) satisfies the original equation (4) for any values of the constants  $C_1, C_2, C_3$  and  $k$ .

In order for equation (16) to be a solution to the problem, it must be subjected to the initial and boundary conditions. Subjecting equation (16) to the boundary conditions for  $x=0$ :

$$\left(\frac{\partial\theta}{\partial x}\right)_{x=0} = 0,$$

finding it:

$$\left(\frac{\partial\theta}{\partial x}\right)_{x=0} = C_1 e^{-ak^2\tau} k \left[ \begin{matrix} C_2 \cos(kx) - \\ -C_3 \sin(kx) \end{matrix} \right]_{x=0} = 0,$$

or:

$$C_2 \cos(0) = C_3 \sin(0),$$

where  $C_2=0$ .

This means that the private solution  $\psi(x) = C_2 \sin(kx)$  must be discarded as not satisfying the specified boundary

conditions. If consider that  $C_2=0$  and denote  $C_1 C_3=A$ , then equation (16) can be written as:

$$\theta = A_1 e^{-ak^2\tau} \cos(kx). \tag{17}$$

Subjecting the particular solution (17) to the boundary condition

$$\left(\frac{\partial\theta}{\partial x}\right)_{x=\delta} = -\frac{\alpha}{\lambda} \theta_{x=\delta}$$

we get

$$-k A e^{-ak^2\tau} \sin(k\delta) = -\frac{\alpha}{\lambda} A_1 e^{-ak^2\tau} \cos(k\delta), \tag{18}$$

from where, after the simplest transformations, we get:

$$\text{ctg}(k\delta) = \frac{k\delta}{\frac{\alpha\delta}{\lambda}}, \tag{19}$$

where  $\alpha\delta/\lambda = Bi$ . If we denote  $k\delta = \mu$ , that last expression can be written as follows:

$$\text{ctg}(\mu) = \frac{\mu}{Bi}. \tag{20}$$

From the analysis of this trigonometric equation, it follows that for each value of  $Bi$ , there is an infinite number of solutions. The simplest equation (20) can be solved graphically.

Denote the left side of equation (20) by  $y_1 = \text{ctg}\mu$ , and the right – through  $y_2 = \mu/Bi$ . The intersection of the cotangensoid  $y_1$  with the line  $y_2$  gives us the value of the roots of the characteristic equation, i.e.  $\mu$  (Fig. 4) [21].

From Fig. 4, it follows that we have an infinite set of values of the quantity, and each subsequent value is greater than the previous one:

$$\mu_1 < \mu_2 < \mu_3 < \dots < \mu_n < \dots$$

It is important to note that each value of the number  $Bi$  corresponds to its own set of roots of equation (20) [22].

The first four roots of the equation (20)  $\mu_1, \mu_2, \mu_3$  and  $\mu_4$  are given for different values of the number  $Bi$  (from 0 to  $\infty$ ).

When  $Bi \rightarrow \infty$  direct line  $y_2 = \mu/Bi$  coincides with the abscissa axis and the roots of the equation will be equal to:

$$\mu_1 = \frac{\pi}{2}, \mu_2 = \frac{3}{2}\pi, \mu_3 = \frac{5}{2}\pi, \dots, \mu_n = (2n-1)\frac{\pi}{2}.$$

When  $Bi \rightarrow 0$  direct line  $y_2 = \mu/Bi$  coincides with the ordinate axis and the tangent of the angle of inclination of the straight line tends to infinity, while the roots of equation (20) are equal to:

$$\mu_1 = 0, \mu_2 = \pi, \mu_3 = 2\pi, \dots, \mu_n = (n-1)\pi,$$

where  $n=1,2,3,\dots$

Therefore, each found root value will correspond to its own particular temperature distribution:

$$\left. \begin{aligned} \theta_1 &= A_1 \cos\left(\mu_1 \frac{x}{\delta}\right) e^{-\mu_1^2 \frac{\alpha \tau}{\delta^2}}, \\ \theta_2 &= A_2 \cos\left(\mu_2 \frac{x}{\delta}\right) e^{-\mu_2^2 \frac{\alpha \tau}{\delta^2}}, \\ &\dots\dots\dots \\ \theta_n &= A_n \cos\left(\mu_n \frac{x}{\delta}\right) e^{-\mu_n^2 \frac{\alpha \tau}{\delta^2}}. \end{aligned} \right\} \quad (21)$$

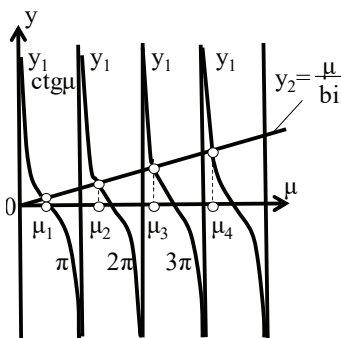


Fig. 4. To solve equation (20)

The obtained partial solutions (21) will satisfy the differential equation for any values of the constants  $A_1, A_2, \dots, A_n$ , but none of these solutions will correspond to the actual temperature distribution at the initial time. However, by superimposing an infinite number of such distributions, with an appropriate choice of  $A_n$  values, it is possible to reproduce any real temperature dependence at the initial time [23].

Based on the above, the general solution can be represented as the sum of an infinite series:

$$\theta_n = \sum_{n=1}^{n \rightarrow \infty} A_n \cos\left(\mu_n \frac{x}{\delta}\right) e^{-\mu_n^2 \frac{\alpha \tau}{\delta^2}}. \quad (22)$$

It is known that if the individual distributions (21) satisfy the differential equation (4) and the boundary conditions (6), then the sum of them also satisfies the same conditions.

The constant  $A_n$  in equation (22) is found from the initial conditions. Subordinating equation (22) to the initial condition, we obtain:

$$\theta_0 = F(x) = \sum_{n=1}^{n \rightarrow \infty} A_n \cos\left(\mu_n \frac{x}{\delta}\right), \quad (23)$$

(23) is the expansion of an even function into a Fourier series with given parameters  $\mu_n$  determined by the characteristic equation (20). For this sequence of numbers, the formula  $1/\mu_n$  is valid:

$$\int_{-\delta}^{+\delta} \cos\left(\mu_n \frac{x}{\delta}\right) \cos\left(\mu_m \frac{x}{\delta}\right) dx = \begin{cases} = 0 & \text{at } n \neq m, \\ \neq 0 & \text{at } n = m. \end{cases} \quad (24)$$

With the help of which you can determine all the coefficients of  $A_n$  in equation (23). To do this, multiply both parts of equation (23) by  $\cos\left(\mu_m \frac{x}{\delta}\right) dx$  and then we integrate the resulting ratio over the thickness of the plate. Then:

$$\int_{-\delta}^{+\delta} F(x) \cos\left(\mu_m \frac{x}{\delta}\right) dx = A_n \int_{-\delta}^{+\delta} \cos^2\left(\mu_m \frac{x}{\delta}\right) dx. \quad (25)$$

For all the other terms in the right-hand side, for which  $n \neq m$ , van. The integral on the right side of the relation (25) is equal to

$$\delta \left(1 + \frac{1}{2\mu_n} \sin 2\mu_n\right).$$

Then:

$$A_n = \frac{\mu_n}{\delta(\mu_n + \sin \mu_n \cos \mu_n)} \int_{-\delta}^{+\delta} F(x) \cos\left(\mu_n \frac{x}{\delta}\right) dx. \quad (26)$$

It follows from equation (26) that  $A_n$  is a function only of the root of the characteristic equation and the initial temperature distribution.

Substituting the obtained expression for the constant  $A_n$  in equation (22), we obtain the final expression for the temperature field during cooling of a homogeneous flat body:

$$\theta = \sum_{n=1}^{n \rightarrow \infty} \frac{\mu_n}{\delta(\mu_n + \sin \mu_n \cos \mu_n)} \times \left[ \int_{-\delta}^{+\delta} F(x) \cos\left(\mu_n \frac{x}{\delta}\right) dx \right] \cos\left(\mu_n \frac{x}{\delta}\right) e^{-\mu_n^2 \frac{\alpha \tau}{\delta^2}}. \quad (27)$$

(27) allows us to obtain the temperature value at any point of a flat body for any time  $\tau$  at any initial temperature distribution  $\Theta$ .

If at the initial time ( $\tau=0$ ) the temperature in the body is evenly distributed (Fig. 2), i.e.  $t_0 - t_f = \Theta_0 = \text{const}$ , then the integral in equation (19) is equal to  $(\Theta_0 2\delta/\mu_n) \sin \mu_n$ . With this in mind, the expression for the constant  $A_n$  takes the form:

$$A_n = \theta_0 \frac{2 \sin \mu_n}{(\mu_n + \sin \mu_n \cos \mu_n)}. \quad (28)$$

Substituting the  $A_n$  value obtained for the case of uniform temperature distribution in a flat body at the initial moment of time into equation (27), we obtain:

$$\theta = \sum_{n=1}^{n \rightarrow \infty} \frac{\theta_0 2 \sin \mu_n}{(\mu_n + \sin \mu_n \cos \mu_n)} \cos\left(\mu_n \frac{x}{\delta}\right) e^{-\mu_n^2 \frac{\alpha \tau}{\delta^2}}. \quad (29)$$

It is advisable to give the temperature field equation (29) a dimensionless form. To do this, divide the right and left sides of equation (29) by  $\theta_0$ . In this case, we denote:

$$\frac{A_n}{\theta_0} = D_n = \frac{2 \sin \mu_n}{(\mu_n + \sin \mu_n \cos \mu_n)}. \quad (30)$$

After these transformations, we get:

$$\frac{\theta}{\theta_0} = \sum_{n=1}^{n \rightarrow \infty} \frac{2 \sin \mu_n}{(\mu_n + \sin \mu_n \cos \mu_n)} \cos\left(\mu_n \frac{x}{\delta}\right) e^{-\mu_n^2 \frac{\alpha \tau}{\delta^2}}. \quad (31)$$

The values included in the temperature field equation (31) are  $\frac{\theta}{\theta_0}, D_n, \mu_n, \frac{\alpha \tau}{\delta^2}, \frac{x}{\delta}$  - are dimensionless and have the following meaning:

- $\frac{\theta}{\theta_0} = \Theta$  – dimensionless temperature;
- $\frac{x}{\delta} = X$  – dimensionless coordinate;
- $\frac{\alpha \tau}{\delta^2} = Fo$  – number Fourier transform, which is a dimensionless time;

-  $D_n$  – dimensionless coefficient.

Taking into account the last notation, equation (23) is written:

$$\Theta = \sum_{n=1}^{n \rightarrow \infty} \frac{2 \sin \mu_n}{(\mu_n + \sin \mu_n \cos \mu_n)} \cos(\mu_n X) e^{-\mu_n^2 Fo}. \quad (32)$$

*Analysis of the resulting solution.* Since,  $\mu_1, \mu_2, \dots, \mu_n$  is a series of increasing numbers, the larger  $\mu$ , the smaller the role of the subsequent member of the series compared to the previous one. Besides, the larger the number  $Fo$ , the faster the members of the series will decrease as the number  $n$  increases.

Numerous studies have shown that even at  $Fo > 0.3$ , the series (32) becomes so rapidly convergent that the temperature distribution is described fairly accurately by the first term of the series:

$$\Theta = \frac{2 \sin \mu_1}{(\mu_1 + \sin \mu_1 \cos \mu_1)} \cos(\mu_1 X) e^{-\mu_1^2 Fo}. \quad (33)$$

Previously designated

$$D_1 = \frac{2 \sin \mu_1}{(\mu_1 + \sin \mu_1 \cos \mu_1)}.$$

Taking this notation into account, equation (33) can be written in the following form:

$$\Theta = D_1 \cos(\mu_1 X) e^{-\mu_1^2 Fo}. \quad (34)$$

The value of  $D_1$  is only a function of the number  $Bi$  and can be calculated and tabulated in advance. In addition, if we consider the temperature for a certain value  $\frac{x}{\delta} = X$ ,  $\cos(\mu_1 X)$

is a function of  $Bi$ .

Specifically for the axis of a flat body  $\frac{x}{\delta} = X = 0$  and  $\cos(\mu_1 0) = 1$ , and for the surface  $\frac{x}{\delta} = X = 1$  and  $\cos(\mu_1 1) = \cos \mu_1$ .

For the axis of a flat body, the product  $D_1 \cos(0)$  is denoted as some function  $N(Bi)$ . Then equation (34) can be written as follows:

$$\Theta_{x=0} = N(Bi) e^{-\mu_1^2 Fo}. \quad (35)$$

For the surface of a flat body, the product  $D_1 \cos \mu_1$  is denoted as some function  $P(Bi)$  and equation (34) is written as:

$$\Theta_{x=1} = P(Bi) e^{-\mu_1^2 Fo}. \quad (36)$$

The functions  $N(Bi)$  and  $P(Bi)$  in equations (35), (36) are tabulated and can be taken from reference books for calculation. Besides, it follows from equations (26) and (27) that for a given coordinate, the dimensionless temperature is only a function of two dimensionless parameters  $Bi$  and  $Fo$ :

$$\Theta_{x=0} = f_1(Bi, Fo)$$

and

$$\Theta_{x=1} = f_2(Bi, Fo).$$

Logarithmizing equation (35), we get:

$$\ln \Theta_{x=0} = \ln N(Bi) - \mu_1^2 Fo. \quad (37)$$

A similar equation can be obtained after the logarithm of (37). It follows from equation (38) that for a given coordinate value and given  $Bi$ , the natural logarithm of the dimensionless temperature depends linearly on time.

It follows from equation (32) that under the conditions of cooling (heating) of the plate for any moment under given boundary conditions, the temperature field has the form of an asymmetric curve with a maximum on the axis of the plate ( $X=0$ ). For each subsequent moment, there will be its curve, monotonically decreasing to the surfaces of the plate. In this case, for any moment, the tangents to the curves at points  $X=\pm 1$  pass through two guide points  $+A$  and  $-A$  located at a distance of  $\pm X_0$  from the plate surface,  $X_0 = 1/Bi$  (Fig. 4).

To prove this important property, consider the temperature field for an arbitrary moment in time  $Fo > 0$ .

Multiplying the boundary condition (6) at  $x = \pm \delta$  by  $\delta/\theta_0$ , we get:

$$-\left[ \frac{\partial \left( \frac{\theta}{\theta_0} \right)}{\partial \left( \frac{x}{\delta} \right)} \right]_{x=\delta} = \frac{\alpha \delta}{\lambda} \left( \frac{\theta}{\theta_0} \right)_{x=\delta}. \quad (38)$$

Writing the last expression in dimensionless quantities, we will have:

$$\left( \frac{\partial \Theta}{\partial X} \right)_{X=1} = -Bi \Theta_{x=1}. \quad (39)$$

Fig. 5 shows [24] that:

$$-\left( \frac{\partial \Theta}{\partial X} \right)_{X=1} = \frac{\Theta_{x=1}}{X_0} = \text{tg } \phi. \quad (40)$$

Comparing expressions (38) and (39), we get:

$$X_0 = \frac{1}{Bi}. \quad (41)$$

It follows from equation (41) that the distance of point  $A$  from the surface is determined by the given unambiguity conditions, which are valid for any moment of time. Consequently, the tangents to all temperature curves at the point of intersection with the surface of the plate and constant boundary conditions will always pass through point  $A$ . This is true not only for the plate, but also for the cylinder, ball, and other geometric shapes.

The proven property of temperature curves makes it possible to determine the nature of the temperature change in the body at a given value of the number  $Bi$ .

#### 4. 2. 3. Determination of the amount of heat released during the cooling process

The amount of heat,  $Q_n$  J, that a flat body gives or receives from both sides during the time from  $\tau=0$  to  $\tau=\infty$

should be equal to the change in the internal energy of the body during the period of its complete cooling (heating):

$$Q_p = 2\delta f \rho c (t_0 - t_f). \tag{42}$$

Then for any period of time from  $\tau=0$  to  $\tau_1$  or, what is the same, from  $Fo$  to  $Fo_1$ , the internal energy of the body will change to:

$$Q = Q_n - Q_1 = 2\delta f \rho c (t_0 - t_f) \left( 1 - \frac{\bar{t}_1 - t_f}{t_0 - t_f} \right). \tag{43}$$

or:

$$Q = Q_n (1 - \bar{\Theta}_1), \tag{44}$$

where

$$\bar{\Theta}_1 = \frac{\bar{t}_1 - t_f}{t_0 - t_f}$$

– the average dimensionless temperature over the thickness of the plate at a time  $\tau_1$ .

From the relations (42), (44), it follows that the calculation of the amount of heat given or received by the plate is reduced to finding the average dimensionless temperature at the moment of time we are interested in. The average dimensionless temperature for the plate layer from the axis of symmetry to the  $X$  plane is found as

$$\bar{\Theta} = \frac{1}{X} \int_0^X \Theta dx \tag{45}$$

in accordance with the mean value theorem.

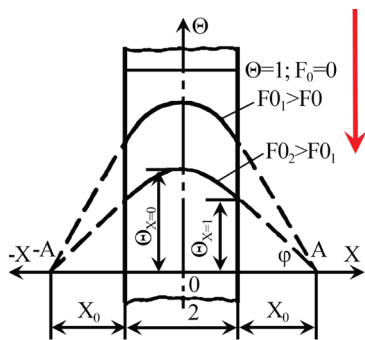


Fig. 5. Change in the temperature field in a flat unlimited wall when it is cooled

If we substitute the value from equation (32) under the sign of the integral in this expression and integrate it in the range from zero to one, we get:

$$\bar{\Theta} = \sum_{n=1}^{\infty} \frac{2 \sin \mu_n}{(\mu_n + \sin \mu_n \cos \mu_n)} \exp(-\mu_n^2 Fo). \tag{46}$$

Substituting in equation (44) the value of the average temperature of the plate calculated by the formula (46) for the moment of time of interest, we get the amount of heat released by the plate into the environment for the considered period.

By  $Bi \rightarrow \infty$  (practically  $Bi > 100$ ) equation (46) takes the form:

$$\bar{\Theta} = \sum_{n=1}^{\infty} \frac{8}{\pi^2 (2n-1)^2} \exp \left[ - \left( \frac{2n-1}{2} \right)^2 \pi^2 Fo \right]. \tag{47}$$

By  $Bi \rightarrow 0$  (practically  $Bi < 100$ ) equation (46) takes the form:

$$\bar{\Theta} = \exp(-Bi Fo). \tag{48}$$

If the values of the number  $Fo > 0.3$  for the plate can be limited to the first term of the series (45), then

$$\bar{\Theta} = \sum_{n=1}^{\infty} \frac{2 \sin \mu_n}{(\mu_n + \sin \mu_n \cos \mu_n)} \exp(-\mu_n^2 Fo). \tag{49}$$

Multiplier  $\frac{2 \sin \mu_n}{(\mu_n + \sin \mu_n \cos \mu_n)}$  depends only on the number  $Bi$  and can be represented as some function  $M(Bi)$ , then equation (49) is written:

$$\bar{\Theta} = M(Bi) \exp(-\mu_n^2 Fo). \tag{50}$$

The function  $M(Bi)$  can be pre-calculated and represented. Then the calculation of the average temperature will be reduced to the calculation of the exponent.

#### 4. 2. 4. Cooling (heating) bodies of finite dimensions

*Cooling of a BP having the form of a parallelepiped.*

Consider the cooling of a parallelepiped in a medium with a constant temperature and a constant heat transfer coefficient and on all its faces [25]. At the initial time ( $\tau=0$ ), all points of the parallelepiped have the same temperature  $t_0$ . A parallelepiped with dimensions  $2\delta_x * 2\delta_y * 2\delta_z$  is homogeneous and isotropic. It is necessary to find the temperature distribution in the parallelepiped for any moment of time, as well as the average temperature necessary to determine the amount of heat supplied (withdrawn).

Let's place the origin in the center of the parallelepiped (Fig. 6).

In this case, the differential equation is written as follows:

$$\frac{\partial t(x, y, z, \tau)}{\partial \tau} = \alpha \nabla^2 t(x, y, z, \tau). \tag{51}$$

Initial conditions ( $\tau=0$ ):

$$t(x, y, z) = t_0 = \text{const}. \tag{52}$$

Under the given conditions, the problem is symmetric with respect to the center of the parallelepiped. If you enter the notation  $\theta = t - t_f$ , then the boundary conditions are written as follows:

a) for the surface at  $\tau > 0$ :  $q = \alpha(t_f - t_c) > 0$

$$\left. \begin{aligned} \pm \lambda \left( \frac{\partial \theta}{\partial x} \right)_{x=\pm\delta_x} &= \alpha \theta_{x=\pm\delta_x}, \\ \pm \lambda \left( \frac{\partial \theta}{\partial y} \right)_{y=\pm\delta_y} &= \alpha \theta_{y=\pm\delta_y}, \\ \pm \lambda \left( \frac{\partial \theta}{\partial z} \right)_{z=\pm\delta_z} &= \alpha \theta_{z=\pm\delta_z}; \end{aligned} \right\} \tag{53}$$

b) in the center of the parallelepiped at  $\tau > 0$ :



$$\left. \begin{aligned} \pm \lambda \left( \frac{\partial \theta}{\partial x} \right)_{x=0, 0 \leq y \leq \delta_y, 0 \leq z \leq \delta_z} &= 0, \\ \pm \lambda \left( \frac{\partial \theta}{\partial y} \right)_{y=0, 0 \leq x \leq \delta_x, 0 \leq z \leq \delta_z} &= 0, \\ \pm \lambda \left( \frac{\partial \theta}{\partial z} \right)_{z=0, 0 \leq x \leq \delta_x, 0 \leq y \leq \delta_y} &= 0. \end{aligned} \right\} \quad (54)$$

Parallelepipeds, cylinders of solved dimensions, and rectangular rods can be considered as bodies formed by the intersection, respectively, of three mutually perpendicular unlimited plates of finite thickness, a cylinder and a plate, and two plates.

It can be proved that the solution of such problems is represented by the product of dimensionless temperatures for bodies of unlimited dimensions, as a result of the intersection of which the body in question was formed.

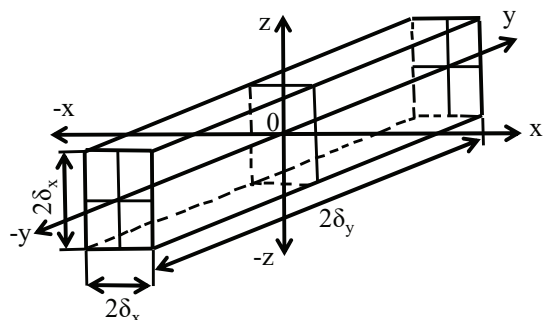


Fig. 6. Cooling of a parallelepiped as a conditional theoretical body similar to a bakery product

As already mentioned, a parallelepiped is formed as a result of the intersection of three mutually perpendicular boundless plates of finite thickness. Therefore, for it, the solution can be represented as the product of dimensionless temperatures for three infinite plates:

$$\Theta = \Theta_x \Theta_y \Theta_z, \quad (55)$$

where:

$$\Theta_x = \frac{t(x, \tau) - t_f}{t_0 - t_f}, \quad \Theta_y = \frac{t(y, \tau) - t_f}{t_0 - t_f}, \quad \Theta_z = \frac{t(z, \tau) - t_f}{t_0 - t_f}.$$

The general solution (40) in the expanded form is written as follows:

$$\Theta = \frac{t(x, y, z, \tau) - t_f}{t_0 - t_f} = \frac{t(x, \tau) - t_f}{t_0 - t_f} \times \frac{t(y, \tau) - t_f}{t_0 - t_f} \cdot \frac{t(z, \tau) - t_f}{t_0 - t_f}. \quad (56)$$

The above solution satisfies both the differential equation and the boundary conditions describing the process of thermal conductivity in a parallelepiped.

Thus, the solution of the problem for the considered body of finite dimensions was reduced to the solution of the problem for an infinite plate of finite thickness. Equation (41) can be represented as:

$$\Theta = F_x \left( \frac{x}{\delta_x}, \frac{\alpha \delta_x}{\lambda}, \frac{\alpha \tau}{\delta_x^2} \right) F_y \left( \frac{y}{\delta_y}, \frac{\alpha \delta_y}{\lambda}, \frac{\alpha \tau}{\delta_y^2} \right) F_z \left( \frac{z}{\delta_z}, \frac{\alpha \delta_z}{\lambda}, \frac{\alpha \tau}{\delta_z^2} \right)$$

or

$$\Theta = F_x (X, Bi_x, Fo_x) F_y (Y, Bi_y, Fo_y) F_z (Z, Bi_z, Fo_z). \quad (57)$$

The multipliers in (41) are calculated by (24). The considered method is known in the theory of thermal conductivity under the name of the theorem on multiplication of solutions. The obtained solution is also valid for finding the average temperature.

The average dimensionless temperature of a parallelepiped is expressed as follows:

$$\bar{\Theta} = \frac{\bar{t}(\tau) - t_f}{t_0 - t_f} = \frac{\bar{t}(\tau)_x - t_f}{t_0 - t_f} = \frac{\bar{t}(\tau)_y - t_f}{t_0 - t_f} = \frac{\bar{t}(\tau)_z - t_f}{t_0 - t_f},$$

or

$$\bar{\Theta} = \bar{\Theta}_x \bar{\Theta}_y \bar{\Theta}_z = \bar{F}_x (Bi_x, Fo_x) \bar{F}_y (Bi_y, Fo_y) \bar{F}_z (Bi_z, Fo_z).$$

In (41), the multipliers are found by (39). Note that the multiplication theorem of solutions is also valid in the more general case, when the coefficients of thermal conductivity are different for different directions, the coefficients of heat transfer on the faces are different.

## 5. Results of research of the critical limits for the critical control points of the production of bakery products

### 5.1. Development of the critical limit of critical control points using the above mathematical model

The bakery product, which has the shape of a parallelepiped with dimensions of 90×140×240 mm, has an initial temperature of 20 °C, and then was placed in an oven with a temperature of 200 °C.

Determine the temperature in the center of the bakery product 20 minutes after loading it into the oven. Physical parameters of BP:

- test density  $p=830-1,100 \text{ kg/m}^3$ ; the density of the bread  $\rho=400 \text{ kg/m}^3$ ;
- coefficient of thermal conductivity  $\lambda=0.15-0.30 \text{ W/(m}\cdot\text{K)}$ ;
- coefficient of thermal conductivity  $a=(12-20) \cdot 10^{-8} \text{ m}^2/\text{s}$ ;
- heat capacity  $c=2,750 \text{ J/(kg}\cdot\text{K)}$ .

Calculation of heating of bakery products – development of a mathematical model of CCP No. 1 for determining the critical limit.

The dimensionless temperature of any point of the parallelepiped (Fig. 7) is equal to the product of the dimensionless temperatures of the three infinite plates whose intersection forms the parallelepiped.

Therefore, the temperature in the center of a bakery product that has the shape of a parallelepiped (Fig. 7) can be calculated by the equation:

$$\frac{t_f - t_c}{t_f - t_0} = \frac{t_f - t_{x=0}}{t_f - t_0} * \frac{t_f - t_{y=0}}{t_f - t_0} * \frac{t_f - t_{z=0}}{t_f - t_0}.$$

The calculation of the temperature in the center of the crumb of bakery products is given in Table 1.

Table 1

Calculation of the temperature in the center of the crumb of bakery products

No.	Using the formulas	Results	Note
1	$ Fo_x = \frac{\alpha * \tau}{\delta_x^2} $	0.12	for a plate thickness $ 2\delta_x=90 $ mm we have $ (\delta_x=45 $ mm=0.045 m)
2	$ Bi_x = \frac{\alpha * \delta_f}{\lambda} $	13.3	
2.1	$ \theta = \frac{t_f - t_{x=0}}{t_f - t_0} $	0.76	dimensionless temperature value
3	$ Fo_y = \frac{\alpha * \tau}{\delta_y^2} $	0.048	for a plate with a thickness of $ 2\delta_x=140 $ mm $ (\delta_x=70 $ mm=0.070 m)
3.1	$ Bi_y = \frac{\alpha * \delta_f}{\lambda} $	20.68	
4	$ \theta = \frac{t_f - t_{y=0}}{t_f - t_0} $	0.85	from dimensionless temperature graph
5	$ Fo_z = \frac{\alpha * \tau}{\delta_z^2} $	0.017	for a plate with a thickness of $ 2\delta_x=240 $ mm $ (\delta_x=120 $ mm=0.120 m)
5.1	$ Bi_z = \frac{\alpha * \delta_f}{\lambda} $	35.45	
5.2	$ \theta_z = \frac{t_f - t_{z=0}}{t_f - t_0} $	0.90	dimensionless temperature
5.3	$ \frac{t_f - t_c}{t_f - t_0} $	0.582	–
5.4	$ t_c = t_f - 0.582(t_f - t_0) $	95.2 °C	the maximum temperature in the center of the crumb of bakery products – that is, the critical limit of CCP No. 1
5.5	$ q = \alpha(t_f - t_c), t_c \frac{\alpha * t_{f-q}}{\alpha} $	99.1 °C	surface temperature, where $ q=6.560 $ Wt/m <sup>2</sup> – heat flow density

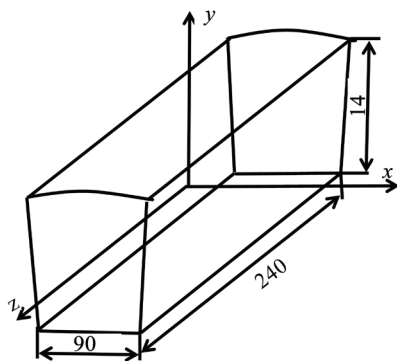


Fig. 7. Diagram of a bakery product in the form of a parallelepiped

**5. 2. Calculation of cooling of bakery products – development of a mathematical model of CCP No.2 for determining the critical limit**

From practice, it is found that after removing the bakery product from the oven, it cools from the initial temperature of 99.1 °C to the ambient temperature, i. e. up to 20 °C within 3 hours.

The cooling temperature of a bakery product that has the shape of a parallelepiped can be calculated by the equation:

$$\frac{t_p - t_f}{t_0 - t_f} = \frac{t_p - t_{x=0}}{t_0 - t_f} * \frac{t_p - t_{y=0}}{t_0 - t_f} * \frac{t_p - t_{z=0}}{t_0 - t_f}$$

Calculation of bakery products with a cooling time of 3 hours or 10800 s is provided in Table 2.

Table 2

Calculation of cooling of bakery products

No.	Using the formulas	Results	Note
1	$ Fo_x = \frac{\alpha * \tau}{\delta_x^2} $	0.64	for a plate thickness $ (2\delta_x=90 $ mm we have $ \delta_x=45 $ mm=0.045), we calculate the Froude and Biot numbers using the formulas
1.1	$ Bi_x = \frac{\alpha * \delta_f}{\lambda} $	13.3	
1.2	$ \theta = \frac{t_f - t_{x=0}}{t_f - t_0} $	0.13	dimensionless temperature value
2	$ Fo_y = \frac{\alpha * \tau}{\delta_y^2} $	0.26	for a plate thickness $ (2\delta_x=140 $ mm we have $ \delta_x=70 $ mm=0.070)
2.1	$ Bi_y = \frac{\alpha * \delta_f}{\lambda} $	20.68	
2.2	$ \theta_y = \frac{t_f - t_{y=0}}{t_f - t_0} $	0.156	a similar finding of the value of the dimensionless temperature from the graph
3	$ Fo_z = \frac{\alpha * \tau}{\delta_z^2} $	0.09	for a plate with a thickness of $ (2\delta_x=240 $ mm $ \delta_x=120 $ mm=0.120)
3.1	$ Bi_z = \frac{\alpha * \delta_f}{\lambda} $	35.45	
3.2	$ \theta_z = \frac{t_f - t_{z=0}}{t_f - t_0} $	0.18	dimensionless temperature
3.3	$ \frac{t_p - t_f}{t_0 - t_f} $	0.036	–
3.4	$ t_p = t_f - 0.036(t_0 - t_f) $	22.84 °C	the maximum temperature in the center of the crumb of bakery products before packaging, that is, the critical limit of CCP No. 2

We determine the temperature values at a distance of  $x_1=15, x_2=30$  and  $x_3=40$  mm from the middle of the BP when cooling for 3 hours or 10.800 s (Table 3).

Based on the results of the calculations, we will plot the temperature distribution over the cross-section of the bakery product (Fig. 8).

Determination of the amount of heat released during cooling of a bakery product.

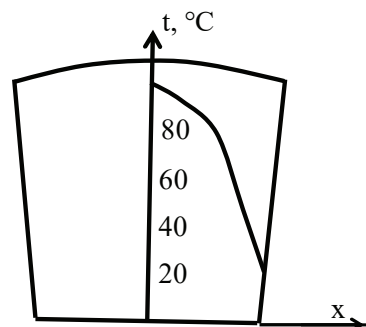


Fig. 8. Temperature distribution graph

Thus, after identifying the risk factor, determining critical limits and establishing monitoring procedures, developing corrective actions [26], i.e. actions that should be taken in cases where the results of monitoring in the CCP show that the process is out of control, a practical HACCP plan of critical control points for bakery production [27, 28] processes is developed (Table 4).

Physical parameters can be used in quantitative interim assessments of manufacturing processes.

The values of these parameters can be correlated with the results of the organoleptic evaluation of the product (semi-finished product) and it becomes possible to establish a kind of lower limit of quality in terms of organoleptic indicators.

Table 3

Calculation of changes in the temperature field of a bakery product

No.	Using the formulas	Results	Note
1	$Fo_{x_1} = \frac{\alpha * \tau}{\delta_{x_1}^2}$	5.76	$x_1=15$ mm
1. 1	$Bi_{x_1} = \frac{\alpha * \delta_{x_1}}{\lambda}$	4.43	–
	$\theta = \sum \frac{2 \sin \mu_n}{\mu_n + \sin \mu_n \cos \mu_n} \cos \left( \mu_n \frac{x_1}{\delta} \right) \exp(-\mu_n^2 * Fo)$	–	the dimensionless temperature
1. 2	N	1.23	$Bi=4.43, Fo_{x_1} = 5.76 > 1$
1. 3	$\mu_1$	1.27	
1. 4	$\mu_n^2$	1.61	
1. 5	$\theta = N \cos \left( \mu_n \frac{x_1}{\delta} \right) \exp(-\mu_n^2 * Fo)$	0.954	
1. 6	$t_{x1} = t_f + 0.954(t_o - t_f)$	95.5 °C	–
2	$Fo_{x_1} = \frac{\alpha * \tau}{\delta_{x_1}^2}$	1.44	$x_2=30$ mm
2. 1	$Bi_{x_1} = \frac{\alpha * \delta_{x_1}}{\lambda}$	8.86	–
2. 2	$t_{x2} = t_f + 0.73(t_o - t_f)$	77.74 °C	$Bi=8.86$ finding it: $N=1.26, \mu_1=1.41, \mu_n^2 = 2$
3	$t_{x3} = t_f + 0.73(t_o - t_f)$	48.7 °C	$x_3=40$ mm

Table 4

Practical plan of HACCP critical control points for the production processes of bakery products (molded bread)

Plan		Do	Check			Act	
CCP No.	Dangerous factor	Critical limit	Monitoring procedure			Correction actions	HACCP records
			Procedure (what will be measured, how, how often)	Frequency (how often)	Responsible		
CCP No. 1 – baking process	Biological hazard factor, the growth of microorganisms in case of non-compliance with the baking temperature according to the recipes. Pathogenic microflora that develops as a result of the violation of temperature and time regimes	$t$ in the center of the crumb 95.2–99.1 °C	Compliance with the technological modes of baking and temperature control. Temperature, Measuring control device: production trusted thermometer	From each batch after a certain period of time.	Baker, gas oven operator	Identification of the causes of nonconformity and their elimination. Calibration and monitoring of equipment. Control of the product, if necessary, its rejection, isolation, and disposal. Additional staff training	Records in the log of control of technological parameters in the furnace, in furnace units. Records of verification of measuring instruments. Records of the results of internal and external audits. Records of confirmation of competence and responsibility of personnel
CCP No. 2 – cooling, finished products, packaging, and labeling (temperature control in the center of the crumb of products before packaging)	Biological hazard, the growth of microorganisms in case of non-compliance with the baking temperature according to the recipes. Pathogenic microflora that develops as a result of the violation of temperature and time regimes	$t$ in the center of the crumb 22.84 °C	Compliance with technological modes of cooling and temperature control. Temperature, Measuring control device: production trusted thermometer	From each batch of bakery products after a certain period of time	Stacker of bakery products	Identification of the causes of nonconformity and their elimination. Calibration and monitoring of equipment. Control of the product, if necessary, its rejection, isolation, and disposal. Additional staff training	Records in the log of control of technological parameters in the furnace, in furnace units. Records of verification of measuring instruments. Records of the results of internal and external audits. Records of confirmation of competence and responsibility of personnel [27]

At the same time, it should be borne in mind that the correlation of the values of individual physical parameters with organoleptic data is often not so linear, since the overall organoleptic quality is due to a combination of many changing factors.

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## 6. Discussion of the results of the implementation of HACCP for the production of bakery products: studies of CCP and development of critical limits

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The obtained mathematical model makes it possible to predict the temperature in the center of geometric bakery products from the period from the oven to cooling. The use of a mathematical model makes it possible to determine in advance the parameters of baking dough in any given article. Critical limits that have been developed and scientifically validated are appropriate to ensure the adequate controls selected for CCPs (Table 4). Thus, the critical limits meet “measurable” criteria. In this case, the critical limit is physical. The results of calculating the change in the temperature field of a bakery product (Table 3) allows you to clearly see the temperature distribution in the center of the crumb (Fig. 8).

The results of theoretical calculation for the critical limit of CCP No. 1 (Table 1) and CCP No. 2 (Table 2) can be used in the practical production of the HACCP plan (Table 4):

1) critical limit of baking (95.2–99.1 °C in the center of the crumb), controls this process, controls the biological hazard factor: the growth of microorganisms when the baking temperature is not observed according to recipes, pathogenic microflora, developing due to violation of temperature and time conditions;

2) critical limit of cooling of bakery products. Cooling of finished products to 22.84 °C at the center of the crumb for safe packaging and labeling, temperature control at the center of the crumb before packing.

Prospects for further research: assessing the change in the state of protein substances with the obtained theoretical calculated data (Table 1), as well as studying the cooling system with the obtained theoretical calculated data (Table 2) to ensure the exclusion of excess drying of bakery products and the formation of condensation moisture on the surface of products inside the package.

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## 7. Conclusions

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1. A critical limit has been developed to control the baking process of a rectangular bakery product (CCT No. 1) – the temperature in the center of the crumb is 95.5–99.1 °C.

2. A critical limit for the cooling process of bakery products (CCT No. 2) has been developed – this is cooling of finished products to 23 °C in the center of the crumb for safe packaging, control of the temperature in the center of the crumb of products before packaging.

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## References

1. Press release on the draft resolution of the Government of the Republic of Kazakhstan “On approval of the National project for the development of the agro-industrial complex of the Republic of Kazakhstan for 2021-2025”. Available at: <https://www.gov.kz/memleket/entities/moa/documents/details/163980?lang=kk>
2. Hazard Analysis and Critical Control Point (HACCP) Tutorial. Available at: <https://studylib.ru/doc/159976/principy-nassr---associaciya-plodoovoshnyh-predpriyatij>
3. Basyuk, A. S., Nelina, V. V., Prohorova, V. V. (2017). *Upravlenie kachestvom na predpriyatii*. Krasnodar: KubGTU, 252.
4. Uazhanova, R., Mannino, S., Tungyshbaeva, U., Kazhymurat, A. (2018). Evaluation of the effectiveness of internal training of personnel in the HACCP system at the bakery enterprise. *Acta Technica*, 63 (1), 1–8.
5. Chernova, E. V., Bychenkova, V. V. (2018). *Obespechenie i kontrol' printsipov NASSR pri proektirovanii i funkcionirovanii predpriyatij*. Sankt-Peterburg: Izd-vo Politekhi, un-ta, 196.
6. Malekjani, N., Jafari, S. M. (2020). Modeling the release of food bioactive ingredients from carriers/nanocarriers by the empirical, semiempirical, and mechanistic models. *Comprehensive Reviews in Food Science and Food Safety*, 20 (1), 3–47. doi: <https://doi.org/10.1111/1541-4337.12660>
7. Anandappa, A. (2013). *Evaluating Food Safety Systems Development and Implementation by Quantifying HACCP Training Durability*. University of Kentucky.
8. Kahindi, B. B. (2016). *Food Safety Management Practices of Small and Medium Sized Food Industry Enterprises in Tanzania*. Western Kentucky University. Available at: <https://digitalcommons.wku.edu/cgi/viewcontent.cgi?article=2566&context=theses>
9. Vayskrobova, E. S. (2011). *Razrabotka integrirovannoy sistemy upravleniya kachestvom i bezopasnost'yu syropopchenyh kolbas*. Magnitogorsk, 23.
10. Valentas Kennet Dzh., Rotshteyn Enrike, R. Singh Pol (2004). *Pishchevaya inzheneriya: Spravochnik s primerami raschetov*. Sankt-Peterburg: Professiya, 848.
11. Uazhanova, R., Tungyshbaeva, U., Kazhymurat, A., Mannino, S. (2018). Evaluation of the Effectiveness of Implementing Control Systems in the Increasing of Food Safety. *Journal of Advanced Research in Dynamical & Control Systems*, 10, 649–656. Available at: <https://elibrary.ru/item.asp?id=38685389>
12. Galun, L. A., Trigubova, L. A., Kirilenko, N. M. (2005). *Standartizatsiya, sertifikatsiya i upravlenie kachestvom v proizvodstve*. Gomel': BTEU, 104.

13. Gudkov, S. A. (2004). Po itogam konferentsii RASKhN «Kachestvo i bezopasnost' sel'skohozyaystvennogo syr'ya i pishchevyh produktov». *Hranenie i pererabotka sel'hozsyr'ya*, 12, 62–68.
14. Ibrahimova, S., Uazhanova, R., Mardar, M., Serikbaeva, A., Tkachenko, N., Zhygunov, D. (2020). Development of recipe composition of bread with the inclusion of juniper using mathematical modeling and assessment of its quality. *Eastern-European Journal of Enterprise Technologies*, 6 (11 (108)), 6–16. doi: <https://doi.org/10.15587/1729-4061.2020.219020>
15. Isachenko, V. P., Osipova, V. A., Sukomel, A. S. (1981). *Teploperedacha*. Moscow: Energiya, 488.
16. Ishevskiy, A. L., Leont'eva, T. I., Gun'kova, P. I. (2019). *Standartizatsii i sertifikatsii pishchevogo syr'ya i produktsii*. Moscow: Mir nauki, 48.
17. Jay, J. M., Loessner, M. J., Golden, D. A. (2005). *Modern Food Microbiology*. Springer, 790. doi: <https://doi.org/10.1007/b100840>
18. Kantere, V. M., Matison, V. A., Sazonov, Yu. S. (2006). *Sistemy menedzhmenta bezopasnosti pishchevoy produktsii na osnove mezhdunarodnogo standartar ISO 22000*. Moscow: Tipografiya RASKhN, 454.
19. Wallace, C. A., Sperber, W. H., Mortimore, S. E. (Eds.) (2018). *Food Safety for the 21st Century*. John Wiley & Sons. doi: <https://doi.org/10.1002/9781119053569>
20. Marriott, N. G., Schilling, M. W., Gravani, R. B. (2018). *Principles of Food Sanitation*. Food Science Text Series. doi: <https://doi.org/10.1007/978-3-319-67166-6>
21. Mortimore, S., Wallace, C. (2013). *HACCP: A Practical Approach*. Springer, 475. doi: <https://doi.org/10.1007/978-1-4614-5028-3>
22. McElhatton, A., Marshall, R. J. (Eds.) (2007). *Food Safety. A Practical and Case Study Approach*. Springer, 312. doi: <https://doi.org/10.1007/978-0-387-33957-3>
23. Rusinovich, A. A., Chernyh, O. Yu., Motuzko, N. S. et. al. (2018). *Sistema analiza riskov i kontrol'nyh kriticheskikh toчек v veterinarnoy deyatel'nosti*. Krasnodar: Kubanskiy gosudarstvennyy agrarniy universitet imeni I.T. Trubilina, 209.
24. Swainson, M. (2019). *Swainson's Handbook of Technical and Quality Management for the Food Manufacturing Sector*. Woodhead Publishing. doi: <https://doi.org/10.1016/c2013-0-16510-5>
25. Varzakas, T., Tzia, C. (Eds.) (2016). *Handbook of Food Processing. Food Safety, Quality, and Manufacturing Processes*. CRC Press, 679. doi: <https://doi.org/10.1201/b19398>
26. Tursunbayeva, Sh. A., Iztayev, A., Magomedov, M., Yakiyayeva, M. A., Muldabekova, B. Zh. (2019). Study of the quality of low-class wheat and bread obtained by the accelerated test method. *J. Periodico Tche Quimica*, 16 (33), 809–822.
27. Uazhanova, R. U., Kazhymurat, A. T., Tungyshbayeva, U. O., Saez, A. C. (2019). Development and implementation of HACCP plan based on FMEA in collagen hydrolizate industry. *Journal of Hygienic Engineering and Design*, 28, 13–21.
28. Yakiyayeva, M., Iztayev, A., Kizatova, M., Maemerov, M., Iztayeva, A., Feydengold, V. et. al. (2016). Influence of ionic, ozone ion-ozone cavitation treatment on safety of the leguminous plants and oil-bearing crops at the storage. *Journal of Engineering and Applied Sciences*, 11 (6), 1229–1234.