

The concept of efficiency is important in economic science; at present, its role in every sector of the economy is growing. Evaluating an enterprise's efficiency makes it possible to implement a correct and profitable strategy of resource allocation, which shows its potential level. Given an annual increase in the number of bankrupt enterprises, the issue of estimating the efficiency of enterprises is relevant for both their owners and managers, as well as for creditors. There are various methods and models for estimating the performance of enterprises. This work has assessed the efficiency of enterprises in the industrial sector over the period of 2017–2018. Stochastic Frontier Analysis is based on the stochastic model of production function. The classic SFA method is based on the production function of the company, which relates the volume of output to the volume of resources consumed. At the same time, the SFA model uses several inputs (volumes of resources consumed) and only one output parameter – the volume of production.

In order to achieve more precise results, a given model has been modified. The model allows several key financial indicators to be taken into consideration as outputs at the same time, based on which the financial activities of the studied economic entities are assessed. The result of the work involving open sources has revealed how the efficiency of different enterprises in the same industry changes over several years. It is shown that the modified Stochastic Frontier Analysis model could be used to assess financial stability and predict bankruptcy.

Keywords: multifactor model, efficiency, stochastic method, bankruptcy, financial stability, panel data

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ADVANCING THE MULTIFACTOR MODEL OF STOCHASTIC FRONTIER ANALYSIS

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1. Introduction

Under a competitive and dynamic market environment, every business faces high and low levels of risk. No entrepreneur guarantees that his/her activities would continue forever, or that his/her goods or services would be in demand. It is quite often that business managers take riskier actions to achieve high profits. Other managers who do not want to take risks and put up with the current situation usually remain afloat and are forced to leave the market after they have lost the competition.

A serious result of underestimating the risk is bankruptcy, which leads to negative consequences not only for the company but also for its employees, other enterprises and institutions, society, and the state.

Bankruptcy is the state of an insolvent enterprise, when an insolvency case is brought to court, or creditors conduct extrajudicial procedures for bankruptcy of the enterprise. The insolvency of an enterprise is the state of the company when it fails to meet its obligations and overdue liabilities exceed more than half of the value of assets on an enterprise's balance sheet.

In today's global market environment, these phenomena are becoming more frequent in all countries. For example, companies of different industries (production, construction, trade, etc.), different sizes, and different organizational and legal forms (private companies, individual enterprises, etc.).

The presence or absence of signs of bankruptcy is determined by the comparison of the volume of monetary obligations and mandatory payments of an enterprise. Cash liabilities include [1] arrears for goods transferred, services rendered, and work performed; mandatory payments to the budget and state non-budget funds. They also include interest-based loans and credits; the amount of damage for damaging creditors' property; the amount of debt due to unfounded enrichment.

The number of bankruptcies of enterprises is constantly increasing. According to data from the Unified Federal Bankruptcy Registry, 41,995 applications for debtors' insolvency were received in 2014 [2]; 52,877 applications were submitted in 2020 [3].

Thus, the issue of estimating the performance efficiency of enterprises is relevant for both creditors and managers. New models and procedures are constantly being devised to predict the bankruptcy of enterprises, which indicates the relevance of the development of new approaches to assess the probability of insolvency of companies.

2. Literature review and problem statement

Models that use multidimensional discriminatory analysis (MDA) have become widespread [4, 5]. These studies have shown that bankrupt enterprises have significantly

different coefficients from well-run enterprises. In such models, the downside is that there is no weight factor for the indicators, as well as there is no final probability coefficient of bankruptcy. Multi-layered perceptron (MLP) of artificial neural networks is used to build a bankruptcy forecasting model in [6]. The downside of using MLP is the lack of mechanisms for identifying the cause, and further improving the model. The author of [7] reported the results of the comparison of genetic algorithms (GA) with the possibilities of logistical regression (LR) and support vector machine (SVM) at medium and small Italian manufacturing enterprises. LR, GA, and SVM were used first in a cumulative sample and then depending on the size and geographic region. The accuracy of the GA model prediction increases when the model is applied according to size and geographic area. Work [8] applies a coefficient method, whose essence is to conduct a financial analysis of an enterprise. The main drawbacks of the method of coefficients are the ambiguous interpretation of results, leading to a decrease in the accuracy of diagnosing bankruptcy, as well as not taking into consideration the industry affiliation. International models are not always applicable in the Russian market due to the specificity of the organization and operation of a number of sectors in the economy. To successfully use these models, weight coefficient adjustments are required. Russian models are implemented with the help of multiple discriminant analysis, which predetermines a qualitative assessment of insolvency and does not make it possible to obtain an accurate quantitative assessment of the probability of bankruptcy risk.

The deterministic approach of Parametric Corrected Least Ordinary Squares (COLS) is distinguished among the parametric methods for analyzing an enterprise's efficiency [9]. This method is based on building a regression model of the efficiency boundary and its parallel shift to the first touch point with one of the observed values. An entity whose observed values lie at the edge of efficiency limit is considered effective while a deviation from the limit is considered ineffective.

There is a parametric method of Stochastic Frontier Analysis (SFA) [10]. It is based on a stochastic model of production function that relates the volume of output to the volume of resources consumed. Several inputs (volumes of resources consumed) are used while the only one output parameter is the volume of production. It is assumed that productivity variation is associated with both performance inefficiency and «noise», so the margin of efficiency is «immersed» in the region of actual performance results. In this case, inefficiency is asymmetrical while an accidental error is subject to symmetrical distribution [11, 12].

Among the Russian studies that use the SFA method and tackle certain sectors of the economy, works [13–19] are worth considering. The analysis reported in paper [13] confirms the applicability of the SFA method for assessing the effectiveness of homeowners' associations (HOAs). The authors performed a modification, which implied that the standard deviations of the semi-normal distribution of the inefficiency factor were considered to be dependent on exogenous factors, that is, the authors of [13] suggested the heteroscedasticity of inefficiency factors. It has been found that the increase in the size of firms leads to an increase in efficiency, as well as the fact that the industry has a positive impact on scale. The most known models in the field of SFA modeling are described in work [14]. The comparison of the results of the evaluation of nine SFA models using a single array of data on the Russian concrete and cement industry. The

results showed that adequate models, such as the four-bug model, the «TRUE» FE model, and the TVD model, could be identified among the selected models, as these models take into consideration trend and heterogeneity. Work [15] examines four models of the production capacity of enterprises that produce and sell household goods. The stochastic boundary model ($\log y, \log L, \log k, \mu$) could be used in the absence of information about inefficiency factors. The stochastic boundary model ($\log y, \log L, \log k, \delta Z$) makes it possible to identify inefficiency factors. Estimates of the inefficiency from these two models are overstated. In the production capacity model, ($\log y, \log L, \log k, s$), the more manageable factors, the less irreparable inefficiency and the higher the production potential. The fourth production capacity model under consideration ($\log y, \log L, \log k, C$) takes into account the general limitation for all observations on the cost of managing inefficiency factors. In the cited study [16], static and dynamic models of the production potential of the Russian regions were built on the basis of the author's methodology. It is also shown that taking into consideration the efficiency factor in the production capacity model significantly improves the differentiation of performance assessments. A two-step analysis presented in paper [17] addresses the impact of technical efficiency (TE) of industrial organizations on the risks for their financial stability. The reported results suggest that TE has a significant negative impact on the likelihood of bankruptcy of large, medium, and most small Russian industrial enterprises.

Paper [18] reports the results of studying and applying the economic methods of DEA and SFA to measure the effectiveness of financial subsidies for farms. The study results show the positive impact of financial subsidies, as well as the impact of non-binding payments, on the examined farms. Two models of dynamic network super-efficiency DEA and SFA have been developed in [19] to treat the relationship between accounting and financial performance indicators for the banking sector. The models not only help make decisions but also map out directions for future research. Paper [20] provides a justification for the feasibility of using DEA and SFA methods to assess the environmental efficiency of urban air in Germany. The reported results highlight the negative impact of urban air pollutants on environmental efficiency and the positive impact of precipitation on efficiency. Paper [21] explores two approaches DEA and SFA to assess the cost-effectiveness and overall productivity of resources and the technical changes in the agricultural enterprises in Iraq. The study result confirmed the hypothesis that agricultural companies are not economically efficient and there is irrational use of resources. Iraqi companies incurred additional production and marketing costs because they did not choose the optimal combination that would provide the production at the lowest cost.

International authors to be considered are [22–31]. Work [22] addresses the impact of intelligent capital (IC) on operational efficiency for Indian financial sector companies. The study results show that all components of IC have a significant impact on the efficiency of a firm. Paper [23] examines the efficiency, performance, and convergence of Norwegian seaports. The authors included other seaports in other Scandinavian countries and the UK in the dataset. That has allowed them to measure the overall effectiveness of Norwegian seaports, as well as the way they operate relative to other comparable seaports. The authors of [24] considered both the economic and environmental results in the forest industry. The results showed that there were no clear

differences in efficiency between China's economic regions, except for North-East China. In addition, the state forestry structure has a significant negative impact on the efficiency of production in China's forest industry. Paper [25] assesses variations in the translog inefficiency model, allowing one-step consideration of factors that affect the inefficiency of India's thermal energy. Work [26] examines the impact of scale of Indonesian banks by assessing cost performance and provides an analysis of the growth in overall factor productivity. The authors conclude that efficiency grew before the Asian crisis, then decreased significantly. Mid-sized banks have proved more efficient than small and large ones, private banks have worked more efficiently than public banks. The authors of [27] reported a study into the impact of the firm's size on Brazil's electricity distribution sector from 1998 to 2005, which was the result of an earlier reform. Firms operate in an environment of increasing economies of scale, in other words, they could increase their productivity by merging. Paper [28] analyzes a change in the overall productivity factor (OPF) in the Chinese economy. Decomposition analysis was used to reveal the impact of cost growth, economies of scale, technical changes, and changes in technical efficiency, on changes in OPE. The cost-effectiveness of the Vietnamese banks is assessed using stochastic boundary analysis, which has a positive, but not significant, correlation with the accounting indicators [29]. The method demonstrates a high consistency in determining the most effective and least effective banks observed between 2005 and 2017. The results of work [30] show that accounting for the systematic differences between commercial, cooperative, and savings banks is very important, as this could avoid misinterpretation of the state of efficiency of the entire German banking sector. The following inputs were used in calculating the efficiency assessment: buildings and branches, depreciation of fixed assets, full employment of personnel, borrowed funds. The volumes and loans of clients, investments in stocks and bonds were considered as outputs. The authors of [31] analyzed the impact of agricultural production growth on grain production in China. The analysis showed that optimization of the distribution of factors of production (especially water and capital) could increase agricultural productivity due to technical efficiency; and there are significant differences in grain production in different provinces. For example, in Beijing and Shanghai, water resources were scarce, although the elasticity of water output was high.

Modern methods for predicting the bankruptcy of enterprises are based on the use of a statistical method, or on an empirical selection of financial coefficients. At the same time, it is expected that combining these approaches could improve the efficiency of the evaluation of enterprises. Our paper reports the use of the SFA method to assess the financial stability of an enterprise based on panel accounting data. The modification of a single-factor SFA model into a multifactor SFA model has been proposed.

3. The aim and objectives of the study

The aim of this work is to build a multifactor model that would make it possible to evaluate the efficiency of enterprise operation by a Stochastic Frontier Analysis method.

To accomplish the aim, the following tasks have been set:

- to modify a single-factor classic SFA model by incorporating multiple output parameters (that is, switching from

one-product to multi-product production function used in the SFA model);

- to check the possibility of using the financial performance indicators of enterprises as input parameters instead of data on the resources consumed and the volume of articles produced.

4. The study materials and methods

Let the output indicator of the i -th entity be characterized by p indicators y_i^k , $k=1, \dots, p$. Then the connection between the output k -th indicator of the i -th entity y_i^k and the input financial indicators x_{ij} can be represented by the following expression:

$$\ln(y_i^k) = \sum_{j=1}^m \beta_j^k \ln(x_{ij}), \tag{1}$$

where m is the number of predictors (indicators of economic and financial activity of an entity) in a regression model; β_j^k are the regression model coefficients.

A deviation of model data from real data is a model's error ϵ_i^k :

$$\epsilon_i^k = \ln(y_i^k) - \sum_{j=1}^m \beta_j^k \ln(x_{ij}), \quad k=1, \dots, p. \tag{2}$$

Represent an accidental error in the following form:

$$\exp(-\hat{u}_i - \hat{\sigma}_i) \leq T_{JMLS} = \exp(-\hat{u}_i) \leq \exp(-\hat{u}_i + \hat{\sigma}_i), \tag{3}$$

where v_i^k is the accidental error related to external causes beyond control of the economic entity activities; u_i^k is the random factor associated with an enterprise's activities, which is termed in the literature [32, 33] as operational inefficiency.

Suppose that the random factor u_i^k of the i -th entity, related to the efficiency of its operation, does not depend on the output indicator number, that is $u_i^k = u_i$. To build a model, one needs to set the probabilistic distributions of random values v_i^k and u_i . Thus, we obtain a multifactor SFA model.

Let the distribution of the probabilities of random values v_i^k and u_i take the following form (4).

$$f_v(v_i^k) = \frac{1}{\sqrt{2\pi}\sigma_v} \cdot \exp\left(-\frac{(v_i^k)^2}{2\sigma_v^2}\right), \quad f_u(u_i) = \lambda \cdot e^{-\lambda u_i}, \quad u_i \geq 0. \tag{4}$$

where, i index is the number of an entity $i=1, \dots, n$ (n is the number of examined entities); v_i^k is the accidental error with parameters $M(v_i^k) = 0$, $M(v_i^k) = \sigma_v^2$, takes into consideration the impact of external factors on the activity of an entity; u_i is the non-negative random error with parameters $M(u_i) = 1/\lambda$.

The quantity u_i takes into consideration the «ineffectiveness» of the i -th entity, and the value $\epsilon_i^k = v_i^k - u_i$ describes a deviation from the boundary of production capacity.

The combined probability density of the random value ϵ_i^k and u_i is equal to (5):

$$f(\epsilon_i^k, u_i) = \frac{\lambda}{\sqrt{2\pi}\sigma_v} \cdot \exp\left(\epsilon_i^k \lambda + \frac{\lambda^2 \sigma_v^2}{2}\right) \times \exp\left(-\frac{(u_i + (\epsilon_i^k + \lambda \sigma_v^2))^2}{2\sigma_v^2}\right). \tag{5}$$

Then the joint density of the vector value ϵ_i and u_i is equal to:

$$f(\epsilon_i, u_i) = \left(\frac{\lambda}{\sqrt{2\pi\sigma_v^2}} \right)^p \cdot \exp\left(\sum_{k=1}^p \left(\epsilon_i^k \lambda + \frac{\lambda^2 \sigma_v^2}{2} \right) \right) \times \exp\left(-\frac{\sum_{k=1}^p (u_i + (\epsilon_i^k + \lambda\sigma_v^2))^2}{2\sigma_v^2} \right). \quad (6)$$

Find the density of the random value ϵ_i :

$$f(\epsilon_i) = \int_0^{\infty} f(\epsilon_i, u_i) du_i = \frac{1}{2} \left(\frac{\lambda}{\sqrt{2\pi\sigma_v^2}} \right)^p \cdot \frac{\sqrt{2\pi\sigma_v^2}}{\sqrt{p}} \cdot e^{-\sum_{k=1}^p \left(\epsilon_i^k \lambda + \frac{\lambda^2 \sigma_v^2}{2} \right)} \times e^{-\frac{\sum_{k=1}^p (\epsilon_i^k + \lambda\sigma_v^2)^2}{2\sigma_v^2}} \cdot \frac{\left(\sum_{k=1}^p (\epsilon_i^k + \lambda\sigma_v^2) \right)^2}{2\sigma_v^2 p} \left(1 - \operatorname{erf} \left(\frac{\sum_{k=1}^p (\epsilon_i^k + \lambda\sigma_v^2)}{\sqrt{2\sigma_v^2 p}} \right) \right). \quad (7)$$

Write down the logarithmic likelihood function:

$$L(\sigma_v, \lambda, \beta) = n \cdot p \cdot \lambda + (n - np) \ln(\sigma_v) + \sum_{i=1}^n \sum_{k=1}^p \left(\lambda \left(\ln(y_i^k) - \sum_{j=1}^m \beta_j^k \ln(x_{ij}) \right) + \frac{\lambda^2 \sigma_v^2}{2} \right) - \frac{1}{2\sigma_v^2} \sum_{i=1}^n \sum_{k=1}^p \left(\lambda \left(\ln(y_i^k) - \sum_{j=1}^m \beta_j^k \ln(x_{ij}) \right) + \lambda\sigma_v^2 \right)^2 + \frac{1}{2\sigma_v^2 p} \sum_{i=1}^n \left(\sum_{k=1}^p \lambda \left(\ln(y_i^k) - \sum_{j=1}^m \beta_j^k \ln(x_{ij}) \right) + \lambda\sigma_v^2 \right)^2 + \sum_{i=1}^n \ln \left(1 - \operatorname{erf} \left[\frac{\sum_{k=1}^p \left(\ln(y_i^k) - \sum_{j=1}^m \beta_j^k \ln(x_{ij}) \right) + \lambda\sigma_v^2}{\sqrt{2\sigma_v^2 p}} \right] \right). \quad (8)$$

Maximizing this function produces the parameters σ_v , λ , β_j^k . Next, calculate the the conditional density of the following probability:

$$f(u_i / \epsilon_i) = \frac{f(u_i, \epsilon_i)}{f(\epsilon_i)}. \quad (9)$$

By substituting (2) and (3) in (4), we obtain:

$$f(u_i / \epsilon_i) = \frac{2\sqrt{p}}{\sqrt{2\pi\sigma_v^2}} \cdot e^{-\frac{\sum_{k=1}^p (\epsilon_i^k + \lambda\sigma_v^2)^2}{2\sigma_v^2}} \cdot \frac{\left(\sum_{k=1}^p (\epsilon_i^k + \lambda\sigma_v^2) \right)^2}{2\sigma_v^2 p} \cdot \frac{\sum_{k=1}^p (u_i + (\epsilon_i^k + \lambda\sigma_v^2))^2}{2\sigma_v^2}}{1 - \operatorname{erf} \left(\frac{\sum_{k=1}^p (\epsilon_i^k + \lambda\sigma_v^2)}{\sqrt{2\pi\sigma_v^2 p}} \right)}. \quad (10)$$

Here, instead of the parameters σ_v , λ , we substitute them with their estimates, found from solving the maximization problem (8), and, instead of ϵ_i^k , its estimate, calculated from formula (2), is used, in which we use β_j^k .

As regards the operational inefficiency indicator, one can obtain the following expression:

$$\hat{u}_i = M(u_i / \epsilon_i) = \frac{\sqrt{2\sigma_v^2}}{\sqrt{p}} \times \left(\frac{1}{\sqrt{\pi}} \frac{e^{-\frac{\left(\sum_{k=1}^p (\epsilon_i^k + \lambda\sigma_v^2) \right)^2}{2\sigma_v^2 p}}}{1 - \operatorname{erf} \left(\frac{\sum_{k=1}^p (\epsilon_i^k + \lambda\sigma_v^2)}{\sqrt{2\sigma_v^2 p}} \right)} - \frac{\sum_{k=1}^p (\epsilon_i^k + \lambda\sigma_v^2)}{\sqrt{2\sigma_v^2 p}} \right). \quad (11)$$

The technical efficiency indicator JMLS is [34, 35]:

$$T_{JMLS} = \exp(-\hat{u}_i). \quad (12)$$

The technical efficiency indicator is in the interval:

$$\exp(-\hat{u}_i - \hat{\sigma}_i) \leq T_{JMLS} = \exp(-\hat{u}_i) \leq \exp(-\hat{u}_i + \hat{\sigma}_i), \quad (13)$$

where $\hat{\sigma}_i = 1/\hat{\lambda}$.

There is no doubt that the practice is interested in the probability that the inefficiency u_i does not exceed the specified value (planned value) up_i :

$$P(u_i \leq up_i) = \int_0^{up_i} f(u_i / \epsilon_i) du_i.$$

The following value then:

$$R_i = 1 - P_i = 1 - \int_0^{-\ln(tp_i)} f(u_i / \epsilon_i) du_i, \quad i = 1, \dots, n, \quad (14)$$

can be interpreted as a risk of not achieving the planned efficiency value $tp_i = \exp(-up_i)$ by the i -th entity. Here, $f(u_i/\epsilon_i)$ is determined from formula (10).

Consider the BC performance assessment [34] $M(e^{-u_i} / \epsilon_i)$.

$$T_{BC} = M(e^{-u_i} / \epsilon_i) = \frac{e^{-\frac{2\sum_{k=1}^p (\epsilon_i^k + \lambda\sigma_v^2) + \sigma_v^2}{2p}}}{1 - \operatorname{erf} \left[\frac{\sum_{k=1}^p (\epsilon_i^k + \lambda\sigma_v^2)}{\sqrt{2\sigma_v^2 p}} \right]} \left(1 - \operatorname{erf} \left[\frac{\sum_{k=1}^p (\epsilon_i^k + \lambda\sigma_v^2) + \sigma_v^2}{\sqrt{2\sigma_v^2 p}} \right] \right). \quad (15)$$

Calculate the second point $M((e^{-u_i})^2 / \epsilon_i)$ of BC efficiency:

$$M((e^{-u_i})^2 / \epsilon_i) = \frac{e^{-\frac{2\sum_{k=1}^p (\epsilon_i^k + \lambda\sigma_v^2) + 2\sigma_v^2}{2p}}}{1 - \operatorname{erf} \left[\frac{\sum_{k=1}^p (\epsilon_i^k + \lambda\sigma_v^2)}{\sqrt{2\sigma_v^2 p}} \right]} \left(1 - \operatorname{erf} \left[\frac{\sum_{k=1}^p (\epsilon_i^k + \lambda\sigma_v^2) + 2\sigma_v^2}{\sqrt{2\sigma_v^2 p}} \right] \right). \quad (16)$$

The variation in BC efficiency is equal to:

$$D_{BC} = \sqrt{M\left(\left(e^{-u_i}\right)^2 / \varepsilon_i\right) - \left(M\left(\left(e^{-u_i}\right)^2 / \varepsilon_i\right)\right)^2}. \quad (17)$$

As a risk of not achieving the target efficiency value, we shall use the value R_i :

$$R_i = \int_0^{T_{p_i}} f_T(T_i) dT_i, \quad (18)$$

where

$$f_T(T_i) = f\left(\left(-\ln(T) / \varepsilon_i\right)\right) \frac{1}{T_i}. \quad (19)$$

Here, $f\left(\left(-\ln(T) / \varepsilon_i\right)\right)$ is the conditional probability density set by formula (10).

5. The results of studying a multifactor SFA model

5. 1. Choosing financial indicators

To analyze financial stability, 70 Russian industrial enterprises were analyzed over the period of 2017–2018 (35 enterprises that went bankrupt and 35 operating enterprises) within the same industry. The output and input variables used represent a system of indicators, which characterizes the financial and economic activities of enterprises in the following groups: liquidity assessment, financial stability assessment, business performance assessment, and profitability assessment. In total, there are about 41 coefficients, 6–12 in each group. Duplicate coefficients were excluded. For example, the inventory turnover factor in days was excluded, but the turnover factor in turnover was included. For further work, it was necessary to select the indicators that are the most significant. After pre-processing the original data, two metrics were selected as output variables:

- 1) a debt ratio shows the share of assets formed as a result of debt financing;
- 2) a working capital agility factor reflects the share of own working capital in an equity.

Explaining variables is a system of 4 indicators:

- 1) profitability ratio;
- 2) financial stability ratio;
- 3) business activity ratio;
- 4) liquidity ratio.

For our study, we use an information-empirical base, which employs data from the documents of mandatory financial reporting: «Accounting balance» and «Financial Results Report». The data were acquired from open sources such as SPARK-Interfax and SCRIN.

5. 2. Results of applying the modified method

Fig. 1 shows the dynamics of the financial and economic activities of bankrupt enterprises (n) and operating enterprises (n), respectively, over the period from 2017 to 2018. Acting enterprises are highlighted in blue and red, and the bankrupt ones are green and purple. The worst enterprises, in terms of efficiency, are those enterprises whose efficiency values are in the range from 0 to 0.2 (in Fig. 1, highlighted in green and purple). The indicators of the bankrupt enterprises descended since 2017 (this is evident from Fig. 1 and Table 1), confirming the status of «bankrupt» enterprises. The closer to zero, the less efficient the enterprise is.

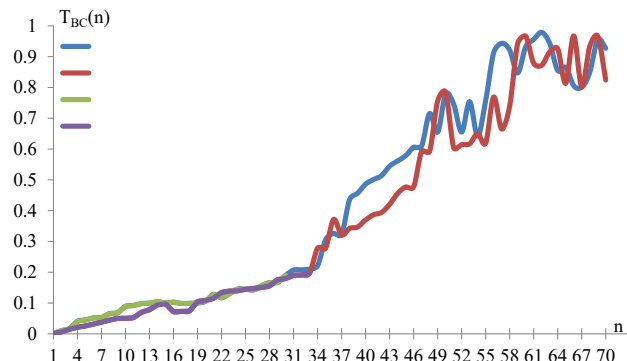


Fig. 1. Enterprise Performance Assessment Chart/compiled by authors

Table 1
Distribution of performance estimates by the SFA method

Efficiency value	2017		2018	
	Number of enterprises	%	Number of enterprises	%
0–0.2	30	42.86	33	47.14
0.2–0.4	7	10	9	12.86
0.4–0.6	8	11.43	6	8.57
0.6–0.8	10	14.28	10	14.29
0.8–1	15	21.43	12	17.14

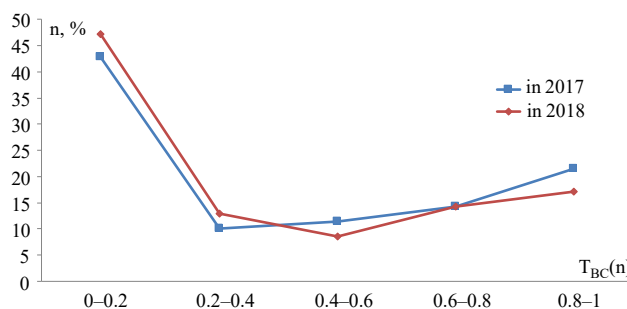


Fig. 2. Efficiency estimates distribution chart

Fig. 2 show the distribution of business performance estimates over 2017–2018. Table 1 and Fig. 2 demonstrate that the share of enterprises with a low efficiency coefficient in 2018 increased while those with a large efficiency coefficient decreased.

6. Discussion of results of studying the SFA multifactor model

The results given in Table 1 (lines 2 and 3) and in Fig. 2 relate to low-efficiency industrial enterprises. 70 enterprises were analyzed. Of these, 35 enterprises were declared bankrupt according to the Court of Arbitration. The results that range from 0 to 0.2 in Fig. 1 demonstrate the correctness and applicability of the proposed modified SFA model to assess the performance efficiency of enterprises. No such calculations have yet been found in the available literature.

The advantage of the proposed SFA model modification is that the initial information for calculating financial indicators is publicly available. Note there are no data in open access on the resources utilized and the articles produced that are used in the classic SFA model. The second advantage is that one can use multiple outputs in the proposed model.

That improves the quality of the results while the classic SFA model employs only one output indicator [12]. Our results generally show the feasibility of the proposed modified SFA method for predicting the bankruptcy of enterprises.

The disadvantage of the proposed model is the fact that financial indicators that accept only positive values should be used to operate the model correctly. This follows from formulas (1), (2), and (8) that include logarithms. That leads to a limitation in the choice of predictors for regression model (1). However, this flaw could be addressed by pre-affinity for the transformation of financial indicators (in this case, the shift conversion).

Further development of the parametric method based on the SFA model may involve the construction of a dynamic model to handle the output indicators of an enterprise under the conditions of restrictions on the inefficiency of its activities.

entities described by financial indicators, rather than the amount of resources used and industrial output.

2. The results of our study also suggest that the SFA multifactor model may be used to diagnose the financial condition of enterprises. Thus, a system of 4 financial indicators is used as input indicators for the modified model. At output, we obtain an efficiency estimate, and if the value of the indicator is in the range from 0 to 0.2, the enterprise is defined as «bankrupt». For successful enterprises, the efficiency indicator is in the range from 0.2 to 1. Among this group of enterprises, 15 enterprises demonstrated an efficiency indicator exceeding 0.8.

It should be noted that this method could be tested in other sectors of the economy, such as construction, trade, agriculture, catering, and many others.

7. Conclusions

1. The analysis of our results has shown that the SFA method could be used to assess the effectiveness of economic

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