

*This paper has substantiated the mechanical-mathematical modeling of the process of fractionation of grain material into fractions. It has been established that this could optimize the process parameters and would make it possible to design new or improve existing working surfaces of centrifugal separators.*

*A mechanical-mathematical model of the pneumatic vibratory centrifugal separation of grain material by density has been improved. This research is based on the method of hydrodynamics of multiphase media. The improved mechanical-mathematical model takes into consideration the interaction between the discrete and continuous phases of grain material by introducing conditions of interaction at the interface of these phases. In the hydrodynamic modeling of the movement of the circular layer of seeds, the coefficient of dynamic viscosity of discrete and continuous phases was taken into consideration.*

*It was established that the pneumatic vibratory centrifugal separation process parameters are critically affected by the circular frequency of rotation of the cylindrical working surface, the frequency and amplitude of its oscillations. As well as such process characteristics as the airflow rate, dynamic viscosity coefficient, the average thickness of a grain material layer, and the mean density of its particles. Rational values for the technical parameters of the grain material pneumatic vibratory centrifugal fractionation process in terms of density have been determined by using the improved mechanical-mathematical model. The amplitude and oscillation frequency of the working surface are in the ranges  $A=(35...50) \cdot 10^{-5}$  m,  $\omega=15.0...15.6$  rad/s. The circular rotation frequency of the working surface,  $\omega=24...25$  rad/s. The airflow rate,  $V=2$  m/s.*

*It was established that using the improved mechanical-mathematical model of fractionation makes it possible to improve the performance of a pneumatic vibratory centrifugal separator by 9%. At the same time, the effectiveness of grain material separation could reach 100%.*

**Keywords:** *mechanical-mathematical model of separation, grain material, seed material, pneumatic vibratory centrifugal separator*

UDC 631.354.024/.028

DOI: 10.15587/1729-4061.2021.236938

# IMPROVING THE MECHANICAL-MATHEMATICAL MODEL OF PNEUMATIC VIBRATION CENTRIFUGAL FRACTIONATION OF GRAIN MATERIALS BASED ON THEIR DENSITY

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Received date 13.05.2021

Accepted date 08.07.2021

Published date 12.07.2021

**How to Cite:** Bredykhin, V., Pak, A., Gurskyi, P., Denisenko, S., Bredykhina, K. (2021). Improving the mechanical-mathematical model of pneumatic vibration centrifugal fractionation of grain materials based on their density. *Eastern-European Journal of Enterprise Technologies*, 4 (1 (112)), 54–60. doi: <https://doi.org/10.15587/1729-4061.2021.236938>

## 1. Introduction

Given the conditions of acute competition among agricultural enterprises, the most important factor related to production is to reduce its cost and improve the quality of operations [1]. This is directly associated with the design and implementation of fundamentally new working bodies and machines, as well as the improvement of existing ones. This factor is of particular importance for the intensification of widespread technological processes such as the separation of grain mixtures [2]. When harvesting, storing, and processing grains, dozens and hundreds of tons of the grain ma-

terial (GM) are separated. Given such a scale, even a slight improvement in the technological process of GM separation into fractions could yield a significant effect in terms of energy- and resource efficiency of this process [3].

The need to improve the efficiency and performance of GM separation machines is also due to the stricter requirements for the quality of grain delivered to grain processing enterprises. The relevance of improving both separation processes and the equipment involved is obvious. Especially so if we take into consideration that the cost of after-harvest processing and storage of grain makes up from 40 to 60 % of the total cost of its production [4].

## 2. Literature review and problem statement

All separators of grain mixtures have one common feature: for fractionation of heterogeneous systems, they use the gravitational field. Making use of the gravitational field predetermines limited specific performance. This is one of the basic important disadvantages of such separators [5]. As a result, in order to find effective techniques for GM separation, interest in centrifugal separators has increased. In such separators, the separation of particles occurs under the influence of inertial forces that far outweigh the gravity that act on these particles. Constant contact with the separating surface increases the likelihood of particles entering the holes, and the high speed of movement of the separated material ensures the high performance of centrifugal separators [6].

The field of inertial forces is used to intensify the process of GM separation not only for size but also the shape, the properties of the particle surface, their density, which is proven by theoretical and experimental studies [7]. Increasing the efficiency of fractionation of heterogeneous systems is achieved as a result of the simultaneous application of centrifugal, Coriolis and gravitational forces [8].

However, centrifugal separators are not widespread at processing plants, even though they have been known for a long time and have been used to solve various tasks [9]. This is explained by the fact that the process of GM separation in centrifugal separators has not been fully investigated. At the same time, there are no scientifically substantiated procedures for determining the structural parameters of these machines, which entails low efficiency of design solutions [10].

Centrifugal separators are typically designed by applying the values of process parameters obtained experimentally. That entails significant errors, as a result of which the increase in energy and resource efficiency is negligibly low [11]. Accordingly, it is a relevant task to employ mechanical-mathematical modeling of the GM separation process. At the same time, there is a need to use different models that would take into consideration the simultaneous effect of centrifugal, Coriolis and gravitational forces.

One of the techniques that has proven effective during the mechanical-mathematical modeling of GM separation process [12] is to apply the concept of the hydrodynamics of multi-phase media. Under a given method, a fluidization medium is characterized by dynamic viscosity, and GM is represented by a two-phase liquid. One phase is discrete, formed by solid particles (seeds). Another phase is the continuous phase formed by the gaseous medium (air). However, the use of this procedure to describe the process of GM fractionation at a pneumatic vibratory centrifugal separator requires appropriate additions and clarifications [13]. That would make it possible to take into consideration as much as possible the interrelation between the physical and mechanical properties of GM and the structural and technological parameters of seed separation process in terms of density.

Thus, the mechanical-mathematical modeling of the process of GM fractionation into fractions is a promising task that could optimize the parameters of the specified process and design new or improve existing working surfaces of centrifugal separators. In this case, one should take into consideration the influence of structural and kinematic parameters of pneumatic vibratory centrifugal separators, the physical and mechanical properties of GM particles, the intralayer processes and forces acting on a layer of GM particles.

## 3. The aim and objectives of the study

The aim of this study is to improve a mechanical-mathematical model of the pneumatic vibratory centrifugal separation of seed materials in terms of seed density, which could improve the efficiency of the process of grain separation into fractions in order to obtain high-quality seed material.

To accomplish the aim, the following tasks have been set:

- to establish the defining parameters for the GM separation process in a fluidized bed in terms of seed density on the working surfaces of the cylindrical shape, and take them into consideration in the mechanical-mathematical model of the process;

- to derive functional dependences between the defining parameters for the process of separating grain materials in a pneumatic vibratory centrifugal layer in the field of action of centrifugal force and the values of the parameters for setting pneumatic vibration centrifuges in order to manage a given process and prove the adequacy of the improved mechanical-mathematical model.

## 4. The study materials and methods

To separate GM into fractions based on the totality of physical and mechanical properties, pneumatic vibration centrifuges are applied, whose working surface is shaped in the form of a body of rotation. The working surfaces of pneumatic vibration centrifuges are cylindrical or conical. The working surfaces taper up or down.

The experimental study of GM separation in a fluidised bed of seeds involved a pneumatic vibration centrifuge whose general view is shown in Fig. 1.



Fig. 1. General view of a pneumatic vibratory centrifugal separator

The separation process proceeds as follows.

The working body of the pneumatic vibratory centrifugal separator is a cylindrical working surface orbiting the axis of rotation. GM through the feeder falls on the spreader, from which, under the action of centrifugal force, is fed to the inner side of the breathable working surface. In this case, a circular layer of seeds is formed, which moves from top to

bottom under the action of gravity and force of inertia from vertical oscillations. A radial airflow is blown through the ring layer of seeds.

The circular layer of seeds, which consists of grain particles of the same size, but different in density, under the action of external forces, is delaminated into several adjacent ones. These adjacent concentric layers consist of grain particles of varying density.

GM particles with a higher natural density are discharged in the unloading tray for the «heavy» fraction through the perforated divider. Grain particles with less density move along the rotor to trays to unload the «light» fraction.

The process of GM separation based on density consists of two phases that occur simultaneously: the stratification and movement of GM.

In the first phase, particles are self-sorted into «heavy» and «light» fractions. Grain particles with higher density (with the same geometric dimensions) are immersed in a layer of GM, and particles with a lower density float to the surface of the layer.

The second phase involves moving the entire layer along the working surface.

The defining factors affecting the parameters of the separation process include the seed density, airflow rate, GM supply, the frequency and amplitude of oscillations of the working surface. It is also necessary to take into consideration the coefficients of anisotropic friction of GM and the working surface of a pneumatic vibration centrifuge.

**5. Results of the mechanical-mathematical modeling of grain material separation in a pneumatic vibratory centrifugal layer**

**5.1. Establishing the defining parameters for the process of the pneumatic vibratory centrifugal separation of grain materials based on their density**

A multiphase media hydrodynamics method is effective to model the GM pneumatic vibratory centrifugal separation process based on seed density. Underlying this approach is the assumption that a layer of GM particles, which differ in density, is simulated by a multiphase structure consisting of the finite number of layers of a discrete component (a GM particle) and a continuous component (air). Such discrete and continuous components are treated as solid interacting environments. This interaction is represented by the appropriate conditions of interaction at the interphase boundary.

When building a model, the cylindrical working surface of radius  $R$  is considered, performing a uniform rotational movement around the vertical axis ( $z$ -axis) at angular velocity  $\omega_1$ . In this case, the surface executes a fluctuation harmonic motion along the same axis at circular frequency  $\omega_2$  and amplitude  $A$ . Two cylindrical coordinate systems with the  $z$ -axis, which coincides with the axis of symmetry of the cylindrical surface, are introduced to describe the movement of the mixture of particles. It is assumed that one of the coordinate systems is rigidly connected to a movable cylindrical surface, and the other is absolute. It is believed that as a result of the effect of an airflow, the field of centrifugal forces and a gravitational field,  $N$ -ring layers of particles were formed, which differ in their air-gravitation and hydrodynamic properties. The movement of each layer of particles is considered as the movement of a continuous environment. We shall introduce the given particle density of the  $n$ -th layer  $\rho_n$  and the average density of particles  $\bar{\rho}_n$ , which form the  $n$ -th layer.

That produces the following:

$$\rho_n = \delta_n \bar{\rho}_n, \tag{1}$$

where  $\delta_n$  is the volumetric share of particles in the  $n$ -th layer (the  $n$ -th discrete phase).

If we assume that  $\rho$  is the reduced density of the continuous phase,  $\bar{\rho}$  is the average density of the gaseous phase that forms the general phase, then we can obtain:

$$\rho = \bar{\rho} \left( 1 - \sum_{n=1}^N \frac{\rho_n}{\bar{\rho}_n} \right) = \bar{\rho} \left( 1 - \sum_{n=1}^N \delta_n \right). \tag{2}$$

One should designate the average thickness of the  $n$ -th circular layer of particles through  $h_n$ , where  $n=1, 2, \dots$ . The index value  $n=1$  corresponds to the circular layer on the cylindrical surface, and  $n=N$  corresponds to the circular layer whose one boundary borders the air.

We study the dynamics of such an  $N$ -phase structure relative to the cylindrical coordinate system, which is rigidly connected to the movable cylindrical surface.

Let  $\vec{\omega}_1$  be a vector of the angular velocity of the cylindrical surface, which is directed along the  $z$ -axis of the cylindrical coordinate system  $Or \times O\varphi \times Oz$  with unit vectors  $\vec{e}_r, \vec{e}_\varphi, \vec{e}_z$ . The absolute acceleration  $\vec{a}_n$  and the speed  $\vec{V}_n$  of the element of the  $n$ -th circular layer (the  $n$ -th discrete phase) can be expressed through the relative acceleration  $\vec{b}_n$  and speed  $\vec{u}_n$  with the help of:

$$\vec{a}_n = \vec{b}_0 + 2\vec{\omega}_1 \times \vec{u}_n + \vec{\omega}_1 \times (\vec{\omega}_1 \times \vec{r}) + \vec{b}_n, \tag{3}$$

$$\vec{V}_n = \vec{u}_0 + \vec{\omega}_1 \times \vec{r} + \vec{u}_n, \tag{4}$$

where  $\vec{b}_0 = -A\omega_2^2 \sin \omega_2 t \vec{e}_z$ ,  $\vec{u}_0 = A\omega_2 \cos \omega_2 t \vec{e}_z$  is, respectively, the acceleration and speed of the longitudinal (along the  $z$ -axis) oscillations of the cylindrical surface;  $\vec{r} = r\vec{e}_r$  is the vector corresponding to the radius of the element under consideration, that is, the  $n$ -th cylindrical layer of particles.

Taking into account (3), (4), the equation that describes the relative motion of the  $n$ -th circular layer of particles can be given in the following form:

$$\rho_n \left( \frac{\partial \vec{u}_n}{\partial t} + (\vec{u}_n, \nabla) \vec{u}_n \right) = -\nabla P_n + \mu_n \Delta (\vec{u}_n + \vec{\omega}_1 \times \vec{r}) + \rho_n \vec{F}_n + \vec{G}_n, \tag{5}$$

$$div \vec{u}_n = 0. \tag{6}$$

where  $n=1, 2, \dots$ ,  $\mu_n$  is the effective coefficient of the dynamic viscosity of the  $n$ -th layer of particles.

For the interaction between a continuous phase and the particles of the  $n$ -th layer:

$$\begin{aligned} \vec{F}_n = & \frac{\bar{\rho}}{2\bar{\rho}_n} \left[ \frac{\partial}{\partial t} (\vec{V} - \vec{u}_n) + (\vec{V} - \vec{u}_n, \nabla) (\vec{V} - \vec{u}_n) + \right. \\ & \left. + 2\vec{\omega}_1 \times (\vec{V} - \vec{u}_n) + \vec{\omega}_1 \times (\vec{\omega}_1 \times \vec{r}) \right] + \\ & + F_n (\vec{V} - \vec{u}_n) \frac{9\bar{\rho}\sqrt{v}}{2\sqrt{\pi a_n \bar{\rho}_n}} \times \\ & \times \int_0^r \left[ \frac{\partial (\vec{V} - \vec{u}_n)}{\partial t} + (\vec{V} - \vec{u}_n, \nabla) (\vec{V} - \vec{u}_n) + \right. \\ & \left. + 2\vec{\omega}_1 \times (\vec{V} - \vec{u}_n) + \vec{\omega}_1 \times (\vec{\omega}_1 \times \vec{r}) \right] (t-r)^{-1/2} dr, \tag{7} \end{aligned}$$

$$\vec{G}_n = -\rho_n g \vec{e}_z - \vec{b}_0 \rho_n - 2\rho_n (\vec{\omega}_1 \times \vec{u}_n) - \vec{\omega}_1 \times (\vec{\omega}_1 \times \vec{r}) \rho_n, \quad (8)$$

where  $\vec{V}$  is the relative velocity of continuous phase;  $\nu$  is the effective coefficient of the kinematic viscosity of continuous phase;  $a_n$  is the equivalent average radius of particles in terms of the volume of the  $n$ -th layer;  $F_n$  is the coefficient characterizing the interaction between a continuous phase and particles of the  $n$ -th layer;  $G_n$  is the reduced gravity force of the layer.

A coefficient characterizing the interaction of the continuous phase with the particles of the  $n$ -th layer can be given as:

$$F_n = \frac{\bar{\rho}}{2\bar{\rho}_n(1-\delta_n)^2 a_n} \left( 1.75V_{0n} + \frac{75\nu\delta_n}{a_n} \right), \quad (9)$$

where  $V_{0n}$  is the average continuous phase speed at the boundary of the  $n$ -th layer  $\left( r = R - \sum_{p=1}^n h_p \right)$ .

In addition to (5), (6), which describe the relative movement of the  $n$ -th discrete phase (the  $n$ -th circular layer of particles), the equation of relative movement of the continuous phase should be considered. When limited to linear approximation, these equations for the  $n$ -th circular layer can be represented in the following form:

$$\rho \frac{\partial \vec{V}}{\partial t} = -(1-\delta)\nabla P + \mu\Delta(\vec{V} + \vec{\omega}_1 \times \vec{r}) - \rho_n \vec{F}_n + \vec{G}, \quad (10)$$

$$\text{div} \vec{u}_n = 0, \quad (11)$$

where  $\mu$  is the effective coefficient of dynamic viscosity of the continuous phase;  $P$  is the excess pressure;  $\delta = \sum_{n=1}^N \delta_n$ .

For  $G$ :

$$G = -\rho g \vec{e}_z - \vec{b}_0 \rho - 2\rho (\vec{\omega}_1 \times \vec{V}) - \vec{\omega}_1 \times (\vec{\omega}_1 \times \vec{r}) \rho. \quad (12)$$

The main difference between these equations is the introduction of additional components for the mass force in (5). Indeed, the transition to relative motion leads to the addition of real mass force (gravity) by force of inertia. Here,  $\vec{b}_0 \rho_n$  is the force of inertia of the gradual movement of the cylindrical surface;  $2(\vec{\omega}_1 \times \vec{u}_n) \rho_n$  is the Coriolis force;  $\vec{\omega}_1 \times (\vec{\omega}_1 \times \vec{r}) \rho_n$  is the centrifugal force.

In equations (5) and (10), the component is  $\Delta(\vec{\omega}_1 \times \vec{r}) = 0$ , because the function  $\vec{\omega}_1 \times \vec{r}$  is harmonic. In addition to equations (5), (6), and (10), (11), the speed  $\vec{u}_n$ ,  $\vec{V}$  fields and pressure  $P_n$  and  $P$  must satisfy the initial and boundary conditions. Namely, the conditions of connection at the interface boundaries of discrete phases and the conditions on the surface of the breathable cylindrical working surface. Without limiting the generality, we assume that for the time  $t \leq 0$ , the field of speeds  $\vec{u}_n$  and  $\vec{V}$ , and the excess pressure  $P_n$  and  $P$  vanish.

$$P|_{t \leq 0} = P_n|_{t \leq 0} = 0, \quad \vec{u}_n|_{t \leq 0} = \vec{V}|_{t \leq 0} = 0. \quad (13)$$

The change in  $\vec{u}_n$ ,  $\vec{V}$ ,  $P_n$ ,  $P$  for time points  $t > 0$  is modeled by equations (5) and (10).

The conditions of connection at the boundary that separates the circular layers of GM (a discrete phase) can be

found as follows. Introduce the stress tensor  $\sigma^n = (\sigma_{ij}^n)_{i,j=1}^3$  for the  $n$ -th circular layer of GM. Components of the stress tensor can be represented in the form:

$$\sigma_{ij}^n = -P_n \delta_{ij} + 2\mu_n e_{ij}^n, \quad n = 1, 2, \dots, N, \quad i, j = 1, \dots, 3. \quad (14)$$

where  $e^n = (e_{ij}^n)_{i,j=1}^3$  is the speed deformation tensor,  $\delta_{ij}$  is the symbol by Kronecker.

In the cylindrical coordinate system, we have:

$$\begin{aligned} e_{11}^n &= \frac{\partial u_z^n}{\partial z}, \quad e_{22}^n = \frac{\partial u_r^n}{\partial r}, \quad e_{33}^n = \frac{1}{r} \frac{\partial u_\phi^n}{\partial \phi} + \frac{u_r^n}{r}, \\ e_{23}^n &= e_{32}^n = \frac{r}{2} \frac{\partial}{\partial r} \left( \frac{u_\phi^n}{r} \right) + \frac{1}{2r} \frac{\partial u_r^n}{\partial \phi}, \\ e_{31}^n &= e_{13}^n = \frac{1}{2r} \frac{\partial u_z^n}{\partial \phi} + \frac{1}{2} \frac{\partial u_\phi^n}{\partial z}, \\ e_{12}^n &= e_{21}^n = \frac{1}{2} \frac{\partial u_r^n}{\partial z} + \frac{1}{2} \frac{\partial u_z^n}{\partial r}. \end{aligned} \quad (15)$$

Substituting (15) in (14) makes it possible to obtain an expression for the components of the stress tensor in the cylindrical coordinate system:

$$\begin{aligned} \sigma_{11}^n &= -P_n + 2\mu_n \frac{\partial u_z^n}{\partial z}, \quad \sigma_{22}^n = -P_n + 2\mu_n \frac{\partial u_r^n}{\partial r}, \\ \sigma_{33}^n &= -P_n + \frac{2\mu_n}{r} \left( \frac{\partial u_\phi^n}{\partial \phi} + \frac{u_r^n}{r} \right), \\ \sigma_{23}^n &= \sigma_{32}^n = \mu_n \left( \frac{1}{r} \frac{\partial u_r^n}{\partial \phi} + \frac{\partial u_\phi^n}{\partial r} - \frac{u_\phi^n}{r} \right), \\ \sigma_{31}^n &= \sigma_{13}^n = \mu_n \left( \frac{1}{r} \frac{\partial u_z^n}{\partial \phi} + \frac{\partial u_\phi^n}{\partial z} \right), \\ \sigma_{12}^n &= \sigma_{21}^n = \mu_n \left( \frac{\partial u_r^n}{\partial z} + \frac{\partial u_z^n}{\partial r} \right). \end{aligned} \quad (16)$$

In (15), (16),  $u_z^n$ ,  $u_r^n$ ,  $u_\phi^n$  are the components of the relative velocity  $\vec{u}_n$ .

The conditions of connection at the boundaries of the circular layers are composed of the continuity of speeds and stresses. Accordingly, at the boundaries of the connection between the  $n$ -th and  $(n+1)$ -th layers, the following speeds must be continuous:

$$\vec{u}_n|_{r=\bar{h}_n} = \vec{u}_{n+1}|_{r=\bar{h}_n}, \quad n = 1, 2, \dots, N-1, \quad (17)$$

where  $\bar{h}_n = R - \sum_{p=1}^n h_p$ .

At these boundaries, the tangential and normal components of the stress tensors of discrete phases should be continuous:

$$\mu_n e_{ij}^n \tau_i n_j|_{r=\bar{h}_n} = \mu_{n+1} e_{ij}^{n+1} \tau_i n_j|_{r=\bar{h}_n}, \quad (18)$$

$$(P_n - 2\mu_n e_{ij}^n n_i n_j)|_{r=\bar{h}_n} = (P_{n+1} - 2\mu_{n+1} e_{ij}^{n+1} n_i n_j)|_{r=\bar{h}_n}. \quad (19)$$

Here, repeated indices mean addition;  $n_i$  and  $\tau_i$  denote the components of the unit vectors, normal and tangent to the boundary of the connection of layers. In addition to the conditions of connection (18), (19), the boundary conditions should be set on the cylindrical surface and at the boundary between a circular layer and air (free boundary). At the free boundary, in case you neglect the influence of an airflow on the GM dynamics, the stress should vanish:

$$\vec{n} \cdot \sigma^N \Big|_{r=R-\sum_{p=1}^n h_p} = 0, \tag{20}$$

where  $\vec{n}$  is the unit vector of the normal to the free boundary;  $\sigma^N$  is the stress tensor of the  $N$ -th circular layer. Boundary conditions on the cylindrical surface can be found as follows. We assume that the velocity of particles located on the cylindrical surface is not zero, and, accordingly, slippage is possible. This means that there is an equality of the tangent stresses of the circular layer interacting with the cylindrical surface, which relates to the unit area. Thus, we assume that the following condition is met on the cylindrical surface ( $r=R$ ):

$$\sigma_{12}^1 \Big|_{r=R} = fN, \quad \sigma_{32}^1 \Big|_{r=R} = fN, \tag{21}$$

where  $N$  is the normal pressure;  $f$  is the sliding friction coefficient.

The normal pressure per unit area is equal to:

$$N = \left| \vec{n} \sigma^1 \vec{n} \right| \Big|_{r=R} = \left| \sigma_{22}^1 \right| \Big|_{r=R}. \tag{22}$$

Considering (22) would make it possible to obtain:

$$\sigma_{12}^1 \Big|_{r=R} = f \left| \sigma_{22}^1 \right| \Big|_{r=R}, \quad \sigma_{32}^1 \Big|_{r=R} = f \left| \sigma_{22}^1 \right| \Big|_{r=R}. \tag{23}$$

Based on the above, the problem of modeling the layered motion of GM on a cylindrical rotating air permeable surface includes the construction of a solution to the system of integrated-differential non-stationary equations (6), (10). They satisfy the initial equation (13) and the boundary conditions (18), (19), (21), (23).

### 5. 2. Proving the adequacy of the improved mechanical-mathematical model of the pneumatic vibratory centrifugal separation of grain

The result of our modeling using the mechanical-mathematical model built is the trajectories of seed layers consisting of particles of different natural density. An example of the derived trajectories is shown in Fig. 2.

To prove experimentally the effectiveness of using the improved mechanical-mathematical model of GM fractionation, we selected the pneumatic vibratory centrifugal separator A1-BCS-100 (manufactured at PAT «Vibroseparator», Zhytomyr, Ukraine), shown in Fig. 3. The selected pneumatic vibratory centrifugal separator was employed to separate the starting GM into fractions, provided that the manufacturer's settings are used and provided that the parameters obtained using the improved mechanical-mathematical model are applied. A GM model was used as the source material, which was a mixture of layered bodies with different density. In this case, the radius of the particles was 2.5 mm, and their density changed discretely, kg/m<sup>3</sup>: 800; 900; 1,000; 1,100; 1,200; 1,300; 1,400.

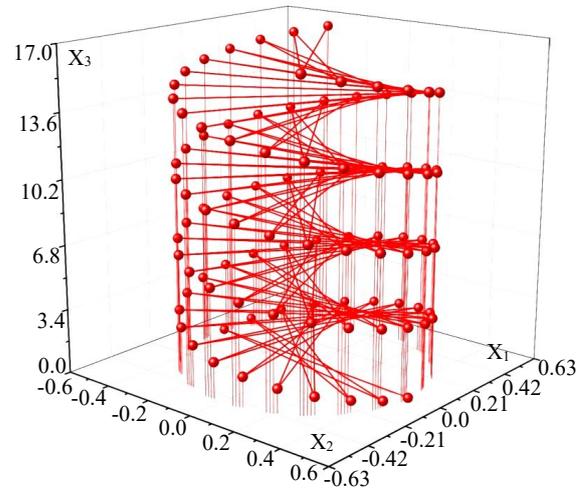


Fig. 2. The trajectory of a grain material particle in the inertial coordinate system



Fig. 3. General view of the modified self-sliding pneumatic vibratory centrifugal separator A1-BCS-100

The result of separating the starting GM at separator parameters specified by the manufacturer is two fractions. The first is a «heavy» fraction, which included layered bodies with a density of  $\rho=1,000...1,400$  kg/m<sup>3</sup>. The second is a «light» fraction with a density of layered bodies  $\rho=800...1,100$  kg/m<sup>3</sup>. When using the parameters determined from the improved mechanical-mathematical model, two fractions were obtained, with a density, respectively, «heavy» –  $\rho=1,100...1,400$  kg/m<sup>3</sup>, «light» –  $\rho=800...1,000$  kg/m<sup>3</sup>.

### 6. Discussion of results of the mechanical-mathematical modeling of the pneumatic vibratory centrifugal separation of grain materials based on their density

The result of our study is the improved mechanical-mathematical model of the pneumatic vibratory centrifugal separation of GM based on seed density. Underlying the study is a method of multi-phase media hydrodynamics where the multiphase structure is represented by a layer of GM, which consists of grain particles (discrete phase) and an airflow (continuous phase). The improved mechanical-mathematical model takes into consideration the interaction of the

discrete and continuous phases of GM by introducing the conditions of interaction at the interphase boundaries, which is expressed by equations (7), (8). During the hydrodynamic modeling of movement of the circular layer of seeds, the coefficient of dynamic viscosity of the discrete and continuous phases was taken into consideration; equation (10).

While generalizing the mechanical-mathematical model, it was found that the following characteristics of the separation process exert a defining effect on the parameters of the pneumatic vibratory centrifugal separation process:

- the circular speed of rotation of the cylindrical working surface around the fixed axis;
- the oscillation frequency of the cylindrical working surface;
- the oscillation amplitude of the cylindrical working surface;
- an airflow rate;
- the coefficient of dynamic viscosity;
- the average thickness of a GM layer;
- the average density of GM particles.

By using the improved mechanical-mathematical model of GM pneumatic vibratory centrifugal fractionation based on density, we have determined the rational values and ranges of values for the technical parameters of the process. Thus, the amplitude and oscillation frequency of the working surface are in the ranges  $A=(35...50) \cdot 10^{-5}$  m,  $\omega=15.0...15.6$  rad/s. The circular rotational frequency of the working surface,  $\omega=24...25$  rad/s. The airflow rate,  $V=2$  m/s. At the same time, the following assumptions were accepted regarding the values of output parameters: air density,  $\rho_n=1.29$  kg/m<sup>3</sup>; GM particle density,  $\rho_n=1,183$  kg/m<sup>3</sup>; friction coefficient of GM particles,  $f=0.45$ ; the equivalent radius of a GM particle,  $a=0.0021$  m. Under such parameters, it is possible to separate GM into two fractions: «heavy» (GM particles with high density) and «light» (GM particles with lower density).

The effectiveness of using the improved mechanical-mathematical model of GM fractionation at the pneumatic vibratory centrifugal separator A1-BCS-100 has been experimentally proven. It was established that when using the parameters recommended by the manufacturer, there is a slight overlap in the density values of the obtained GM fractions. Thus, the upper limit of the «light» fraction is 1,100 kg/m<sup>3</sup>, and the lower limit of the «heavy» fraction is 1,000 kg/m<sup>3</sup>. That is, layer-shaped model bodies with a density of 1,000 and 1,100 kg/m<sup>3</sup> are included in both the «light» and «heavy» fractions of the GM model. That is, there is a separation of the GM model at the efficiency of cleaning of 60...80 %, as predicted by the manufacturer of the pneumatic vibratory centrifugal separator.

When applying the parameters determined from the improved mechanical-mathematical model, no such overlap in

the range of the investigated density of fractions is observed. That is, for a given model of GM, components are separated with an efficiency of 100 %.

It should be noted that a given GM model does not take into consideration the existence of fractions with intermediate density, as well as different shapes of actual GM. However, the advantages of using the improved mechanical-mathematical model for calculating the rational parameters of the pneumatic vibratory centrifugal separator are obvious.

It should also be noted that the performance of the pneumatic vibratory centrifugal separator when applying the parameters determined from the improved mechanical-mathematical model was 60 t/h. At the same time, the performance of the separator based on the parameters specified by the manufacturer was 54 t/h.

Thus, taking into consideration the interaction between the discrete and continuous phases of GM and their coefficient of dynamic viscosity in the improved mechanical-mathematical model of fractionation makes it possible to increase the performance of the pneumatic vibratory centrifugal separator by 9 %. At the same time, the effectiveness of GM separation could reach 100 %.

The likely prospect of further research is the automation of pneumatic vibratory centrifugal separators configuration using the improved mechanical-mathematical model.

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## 7. Conclusions

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1. We have improved the mechanical-mathematical model of grain material fractionation at pneumatic vibratory centrifugal separators, which is based on the theory of movement and the hydrodynamics of continuous multiphase media. The model took into consideration the interaction between the discrete and continuous phase of the material in terms of the mutual movement of particle layers, their different density, and the interaction between these layers and a continuous phase (air). The instantaneous hydrodynamic resistance, coefficients of dynamic viscosity of discrete and continuous phase of grain material are taken into consideration.

2. We have derived the functional dependences between the defining parameters of the pneumatic vibratory centrifugal separation process of grain material based on seed density and the structural-kinematic parameters of the pneumatic vibratory centrifugal separator. The effectiveness of using the improved mechanical-mathematical model was proven while choosing the rational parameters for the pneumatic vibratory centrifugal separator. Efficiency implies increasing the performance of the separator by 9 % while the efficiency of grain material separation could reach 100 %.

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