

Sequencing is the most impact factor in many production areas, such as assembly lines, batch production, Travelling Salesman Problem (TSP), product sequences, process sequences, etc. The flow and analysis from one item to another can be presented by the square matrix in which the number of rows or columns is equal to the number of manipulated items, this special matrix form is called "From-To matrix". The matrix suffers from many drawbacks when it is applied to determine the optimal sequences, such as the number of variables must be as small as possible, there is no flexibility to determine the start or the end sequence to find the best sequencing with some conditions. Also, there is no possibility to add relations to point a variable as wanted or prevented from the sequence. In this paper, we solve the From-To matrix by binary linear programming (BLP).

The proposed BLP approach has been applied in Ur company to solve the From-To matrix. This company has a production line that can manufacture four products: A, B, C, and D, the setup time matrix is considered as From-To matrix and the goal of this company is to get an optimum sequence of products with minimum time. The solution of state transition of the From-To matrix using BLP can be formulated in the following five model cases according to transition requirement condition and desired: the first case gives all possible sequence items, the second case lists the sequence items when the first sequence is known, the third case lists the sequence items when the last sequence is known, the fourth case gives all possible sequence items with a condition that prevents occurring of an undesired sequence, and the fifth case gives all possible sequence items with the condition of a wanted occurring of the desired sequence.

Furthermore, we found the optimum sequences for states by determining the start or end sequences, and also add the wanted relations or prevented. The mathematical formulas for the number of all sequences under some conditions are derived and proved

**Keywords:** optimal sequence; binary linear programming; From-To matrix, exact solution

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# DEVISING AN EFFICIENT APPROACH TO DETERMINE THE OPTIMAL SEQUENCE OF FROM-TO MATRIX

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## 1. Introduction

The sequencing problem is the most impact factor in many applications, such as scheduling problem, Travelling Salesman Problem (TSP), products sequences, process sequences, etc. The sequencing problem is a specialized scheduling problem in which an ordering of products completely determines a schedule, several works dealing with scheduling issues are presented in the optimization. Recently, a number of studies have been carried out on the permutation flow scheduling problem under the constraint of setup time. Two types of setup time are distinguished: the first depends on the sequence of jobs on the same machine, called the sequence-dependent setup time, and the second is qualified as a setup time independent of the sequence of jobs on the same machine, called the sequence-independent setup time [1–3].

The probability method to find the best sequencing is by finding all possible sequences and choosing the sequence that has the minimum or maximum length depending on the criteria in the From-To matrix. This method is generally used in small matrices because the number of possibilities is equal to  $n$ , where  $n$  is the number of columns or rows of the matrix. Another drawback in the From-To matrix is that there is no flexibility to determine the start or the end sequence to find the best

sequencing with some conditions. Also, there is no possibility to add relations to point a variable as wanted or prevented from the sequence. Therefore, one needs to construct an efficient procedure for solving these kinds of problems.

## 2. Literature review and problem statement

The paper [2] introduced the modified assignment method based on the goal programming method for determining the optimal products sequence-dependent setup cost and/or setup time in single demand that consist of multi-products. This method can be finding the optimum solution directly, also help the change in priority of goals of the company lead to multiple alternatives that can help the decision-maker to select the best alternatives to minimize setup cost and/or setup time. The limitations of this approach are that there is no flexibility to determine the start or the end sequence to find the best sequencing with some conditions. Also, there is no possibility to add relations to point a variable as wanted or prevented from the sequence. The paper [4] introduced efficient production scheduling and sequencing to achieve the overall material supply, production, and distribution efficiency around the mixed-model assembly line in a supply chain, where production

scheduling and finished goods distribution have been increasingly considered in an integrated manner to achieve an overall best efficiency. This study introduces a heuristic procedure to achieve an integrated consideration of production scheduling and product distribution with production smoothing for the automobile just-in-time production assembly line. A meta-heuristic procedure is also developed for improving the heuristic solution. Also, this study presents plant assignment for a product with multiple product options as a prior step to scheduling and sequencing for a mixed-model assembly line. The drawback of this approach is that the solution will be heuristic not exact.

The paper [5] formulated sequencing and lot sizing with non-triangular setup times based on the Asymmetric Traveling Salesman Problem (ATSP) at an animal feed plant. To solve the model, optimal solution methods based on iterative subtour elimination and patching are developed. This study also presents a modelling and solution approach for the Anifeed's lot sizing and scheduling problem based on two formulations related to the Asymmetric Travelling Salesman Problem (ATSP), and tests both in the context of Anifeed's production environment. Furthermore, this study motivates future and more thorough research into factors impacting the performance of the models and methods. The disadvantage of this method is that setup times are sequence-dependent and may also be asymmetric and non-triangular.

The paper [6] developed a mathematical programming model to minimize the total flow time on the Flow-Shop Group Scheduling (FSGS) problem for solving large-size issues. After having defined a wide benchmark of test cases arisen from real-world manufacturing environments, the authors fulfilled an extensive comparison among the proposed meta-heuristics, from which the outperforming results of the ant colony approach clearly emerged. Also, this study shows that the HACO algorithm has a superior performance compared to the best available algorithm based on a memetic algorithm with an average percentage deviation of around 1.0% from the lower bound. The advantage of this approach is that the proposed ACO algorithm can be applied to efficiently solve problems larger in size than the ones already considered by tuning the parameters of the ACO algorithm. The disadvantage of this method is that the solution obtained using this algorithm is heuristic not exact.

The paper [7] analyzed a flow shop sequence-dependent group scheduling problem with limited inter-operational buffer capacity truly observed in the inspection department of a company producing electronic devices. The authors proposed a matrix-encoding (GA). The disadvantage of this approach is that setup times are sequence-dependent and the solution is heuristic.

The paper [8] used the travelling salesperson problem in many other fields such as logistics, planning and manufacturing, where the ultimate goal is to find the optimal path given a set of distances. This study presents different approaches to solving scheduling problems with batching and sequence-dependent setups. These solution approaches range from exact methodologies to heuristic methodologies. These various solution approaches can be used to improve performance metrics such as the number of tardy jobs, maximum lateness, and deviation from job due dates. The disadvantage of this approach is that the solution is heuristic and the algorithm is not a polynomial-time algorithm.

The paper [9] presented a genetic algorithm-based Travelling Salesman Problem with Precedence Constraints Approach (TSPPCA) to minimize completion time. The main

advantage of this method is that it is used to solve the sequencing problems for multiple demands with multi-product. The limitations of this approach are that there is no flexibility to determine the start or the end sequence to find the best sequencing with some conditions and the solution is heuristic.

The paper [10] deals with a sequencing problem that arises when there are multiple repair actions available to fix a broken man-made system and the true cause of the system failure is uncertain, where a binary integer programming formulation for the problem is proposed. This can be used to solve the problem directly or to compute lower bounds of the minimum expected cost using linear programming relaxation. The disadvantage of this approach is that computing lower bounds of the minimal expected cost and computing initial feasible solutions are performed using some greedy algorithms.

Linear programming is a mathematical model used extensively in decision making, a process of allocating available resources to maximize profits or minimize costs in business operations.

In binary linear programming, all the decision variables are binary. A binary linear programming model can be formulated as follows [11, 12]:

$$\text{MinZ or MaxZ} = \sum_{j=1}^q C_j X_j.$$

Subject to:

$$\sum_{i=1}^m a_{ij} X_j \begin{cases} \leq \\ \geq \\ = \end{cases} b_i, \text{ for } i=1, 2, \dots, m,$$

$$X_j = 0 \text{ or } 1, j=1, 2, \dots, q,$$

$$a_{ij}, b_i \text{ and } C_j \in R,$$

where  $q$  – number of variables;  $m$  – number of constraints.

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### 3. The aim and objectives of the study

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The aim of the study is to determine the optimal sequence of the From-To matrix using an efficient approach and demonstrate it using a case study.

To achieve this aim, the following objectives are accomplished:

- to choose a mathematical formulation so that the obtained optimal solution is exact;
- to check the efficiency of the presented approach by selecting a specific case study and use some programming languages or optimization solver to find the solution of the presented case study.

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### 4. Materials and methods

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#### 4.1. Binary linear programming

Linear programming is a mathematical model used extensively in decision making, a process of allocating available resources to maximize profits or minimize costs in business operations.

In binary linear programming, all the decision variables are binary. A binary linear programming model can be formulated as follows [11, 12]:

$$\text{Min}Z \text{ or } \text{Max}Z = \sum_{j=1}^q C_j X_j.$$

Subject to:

$$\sum_{i=1}^m a_{ij} X_j \begin{cases} \leq \\ \geq \\ = \end{cases} b_i, \text{ for } i=1, 2, \dots, m,$$

$$X_j = 0 \text{ or } 1, j=1, 2, \dots, q,$$

$$a_{ij}, b_i \text{ and } C_j \in R,$$

where  $q$  – number of variables;  $m$  – number of constraints.

#### 4. 2. Binary linear programming for From-To matrix

The transformation from one item to another can be presented by the matrix in which the number of rows is equal to the number of columns and equal to the number of items. This is called From-To matrix as shown in Table 1.

Table 1

From-To matrix

From \ To	1	2	...	$n$
1	$C_{11}$	$C_{12}$	...	$C_{1n}$
2	$C_{21}$	$C_{22}$	...	$C_{2n}$
$\vdots$	$\vdots$	$\vdots$	...	$\vdots$
$n$	$C_{n1}$	$C_{n2}$	...	$C_{nn}$

### 5. Results of determining the optimal sequence of the From-To matrix using an efficient approach

#### 5. 1. Binary linear programming (BLP) approach

The solution of state transition of the From-To matrix using BLP can be formulated in the following five model cases according to transition requirement condition and desired:

- case 1 gives all possible sequence items;
- case 2 lists the sequence items when the first sequence is known;
- case 3 lists the sequence items when the last sequence is known;
- case 4 gives all possible sequence items with a condition that prevents occurring of an undesired sequence;
- case 5 gives all possible sequence items with the condition of a wanted occurring of the desired sequence.

##### 5. 1. 1. Case number one: all possible sequence items

The number of sequences ( $ns$ ) equals to  $(n!)$  and the mathematical formulas for BLP to determine the optimal sequence are as follows:

$$\text{Min}Z \text{ (or } \text{Max}Z) = \sum_{i=1}^n \sum_{j=1}^n C_{ij} X_{ij}.$$

Subject to:

$$\sum_{j=1}^n X_{ij} \leq 1, \text{ for all } i=1, 2, \dots, n,$$

$$\sum_{i=1}^n X_{ij} \leq 1, \text{ for all } j=1, 2, \dots, n,$$

$$X_{ij} + X_{ji} \leq 1 \text{ for } i \neq j,$$

$$X_{ij} = \begin{cases} \infty & \text{if objective function} \\ & \text{is Minimization,} \\ -\infty & \text{if objective function} \\ & \text{is Maximization,} \end{cases} \text{ for } i = j,$$

$$\sum_{i=1}^n \sum_{j=1}^n X_{ij} = N - 1,$$

$$X_{ij} = 0 \text{ or } 1, j=1, 2, \dots, n.$$

From-To matrix is a square matrix, therefore  $n=m$  and using  $n$  only that indicates the number of rows or columns. The coefficient of a decision variable in the objective function ( $C_{ij}$ ) is given from the From-To matrix, while its constraint is equal to one.

Equations (1), (2) described the constraint of each row and each column, which is smaller than or equal to one, respectively. Equation (3) described the constraint, which is smaller than or equal to one to represent transaction between two items that indicate the number for its row equal to indicate number column to other item and reflect, its prevent select the same relation when determining the sequence, the intersection point of the same items represents an infinite amount, to be ruled out selected the sequence. Equation (4) described the constraint for the number of transactions between items that is equal to  $n-1$ .

##### 5. 1. 2. Case number two: the sequence items when the first sequence is known

The mathematical formulas for BLP to determine the optimal sequence when the first sequence is known are as follows:

$$\text{Min}Z \text{ (or } \text{Max}Z) = \sum_{i=1}^n \sum_{j=1}^n C_{ij} X_{ij}.$$

Subject to:

$$\sum_{j=1}^n X_{ij} = 1, \text{ for } i, \text{ indicate that the row item will be the first sequence.}$$

$$\sum_{j=1}^n X_{ij} \leq 1, \text{ for all } i=1, 2, \dots, n-1,$$

$$\sum_{j=1}^n X_{ij} = 0, \text{ for } j, \text{ indicate that the column item will be the first sequence}$$

$$\sum_{i=1}^n X_{ij} \leq 1, \text{ for all } j=1, 2, \dots, n-1, \text{ for all } j=1, 2, \dots, n,$$

$$X_{ij} + X_{ji} \leq 1 \text{ for } i \neq j,$$

$$X_{ij} = \begin{cases} \infty & \text{if objective function} \\ & \text{is Minimization,} \\ -\infty & \text{if objective function} \\ & \text{is Maximization,} \end{cases} \text{ for } i = j,$$

$$\sum_{i=1}^n \sum_{j=1}^n X_{ij} = N - 1,$$

$$X_{ij}=0 \text{ or } 1, j=1, 2, \dots, n.$$

The number of sequences in this case is  $(n1)!$ .

### 5. 1. 3. Case number three: the sequence items when the last sequence is known

The mathematical formulas for BLP to determine the optimal sequence when the last sequence is known are as follows:

$$\text{Min}Z(\text{or Max}Z) = \sum_{i=1}^n \sum_{j=1}^n C_{ij} X_{ij}.$$

Subject to:

$$\sum_{j=1}^n X_{ij} = 0, \text{ for } i, \text{ indicate that the row item will be the}$$

last sequence.

$$\sum_{j=1}^n X_{ij} \leq 1, \text{ for all } i=1, 2, \dots, n-1,$$

$$\sum_{i=1}^n X_{ij} = 1, \text{ for } j, \text{ indicate that the column item will be the}$$

last sequence

$$\sum_{i=1}^n X_{ij} \leq 1, \text{ for all } j=1, 2, \dots, n-1,$$

$$X_{ij} + X_{ji} \leq 1 \text{ for } i \neq j.$$

$$X_{ij} = \begin{cases} \infty & \text{if objective function} \\ & \text{is Minimization,} \\ -\infty & \text{if objective function} \\ & \text{is Maximization,} \end{cases} \text{ for } i = j,$$

$$\sum_{i=1}^n \sum_{j=1}^n X_{ij} = N - 1,$$

$$X_{ij}=0 \text{ or } 1, j=1, 2, \dots, n.$$

The number of sequences in this case is  $(n-1)!$ .

### 5. 1. 4. Case number four: all possibilities of sequence items with some prevented relation

The mathematical formulas for BLP to determine the optimal sequence are as follows:

$$\text{Min}Z(\text{or Max}Z) = \sum_{i=1}^n \sum_{j=1}^n C_{ij} X_{ij},$$

Subject to:

$$\sum_{j=1}^n X_{ij} \leq 1, \text{ for all } i=1, 2, \dots, n,$$

$$\sum_{i=1}^n X_{ij} \leq 1, \text{ for all } j=1, 2, \dots, n,$$

$$X_{ij} + X_{ji} \leq 1 \text{ for } i \neq j$$

$$X_{ij} = \begin{cases} \infty & \text{if objective function} \\ & \text{is Minimization,} \\ -\infty & \text{if objective function} \\ & \text{is Maximization,} \end{cases} \text{ for } i = j,$$

$$\sum_{i=1}^n \sum_{j=1}^n X_{ij} = N - 1,$$

$X_{ij}=0$  for  $i$  and  $j$  prevented relation index,

$$X_{ij}=0 \text{ or } 1, j=1, 2, \dots, n.$$

The number of sequences in this case is

$$nS = \begin{cases} n! - [(n-1)! \times R_p] & \text{if } R_p = 1 \text{ or } R_p > 1 \\ & \text{and } R_p \text{ is connected relations,} \\ n! - [(n-1)! \times R_p] + R_p & \text{if } R_p > 1 \\ & \text{and } R_p \text{ is unconnected relations,} \end{cases}$$

where  $R_p$  is the number of relation that prevents some sequences from occurring.

### 5. 1. 5. Case number four: all possibilities of sequence items with some wanted relation

The mathematical formulas for BLP to determine the optimal sequence are as follows:

$$\text{Min}Z(\text{or Max}Z) = \sum_{i=1}^n \sum_{j=1}^n C_{ij} X_{ij}.$$

Subject to:

$$\sum_{j=1}^n X_{ij} \leq 1, \text{ for all } i=1, 2, \dots, n,$$

$$\sum_{i=1}^n X_{ij} \leq 1, \text{ for all } j=1, 2, \dots, n,$$

$$X_{ij} + X_{ji} \leq 1 \text{ for } i \neq j,$$

$$X_{ij} = \begin{cases} \infty & \text{if objective function} \\ & \text{is Minimization,} \\ -\infty & \text{if objective function} \\ & \text{is Maximization,} \end{cases} \text{ for } i = j,$$

$$\sum_{i=1}^n \sum_{j=1}^n X_{ij} = N - 1(4),$$

$X_{ij}=1$  for  $i$  and  $j$  wanted relation index

$$X_{ij}=0 \text{ or } 1, j=1, 2, \dots, n.$$

The number of sequences in this case is  $(n-Rw)!$ , where  $Rw$  is the number of wanted relation and  $0 < Rw \leq n-1$ , the wanted relation must not conflict for example  $A-B, B-C$  where  $A$  is related with the first relation for  $B$  then  $B$  is related with  $C$ .

## 5. 2. Case study and optimization solver

The proposed BLP approach was applied in Ur company to solve the From-To matrix. This company has a production line that can manufacture four products:  $A, B, C,$  and  $D$ , the setup time matrix is considered as From-To matrix as shown in Table 2 and the goal of this company is to get an optimum sequence of products with minimum time.

The number of all possibilities of sequences is equal to  $(n)! = 4! = 24$  as shown in Table 3.

Table 2

Setup time matrix

From \ To	A	B	C	D
A	0	91	102	210
B	50	0	80	110
C	20	35	0	70
D	30	40	65	0

Table 3

Possibilities for four products

No.	Products Sequence	Sequence Time
1	A-B-C-D	91+80+70=241
2	A-B-D-C	91+110+65=266
3	A-C-B-D	102+35+110=247
4	A-C-D-B	102+70+40=212
5	A-D-B-C	210+40+80=330
6	A-D-C-B	210+65+35=310
7	B-A-C-D	50+102+70=222
8	B-A-D-C	50+210+65=325
9	B-C-A-D	80+20+219=310
10	B-C-D-A	80+70+30=180
11	B-D-A-C	110+30+102=242
12	B-D-C-A	110+65+20=195
13	C-A-B-D	20+91+110=221
14	C-A-D-B	20+210+40=270
15	C-B-A-D	35+50+210=295
16	C-B-D-A	35+110+30=175
17	C-D-A-B	70+30+91=191
18	C-D-B-A	70+40+50=160
19	D-A-B-C	30+91+80=201
20	D-A-C-B	30+102+35=167
21	D-B-A-C	40+50+102=192
22	D-B-C-A	40+80+20=140
23	D-C-A-B	65+20+91=176
24	D-C-B-A	65+35+50=150

To determine the optimal sequence of products, which depends on the setup matrix, the case study will be applied for the five cases described above as follows:

1. First case.

The problem can be formulated using the programming language winQSB and a binary linear programming method. The infinite amount is replaced by the number (1000) or any large number, then the mathematical model will be as follows:

$$\text{Min} Z = 1000X_{11} + 91X_{12} + 102X_{13} + 210X_{14} + 50X_{21} + 1000X_{22} + 80X_{23} + 110X_{24} + 20X_{31} + 35X_{32} + 1000X_{33} + 70X_{34} + 30X_{41} + 40X_{42} + 65X_{43} + 1000X_{44}$$

S. t.

$$\begin{aligned} X_{11} + X_{12} + X_{13} + X_{14} &\leq 1, \\ X_{21} + X_{22} + X_{23} + X_{24} &\leq 1, \\ X_{31} + X_{32} + X_{33} + X_{34} &\leq 1, \\ X_{41} + X_{42} + X_{43} + X_{44} &\leq 1, \\ X_{11} + X_{21} + X_{31} + X_{41} &\leq 1, \\ X_{12} + X_{22} + X_{32} + X_{42} &\leq 1, \\ X_{13} + X_{23} + X_{33} + X_{43} &\leq 1, \\ X_{14} + X_{24} + X_{34} + X_{44} &\leq 1, \end{aligned}$$

$$\begin{aligned} X_{12} + X_{21} &\leq 1, \\ X_{13} + X_{31} &\leq 1, \\ X_{14} + X_{41} &\leq 1, \\ X_{23} + X_{32} &\leq 1, \\ X_{24} + X_{42} &\leq 1, \\ X_{34} + X_{43} &\leq 1. \end{aligned}$$

$$X_{11} + X_{12} + X_{13} + X_{14} + X_{21} + X_{22} + X_{23} + X_{24} + X_{31} + X_{32} + X_{33} + X_{34} + X_{41} + X_{42} + X_{43} + X_{44} = 4.$$

$$X_{ij} = 1 \text{ or } 0.$$

The result is as follows:

$$X_{42} = X_{23} = X_{31} = 1.$$

This means that the optimal sequence is:

$$D-B-C-A = 40 + 80 + 20 = 140 \text{ hours.}$$

The optimal sequence takes No. 22 in Table 2 and it considers less time for the sequence of products, and the number of sequences is equal to  $n! = 24$  according to the first state.

2. Second case.

The number of sequences at this state is equal to  $(n-1)! = 3! = 6$ , where the first sequence is known and from Table 2, No. 1 to 6 start with product A, No. 7 to 12 start with product B, No. 13 to 18 start with product C and No. 19 to 24 start with product D.

The result for the optimal sequence of the product when must start with A, as follows:

$$X_{13} = X_{34} = X_{42} = 1.$$

This means that the optimal sequence that takes No. 4 in Table 2:

$$A-C-D-B = 102 + 70 + 40 = 212 \text{ hours.}$$

3. Third case.

The number of sequences at this state is equal to  $(n-1)! = 3! = 6$ , where the last sequence is known and from Table 2, the result for the sequence of the product when must be last with A, No. (4, 6, 14, 17, 20 and 23), the optimal sequence that takes No. 20 in Table 2 is as follows:

$$X_{41} = X_{13} = X_{32} = 1.$$

This means that the optimal sequence is:

$$D-A-C-B = 30 + 102 + 35 = 167 \text{ hours.}$$

4. Fourth case.

The number of sequences when two relations prevented are B-C, D-A equals to

$$(n! - [(n-1)! \times R_p]) + R_p = 24 - (6 \times 2) + 2 = 14,$$

the result for the sequence of the product is No. (2, 3, 4, 6, 7, 8, 12, 13, 24, 15, 16, 17, 23, 24), the optimal sequence that takes No. 24 in Table 2 is as follows:

$$X_{43} = X_{32} = X_{21} = 1.$$

This means that the optimal sequence is:

$$D-C-B-A=65+35+50=150 \text{ hours.}$$

5. Fifth case.

The number of sequences when two relations wanted are  $B-C, D-A$  equals to  $(n-R_w)!=(4-2)!=2!=2$ , the result for the sequence of the product is No. (10 and 19), the optimal sequence that takes No. 10 in Table 2 is as follows:

$$X23=X34=X41=1.$$

This means that the optimal sequence is:

$$B-C-D-A=80+70+30=180 \text{ hours.}$$

### 5. 3. Comparison

In order to show the novelty of our work, we compare the results of our approach with previous related works in Table 4 as follows:

Table 4

#### Comparison

Previous research	Our work
Heuristics solution is obtained for sequencing problems in most cases	Exact solution is obtained for sequencing problems
The most important drawback of existing methods used to solve the sequencing problems is that the sequence must have a few products and dependent setup cost or setup time	It can deal with different classes of sequence problems and it is not affected by the number of products in the sequence
“From-To matrix”. The matrix suffers from many drawbacks when it is applied to determine the optimal sequences; such as the number of variables must be as small as possible, there is no flexibility to determine the start or the end sequence to find the best sequencing with some conditions. Also, there is no possibility to add relations to point a variable as wanted or prevented from the sequence	The binary linear programming (BLP) approach solved the From-To matrix, where we found the optimum sequences for states by determining the start or end sequence, and also add the desired relations: wanted or prevented

### 6. Discussion of experimental results of devising an efficient approach for determining the optimal sequence of the From-To matrix

The scheduling and sequencing decisions consider operations decisions; we have presented an exact method to determine the optimal sequences, which have the minimum or maximum length depending on the criteria in the From-To matrix using binary linear programming. This method is applied in Ur company that has a production line that

can manufacture four products:  $A, B, C,$  and  $D$ , in which the setup time matrix is considered as From-To matrix. The formulation of the From-To matrix for the presented case study is shown in Table 2. Using this formulation, we calculate the number of all possibilities of sequences which is equal to  $(n)!=4!=24$  as shown in Table 3.

After that, to determine the optimal sequence of products, which depends on the setup matrix, we have described five different cases based of our data in Table 2. For the first case, we have seen that the optimal sequence is:  $DBCA=40+80+20=140$  hours and the optimal sequence takes No. 22 in Table 2 and it considers less time for the sequence of products, and the number of sequences is equal to  $n!=24$  according to the first state. For the second case, we have noticed that the optimal sequence that takes No. 4 in Table (2) is as follows:  $A-C-D-B=102+70+40=212$  hours. Also, we have seen that the optimal sequences for the remaining three cases are respectively as follows: the optimal sequence for the third case is:  $D-A-C-B=30+102+35=167$  hours, the optimal sequence for the fourth case is:  $D-C-B-A=65+35+50=150$  hours and the optimal sequence for the fifth case is:  $B-C-D-A=80+70+30=180$  hours. As noted, this method can find the optimum sequences for the five states by determining the start or the end sequence, and also add wanted or prevented relations. Moreover, the mathematical formulas for the number of all sequences under some conditions can be described. The limitation of our approach is that it does not cover the due date for the given items. Moreover, the disadvantage of the presented approach is that it works on static environment not dynamic.

### 7. Conclusions

1. A mathematical formulation using the binary linear programming approach has been successfully presented to determine the optimal sequence of the From-To matrix so that the obtained optimal solution is exact. Where the number of sequences for the presented five cases is calculated respectively as 24, 6, 6, 14, 2.

2. The efficiency of the presented approach has been shown by selecting a specific case study. Which has a production line that can manufacture four products:  $A, B, C,$  and  $D$ , in which the setup time matrix is considered as From-To matrix. As noted, this method can find the optimum sequences for five states by determining the start or the end sequence, and also add relations wanted or prevented. Also, the mathematical formulas have been described for the number of all sequences under some conditions. Moreover, the optimal solution for the model has been found using a Win QSB solver.

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