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CONSTRUCTION OF MATHEMATICAL MODELS OF THE STRESSED-STRAINED STATE OF A MATERIAL WITH A POROUS WATER-SATURATED BASE UNDER DYNAMIC LOAD

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Materials of beams, plates, slabs, strips have been commonly applied in various fields of industry and agriculture as flat elements in the structures for machinery and construction. They are associated with the design of numerous engineering structures and facilities, such as the foundations of various buildings, airfield and road surfaces, floodgates, including underground structures.

This paper reports a study into the interaction of the material (of beams, plates, slabs, strips) with the deformable base as a three-dimensional body and in the exact statement of a three-dimensional problem of mathematical physics under dynamic loads.

The tasks of studying the interaction of a material (beams, plates, slabs, strips) with a deformable base have been set. A material lying on a porous water-saturated viscoelastic base is considered as a viscoelastic layer of the same geometry. It is assumed that the lower surface of the layer is flat while the upper surface, in a general case, is not flat and is given by some equation.

Classical approximate theories of the interaction of a layer with a deformable base, based on the Kirchhoff hypothesis, have been considered. Using the well-known hypothesis by Timoshenko and others, the general three-dimensional problem is reduced to a two-dimensional one relative to the displacement of points of the median plane of the layer, which imposes restrictions on external efforts. In the examined problem, there is no median plane. Therefore, as the desired values, displacements and deformations of the points in the plane have been considered, which, under certain conditions, pass into the median plane of the layer.

It is not possible to find a closed analytical solution for most problems while experimental studies often turn out to be time-consuming and dangerous processes

Keywords: construction of mathematical models, interaction of material with base, dynamic load, boundary condition, general solution

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1. Introduction

The current stage in the development of mechanics, including determining the stressed-strained state (SSS) of structures, is associated with the widespread use of mathematical methods. Practice puts forward the tasks of multivariate studies into two-dimensional and three-dimensional systems, which could be sometimes adequately solved only through mathematical modeling. As a rule, it is not possible to find a closed analytical solution for most problems while experimental research often turns out to be a time-consuming and dangerous process.

Studies on this problem from the 20^{th} century show that, first, they were conducted to calculate statistical problems related to the strength of structures with an elastic base; second, the calculations of structural elements disregarded the influence exerted by the elastic-plastic properties of the base. Based on those studies, there is a need to construct mathematical models of dynamic bending of the beam and slab on elastic, elastic-plastic, and viscous bases, taking into consideration the influence of the temperature of structural elements and their base.

Modeling of the general theory of interaction of material (for beams, plates, slabs, strips) with a deformable base as a three-dimensional body and using the exact statement of a three-dimensional mathematical problem under dynamic loads, is investigated. The following issues are addressed:

1. Statement of the problem of interaction of the material (for beams, plates, slabs, strips) with the deformable base and the interaction of the material (in viscoelastic beams, plates, slabs, strips) lying on a porous water-saturated viscoelastic base, in an exact three-dimensional linear statement, considered as a viscoelastic layer of the same geometry. It is assumed that the lower surface of the layer is flat while the upper surface, in a general case, is not flat and is given by some equation.

2. General solution to the problem. Kirchhoff's hypothesis underlies the classical approximate theories of the interaction of a layer with a deformable base. Using the well-known hypothesis by Timoshenko and others, the general three-dimensional problem is reduced to a two-dimensional one relative to the displacement of points of the median plane of the layer, which imposes restrictions on external

efforts. In the considered problem, there is no median plane. Therefore, as the desired values, displacements and deformations of the points in the plane are considered, which, under certain conditions, pass into the median plane of the layer. In the study of wave processes in deformable media, or when solving problems of interaction of the layer with the deformable base, methods of mathematical physics are used.

It is on the basis of the problem under consideration that it is possible to devise the most effective methods for assessing and predicting the operational and technical condition of structures, to work out effective ways to protect against negative influences, to assess the effectiveness of new non-traditional structures in any industry. Problems that take into consideration the interactions of non-elastic media with a deformable base or the interaction of non-elastic media lying on a porous water-saturated viscoelastic base are important and relevant.

2. Literature review and problem statement

The problem of the interaction of beams and slabs on a deformable base attracts the attention of numerous researchers. In the second half of the 20th century, that is, 1950–1990, a fairly large number of different models of the base were proposed. Each of them has its disadvantages and advantages, leads to different results in terms of convergence with experiment. And, apparently, there is no need to make contrasts since each model has the right to exist. However, in each case, one should set the boundaries of the use of each of them.

Studies in world science up to 2021 have considered the propagation of waves in deformable layered media under the influence of intense loads. The dynamic problem of bending of the cantilever beam located in a resisting environment under elastic-plastic deformations is solved in [1]. The method of solving this problem was based on the introduction of some fictitious boundary between the elastic and plastic zones. A detailed overview of the dynamic calculation of structures is given in [2].

Work [2] reports mathematical models of interaction of a beam (plate, slab, strip) with a deformable base under dynamic loads. It was revealed that accounting for wave processes in deformable media and the interaction of two deformable media under dynamic mobile loads reveals the strength characteristics of the structure. In [3], the basic conclusions on modeling the propagation of blast waves in multilayered inhomogeneous half-space were drawn. In [4], the interactions of two deformable media under dynamic mobile loads were investigated. It was revealed that at the interaction of two media under the action of dynamic moving loads, the strength characteristics of the structure increase. In [5], a model calculation of the interaction of beams (plates and strips) with a deformable multilayer base under dynamic loads [6] was carried out. The solution to the problem on the impact of the mobile load, taking into consideration the heterogeneity of the medium, was derived. In [7], the influence of the mobile pressure load of unchanged profile on a pipeline with the deformable base was investigated; the strength characteristics of the structure were revealed.

The review of [2–7] shows that the propagation of waves in deformable media and the interaction of two deformable media under dynamic mobile loads were investigated. It is demonstrated that accounting for wave processes in deformable media and the interaction of two deformable media under dynamic mobile loads significantly reveals the strength characteristics of the structure. It was found that in order to determine the strength characteristics of a structure at the interaction of two deformable media under the action of dynamic moving loads, the specification of the state diagram $P=P(\varepsilon)$ is essential. This is a global task in modern industry, construction, agriculture, and military affairs.

Paper [8] reports an experimental study of the deformability of media during the propagation of ultrasonic waves and acoustic emission of rock salt at triaxial compression. The study does not consider the propagation of waves in deformable media and their interaction under dynamic mobile loads. In [9], the propagation of natural waves on a multilayer viscoelastic cylindrical body hosting the surface of the weakened mechanical contact was investigated; however, the strength characteristics of the object under study were not considered. The authors of [10] built the models of propagation of elastic waves during exploratory drilling on the island of artificial ice. In [11], by applying the variational principle, the propagation of a flat wave in the thermoelastic medium with double porosity was considered according to the Lord-Shulman theory. Papers [10, 11] disregarded the strength characteristics of the object under study, and the interactions of the media were not taken into consideration. Study [12] investigated the propagation of transverse waves through parallel joints of rocks at stress on the spot but the wave and strength characteristics of the object under study were not considered. In [13], single waves in power deformable pipelines with the laminar or turbulent fluid flow were investigated but the wave and strength characteristics of the object under study were not considered; the interactions of media were not taken into consideration. Paper [14] examined the mathematical modeling of the Stoneleigh wave in a transversal-isotropic thermoelastic medium. In [15], the propagation of S-waves through parallel joints of rocks at stress on site was investigated. Papers [14, 15] disregarded the wave and strength characteristics of the object under study; the interactions of media were not taken into consideration. In [16], the propagation of an oblique transverse wave in finitely deformable layered composites was studied but the interactions of media were not taken into consideration. In [17], a method of mathematical modeling was used for the Stoneleigh wave in a transverse-isotropic thermoelastic medium but the strength characteristics of the object under study were not considered; the interactions of the media were not taken into consideration. The authors of [18] suggested three-dimensional modeling of the influence of the bulge on the control over the propagation of nonlinear waves of stresses caused by the explosive load but the wave and strength characteristics of the object under study and the interaction of media were not considered. Paper [19] gives a parametric assessment of dispersed viscoelastic layered media to monitor the state of structures; the wave and strength characteristics of the object under study were not considered; the interactions of media were disregarded.

In [20], the propagation of a Love wave in functional-gradient media with an electrode boundary and a sharply thickened imperfect interface was investigated; the deformations of the interacting media under intense load were not taken into consideration. Study [21] reports the microscopic instabilities and the propagation of elastic waves in finitely deformable layered materials with compressible hyperelastic phases. In [22], the result of studying the thermomechanical behavior of multilayer media is given on the basis of the Lord-Shulman model; the interactions of media were not considered. Paper [23] shows the propagation of SH-waves in two anisotropic layers associated with isotropic half-space under the influence of gravity; there are no results of studies into wave propagation in deformable media and the interaction of two deformable media under dynamic mobile intense loads. In [24], three-dimensional numerical modeling of methane and air combustion in inert porous media on a pore scale under conditions of propagation of the combustion wave in the medium up and downstream was considered; there are no results of studies into wave propagation in deformable media and the interaction of two deformable media under dynamic mobile intense loads. In [25], the effect of heat load on cattle, the mechanisms of microcrack initiation in stainless steel, the hermetic-mechanical behavior of multilayer media based on the Lord-Schulman model is investigated but there are no results of studies of wave propagation in deformable media and the interaction of two deformable media under dynamic mobile intense loads. Paper [26] considers the oblique propagation of gravitational waves during sudden stratospheric warming but does not consider issues related to the deformation of interacting media under intense load. The authors of [27] show the influence of small defects on the fatigue strength of martensitic stainless steel; the propagation of waves in deformable media and the interaction of two deformable media under dynamic loads were not investigated. In [28], three-dimensional numerical simulation of methane and air combustion in inert porous media on a pore scale under conditions of propagation of the combustion wave in the medium up and downstream is reported but there are no results of studies into wave propagation in deformable media and the interaction of two deformable media under dynamic mobile intense loads. There is no accounting for wave processes in deformable media and the interaction of two deformable media under dynamic mobile loads, thus the essential strength characteristic of the structure was disregarded.

The reason for this is the parameters of materials associated with finding a closed analytical form of the relationship between pressure and deformation under dynamic loads. For most problems, it is not possible to find a closed analytical solution while experimental research is often time-consuming and dangerous. An option for overcoming the difficulties may be the use of the method of mathematical physics. This approach was employed in work [2] but the model of linear deformations of interacting media disregarded the processes occurring in the medium with a decrease in the dynamic load.

The above allows us to assert that it is expedient to conduct a study to build a mathematical model of the stressed-strained state of the material with a deformable porous water-saturated base under dynamic load.

3. The aim and objectives of the study

The aim of this work is to construct mathematical models of the interaction of the material (in the viscoelastic beam, plates, slabs, strips) with a deformable base under dynamic loads. This would make it possible to more accurately determine the state of the investigated interacting object with a deformable base under dynamic loads; the results could be used when tackling technical and technological issues in the industry, construction, agriculture, etc.

To accomplish the aim, the following tasks have been set:

– to state the problem of the dynamic interaction of the material (in the viscoelastic beam, plates, slabs, strips) with a deformable base;

– to derive a general solution to the problem based on the classical approximate theory of the interaction of the layer with the deformable base.

4. The study materials and methods

The object of the study is the interaction of the material with the deformable base under dynamic loading.

The main hypothesis of the study is that one should consider that the process obeys natural laws, such as the conservation of the amount of motion, energy, continuity, mass, as well as initial and boundary conditions.

The methods of mathematical physics were used taking into consideration the following laws: preservation of the amount of motion, energy, continuity, mass, as well as the initial and boundary conditions of the dynamic interaction of the material with the deformable base.

Wave problems for deformable media described by equations of motion for viscoelastic media, the behavior of which is described by partial differential equations of the fourth or higher order, have been solved. Constructing general solutions to the equations of motion is a difficult mathematical task, and this complexity is exacerbated by various types of boundary conditions. Using the methods of mathematical physics, general solutions to the problem have been obtained on the basis of the classical approximate theory of the interaction of the layer with the deformable base.

We accepted assumptions that the process is described by the equations of motion, and the medium is viscous.

Our adopted simplification implies that the general solutions to the problem were derived on the basis of the classical approximate theory.

5. Results of studying the dynamic interaction of the material (viscoelastic beam, plates, slabs, strips) with a deformable base

5. 1. Problem statement.

We assume that the lower surface of the layer is flat while the upper surface is not flat, and is given by the following equation from [2]

$$z = F(x,y).$$

The parameters of the layer material are to be indicated by the index "o", and those of the base – by the index "1".

The dependences of stresses $\sigma_{ij}^{(0)}$ on deformation $\varepsilon_{ij}^{(0)}$ at points in the layer are described by linear operator equations, that is, we propose giving them in the form of Boltzmann relations

$$\sigma_{jj}^{(0)} = L_0\left(\varepsilon^{(0)}\right) + 2M_0\left(\varepsilon_{jj}^{(0)}\right),\quad \sigma_{ij}^{(0)} = M_0\left(\varepsilon_{ij}^{(0)}\right),$$

$$(i \neq j),\quad (i,j = x,y,z), \tag{1}$$

where viscoelastic operators L_0 and M_0 are the linear integrated operators in the following form

$$L_0(\tau)=\lambda_0\left[\tau(t)-\int_0^t f_1(t-\xi)\tau(\xi)\mathrm{d}\xi\right],$$

$$M_0(\tau)=\mu_0\left[\tau(t)-\int_0^t f_2(t-\xi)\tau(\xi)\mathrm{d}\xi\right], \tag{2}$$

$f_j(t)$ are the kernels of viscous operators; λ_0, μ_0 are the elastic constants or Lamé coefficients.

In a general case, the kernels of operators are arbitrary and different, that is, the Poisson coefficients of the material of the layer are variable while the kernels of the operators are such that there exists a resolvent of integrated operators (2).

Introduce potentials Φ_0 and $\bar{\Psi}_0$ of the longitudinal and transverse waves [1, 6]

$$\bar{U}_0=\mathrm{grad}\Phi_0+\mathrm{rot}\bar{\Psi}_0, \tag{3}$$

where $\bar{U}_0$ is the vector of moving the points of the layer; the equations of motion of the layer takes the following form

$$N_0(\Delta\Phi_0)=\rho_0\frac{\partial^2\Phi_0}{\partial t^2},\quad M_0(\Delta\bar{\Psi}_0)=\rho_0\frac{\partial^2\bar{\Psi}_0}{\partial t^2}, \tag{4}$$

where the operator N_0 is Δ – the three-dimensional Laplace operator

$$\Delta=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}.$$

Considering Helmholtz's theorem, in the absence of internal sources, the vector potentials $\bar{\Psi}$ of transverse waves must satisfy the following condition from [2, 6]

$$\mathrm{div}\bar{\Psi}_0=0. \tag{5}$$

Condition (5) is a closing condition for determining the components of the $\bar{\Psi}_0$. potential vector.

The base is considered as a porous water-saturated soil. The dependences of stresses $\sigma_{ij}^{(1)}$ and $\sigma^{(1)}$ on deformations $\varepsilon_{ij}^{(1)}$ and $\varepsilon^{(2)}$ in the solid and liquid components are described by the following ratios according to the generalized model by M. Bio [7]

$$\sigma_{jj}^{(1)}=L_1\left(\varepsilon^{(1)}\right)+2M_1\left(\varepsilon_{jj}^{(1)}\right)+Q\left(\varepsilon^{(2)}\right),\quad \sigma_{ij}^{(1)}=M_1\left(\varepsilon_{ij}^{(1)}\right), \tag{6}$$

$$\sigma^{(1)}=R\left(\varepsilon^{(2)}\right)+Q\left(\varepsilon^{(1)}\right),\quad (i\neq j,\ i,j=x,y,z),$$

where index "1" is for the solid component, index «2» – for the liquid component; operators L_1, M_1, Q, R are the linear integrated operators of type (2), that is

$$L_1(\tau)=\lambda_1\left[\tau(\xi)-\int_0^t f_{11}(t-\xi)\tau(\xi)\mathrm{d}\xi\right],$$

$$M_1(\tau)=\mu_1\left[\tau(\xi)-\int_0^t f_{12}(t-\xi)\tau(\xi)\mathrm{d}\xi\right],$$

$$Q(\tau)=Q_1\left[\tau(\xi)-\int_0^t f_{13}(t-\xi)\tau(\xi)\mathrm{d}\xi\right],$$

$$R(\tau)=R_1\left[\tau(\xi)-\int_0^t f_{14}(t-\xi)\tau(\xi)\mathrm{d}\xi\right].$$

Similarly to (3), introduce the Φ_j and $\bar{\Psi}_j$ potentials for both the solid and liquid base components

$$\bar{U}_1=\mathrm{grad}\Phi_1+\mathrm{rot}\bar{\Psi}_1,\quad \mathrm{div}\bar{\Psi}_1=0,$$

$$U_2=\mathrm{qrad}\Phi_2+\mathrm{rot}\Psi_2,\quad \mathrm{div}\bar{\Psi}_2=0, \tag{7}$$

where $\bar{U}_1$ and $\bar{U}_2$ are the point displacement vectors of the solid and liquid components, respectively.

The motion of the porous water-saturated base is described by the following equations from [5, 7, 8]

$$N_1(\Delta\Phi_1)+Q(\Delta\Phi_2)=\rho_{11}\frac{\partial^2\Phi_1}{\partial t^2}+\rho_{12}\frac{\partial^2\Phi_2}{\partial t^2}, \tag{8}$$

$$Q(\Delta\Phi_1)+R(\Delta\Phi_2)=\rho_{12}\frac{\partial^2\Phi_1}{\partial t^2}+\rho_{22}\frac{\partial^2\Phi_2}{\partial t^2},$$

$$M_1\left(\Delta\bar{\Psi}_1\right)=\frac{\rho_{11}\rho_{22}-\rho_{12}^2}{\rho_{22}}\frac{\partial^2\bar{\Psi}_1}{\partial t^2},\ \bar{\Psi}_2=-\frac{\rho_{12}}{\rho_{22}}\bar{\Psi}_1, \tag{9}$$

where

$$N_1=L_1+2M_1,\quad \Delta\Phi_1=\varepsilon^{(1)},\quad \Delta\Phi_2=\varepsilon^{(2)};$$

Δ is the three-dimensional Laplace operator; Φ_1, Φ_2 are the potentials of longitudinal waves of the solid and liquid components; $\varepsilon^{(1)}$, $\varepsilon^{(2)}$ are, respectively, deformations, while

$$\rho_{11}\rho_{22}-\rho_{12}^2>0,\quad \rho_{12}<0,\quad \rho_{11}=(1-k_0)\rho_s,\quad \rho_{22}=k_0\rho_f-\rho_{12};$$

k_0 is the porosity of the medium; ρ_s, ρ_f are the densities of solid and liquid components.

Assuming that non-stationary forces act on the layers, both on the upper and lower ones, and the forces on the lower surface can be caused by internal, in particular, seismic waves.

Boundary conditions on the upper surface of the layer $z=F(x,y)$ take the following form from [9–11]

$$\sigma_{nn}^{(0)}=f_n^{(0)}(x,y,t),\quad \sigma_{ns_j}^{(0)}=f_{ns_j}^{(0)}(x,y,t),\quad (j=1,2), \tag{10}$$

where σ_{nn} and σ_{ns_j} are the normal and tangential stresses; n is the normal to the surface $F(x,y)$; s_j is the orthogonal directions in the tangent plane drawn to the point of the surface of the layer.

The stresses σ_{nn} and σ_{ns_j} are expressed by the following formulas through the stresses σ_{ij} in Cartesian coordinates [12–15]

$$\begin{aligned}\sigma_{nn}&=\sigma_{xx}l_0^2+\sigma_{yy}m_0^2+\sigma_{zz}n_0^2+\\&+2\sigma_{xy}l_0m_0+2\sigma_{xz}l_0n_0+2\sigma_{yz}m_0n_0,\end{aligned} \tag{11}$$

$$\begin{aligned}\sigma_{ns_j}&=\sigma_{xx}l_0l_j+\sigma_{yy}m_0m_j+\sigma_{zz}n_0n_j+\sigma_{xy}\left(l_0m_j+l_jm_0\right)+\\&+\sigma_{xz}\left(l_0n_j+l_jn_0\right)+\sigma_{yz}\left(m_0n_j+m_jn_0\right),\end{aligned}$$

where (l_0, m_0, n_0) are the guide cosines of normal n; (l_j, m_j, n_j) are the guide cosines of orthogonal coordinates s_j and are equal to

$$l_0=-\frac{F_x'}{\Delta_0},\quad m_0=-\frac{F_y'}{\Delta_0},\quad n_0=\frac{1}{\Delta_0},$$

$$\Delta_0=\sqrt{1+\left(F_x'\right)^2+\left(F_y'\right)^2},\ l_1=\frac{1}{\Delta_1},\quad m_1=\frac{\mathrm{tg}\alpha_0}{\Delta_1},$$

$$n_1=\frac{F'_x+F'_y\mathrm{tg}\alpha_0}{\Delta_1},\quad \Delta_1=\sqrt{\cos^{-2}\alpha_0+\left(F'_x+F'_y\mathrm{tg}\alpha_0\right)^2},$$

in this case, the function $F(x,y)$ describes the upper surface of the layer.

Next, it was assumed that the partial derivatives from the function $F(x,y)$ do exist and are continuous almost everywhere except for points of the first kind, which are nothing more than for counting.

At the contact boundary $z=-h$, in the absence of friction, we have the following boundary conditions [16–18]

$$\sigma_{zz}^{(0)}=k_0\sigma_{zz}^{(2)}+\left(1-k_0\right)\sigma^{(1)}+f_z^{(1)}\left(x,y,z\right),$$

$$\sigma_{xz}^{(0)}=\sigma_{yz}^{(0)}=0,$$

$$\sigma_{xz}^{(1)}+f_{xz}^{(1)}\left(x,y,z\right)=0,\quad \sigma_{yz}^{(1)}+f\left(x,y,z\right)=0,$$

$$w_0=w_1+f_1^{(1)}\left(x,y,z\right),\quad w_0=w_2+f_2^{(1)}\left(x,y,z\right),\tag{12}$$

and, at infinity $z=\infty$, perturbation tends to zero.

In (12), the functions $f_j^{(1)}$, $f_z^{(1)}$, $f_{iz}^{(1)}$ are not independent but are determined through the parameters of internal sources, in particular, the parameters of waves from an earthquake.

Conditions (10) for the weakly curved surface $z=F(x,y)$ of the layer, that is, when the derivatives above the first order are neglected while the products of the derivatives of the first order are also neglected, are simplified to take the following form

$$\sigma_{zz}^{(0)}-2\left(F'_x\sigma_{xz}^{(0)}+F'_y\sigma_{yz}^{(0)}\right)=f_n^{(0)}\left(x,y,t\right),$$

$$F'_x\left(\sigma_{zz}^{(0)}-\sigma_{xy}^{(0)}\right)-F'_y\sigma_{xz}^{(0)}+\sigma_{xz}^{(0)}=f_{ns_1}^{(0)}\left(x,y,t\right),$$

$$F'_y\left(\sigma_{zz}^{(0)}-\sigma_{yy}^{(0)}\right)-F'_x\sigma_{xy}^{(0)}+\sigma_{yz}^{(0)}=f_{ns_2}^{(0)}\left(x,y,t\right).\tag{13}$$

The initial conditions of the problem are zero [19–21], that is

$$\frac{\partial\Phi_0}{\partial t}=\Phi_0=\frac{\partial\bar{\Psi}_0}{\partial t}=\bar{\Psi}_0=0,$$

$$\frac{\partial\Phi_j}{\partial t}=\Phi_j=\frac{\partial\bar{\Psi}_j}{\partial t}=\bar{\Psi}_j=0,\quad \left(j=1,2\right),\quad t=0.\tag{14}$$

The displacements u, v, w, the deformations ε_{ij}, and the stresses σ_{ij} in the Cartesian coordinates through the potentials Φ and $\bar{\Psi}$ of the longitudinal and transverse waves are determined from the following formulas from [22–24] for displacements

$$u=\frac{\partial\Phi}{\partial x}+\frac{\partial\Psi_3}{\partial y}-\frac{\partial\Psi_2}{\partial z},\quad v=\frac{\partial\Phi}{\partial y}+\frac{\partial\Psi_1}{\partial z}-\frac{\partial\Psi_3}{\partial x},$$

$$w=\frac{\partial\Phi}{\partial z}+\frac{\partial\Psi_2}{\partial x}-\frac{\partial\Psi_1}{\partial y},\tag{15}$$

for strains

$$\varepsilon_{xx}=\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Psi_3}{\partial x\partial y}-\frac{\partial^2\Psi_2}{\partial x\partial z},$$

$$\varepsilon_{yy}=\frac{\partial^2\Phi}{\partial y^2}+\frac{\partial^2\Psi_1}{\partial y\partial z}-\frac{\partial^2\Psi_3}{\partial x\partial y},$$

$$\varepsilon_{zz}=\frac{\partial^2\Phi}{\partial z^2}+\frac{\partial^2\Psi_2}{\partial x\partial z}-\frac{\partial^2\Psi_1}{\partial y\partial z},\tag{16}$$

$$\varepsilon_{xy}=2\frac{\partial^2\Phi}{\partial x\partial y}+\frac{\partial^2\Psi_1}{\partial x\partial z}-\frac{\partial^2\Psi_2}{\partial y\partial z}+\frac{\partial^2\Psi_3}{\partial y^2}-\frac{\partial^2\Psi_3}{\partial x^2},$$

$$\varepsilon_{xz}=2\frac{\partial^2\Phi}{\partial x\partial z}+\frac{\partial^2\Psi_3}{\partial y\partial z}-\frac{\partial^2\Psi_3}{\partial z^2}+\frac{\partial^2\Psi_2}{\partial x^2}-\frac{\partial^2\Psi_1}{\partial x\partial y},$$

$$\varepsilon_{yz}=2\frac{\partial^2\Phi}{\partial y\partial z}+\frac{\partial^2\Psi_2}{\partial x\partial y}-\frac{\partial^2\Psi_3}{\partial x\partial z}+\frac{\partial^2\Psi_1}{\partial z^2}-\frac{\partial^2\Psi_1}{\partial y^2},$$

for stresses

$$\sigma_{xx}=L\left(\Delta\Phi\right)+2M\left(\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Psi_3}{\partial x\partial y}-\frac{\partial^2\Psi_2}{\partial x\partial z}\right),$$

$$\sigma_{yy}=L\left(\Delta\Phi\right)+2M\left(\frac{\partial^2\Phi}{\partial y^2}+\frac{\partial^2\Psi_1}{\partial y\partial z}-\frac{\partial^2\Psi_3}{\partial x\partial y}\right),$$

$$\sigma_{zz}=L\left(\Delta\Phi\right)+2M\left(\frac{\partial^2\Phi}{\partial z^2}+\frac{\partial^2\Psi_2}{\partial x\partial z}-\frac{\partial^2\Psi_1}{\partial y\partial z}\right),$$

$$\sigma_{xy}=M\left(2\frac{\partial^2\Phi}{\partial x\partial y}+\frac{\partial^2\Psi_1}{\partial x\partial z}-\frac{\partial^2\Psi_2}{\partial y\partial z}+\frac{\partial^2\Psi_3}{\partial y^2}-\frac{\partial^2\Psi_3}{\partial x^2}\right),\tag{17}$$

$$\sigma_{xz}=M\left(2\frac{\partial^2\Phi}{\partial x\partial z}+\frac{\partial^2\Psi_3}{\partial y\partial z}-\frac{\partial^2\Psi_2}{\partial z^2}+\frac{\partial^2\Psi_2}{\partial x^2}-\frac{\partial^2\Psi_1}{\partial x\partial y}\right),$$

$$\sigma_{yz}=M\left(2\frac{\partial^2\Phi}{\partial y\partial z}+\frac{\partial^2\Psi_2}{\partial x\partial y}-\frac{\partial^2\Psi_3}{\partial x\partial z}+\frac{\partial^2\Psi_1}{\partial z^2}-\frac{\partial^2\Psi_1}{\partial y^2}\right).$$

Thus, the exact three-dimensional problem on the motion of a viscoelastic layer of variable thickness lying on a deformable porous water-saturated soil is reduced to solving the integrated-differential equations of motion (4) and (8) in the potentials Φ_0, Φ_1, Φ_2, and $\bar{\Psi}_0$, $\bar{\Psi}_1$, $\bar{\Psi}_2$. The problem is reduced under boundary conditions (12), (13) conditions, under the formulated constraints and zero initial conditions (14).

5. 2. A general solution to the problem on the interaction of the layer with the deformable base

Within the framework of the set goal, the following problem is stated and solved:

– to derive a general solution to the problem based on the classical approximate theory of the interaction of the layer with the deformable base.

In this statement of the problem, the median plane is absent. Therefore, as the desired values, we shall consider the displacements and deformations of the points of the plane $z=0$, which, at $F(x, y)=h$, passes into the median plane of the layer.

In the study of wave processes in linear deformable media or when solving problems of interaction of the layer with the deformable base, mathematical methods are used.

The problem is solved in an exact three-dimensional statement, applying the Fourier transform along the x, y coordinates, as well as the Laplace transform for time t.

A general solution to the problem under zero initial conditions is found, assuming [25–27]

$$\Phi_j=\int_0^\infty\left.\begin{matrix}\sin kx\\-\cos kx\end{matrix}\right\}\mathrm{d}k\int_0^\infty\left.\begin{matrix}\sin qy\\-\cos qy\end{matrix}\right\}\mathrm{d}q\int_{(l)}\Phi_j^{(0)}e^{pt}\mathrm{d}p,$$

$$\Psi_{1j}=\int_0^\infty \left.\begin{matrix}\sin kx\\-\cos kx\end{matrix}\right\}\mathrm{d}k\int_0^\infty \left.\begin{matrix}\cos qy\\ \sin qy\end{matrix}\right\}\mathrm{d}q\int_{(l)}\Psi_{1j}^{(0)}e^{pt}\mathrm{d}p,$$

$$\Psi_{2j}=\int_0^\infty \left.\begin{matrix}\cos kx\\ \sin kx\end{matrix}\right\}\mathrm{d}k\int_0^\infty \left.\begin{matrix}\sin qy\\-\cos qy\end{matrix}\right\}\mathrm{d}q\int_{(l)}\Psi_{2j}^{(0)}e^{pt}\mathrm{d}p,$$

$$\Psi_{3j}=\int_0^\infty \left.\begin{matrix}\cos kx\\ \sin kx\end{matrix}\right\}\mathrm{d}k\int_0^\infty \left.\begin{matrix}\cos qy\\ \sin qy\end{matrix}\right\}\mathrm{d}q\int_{(l)}\Psi_{3j}^{(0)}e^{pt}\mathrm{d}p. \quad (18)$$

however, due to the formulated limitations on the functions of external forces, the functions $\Phi_j^{(0)}$, $\Psi_{ij}^{(0)}$ are negligible outside the domain $|k|\le k_0$, $|q|\le q_0$, $|\mathrm{Im}\,p|\le\omega_0$, and expressions (18) can be differentiated under the sign of the integral.

Substituting (18) in the equations of motion (3) and (8), for $\Phi_j^{(0)}$, $\Psi_{ij}^{(0)}$, we obtained ordinary differential equations [28, 29]

$$\frac{d^2\Phi_0^{(0)}}{dz^2}-\alpha_0^2\Phi_0^{(0)}=0,\ \frac{d^2\Psi_{i0}^{(0)}}{dz^2}-\beta_0^2\Psi_{i0}^{(0)}=0, \quad (19)$$

$$N_1^{(0)}\left[\frac{d^2\Phi_1^{(0)}}{dz^2}-\left(k^2+q^2\right)\Phi_1^{(0)}\right]+$$
$$+Q_0\left[\frac{d^2\Phi_2^{(0)}}{dz^2}-\left(k^2+q^2\right)\Phi_2^{(0)}\right]=$$
$$=\rho^2\left[\rho_{11}\Phi_1^{(0)}+\rho_{12}\Phi_2^{(0)}\right],$$

$$Q_0\left[\frac{d^2\Phi_1^{(0)}}{dz^2}-\left(k^2+q^2\right)\Phi_1^{(0)}\right]+$$
$$+R_0\left[\frac{d^2\Phi_2^{(0)}}{dz^2}-\left(k^2+q^2\right)\Phi_2^{(0)}\right]=$$
$$=\rho^2\left[\rho_{12}\Phi_1^{(0)}+\rho_{22}\Phi_2^{(0)}\right],$$

$$\frac{d^2\Psi_{i1}^{(0)}}{dz^2}-\beta_1^2\Psi_{i1}^{(0)}=0,\quad \Psi_{i2}^{(0)}=\nu_1\Psi_{i1}^{(0)},\quad \nu_1=-\frac{\rho_{12}}{\rho_{22}}, \quad (20)$$

where

$$\alpha_0^2=\left(k^2+q^2\right)+\rho_0\frac{p^2}{N_0^{(0)}},\quad \beta_0^2=\left(k^2+q^2\right)+\rho_0\frac{p^2}{M_0^{(0)}},$$

$$\beta_1^2=\left(k^2+q^2\right)+\rho_1\frac{p^2}{M_1^{(0)}},\quad \rho_1=\left(\rho_{11}\rho_{22}-\rho_{12}^2\right)\rho_{22}^{-1}, \quad (21)$$

$N_j^{(0)}$, $M_j^{(0)}$, Q_0, R_0 are the Laplace-transformed operators N_j, M_j, Q, R.

The transformed values of displacements at the points of the layer and the base are expressed through the following formulas

$$u_0^{(0)}=k\Phi_0^{(0)}-\frac{d\Psi_{20}^{(0)}}{dz}-q\Psi_{30}^{(0)},$$

$$v_0^{(0)}=q\Phi_0^{(0)}+\frac{d\Psi_{10}^{(0)}}{dz}+k\Psi_{30}^{(0)},$$

$$w_0^{(0)}=\frac{d\Phi_0^{(0)}}{dz}+q\Psi_{10}^{(0)}-k\Psi_{20}^{(0)}, \quad (22)$$

$$u_1^{(0)}=k\Phi_1^{(0)}-\frac{d\Psi_{21}^{(0)}}{dz}-q\Psi_{31}^{(0)},$$

$$v_1^{(0)}=q\Phi_1^{(0)}+\frac{d\Psi_{11}^{(0)}}{dz}+k\Psi_{31}^{(0)},$$

$$w_1^{(0)}=\frac{d\Phi_1^{(0)}}{dz}+q\Psi_{11}^{(0)}-k\Psi_{21}^{(0)},$$

$$u_2^{(0)}=k\Phi_2^{(0)}-\nu_1\left(\frac{d\Psi_{21}^{(0)}}{dz}+q\Psi_{31}^{(0)}\right),$$

$$v_2^{(0)}=q\Phi_2^{(0)}-\nu_1\left(\frac{d\Psi_{11}^{(0)}}{dz}+k\Psi_{31}^{(0)}\right),$$

$$w_2^{(0)}=\frac{d\Phi_0^{(0)}}{dz}+\nu_1\left(q\Psi_{11}^{(0)}-k\Psi_{21}^{(0)}\right). \quad (23)$$

Equations (20) can be represented as separated equations. To this end, assume

$$\Phi_1^{(0)}=\varphi^{(0)},\quad \Phi_2^{(0)}=\gamma\varphi^{(0)}. \quad (24)$$

Substituting (24) in equation (20), we obtain

$$\frac{d^2\varphi^{(0)}}{dz^2}-\left(k^2+q^2\right)\varphi^{(0)}=\frac{p^2\left(\rho_{11}+\gamma\rho_{12}\right)}{N_1^{(0)}+\gamma Q_0}\varphi^{(0)},$$

$$\frac{d^2\varphi^{(0)}}{dz^2}-\left(k^2+q^2\right)\varphi^{(0)}=\frac{p^2\left(\rho_{12}+\gamma\rho_{22}\right)}{Q_0+\gamma R_0}\varphi^{0}. \quad (25)$$

Equating the right-hand sides in (25), to find γ, the following algebraic equation is built.

$$\left(\rho_{22}Q_0-\rho_{12}R_0\right)\gamma^2-\left(\rho_{11}R_0-\rho_{22}N_1^{(0)}\right)\gamma-$$
$$-\left(\rho_{11}Q_0-\rho_{12}N_1^{(0)}\right)=0, \quad (26)$$

denoting the roots of equation (26) through γ_1, γ_2, for Φ_1 and Φ_2, we have

$$\Phi_1^{(0)}=\varphi_1^{(0)}+\varphi_2^{(0)},\quad \Phi_2^{(0)}=\gamma_1\varphi_1^{(0)}+\gamma_2\varphi_2^{(0)}, \quad (27)$$

in this case, $\varphi_1^{(0)}$ and $\varphi_2^{(0)}$ satisfy the following equations

$$\frac{d^2\varphi_1^{(0)}}{dz^2}-\alpha_1^2\varphi_1^{(0)}=0,\quad \frac{d^2\varphi_2^{(0)}}{dz^2}-\alpha_2^2\varphi_2^{(0)}=0, \quad (28)$$

where

$$\alpha_j^2=\left(k^2+q^2\right)+\frac{p^2\left(\rho_{12}\gamma_j+\rho_{11}\right)}{N_1^{(0)}+\gamma_j Q_0}.$$

The γ_1 and γ_2 roots of equation (26) are

$$\gamma_{1,2}=\left\{\begin{matrix}\left(\rho_{11}R_0-\rho_{22}N_1^{(0)}\right)\pm\\ \pm\left[\begin{matrix}\left(\rho_{11}R_0-\rho_{22}N_1^{(0)}\right)^2+\\ +4\left(\rho_{22}Q_0-\rho_{12}R_0\right)\left(\rho_{11}Q_0-\rho_{12}N_1^{(0)}\right)\end{matrix}\right]^{1/2}\end{matrix}\right\}\times$$
$$\times\left[2\left(\rho_{22}Q_0-\rho_{12}R_0\right)\right]^{-1}. \quad (29)$$

General solutions to ordinary differential equations (19) and (28) of the second order with constant coefficients are found by known mathematical methods through characteristic equations; these solutions take the following form [30]

$$\Phi_0^{(0)} = A_1 ch(\alpha_0 z) + A_2 sh(\alpha_0 z),$$

$$\Psi_{10}^{(0)} = B_1 sh(\beta_0 z) + B_2 ch(\beta_0 z),$$

$$\Psi_{20}^{(0)} = C_1 sh(\beta_0 z) + C_2 ch(\beta_0 z),$$

$$\Psi_{30}^{(0)} = D_1 ch(\beta_0 z) + D_2 sh(\beta_0 z), \qquad (30)$$

for a layer where A_j, B_j, C_j, D_j are the arbitrary integration constants, and, for a base (under attenuation conditions $z=\infty$),

$$\varphi_1^{(0)} = A_0 e^{\alpha_1 z}, \quad \varphi_2^{(0)} = B_0 e^{\alpha_2 z},$$

$$\Psi_{11}^{(0)} = C_0 e^{\beta_1 z}, \quad \Psi_{21}^{(0)} = D_0 e^{\beta_1 z}, \quad \Psi_{31}^{(0)} = E_0 e^{\beta_1 z}, \qquad (31)$$

In this case, the arbitrary integration constants A_j, B_j, C_j, D_j, C_0, D_0, E_0, due to the solenoidality of (3) and (4), are related through the following dependences

$$kB_j + qC_j + \beta_0 D_j = 0, \quad kC_0 + qD_0 + \beta_1 E_0 = 0, \quad (j=1,2). \qquad (32)$$

For the transformed layer point movements, we have the following representation

$$\begin{aligned} u_0^{(0)} &= k\left[A_1 ch(\alpha_0 z) + A_2 sh(\alpha_0 z)\right] - \\ &-\beta_0\left[C_1 ch(\beta_0 z) + C_2 sh(\beta_0 z)\right] - \\ &-q\left[D_1 ch(\beta_0 z) + D_2 sh(\beta_0 z)\right], \\ v_0^{(0)} &= q\left[A_1 ch(\alpha_0 z) + A_2 sh(\alpha_0 z)\right] + \\ &+\beta_0\left[B_1 ch(\beta_0 z) + B_2 sh(\beta_0 z)\right] + \\ &+k\left[D_1 ch(\beta_0 z) + D_2 sh(\beta_0 z)\right], \\ w_0^{(1)} &= \alpha_0\left[A_1 sh(\alpha_0 z) + A_2 ch(\alpha_0 z)\right] + \\ &+q\left[B_1 sh(\beta_0 z) + B_2 ch(\beta_0 z)\right] - \\ &-k\left[C_1 sh(\beta_0 z) + C_2 ch(\beta_0 z)\right]. \end{aligned} \qquad (33)$$

Expanding the hyperbolic functions (33) into power series for z for the transformed values of the movements of layer points $u_0^{(0)}$, $v_0^{(0)}$, $w_0^{(0)}$

$$u_0^{(0)} = \sum_{n=0}^{\infty}\left\{\begin{aligned} &\left[k\alpha_0^{2n}A_1 - \beta_0^{2n+1}C_1 - q\beta_0^{2n}D_1\right]\frac{z^{2n}}{(2n)!} + \\ &+\left[k\alpha_0^{2n+1}A_2 - \beta_0^{2n+2}C_2 - q\beta_0^{2n+1}D_2\right]\frac{z^{2n+1}}{(2n+1)!} \end{aligned}\right\},$$

$$v_0^{(0)} = \sum_{n=0}^{\infty}\left\{\begin{aligned} &\left[q\alpha_0^{2n}A_1 + \beta_0^{2n+1}B_1 + k\beta_0^{2n}D_1\right]\frac{z^{2n}}{(2n)!} + \\ &+\left[q\alpha_0^{2n+1}A_2 + \beta_0^{2n+2}B_2 + k\beta_0^{2n+1}D_2\right]\frac{z^{2n+1}}{(2n+1)!} \end{aligned}\right\}, \qquad (34)$$

$$w_0^{(0)} = \sum_{n=0}^{\infty}\left\{\begin{aligned} &\left[\alpha_0^{2n+2}A_1 + (qB_1 - kC_1)\beta_0^{2n+1}\right]\frac{z^{2n+1}}{(2n+1)!} + \\ &+\left[\alpha_0^{2n+1}A_2 + (qB_2 - kC_2)\beta_0^{2n}\right]\frac{z^{2n}}{(2n)!} \end{aligned}\right\}.$$

Similar representations for the transformed stress values

$$\sigma_{zz}^{(0)} = M_{10}\sum_{n=0}^{\infty}\left\{\begin{aligned} &\left[\begin{aligned} &\alpha_0^{2n}\left(\beta_1^2 + k^2 + q^2\right)A_1 + \\ &+2\beta_0^{2n+1}\left(qB_1 - kC_1\right) \end{aligned}\right]\frac{z^{2n}}{(2n)!} + \\ &+\left[\begin{aligned} &\alpha_0^{2n+1}\left(\beta_1^2 + k^2 + q^2\right)A_2 + \\ &+2\beta_0^{2n+2}\left(qB_2 - kC_2\right) \end{aligned}\right]\frac{z^{2n+1}}{(2n+1)!} \end{aligned}\right\},$$

$$\sigma_{xx}^{(0)} = $$

$$= M_{10}\sum_{n=0}^{\infty}\left\{\begin{aligned} &\left\{\begin{aligned} &\left[\frac{L_{10}}{M_{10}}\left(\alpha_0^2 - k^2 - q^2\right) - 2k^2\right]\alpha_0^{2n}A_1 + \\ &+2k\beta_0^{2n}\left[\beta_0 C_1 + qD_1\right] \end{aligned}\right\}\times \\ &\times\frac{z^{2n}}{(2n)!} + \\ &+\left\{\begin{aligned} &\left[\frac{L_{10}}{M_{10}}\left(\alpha_0^2 - k^2 - q^2\right) - 2k^2\right]\alpha_0^{2n+1}A_2 + \\ &+2k\beta_0^{2n+1}\left[\beta_0 C_2 + qD_2\right] \end{aligned}\right\}\times \\ &\times\frac{z^{2n+1}}{(2n+1)!} \end{aligned}\right\},$$

$$\sigma_{yy}^{(0)} = $$

$$= M_{10}\sum_{n=0}^{\infty}\left\{\begin{aligned} &\left\{\begin{aligned} &\left[\frac{L_{10}}{M_{10}}\left(\alpha_0^2 - k^2 - q^2\right) - 2q^2\right]\alpha_0^{2n}A_1 - \\ &-2q\beta_0^{2n}\left[\beta_0 B_1 + kD_1\right] \end{aligned}\right\}\times \\ &\times\frac{z^{2n}}{(2n)!} + \\ &+\left\{\begin{aligned} &\left[\frac{L_{10}}{M_{10}}\left(\alpha_0^2 - k^2 - q^2\right) - 2q^2\right]\alpha_0^{2n+1}A_2 - \\ &-2q\beta_0^{2n+1}\left[\beta_0 B_2 + kD_2\right] \end{aligned}\right\}\times \\ &\frac{z^{2n+1}}{(2n+1)!} \end{aligned}\right\}, \qquad (35)$$

$$\sigma_{xy}^{(0)} = M_{10}\sum_{n=0}^{\infty}\left\{\begin{aligned} &\left[\begin{aligned} &2qk\alpha_0^{2n}A_1 + \\ &+\beta_0^{2n+1}\left(kB_1 - qC_1\right) + \\ &+\left(k^2 - q^2\right)\beta_0^{2n}D_1 \end{aligned}\right]\frac{z^{2n}}{(2n)!} + \\ &+\left[\begin{aligned} &2qk\alpha_0^{2n+1}A_2 + \\ &+\beta_0^{2n+2}\left(kB_2 - qC_2\right) + \\ &+\left(k^2 - q^2\right)\beta_0^{2n+1}D_2 \end{aligned}\right]\frac{z^{2n+1}}{(2n+1)!} \end{aligned}\right\},$$

$$\sigma_{yz}^{(0)} = M_{10}\sum_{n=0}^{\infty}\left\{\begin{aligned} &\left[\begin{aligned} &2q\alpha_0^{2n+2}A_1 + \\ &+\left(\beta_0^2 + q^2\right)\beta_0^{2n+1}B_1 - \\ &-k\left(qC_1 - \beta_0 D_1\right)\beta_0^{2n+1} \end{aligned}\right]\frac{z^{2n+1}}{(2n+1)!} + \\ &+\left[\begin{aligned} &2q\alpha_0^{2n+1}A_2 + \\ &+\left(\beta_0^2 + q^2\right)\beta_0^{2n}B_2 - \\ &-k\left(qC_2 - \beta_0 D_2\right)\beta_0^{2n} \end{aligned}\right]\frac{z^{2n}}{(2n)!} \end{aligned}\right\},$$

$$\sigma_{xz}^{(0)} = M_{10}\sum_{n=0}^{\infty}\left\{\begin{matrix}\left[\begin{matrix}2k\alpha_0^{2n+2}A_1 + \\ +q\left(kB_1-\beta_0 D_1\right)\beta_0^{2n+1} - \\ -\beta_0^{2n+1}\left(\beta_0^2+k^2\right)C_1\end{matrix}\right]\dfrac{z^{2n+1}}{(2n+1)!} + \\ +\left[\begin{matrix}2k\alpha_0^{2n+1}A_2 + \\ +q\left(kB_2-\beta_0 D_1\right) - \\ -\beta_0^{2n}\left(\beta_0^2+k^2\right)C_2\end{matrix}\right]\dfrac{z^{2n}}{(2n)!}\end{matrix}\right\},$$

Introduce the following auxiliary functions

$$U_0 = kA_1 - \left(\beta_0 C_1 + qD_1\right),$$

$$U_{10} = k\alpha_0 A_2 - \beta_0\left(\beta_0 C_2 + qD_2\right),$$

$$V_0 = qA_0 + \left(\beta_0 B_1 + kD_1\right),$$

$$V_{10} = k\alpha_0 A_2 + \beta_0\left(\beta_0 B_2 + qD_2\right),$$

$$W_0 = \alpha_0^2 A_1 + \beta_0\left(qB_1 - kC_1\right),$$

$$W_{10} = \alpha_0 A_2 + \left(qB_2 - kC_2\right), \tag{36}$$

which are the coefficients at z in the zero and first powers in decompositions (34), while U_0, V_0, W_{10} are the movements of points of the plane z=0; U_{10}, V_{10}, W_0 are the deformations of these same points in the direction of the z axis.

Expressing the integration constants A_j, B_j, C_j, D_j through U_0, V_0, W_0, U_{10}, V_{10}, W_{10} taking into consideration dependence (32), we obtain

$$A_1 = \frac{W_0 - kU_0 - qV_0}{\alpha_0^2 - k^2 - q^2},$$

$$B_1 = \frac{kq\left(\beta_0^2-\alpha_0^2\right)U_0 + \left(\alpha_0^2\beta_0^2 - k^2\beta_0^2 - q^2\alpha_0^2\right)V_0 - q\left(\beta_0^2-k^2-q^2\right)W_0}{\beta_0\left(\alpha_0^2-k^2-q^2\right)\left(\beta_0^2-k^2-q^2\right)},$$

$$C_1 = \frac{\left(k^2\alpha_0^2 + \beta_0^2 q^2 - \beta_0^2\alpha_0^2\right)U_0 + kq\left(\alpha_0^2-\beta_0^2\right)V_0 + k\left(\beta_0^2-k^2-q^2\right)W_0}{\beta_0\left(\alpha_0^2-k^2-q^2\right)\left(\beta_0^2-k^2-q^2\right)},$$

$$D_1 = \frac{qU_0 - kV_0}{\beta_0^2-k^2-q^2}, \quad A_2 = \frac{\beta_0^2 W_{10} - kU_{10} - qV_{10}}{\alpha_0\left(\alpha_0^2-k^2-q^2\right)}, \tag{37}$$

$$B_2 = \frac{\left(\beta_0^2-k^2-\alpha_0^2\right)V_{10} + q\left(k^2+q^2\right)W_{10} - 2kqU_{10}}{\left(\beta_0^2-k^2-q^2\right)},$$

$$C_2 = \frac{kW_{10} - U_{10}}{\left(\beta_0^2-k^2-q^2\right)}, \quad D_2 = \frac{qU_{10} - kV_{10}}{\beta_0\left(\beta_0^2-k^2-q^2\right)}.$$

Moving in (34) from the integration constants A_j, B_j, C_j, D_j to the auxiliary quantities in (33) taking into consideration condition (32), that is substituting (33) in (34), we obtain

$$u_0^{(0)} = \sum_{n=0}^{\infty}\left\{\begin{matrix}\left[\begin{matrix}\left(\beta_0^{2n} - k^2 C_{10}Q_{1n}^{(0)}\right)U_0 - \\ -kC_{10}Q_{1n}^{(0)}\left(qV_0 - W_0\right)\end{matrix}\right]\dfrac{z^{2n}}{(2n)!} + \\ +\left[\begin{matrix}\left(\beta_0^{2n} + k^2 D_{10}Q_{1n}^{(0)}\right)U_{10} + \\ +kD_{10}Q_{1n}^{(0)}\left(qV_{10} - \beta_0^2 W_{10}\right)\end{matrix}\right]\dfrac{z^{2n+1}}{(2n+1)!}\end{matrix}\right\},$$

$$v_0^{(0)} = \sum_{n=0}^{\infty}\left\{\begin{matrix}\left[\begin{matrix}\left(\beta_0^{2n} - q^2 C_{10}Q_{1n}^{(0)}\right)V_0 - \\ -qC_{10}Q_{1n}^{(0)}\left(kV_0 - W_0\right)\end{matrix}\right]\dfrac{z^{2n}}{(2n)!} + \\ +\left[\begin{matrix}\left(\beta_0^{2n} + q^2 D_{10}Q_{1n}^{(0)}\right)V_{10} + \\ +qD_{10}Q_{1n}^{(0)}\left(kU_{10} - \beta_0^2 W_{10}\right)\end{matrix}\right]\dfrac{z^{2n+1}}{(2n+1)!}\end{matrix}\right\}, \tag{38}$$

$$w_0^{(0)} = \sum_{n=0}^{\infty}\left\{\begin{matrix}\left[\begin{matrix}\left(\beta_0^{2n} + \alpha_0^{2n} C_{10}Q_{1n}^{(0)}\right)W_0 - \\ -\alpha_0^2 C_{10}Q_{1n}^{(0)}\left(kU_0 + qV_0\right)\end{matrix}\right]\dfrac{z^{2n+1}}{(2n+1)!} + \\ +\left[\begin{matrix}\left(\beta_0^{2n} - \beta_0^{2n} D_{10}Q_{1n}^{(0)}\right)W_{10} + \\ +D_{10}Q_{1n}^{(0)}\left(kU_{10} + qV_{10}\right)\end{matrix}\right]\dfrac{z^{2n}}{(2n)!}\end{matrix}\right\},$$

where

$$C_{10} = 1 - N_0^{(0)}\left[M_0^{(0)}\right]^{-1}, \quad D_{10} = 1 - \left[N_0^{(0)}\right]^{-1} M_0^{(0)},$$

$$Q_{1n}^{(0)} = \sum_{n=0}^{\infty}\alpha_0^{2(n-m-1)}\beta_0^{2m}, \quad Q_{10}^{(0)} \equiv 0, \quad Q_{11}^{(0)} \equiv 1.$$

Converting expressions (38) to k, q, p, or moving to the true movements u_0, v_0, w_0 of the points of the layer through the movements and deformations U, V, W, U_1, V_1, W_1 – points of the plane z=0 in the direction of the z coordinate, we obtained [2, 31, 32]

$$u_0 = \sum_{n=0}^{\infty}\left\{\begin{matrix}\left[\begin{matrix}\left(\lambda_2^{(n)} + \dfrac{\partial^2}{\partial x^2}C_1 Q_{1n}\right)U + \\ +C_1 Q_{1n}\dfrac{\partial}{\partial x}\left(\dfrac{\partial V}{\partial y} + W\right)\end{matrix}\right]\dfrac{z^{2n}}{(2n)!} + \\ +\left[\begin{matrix}\left(\lambda_2^{(n)} - D_1 Q_{1n}\dfrac{\partial^2}{\partial x^2}\right)U_1 - \\ -D_1 Q_{1n}\dfrac{\partial}{\partial x}\left(\dfrac{\partial V_1}{\partial y} + \lambda_2^{(1)}W_1\right)\end{matrix}\right]\dfrac{z^{2n+1}}{(2n+1)!}\end{matrix}\right\},$$

$$v_0 = \sum_{n=0}^{\infty}\left\{\begin{matrix}\left[\begin{matrix}\left(\lambda_2^{(n)} + C_1 Q_{1n}\dfrac{\partial^2}{\partial y^2}\right)V + \\ +C_1 Q_{1n}\dfrac{\partial}{\partial y}\left(\dfrac{\partial U}{\partial x} + W\right)\end{matrix}\right]\dfrac{z^{2n}}{(2n)!} + \\ +\left[\begin{matrix}\left(\lambda_2^{(n)} - D_1 Q_{1n}\dfrac{\partial^2}{\partial y^2}\right)V_1 - \\ -D_1 Q_{1n}\dfrac{\partial}{\partial y}\left(\dfrac{\partial U_1}{\partial x} + \lambda_2^{(1)}W_1\right)\end{matrix}\right]\dfrac{z^{2n+1}}{(2n+1)!}\end{matrix}\right\}, \tag{39}$$

$$w_0 = \sum_{n=0}^{\infty}\left\{\begin{matrix}\left[\begin{matrix}C_1 Q_{1n}\lambda_1^{(1)}\left(\dfrac{\partial U}{\partial x} + \dfrac{\partial V}{\partial y}\right) + \\ +\left(C_1 Q_{1n}\lambda_2^{(1)} + \lambda_2^{(n)}\right)W\end{matrix}\right]\dfrac{z^{2n+1}}{(2n+1)!} + \\ +\left[\begin{matrix}\left(\lambda_2^{(n)} - \lambda_2^{(1)}D_1 Q_{1n}\right)W_1 - \\ -D_1 Q_{1n}\left(\dfrac{\partial U_1}{\partial x} + \dfrac{\partial V_1}{\partial y}\right)\end{matrix}\right]\dfrac{z^{2n}}{(2n)!}\end{matrix}\right\},$$

where

$$C_1 = 1 - N_0 M_0^{-1}, \quad D_1 = 1 - N_0^{-1} M_0,$$

$$Q_{1n} = \sum_{m=0}^{n-1} \lambda_1^{2(n-m-1)} \lambda_2^{(m)},$$

$$\lambda_2^{(1)} = \left[\rho_1 M_0^{-1} \left(\frac{\partial^2}{\partial t^2} \right) - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} \right],$$

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}.$$

In this case, the $\lambda_1^{(1)}$ and $\lambda_2^{(1)}$ operators are the two-dimensional integrated-differential equations of the propagation of longitudinal and transverse waves in the plane $z=0$.

Expressions (39) have been derived only from solving the equation of motion (4) under zero initial conditions and are the general solutions to the problem about the layer.

To find unknown U, V, W, U_1, V_1, W_1 and the base parameters A_0, B_0, C_0, D_0, E_0, we have the boundary conditions (12) and (13).

6. Discussion of results of studying the stressed-strained state of the material with a porous water-saturated base under dynamic load

This study has stated the problem, constructed mathematical models of the interaction of the material (for the viscoelastic beam, plates, slabs, strips) with a deformable base under dynamic loads. The problem is reduced to solving the integrated-differential equations of motion (4) and (8) in the potentials Φ_0, Φ_1, Φ_2 and $\bar{\Psi}_0$, $\bar{\Psi}_1$, $\bar{\Psi}_2$ under the boundary conditions (12), (13), under formulated constraints and zero initial conditions (14).

We have considered the problem of propagation of a plastic wave in a two-layer medium with a flatly parallel interface boundary under the influence of an intense load of a falling profile moving along its upper boundary with a constant over seismic speed D.

The two-layer medium consists of a soft layer of soil with a thickness of h with an elastic deformable base. The soil is modeled by an inelastic ideal medium with linear compressibility and linear irreversible unloading. Consequently, the resistance of the medium to shear forces is neglected. According to this statement, the influence of deformation of the base and load profile on the distribution of dynamic parameters of the layer and the contact surface has been investigated. Let a monotonously decreasing normal load with speed D moves along the upper boundary of the layer with an elastic base; the exceeding speed of the propagation of waves does not change. The material of the layer has such a property that when loading and unloading, the relationship between the pressure P and the volumetric deformation ε is linear and irreversible, the angle of inclination E_2 of the unloading branch of the P~ε diagram exceeds the angle of inclination E_1 of the loading branch, that is $E_1<E_2$.

Based on the results of studying the construction of mathematical models of wave propagation in a multilayer, in particular heterogeneous, half-space, taking into consideration irreversible processes within the framework of an ideal nonlinear-compressible and linear-elastic medium, the following conclusions can be drawn:

1. The problem has been stated and an analytical solution to the problem of the propagation of a plastic wave in half-space has been constructed for the case where the dependence between pressure and volumetric deformation during loading and unloading is linear but different. Based on the analysis of the results, it has been shown that if the mobile load acting at the boundary of the half-space has a monotonous-decreasing profile, then the medium is unloaded in the perturbation region, and the oblique compression wave is obtained by the load-unloading wave. The pressure of the medium against the background of this wave, depending on the depth of the half-space, decreases slower than on the free surface. In the case when the relationship between P and when the medium is loaded is taken to be nonlinear and impact, which corresponds to the propagation in the medium of a two-dimensional shock wave, the pressure in the perturbation region, compared with the linear case, is somewhat overestimated.

2. We have investigated the problem of propagation of a plastic wave in a two-layer medium with densities ρ_1, ρ_2 for the case when the state diagram $P=P(\varepsilon)$ of the first medium (soil) is impact-induced. Additionally, when loading, it takes the form $P(\varepsilon)=a_1\varepsilon+a_1\varepsilon^2$, and the second medium (black rock or gasket) is elastic or rigid plastic. The problem is solved analytically by both direct and inverse methods, taking into consideration wave processes in the second medium and without taking them into consideration.

Thus, our study of the stressed-strained state of a medium under the influence of mobile load confirms the need and importance of taking into consideration nonlinear, irreversible, wave processes. Clarification of the accounting for nonlinearity, irreversibility of the object under study during wave processes is the main direction for the mechanics of deformable media. In our work on studying the stressed-strained state of the medium under the influence of mobile load, when considering the object with a porous water-saturated viscoelastic medium, we have determined the scope and boundaries of the applicability of the results, the conditions that must be met, as well as the solution robustness.

7. Conclusions

1. We have stated the problem of the interaction of the material (for beams, plates, slabs, strips) with the deformable base. The material (of the viscoelastic beam, plate, slab, strip) lying on a porous water-saturated viscoelastic base is considered as a viscoelastic layer of the same geometry. It was assumed that the lower surface of the layer is flat while the upper surface is not flat and is given by the equation $z=F(x,y)$. Thus, the exact three-dimensional problem of the motion of the viscoelastic layer of variable thickness lying on the deformable porous water-saturated soil is reduced to solving the integrated-differential equations of motion (4) and (8) in the potentials Φ_0, Φ_1, Φ_2 and $\bar{\Psi}_0$, $\bar{\Psi}_1$, $\bar{\Psi}_2$. It has a solution under the boundary conditions (12), (13), under formulated restrictions and zero initial conditions (14).

2. The equations for a general solution to the problem of interaction of the layer with the deformable base have been derived. We have built the classical approximate equations of the theory of interaction of a layer with a deformable base, based on Kirchhoff's hypothesis. Using the well-known hypotheses by Timoshenko and others, the general three-dimensional problem is reduced to a two-dimensional one relative to the displacements of points of the median plane of the layer, which imposes restrictions on external

conditions. In the present problem, there is no median plane. Therefore, the displacement and deformation of the points of the plane z=0, which, at $F(x, y)=h$, passes into the median plane of the layer, have been considered as the desired quantities. Wave processes in the linear deformable media, as well as in solving problems of interaction of the layer with the deformable base, have been investigated using mathematical methods. The derived general equations of the interaction of the layer with the deformable base are complex in structure, contain derivatives of any order for coordinates and time.

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