

*This paper reports the assessment of the influence of dynamic motion parameters on the formation and disappearance at the cylindrical surface of the chamber of the rotating drum of the near-wall layer of non-loose granular fill.*

*Based on the results of experimental visualization of the flow, the effect of solidity on the behavior of granular fill was revealed. The hydrodynamic effect of fill quasi-liquefaction under the action of solidity has been established, which involves the occurrence of a connecting interaction between adjacent layers and the surface of the chamber. Conversion of shear circulation flow to homogeneous dense clustered stream with slipping and rolling without relative movement of particles was detected.*

*The hydrodynamic characteristics of circulation flow transition to the near-wall layer mode during rotation acceleration have been defined. Such a transition is implemented by smoothly increasing the thickness of the layer when the rest of the fill is circulated at the bottom of the chamber.*

*The effect of the rheological hysteresis of the movement of the rotating chamber fill, caused by quasi-liquefaction of non-loose granular environment, has been established. The effect implies exceeding the speed limit  $\omega_{fl}$  in the formation of a near-wall layer, at rotation acceleration, above the boundary  $\omega_{dl}$  of the layer disappearance when the rotation slows down. The manifestation of hysteresis mainly increases with an increase in Reynolds number. The intensity of increased hysteresis manifestation increases with a decrease in the degree of filling the chamber. The value of the Froud number for the  $\omega_{fl}$  and  $\omega_{dl}$  boundaries increases with the increase in  $Re$ . It has been established that at the relative particle size of the dispersed fill  $\psi_{dc} \approx (0.065-1.04) \cdot 10^{-3}$  and  $Re=30-500$ ,  $Fr=1-2.9$ , for the  $\omega_{fl}$  boundary, and  $Fr=0.5-1.4$ , for the  $\omega_{dl}$  boundary. The  $Fr$  value for the  $\omega_{fl}$  limit was found to exceed this value for the  $\omega_{dl}$  boundary by 1.6–2.1 times.*

*The established effects make it possible to substantiate the rational parameters for the grinding process in drum-roll mills*

*Keywords: drum-roll mill, non-loose granular fill, formation of a wall layer, rheological hysteresis*

# ESTABLISHING THE CONDITIONS FOR THE FORMATION OF A NEAR-WALL LAYER OF SOLID GRANULAR FILL OF A ROTATING DRUM

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## 1. Introduction

Drum-type machines are widely used in many industries to process various flowing materials. The versatility of such equipment relates to the possibility of performing various technological processes when varying operational parameters.

The motion modes of the medium processed in a rotating drum chamber are determined by the ratio of gravitational and distorted, due to relative movement, centrifugal force field. Under the condition of a low rotational speed, the action of the gravitational field prevails, which causes the circulating mode of movement of the medium at the bottom of the chamber. At high angular velocity, the centrifugal field generates a near-wall motion mode on the cylindrical surface of the chamber.

To implement a series of drum machine workflows, the conditions for switching modes of movement of the processed medium are decisive. These conditions are especially relevant for drum-roll mills (Horomill) [1]. Such grinding implies the action of pressure, with the help of a crushing roll, on

the free surface of the near-wall layer of crushed material on the cylindrical surface of a horizontally rotating chamber. At the same time, the determining parameter of the process implementation is the value of the drum rotation speed. At a low speed value, the formation of a near-wall layer of the material is not completed, causing a decrease in productivity and grinding thinness. Instead, with an inflated speed value, the dynamic load on the equipment and the energy intensity of the drum rotation drive increase. It appears that the rational one is the minimum value of the rotation speed, which is necessary and sufficient for the formation and preservation of a stable near-wall layer of material during grinding.

At the same time, the conditions of transition of motion modes are established only for the most common types of processed media – non-loose granular material and Newtonian liquid. Instead, in drum-roll mills, mainly fine granular materials are crushed, which show the properties of solidity. The rheological behavior of such a medium in a rotating chamber is not yet sufficiently elucidated and the conditions for switching current modes are uncertain.

The task of predicting the effect of dynamic motion parameters on the conditions of formation and disappearance on the surface of the rotating chamber of a solid granular fill appears to be quite relevant.

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## 2. Literature review and problem statement

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The results of studies into the behavior of solid granular fill of the rotating drum chamber are limited to the consideration of features of the effect of particle adhesion on the circulation mode of movement.

To solve this task, several studies applied numerical methods. The authors of [2] used a discrete element method (DEM) to investigate the effect of adhesion between particles on patterns of fill movement in the cross-section of the chamber. It has been shown that an increase in solidity causes the occurrence of surface tension of granular fill. At the same time, a decrease in concavity, the trend to the flat, and the occurrence of a convex shape of a free fill surface were established. An increase in the thickness of the shear layer and the distance from the free surface to the circulation core was detected. The transition of the shear circulation mode to the rolling mode has been observed. DEM with experimental verification was used in [3] to assess the effect of solidity on fill motion modes. Increased convexity of the free surface shape, the increase in the thickness of the shear layer, and the transition from shear circulation to rolling have been established. The authors of [4, 5] employed DEM and experimentally detected the transition of the fill movement mode with periodic collapse into continuous rolling. A decrease in the mutual pushing of particles due to the dissipation of kinetic energy with increased solidity was established. An increase in the uniformity of the granular flow due to the increased density of particle packing was detected. The free movement of individual particles, due to the clustering of the flow, is converted to the movement of clustered groups. Due to increased internal resistance of the current, the speed of movement and shear of the flow decreases. Motion mode with periodic collapse, at low rotational speed, is converted to continuous rolling. The thickness of the shear layer and the concavity of the free surface have been established. The effect of solidity on the modes of movement of wet granular fill was studied by DEM in [6]. It was found that with increased solidity, the thickness and uniformity of the shear layer increase while the flow rate decreases, as well as the media pseudo-temperature and the reciprocal pushing of particles.

The effect of solidity on fill motion modes was also studied experimentally. The authors of [7] investigated patterns of the granular-fluid mixture movement under the periodic collapse mode during the slow rotation of the chamber. The mode of the periodic collapse of the free surface at slow rotation was considered. They established a decrease in the angle of inclination of the free surface and an increase in the thickness of the shear layer with a significant increase in solidity. The position of the free surface and the nature of the circulation of the center of masses were studied in [8]. Sliding slip conversion of fill into the slip with clutch rolling has been detected. The formation of microcracks in a solid fill was established, caused by shear stresses. The authors of [9] experimentally detected the transition of the shear circulation mode of non-loose fill into the plunger flow from rolling without the relative movement of particles. At the same time, the thickness of the shear layer increases, the mobility of the

particles decreases, and the concave shape of the free surface turns almost flat. The effect of adhesion on the movement of wet granular fill was experimentally investigated in [10]. Increased internal movement resistance and decreased flow shear with increased solidity were detected. A decrease in the push between particles due to the dissipation of kinetic energy has been established. The increase in shear layer height, decrease in flow rate and amplification of the concave shape of a free surface are shown.

However, the data reported did not make it possible to establish the conditions for the formation of a near-wall layer of solid granular fill on the surface of the rotating drum chamber.

The manifestation of the rheological properties of the non-loose granular material as the medium processed by drum-type machines seems rather complicated. However, it can be assumed that its hydrodynamic behavior is similar to that of simpler, in terms of rheological characteristics, solid granular material, and Newtonian fluid. Instead, the predicted similar flow modes in the rotating chamber of separately non-loose granular and liquid fill are relatively well understood.

There are known attempts to determine the speed of rotation of the drum in the formation and disappearance of the near-wall layer of non-loose granular fill on the smooth cylindrical surface of the chamber. The authors of [11] experimentally established a significant increase in such a rotational speed with a reduction in the degree of filling the chamber. Based on the experiment and numerical modeling by DEM, the authors of [12] proposed to take into consideration the effect of the angle of natural loading slope on the maximum rotational speed. Two rotation speed limits were established – at the beginning and end of the formation or destruction of the near-wall layer. The authors of [13] experimentally found the effect of the ratio of the density of the liquid and solid phase of two-fraction fill on the rotational speed in the formation of a near-wall layer. The method of numerical hydrodynamic modeling was used in [14] to assess the effect of drum length and the nonsphericity of the fill particle shape on the value of the maximum rotational speed. However, there were no reliable conditions for the formation of a near-wall fill layer on the chamber surface.

Instead, the nature of the established motion mode of non-loose granular fill in the cross-section of the chamber of a stationary rotating drum is three-phase [15]. In the pattern of movement in the cross-section of the chamber, three zones of the current can be distinguished: solid-state, non-free fall, and shear layer. The solid-state zone [16] is characterized by quasi-solid motion without the relative movement of fill particles along with the cylindrical surface of the chamber. In the zone of non-free fall, particles are separated from the solid-state zone, followed by a fall with interaction. In the shear zone [17], there is a rapid gravitational fill flow near a free surface.

The implementation of the three zones of the current is determined by the speed of rotation of the drum  $\omega$ . At slow rotation, the solid-state zone prevails, the zone of non-free falling is weakly expressed, and the zone of a sliding layer has insignificant thickness. As  $\omega$  increases, the particles of the non-free fall zone and the shear layer increase due to the solid-state zone. With the subsequent increase of  $\omega$ , the share of the non-free fall zone reaches the maximum value, and the shear layer disappears. When  $\omega$  approaches the limit value, the non-free fall zone passes into a solid-state near-wall layer.

The characteristic modes of movement of the non-loose fill of the rotating drum (Fig. 1) are characterized by combinations of the three specified zones of the current [18].

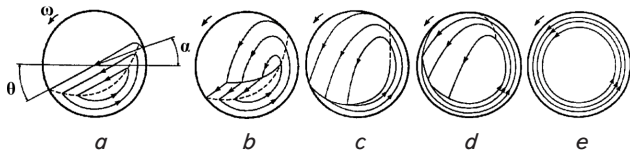


Fig. 1. Schemes of motion modes of non-loose granular fill of the rotating drum chamber with an increase in rotation speed  $\omega$  (from [18]): *a* – mode without tossing; *b* – mode with partial tossing; *c* – full-toss mode; *d* – incomplete centrifugation mode; *e* – near-wall layer mode (centrifugation);  $\theta$  – the average angle of inclination of the free fill surface to the horizontal;  $\alpha$  – fill angle lift

Under the motion mode without tossing (Fig. 1, *a*) the solid-state and shearing zones occur. Under the partial tossing mode (Fig. 1, *b*), all three zones are implemented. Under a full-toss mode (Fig. 1, *c*), the shear layer disappears. Under an incomplete centrifugation mode (Fig. 1, *d*), a near-wall layer begins to form with a gradual decrease in the non-free fall zone. The shear layer mode (Fig. 1, *a*) is characterized only by a solid-state zone. The formation and disappearance on the surface of the chamber of the rotating drum of the near-wall layer of non-loose granular fill are associated with mutual transitions of modes at full tossing (Fig. 1, *c*), incomplete (Fig. 1, *d*) and full (Fig. 1, *e*) centrifugation.

The authors of [18] experimentally determined the conditions for the existence of a near-wall layer of non-loose granular material in a wide range of variations in system parameters. Two-dimensional steady fill movement was considered in the cross-section of the cylindrical chamber of radius  $R$  of the drum permanently rotating around the horizontal axis at angular velocity  $\omega$ . As dynamic similarity criteria, the Froud  $Fr$  and Reynolds  $Re$  numbers on the cylindrical surface of the chamber were taken.  $Fr = (\omega^2 R) / g$  and  $Re = (\omega R^2) / \nu$  ( $g$  – gravitational acceleration,  $\nu$  – analog of the kinematic viscosity of granular fill) were adopted as expressions for dynamic criteria. The geometric and frictional characteristics of the non-loose granular material used for the experiment approximately corresponded to the parameters of intra-chamber fill of drum-type machines. Comparative analysis with patterns of Newtonian fluid flow in the rotating chamber made it possible to approximately estimate the value of the analog of the kinematic viscosity of granular fill –  $\nu \approx 10^{-3} \text{ m}^2/\text{s}$ . As a geometric criterion of similarity, the degree of filling the chamber with loading  $\kappa = \omega / (\pi R^2 L)$  ( $\omega$  – the volume of granular fill at rest,  $L$  – the length of the chamber) was taken.

Fig. 2 shows in the coordinates  $Re$  and  $Fr$  for four discrete values  $\kappa$  the experimental boundaries of transition modes of movement of non-loose granular fill [18].

The boundary  $\omega_{fbdl}$  corresponds to the beginning of the formation of the near-wall layer with an increase in the rotational speed  $\omega$  and, at the same time, the completion of the destruction of the layer while reducing  $\omega$ . The boundary  $\omega_{bfedl}$  corresponds to the mutual transition of the modes of motion with full toss (Fig. 1, *c*) and incomplete centrifugation (Fig. 1, *d*). The zone above the  $\omega_{fbdl}$  boundary corresponds to the mode of incomplete centrifugation (Fig. 1, *d*), and below – the mode with full tossing (Fig. 1, *c*). The position of the  $\omega_{bfedl}$  bound-

ary practically does not depend on the degree of filling the chamber with loading  $\kappa$ .

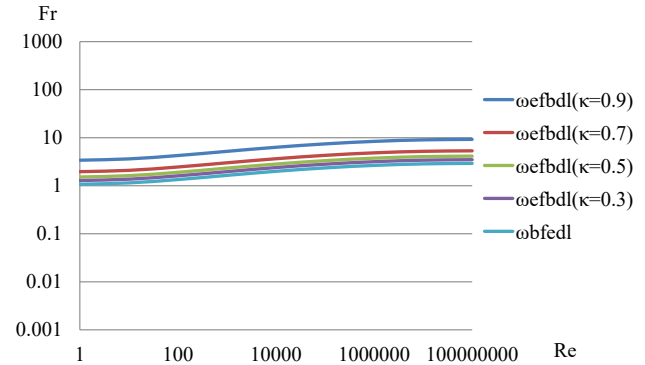


Fig. 2. Experimental boundaries of transition of motion modes of non-loose granular fill of rotating drum for  $\kappa=0.3; 0.5; 0.7; \text{ and } 0.9$  (from [18]):  $\omega_{bfedl}$  – the limit of mutual transition of motion modes with the full toss and incomplete centrifugation;  $\omega_{fbdl}$  – the limit of mutual transition of incomplete centrifugation and near-wall layer modes

The  $\omega_{fbdl}$  boundary corresponds to the completion of the formation of the near-wall layer with an increase in the rotational speed  $\omega$  and, at the same time, the beginning of the destruction of the layer while reducing  $\omega$ . The  $\omega_{bfedl}$  boundary corresponds to the mutual transition of incomplete centrifugation modes (Fig. 1, *d*) and the near-wall layer (centrifugation) (Fig. 1, *e*). The position of the  $\omega_{fbdl}$  boundary is determined by the value  $\kappa$ . A zone above the  $\omega_{fbdl}$  boundary corresponds to the movement mode in the form of a near-wall layer (Fig. 1, *e*), and below – the incomplete centrifugation mode (Fig. 1, *d*).

The authors of [18], for comparison, provided the conditions for the existence of the near-wall layer of the liquid filling of the chamber in a wide range of variations of system parameters.

Fig. 3 shows in the coordinates  $Re$  and  $Fr$  for five discrete values  $\kappa$  the boundaries of transition of flow modes established by an experimental-analytical method. The boundary  $\omega_{fl}$  corresponds to the formation of a near-wall layer at the acceleration of the rotation of the drum. The  $\omega_{dl}$  boundary corresponds to the destruction of the layer when the rotation slows down.

When comparing the results from determining the boundaries of the transition of motion modes in a rotating chamber of non-loose granular material and Newtonian liquid, their fundamental differences were found. They involve the occurrence of a hysteresis transition of fluid flow modes and the practical absence of such a transition for loose material.

The values of the boundaries of the transition of motion modes of these two common rheological types of processed media depend significantly on  $Re$  and  $\kappa$ . The comparative analysis of Fig. 2, 3 testified to the identity of individual conditions for the existence of the near-wall layer of these two media when  $Re$  approaches the limit values [18]. At  $Re \rightarrow \infty$ , the  $\omega_{fbdl}$  limits for granular material and  $\omega_{dl}$  for liquid comply with  $Fr \rightarrow 3 / (1 - \kappa)^{0.5}$ , and the boundary  $\omega_{bfedl}$  for granular material –  $Fr \rightarrow 3$ . At  $Re \rightarrow 0$ , the  $\omega_{fbdl}$  boundary for granular material and the estimated limit for liquid, according to [19, 20], meet the condition  $Fr \rightarrow 1 / (1 - \kappa)^{0.5}$ , and the  $\omega_{bfedl}$  limit – for granular material –  $Fr \rightarrow 1$ .

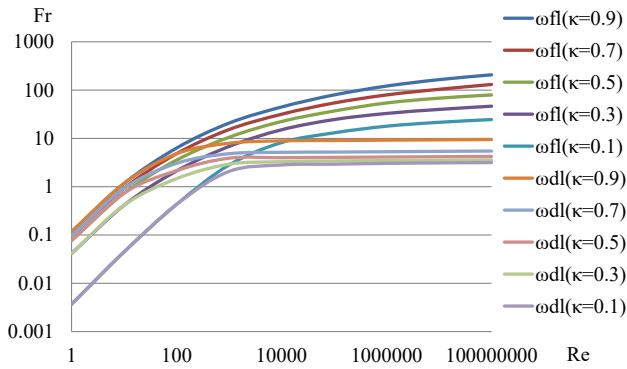


Fig. 3. Experimental-analytical boundaries of transition of Newtonian fluid flow modes in a rotating chamber for  $\kappa=0.1; 0.3; 0.5; 0.7; \text{ and } 0.9$  (from [18]):  $\omega_{fl}$  – the boundary of formation of the near-wall layer mode at rotation acceleration;  $\omega_{dl}$  – the limit of destruction of the near-wall layer mode when the rotation slows down

Therefore, the rheological hysteresis of fluid flow has a pronounced manifestation, and the possible friction hysteresis of the movement of loose granular fill is little noticed. Instead, the technological application of the rheological hysteresis transition of motion modes in the formation of the near-wall layer of the processed intra-chamber fill seems quite relevant.

The first known experimental analysis of the conditions of transition of the flow modes of low-viscous Newtonian liquid with a small filling of the rotating drum chamber is given in [21, 22]. The authors established the phenomenon of active manifestation of rheological hysteresis of viscous current, which accompanies the formation and disappearance of the near-wall layer on the cylindrical surface of the chamber. It turned out that the speed value, when the rotation and formation of the layer are accelerated, can significantly exceed this value when the rotation and destruction of the layer are slowed down. Work [23] reported a numerical calculation with experimental verification of the stability conditions of the thin near-wall layer of real liquid on the surface of the rotating chamber. It has been shown that with increasing viscosity, the rheological hysteresis of flow modes may not occur. The discrepancy between the results of the analytical model and experimental data regarding the conditions of existence of the liquid near-wall layer in the case of viscosity variation was found in [24]. The authors of [25] performed a numerical calculation with experimental verification of the rheological hysteresis of high and low viscosity liquid. The discrepancy between numerical and experimental data for the conditions of existence of the near-wall layer in the case of low-viscous liquids was detected. The experimental analysis of flow modes in [26] showed the absence of a manifestation of rheological hysteresis in the case of a small degree of filling the chamber and high viscosity. The authors of [27] experimentally established a significant effect exerted on the manifestation of hysteresis of the viscous current of the degree of filling the chamber and internal friction of the medium.

The comparative analysis of the analytical and experimental results [28] made it possible to identify features of the manifestation of the rheological hysteresis of the Newtonian current in the chamber of the rotating drum. The authors compared in the coordinates Re and Fr the plots of the boundaries of the transition of flow modes for  $\kappa=0.5$  (Fig. 4), obtained analytically in [29] and [19, 20] and experimentally.

Inclined bar lines correspond to the chambers of the rotating drum of an unchanging radius.

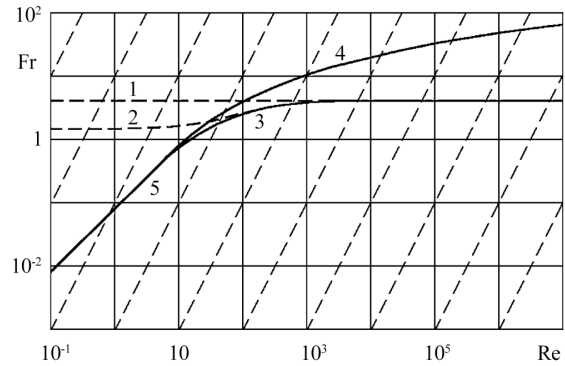


Fig. 4. The diagram of transition of Newtonian liquid flow modes in the chamber of the rotating drum for  $\kappa=0.5$  (from [28]): 1 – the estimated limit of destruction of the near-wall layer of the ideal completely non-viscous liquid when the rotation slows down, from [29]; 2 – the estimated limit of destruction of the real liquid layer when the rotation slows down, from [19, 20]; 3 – the experimental boundary of destruction of the layer during the rotational slowdown; 4 – the experimental boundary of layer formation at the acceleration of rotation; 5 – a common experimental boundary for the formation of a layer, while accelerating rotation, or destroying a layer, while the rotation is slowing down

An increase in the manifestation of rheological hysteresis with an increase in the degree of filling the chamber with liquid  $\kappa$  was revealed in [28]. In addition, the authors of [30] established a significant influence exerted by Reynolds number on the current modes in the rotating chamber. In particular, an intensive manifestation of hysteresis was experimentally registered in the case of large Re [28] and the jump-like transition of flow modes (plots 3, 4 in Fig. 4). In this case, there are wave-like disturbances of a free surface during the destruction of the layer. Instead, hysteresis does not appear at small Re. At the same time, secondary circulating currents are formed in the form of a roller [31] (Fig. 5) and smooth implicitly pronounced transitions of motion modes (plot 5 in Fig. 4).

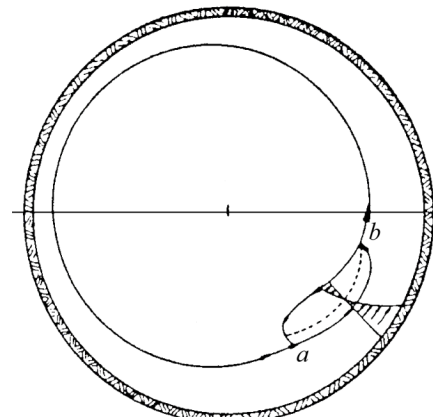


Fig. 5. Schematic of the pattern of movement in the cross-section of the rotating chamber during the evolution of the secondary circulation flow in the form of a roller on the free surface of the near-wall layer (from [31])

In this case, there is a significant deformation of the free surface when the layer is formed or disappeared and the boundaries of the mutual transitions of the modes coincide.

However, the obtained results of the manifestation of hysteresis transitions of motion modes relate only to the single-phase viscous liquid fill medium of the rotating chamber. Instead, the manifestation of the possible hysteresis of the high-speed boundaries of formation and disappearance of the near-wall layer of the two-phase granular fill remains unclear.

However, it was found that the behavior of granular fill of the rotating drum has a pronounced unstable character [32]. Such instability is often manifested in the occurrence of clustered groups of particles in the form of striped formations. In work [33], the manifestation of friction hysteresis was observed during the transition of the movement of loose granular material from an unstructured state to a structured one. With a small  $\kappa$ , a high-speed hysteresis mutual transition of motion modes in the form of a homogeneous near-wall layer and with radial-symmetrical rings was detected.

An overview of the implementation of nonlinear transitions of various motion modes of loose granular fill of the rotating chamber, mainly in application to drum mills, is given in [34]. It is shown that such transitions are implemented in the form of friction hysteresis, due to the manifestation of complex, far from studied, rheological properties of the intra-chamber medium.

One of the manifestations of friction hysteresis was investigated, which involves the transition of the angles of inclination of the free fill surface of the slow-rotating chamber before and after the collapse. The numerical and experimental analysis of the implementation of such a hysteresis transition at a low rotational speed was carried out in [35]. The results of numerical modeling and experiments of hysteresis transition of the angles of inclination of the free surface at the beginning and after the completion of the collapse were compared in [36]. It has been shown that such friction transitions are caused by the emergence of solid-state, quasi-liquefied, and quasi-gaseous rheological states in certain loading zones. It was found that the implementation of individual rheological states is due to the rate of shear flows of the loose granular fill of the rotating drum.

Another manifestation of the friction hysteresis of the movement of the loose granular fill of the rotating drum was also considered. It involves the mutual transition of motion modes with the periodic collapse of the free surface and continuous flow at a considerable speed of rotation. The authors of [37] established the manifestation of such hysteresis based on the use of a numerical model describing the partial quasi-liquefaction of the granular medium. An experimental study of this hysteresis was reported in [38]. The authors of [39] experimentally studied the effect of the geometrical and physical properties of fill particles on the manifestation of hysteresis. The experimental analysis of differences in the implementation of such hysteresis in dry granular materials and suspensions is given in [40]. It has been shown that the manifestation of hysteresis in suspensions has a frictional rheological character and is caused by inter-part friction.

The authors of [41] experimentally studied the speed limits of the existence of a near-wall layer of loose granular fill of the rotating drum chamber. The influence of physical properties, sphericity, and density of particle material, and internal and external friction was considered. Friction hysteresis of the rotational speed value was registered during

the formation or destruction of the fill near-wall layer, depending on the direction of change in speed. The speed limit at direct transition, during acceleration of rotation, exceeds this value in reverse transition, during the slow rotation. At the same time, the value of the limit speed of drum rotation  $\omega$  exceeds the critical value  $\omega_c = (g/R)^{0.5}$ . The dependence of the manifestation of hysteresis on the physical properties, sphericity, and density of particle material, as well as internal and external friction of fill, has been established. Increased manifestation of hysteresis with a decrease in surface roughness, an increase in the nonsphericity of the shape of particles, and a decrease in fill friction were detected. However, with a degree of filling the chamber with a load exceeding 0.3, the manifestation of hysteresis was weakly expressed. In this case, the difference between the speed limits of the formation and destruction of the near-wall layer was notably small. However, the issues related to the manifestation of the rheological hysteresis of the movement of the solid granular fill of the rotating chamber remain unresolved. The reason for this is the increased difficulty in taking into consideration the adhesion properties of particles and the surface of the chamber, which cause ambiguous behavior of such a medium.

Several works investigated manifestations of rheological hysteresis transitions of individual modes of movement of the solid granular fill of the rotating drum. The manifestation of hysteresis of the transition of the angles of inclination of the free fill surface of a slowly rotating chamber before and after the collapse was experimentally studied in [42]. The significant impact of particle adhesion on the dynamics of the current and such manifestation of hysteresis was shown. Increasing solidity increases the angle of inclination of the free surface and the thickness of the layer collapsed and increases the dissipation of the kinetic fill energy. Hysteresis transitions of motion modes with the periodic collapse of the free surface and continuous flow at a considerable rotational speed were experimentally investigated in [43, 44]. It has been shown that the increase in particle adhesion increases the manifestation of hysteresis due to the emergence of the phenomenon of clustering of the granular stream. However, the effect of rheological hysteresis on the conditions of existence in the rotating chamber of the near-wall fill, due to objective difficulties, was not clarified.

Thus, the obtained data from analytical and numerical modeling and experiments showed the similarity of the behavior of the solid granular fill of the rotating drum with the currents of other mediums. Close in modes of movement are the loose granular material and Newtonian liquid, the behavior of which in the chambers of drum-type machines is quite well studied. Similar to these three types of media are the circulating motion modes and manifestations of hysteresis regime transitions. However, these results apply only to small rotational speeds and low dynamics of the interaction process of the medium with the drum chamber. In addition, the effect of the adhering properties of particles on the modes of movement of non-loose granular fill remains unexplained due to the increased complexity of the manifestation of rheological characteristics.

No models have been built to determine the effect of adhesion on the conditions for the formation of a near-wall layer of the solid granular fill. This is due to the insurmountable difficulties of analytical and numerical modeling and the increased complexity of the hardware experimental investigation of the behavior of the granular fill of a high-speed rotating drum. The lack of such models is especially negative

in the case of the implementation of the innovative grinding process in drum-roll mills.

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### 3. The study materials and methods

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The purpose of this work is to determine the effect of dynamic motion parameters on the conditions of formation and destruction on the surface of the chamber of a near-wall layer of the solid granular fill of the rotating drum. That would make it possible to predict the effectiveness of implementing the processing of solid granular materials in drum-type machines with the formation of a near-wall layer of the intra-chamber fill.

To accomplish the aim, the following tasks have been set:

- to identify the hydrodynamic characteristics of the flow of solid granular fill in the chamber of a rotating drum during the mutual transition of circulation and near-wall motion modes;

- to estimate the values of the dynamic parameters of movement that meet the conditions for the formation and disappearance of the near-wall layer of solid granular fill on the surface of the rotating drum chamber.

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### 4. The study materials and methods

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#### 4. 1. Procedure for identifying the hydrodynamic characteristics of the flow of solid granular fill in the formation of a near-wall layer

As a research method, experimental physical visualization of motion patterns of the main types of flowing fill of the rotating drum chamber was adopted. With the help of video recording through the transparent end wall of the chamber, patterns of the steady fill movement were recorded in a stationary rotating drum. The two-dimensional current in the plane, which is perpendicular to the axis of the chamber, was considered. It was believed that the influence of the end walls on the fill movement can be neglected since the length of the cylindrical chamber significantly exceeds its length.

Circulation motion modes were studied, before the formation of the near-wall layer, for three types of flowing fill – loose granular, Newton liquid, and solid granular. To compare the peculiarities of flow modes caused by the difference in the rheological properties of media, the same geometric and dynamic conditions of movement were implemented. The drum chambers had the same  $R$  radius and a close degree of filling with load  $\kappa=0.4-0.5$ . The kinematic viscosity of the liquid modeling Newton liquid fill was  $\nu_l=10^{-3}$  m<sup>2</sup>/s. The approximate value of the accepted analog of kinematic viscosity of the used loose and solid granular fill was approaching the same value –  $\nu_g\approx 10^{-3}$  m<sup>2</sup>/s. A capillary viscosimeter was used to measure the kinematic viscosity of the liquid.

The geometrical characteristics of granular fill particles were evaluated by their relative size in the chamber,  $\psi_d=d/2R$  ( $d$  – the average absolute particle size). The relative size of spherical-shaped particles for loose granular fill was  $\psi_{dc}=0.0104$ . The relative particle size of the dispersed solid fill, which was cement, was  $\psi_{dc}\approx 0.13\cdot 10^{-3}$ . A laser-type analyzer was used to measure the particle size of granular materials.

For further comparative analysis, 4–5 motion patterns of each type of fill were acquired in ascending rotation speed.

In addition, we investigated the peculiarities of completing the transition of the circulation mode of the solid granular fill to the near-wall layer mode during rotation acceleration. 3 drums with chambers of different radius were used. The relative dimensions of the dispersed cement particles in these chambers were  $\psi_{dc}\approx 0.065\cdot 10^{-3}$ ;  $0.092\cdot 10^{-3}$ ;  $0.13\cdot 10^{-3}$ . Discrete values for the chamber's filling rate were  $\kappa=0.3$ ;  $0.5$ ;  $0.7$ ; and  $0.9$ . Laboratory beakers were used to dosage a portion of the flowing fill of the drum chamber. The volume of granular fill was determined at rest, without compaction when filling the measuring chamber.

#### 4. 2. Procedure for evaluating dynamic parameters of movement in the formation of a near-wall layer of solid granular fill

As a research method, experimental physical visualization of transition modes of movement of the solid granular fill of the rotating drum chamber was adopted. With the help of direct visual observation through the transparent end wall of the chamber, mutual transitions of fill motion modes were recorded with a smooth change in the speed of rotation. The transition of the circulation mode to the mode in the form of a near-wall layer was recorded with a smooth increase in the speed of rotation. The reverse transition of the mode in the form of a near-wall layer to the circulation mode was recorded with a smooth decrease in the rotational speed.

We registered the value of the angular velocity  $\omega_{fb}$ , which corresponds to the formation of the near-wall layer at the acceleration of rotation, and the speed  $\omega_{dl}$ , which corresponds to the destruction of the layer when the rotation slows down. A stroboscopic type tachometer was used to measure the speed of rotation of the filled drum. The value of the speed of stationary rotation of the drum during the steady movement of the flowing fill of the chamber was constantly checked for convincing in the correctness of measurements. When using error spreading analysis, the speed measurement error was approximately  $\pm 3\%$ . The assessment was carried out by measuring the stationary rotation speed 5 times for one steady fill motion mode.

Cement was taken as a solid granular material of intra-chamber fill.

9 drums with chambers of different radii were used, the values of which form a sequence of numbers that constitute a geometric progression with the denominator  $2^{0.5}$ . The relative dimensions of dispersed solid fill particles in these chambers were  $\psi_{dc}\approx 0.065\cdot 10^{-3}$ ;  $0.092\cdot 10^{-3}$ ;  $0.13\cdot 10^{-3}$ ;  $0.18\cdot 10^{-3}$ ;  $0.26\cdot 10^{-3}$ ;  $0.37\cdot 10^{-3}$ ;  $0.52\cdot 10^{-3}$ ;  $0.74\cdot 10^{-3}$ ;  $1.04\cdot 10^{-3}$ .

To detect the possible manifestation of the rheological hysteresis of the transition of modes of movement of solid fill, a considerable value of the degree of filling the chamber was taken –  $\kappa\geq 0.3$ . The fill state values were  $\kappa=0.3$ ;  $0.5$ ;  $0.7$ ; and  $0.9$ .

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### 5. Results of studying the formation of a near-wall layer of solid granular fill

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#### 5. 1. Results of identifying the hydrodynamic characteristics of the flow of solid granular fill in the formation of a near-wall layer

The determined patterns of the steady circulation mode of movement of three types of flow fill in the chamber of a stationary rotating drum are shown in Fig. 6–9. Motion

patterns for the loose granular fill are shown in Fig. 6, for Newton liquid – in Fig. 7, 8, for a solid granular one – in Fig. 9.

The experimental patterns of movement in Fig. 7–9 characterize the effect of the rheological properties of three types of fill on the mutual transition of the circulating mode of the current into the near-wall layer.

Our patterns of completing the transition of the circulation mode of the solid granular fill to the near-wall layer mode during the acceleration of rotation are shown in Fig. 10–13. Motion patterns for the degree of filling the chamber  $\kappa=0.3$  are shown in Fig. 10; for  $\kappa=0.5$  – in Fig. 11; for  $\kappa=0.7$  – in Fig. 12; and for  $\kappa=0.9$  – in Fig. 13.

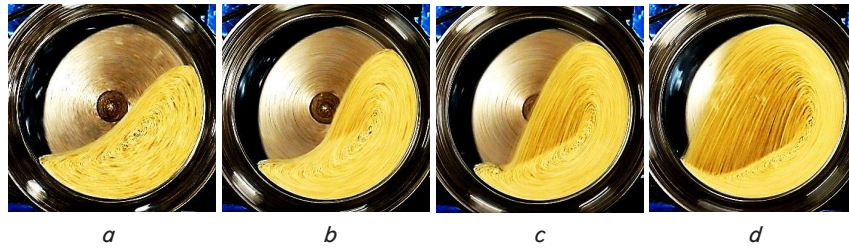


Fig. 6. Patterns of the circulation mode of motion of loose granular fill at the relative particle size  $\psi_{dc}=0.0104$  and the degree of filling the chamber  $\kappa=0.4$ : *a* –  $Re=32.4$ ;  $Fr=0.09$ ; *b* –  $Re=75.7$ ;  $Fr=0.49$ ; *c* –  $Re=91.9$ ;  $Fr=0.723$ ; *d* –  $Re=108$ ;  $Fr=1$

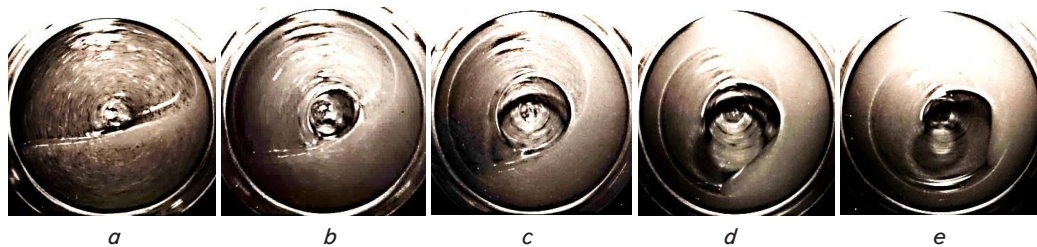


Fig. 7. Patterns of circulation mode of Newton liquid fill at the degree of filling the chamber  $\kappa=0.5$ : *a* –  $Re=43.5$ ;  $Fr=0.162$ ; *b* –  $Re=88.2$ ;  $Fr=0.666$ ; *c* –  $Re=132$ ;  $Fr=1.48$ ; *d* –  $Re=176$ ;  $Fr=2.66$ ; *e* –  $Re=203$ ;  $Fr=3.54$

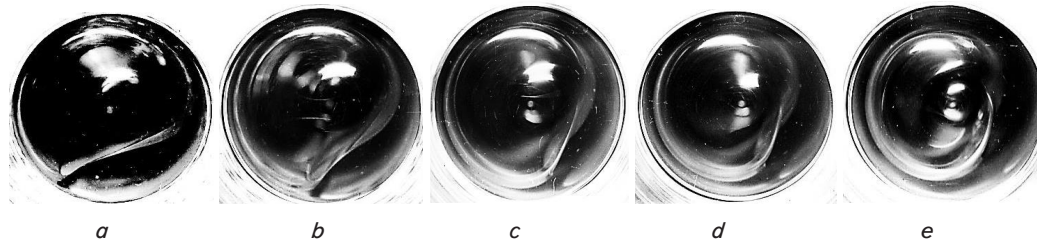


Fig. 8. Motion patterns of the secondary flow of Newtonian liquid in a rotating chamber at  $\kappa=0.3$ : *a* –  $Re=1.54$ ;  $Fr=0.0131$ ; *b* –  $Re=2.94$ ;  $Fr=0.0473$ ; *c* –  $Re=4.41$ ;  $Fr=0.107$ ; *d* –  $Re=5.81$ ;  $Fr=0.185$ ; *e* –  $Re=7.28$ ;  $Fr=0.29$

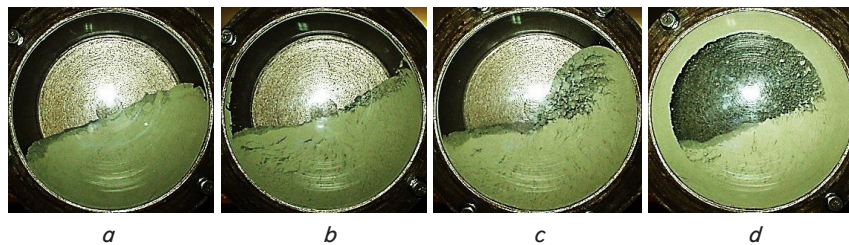


Fig. 9. Patterns of the circulating motion of the solid granular fill at the relative particle size  $\psi_{dc}\approx 0.13\cdot 10^{-3}$  and the degree of filling the chamber  $\kappa=0.4$ : *a* –  $Re=75.7$ ;  $Fr=0.49$ ; *b* –  $Re=115$ ;  $Fr=1.12$ ; *c* –  $Re=137$ ;  $Fr=1.61$ ; *d* –  $Re=153$ ;  $Fr=2.02$

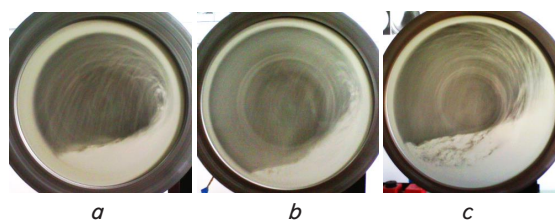


Fig. 10. Patterns of completing the transition of the circulation mode of the solid granular fill to the near-wall layer mode during the acceleration of rotation at  $\kappa=0.3$ : *a* –  $\psi_{dc}\approx 0.13\cdot 10^{-3}$ ;  $Re=156$ ;  $Fr=2.09$ ; *b* –  $\psi_{dc}\approx 0.092\cdot 10^{-3}$ ;  $Re=276$ ;  $Fr=2.3$ ; *c* –  $\psi_{dc}\approx 0.065\cdot 10^{-3}$ ;  $Re=504$ ;  $Fr=2.72$

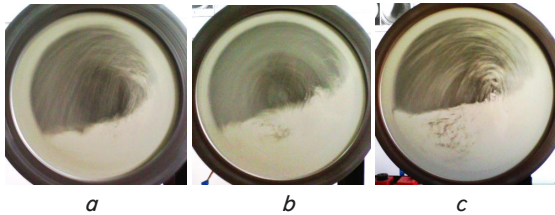


Fig. 11. Patterns of completing the transition of the circulation mode of the solid granular fill to the near-wall layer mode during the acceleration of rotation at  $\kappa=0.5$ :  $a - \psi_{dc} \approx 0.13 \cdot 10^{-3}$ ;  $Re = 141$ ;  $Fr = 1.7$ ;  $b - \psi_{dc} \approx 0.092 \cdot 10^{-3}$ ;  $Re = 247$ ;  $Fr = 1.85$ ;  $c - \psi_{dc} \approx 0.065 \cdot 10^{-3}$ ;  $Re = 439$ ;  $Fr = 2.06$

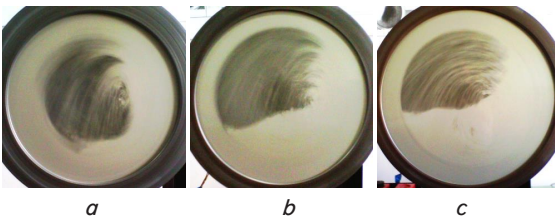


Fig. 12. Patterns of completing the transition of the circulation mode of the solid granular fill to the near-wall layer mode during the acceleration of rotation at  $\kappa=0.7$ :  $a - \psi_{dc} \approx 0.13 \cdot 10^{-3}$ ;  $Re = 138$ ;  $Fr = 1.64$ ;  $b - \psi_{dc} \approx 0.092 \cdot 10^{-3}$ ;  $Re = 241$ ;  $Fr = 1.76$ ;  $c - \psi_{dc} \approx 0.065 \cdot 10^{-3}$ ;  $Re = 419$ ;  $Fr = 1.88$

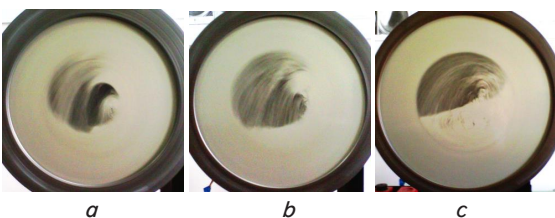


Fig. 13. Patterns of completing the transition of the circulation mode of the solid granular fill to the near-wall layer mode during the acceleration of rotation at  $\kappa=0.9$ :  $a - \psi_{dc} \approx 0.13 \cdot 10^{-3}$ ;  $Re = 148$ ;  $Fr = 1.87$ ;  $b - \psi_{dc} \approx 0.092 \cdot 10^{-3}$ ;  $Re = 262$ ;  $Fr = 2.08$ ;  $c - \psi_{dc} \approx 0.065 \cdot 10^{-3}$ ;  $Re = 472$ ;  $Fr = 2.38$

The experimental patterns of movement in Fig. 10–13 characterize the peculiarities of the flow of solid granular fill at the end of the transition of the circulating layer to the near-wall layer mode during the acceleration of rotation.

### 5. 2. Results of evaluating the dynamic parameters of movement in the formation of a near-wall layer of solid granular fill

The plots of the obtained results from determining experimentally the boundaries of the mutual transition of the circulation and the mode of the near-wall layer of motion of the solid granular fill of the rotating drum are shown in Fig. 14–17. We established the limit of formation of the near-wall layer mode at the acceleration of rotation  $\omega_{fl}$  and the limit of destruction of the near-wall layer mode at the time of slow rotation  $\omega_{dl}$ . The plots of the limits for the degree of filling of the chamber  $\kappa=0.3$  are shown in Fig. 14; for  $\kappa=0.5$  – in Fig. 15; for  $\kappa=0.7$  – in Fig. 16; and for  $\kappa=0.9$  – in Fig. 17.

As an approximation for the dependences of  $\omega_{fl}$  and  $\omega_{dl}$  boundaries, power functions were selected in the  $Re$  and  $Fr$  coordinates. In the logarithmic scales  $Re$  and  $Fr$ , such power dependences accept a linear shape (Fig. 14–17).

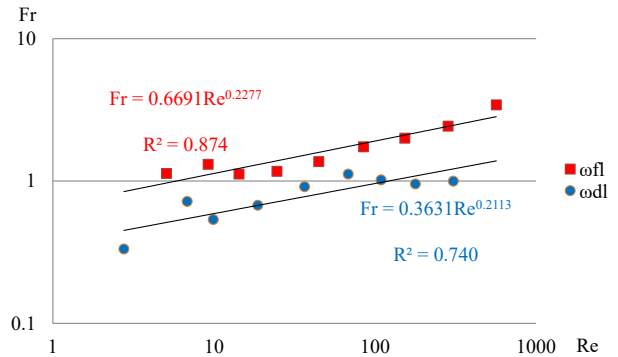


Fig. 14. Experimental boundaries of transition of the motion modes of solid granular fill of a rotating drum  $\omega_{fl}$  and  $\omega_{dl}$  for the degree of filling a chamber  $\kappa=0.3$

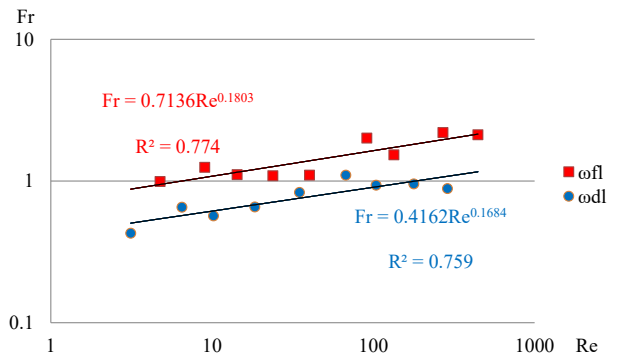


Fig. 15. Experimental boundaries of transition of the motion modes of bound granular fill of rotating drum  $\omega_{fl}$  and  $\omega_{dl}$  for the degree of filling chamber  $\kappa=0.5$

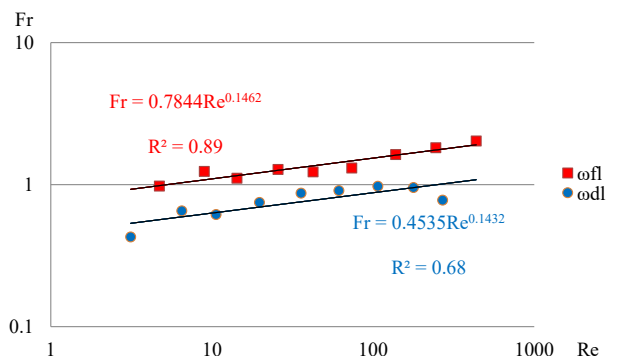


Fig. 16. Experimental boundaries of transition of the motion modes of solid granular fill of a rotating drum  $\omega_{fl}$  and  $\omega_{dl}$  for the degree of filling a chamber  $\kappa=0.7$

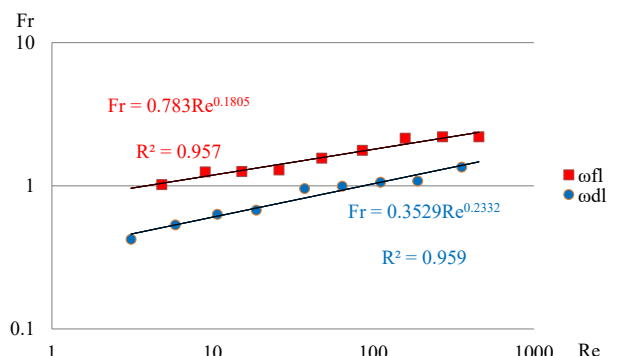


Fig. 17. Experimental boundaries of transition of the motion modes of solid granular fill of a rotating drum for  $\omega_{fl}$  and the  $\omega_{dl}$  degree of filling a chamber  $\kappa=0.9$



The experimental dependences of the boundaries of transition of  $\omega_{fl}$  and  $\omega_{dl}$  flow modes characterize the influence of dynamic motion parameters on the formation and destruction of the near-wall layer of solid granular fill.

## 6. Discussion of results of studying the current conditions in the formation of a near-wall layer of solid granular fill

Our results of the experimental visualization of the current, as well as the numerical data, have made it possible to qualitatively and quantitatively assess the effect of solidity on the formation of a near-wall layer of granular fill.

The visual qualitative analysis of motion patterns in Fig. 6–9 clearly demonstrates the peculiarities of the implementation of the circulating flow of the solid granular fill compared to the loose and liquid ones. It was found that the solidity properties significantly enhance the effect of quasi-liquefaction of the granular fill of the rotating chamber. This liquefaction of the solid granular material is caused by adhesion between the layers and the surface of the chamber, by analogy with the adhesion of layers and sticking to a solid wall for liquid. That weakens the similarity of the behavior of the solid granular fill (Fig. 9) with the movement of loose granular material (Fig. 6) and increases the similarity with the flow of liquid (Fig. 7, 8).

Patterns of the circulating mode of solid granular fill (Fig. 9) differ significantly from motion patterns for the loose one (Fig. 1, 6). As a result of adhesion between the particles of a solid granular fill, a three-phase motion mode characteristic of the loose one is distorted. In particular, when a solid fill moves (Fig. 9), there are no modes of movement with partial tossing (Fig. 1, *b*), with full tossing (Fig. 1, *c*), and incomplete centrifugation (Fig. 1, *d*). The share of the non-free fall zone for a solid granular fill (Fig. 9, *c*) is much smaller than such a fraction for the loose one (Fig. 6, *d*). The angles  $\alpha$  and the inclination of a free surface  $\theta$  for a solid granular fill (Fig. 9) are significantly smaller than such angles for the loose one (Fig. 1, *a*, 6).

However, the most important is the fundamental difference between the hydrodynamic characteristics of the current in the formation of a near-wall layer of the solid granular fill from such characteristics for the loose one. The formation of a near-wall layer of the solid fill has been determined (Fig. 9, *d*, 10–13) to occur without the emergence of a mode of movement with a full toss characteristic of the loose one (Fig. 1, *d*).

At the same time, the patterns of the circulating mode of the movement of the solid granular fill (Fig. 9) are significantly approaching the patterns of the fluid flow (Fig. 7, 8). The adhesion of the solid granular fill causes a decrease in the mobility and mutual pushing of particles due to the dissipation of kinetic energy. The non-free relative movement of individual particles, due to the clustering of the stream, is converted to the movement of individual cluster groups. The uniformity of the granular flow increases due to the increased density of particle packing. The shear circulating motion of a granular fill is transformed into a quasi-plunger flow with slipping and rolling without the relative motion of particles (Fig. 9) similar to the flow of liquid (Fig. 7, 8). Shear stresses that occur in such a dense homogeneous clustered granular stream appear in the form of microcracks (Fig. 9). The thickness of the descending layer increases, as well as the distance from the free surface to the center of the fill circulation.

The occurrence of an analog of the surface tension of a solid fill causes a decrease in the concavity and an increase in the convex shape of a free surface (Fig. 9). The zone of non-free fall of the solid granular fill (Fig. 9, *c*) is transformed into a vortex roller, as in the flow of liquid (Fig. 7, *b*, *c*, 8).

Our analysis of Fig. 7–13 reveals the similarity of hydrodynamic conditions of transition of circulation mode of movement to the near-wall layer mode for the solid granular and liquid fill. With rotation acceleration, the thickness of the near-wall layer of the solid fill smoothly increases with the circular movement of the rest in the lower part of the chamber (Fig. 9, *g*, 10–13), similar to the flow of liquid (Fig. 7, *d*, *e*, 8).

The quantitative analysis of plots in Fig. 14–17 makes it possible to identify patterns of change in the dynamic parameters of movement in the formation and destruction of the near-wall layer of solid granular fill.

A persistent manifestation of the rheological hysteresis of the movement of the solid fill of the rotating chamber, which is inherent in the flow of liquid fill, was registered. Hysteresis is to exceed the value of the speed limit of the formation of the near-wall layer at the acceleration of rotation of  $\omega_{fl}$  over the value of the boundary of destruction of the layer when the rotation  $\omega_{dl}$  is slowed down. In particular, the value of the Froude number for the boundary  $\omega_{fl}$  exceeds that value for the  $\omega_{dl}$  boundary. It was found that the manifestation of hysteresis, as the difference between the boundaries of the Fr value for  $\omega_{fl}$  and  $\omega_{dl}$ , is mainly enhanced with the increase in the Reynolds number. The intensity of hysteresis amplification increases with a decrease in the degree of filling the chamber  $\kappa$ . An increase in the value of Fr separately for each boundary  $\omega_{fl}$  and  $\omega_{dl}$  with increasing Re was also revealed.

In the considered range  $Re=30-500$ , the Froude number's value Fr for the  $\omega_{fl}$  boundary is  $Fr=1-2.9$ , and for the  $\omega_{dl}$  boundary –  $Fr=0.5-1.4$ . The Fr value for the  $\omega_{dl}$  boundary exceeds 1 at  $Re=100-250$ . Exceeding the Fr value for the  $\omega_{fl}$  boundary above this value for the  $\omega_{dl}$  boundary is 1.6–2.1 times.

Our results on the effect of rheological hysteresis on the movement of solid granular fill have made it possible to increase the efficiency of the grinding process in drum-roll mills. It turned out that during the period of material supply to the chamber and the formation of the near-wall layer, the rotation speed must be maintained not lower than the boundary of layer formation when accelerating the rotation  $\omega_{fl}$ . Instead, during grinding by rolling under the roll, the rotation speed can be reduced not lower than the limit of the destruction of the layer when the rotation  $\omega_{dl}$  slows down. That makes it possible to significantly reduce the energy intensity of the grinding, as well as the dynamic load, due to the rotation of the drum at the minimum possible speed sufficient to preserve the near-wall layer of the material.

The applicability of our results is predetermined by the following main accepted restrictions. The solid granular material of an intra-chamber fill was modeled by cement with the accepted analog of kinematic viscosity of  $\nu_g \approx 10^{-3} \text{ m}^2/\text{s}$ . 9 drums with chambers of different radii were used at the ranges of changing the relative particle size of the dispersed solid fill  $\psi_{dc} \approx (0.065-1.04) \cdot 10^{-3}$  and Reynolds number  $Re=30-500$ . The volumetric filling rate of the drum chamber with particles of a solid granular fill in a state of free rest accepted discrete values  $\kappa=0.3; 0.4; 0.5; 0.7; \text{ and } 0.9$ .

The disadvantages of the applied approach of assessing the effect of solidity on the formation of a near-wall layer of

a granular fill include the failure to take into consideration the bifurcation values of the control parameters of the drum machine.

In the future, it is advisable to elucidate the qualitative and quantitative impact on the dynamic and technological parameters of the process of possible self-oscillations of fill. That could make it possible to establish rational conditions for the implementation of the process of grinding solid granular materials in drum-roll mills if the fill pulsations are self-excited.

## 7. Conclusions

1. It was found that the solidity of the granular fill of the rotating drum weakens the similarity of its behavior with the movement of the loose fill and increases the similarity with the flow of liquid. In this case, the three-phase mode of movement of the granular fill in the rotating chamber is distorted. The transition of the circulation flow to the near-wall layer mode, during the acceleration of rotation, occurs by smoothly increasing the thickness of the layer when the rest of the fill is circulated at the bottom of the chamber. This is

due to adhesion between the layers of the solid granular fill and the surface of the chamber. That is predetermined by the manifestation of the established hydrodynamic effect of quasi-liquefaction of granular fill under the influence of solidity.

2. It was established that the speed limit  $\omega_{fl}$  of the formation of a near-wall layer of the solid granular fill, at the acceleration of rotation, exceeds the limit  $\omega_{dl}$  of the layer destruction when the rotation slows down. The difference between the Froude number for these boundaries increases predominantly as Reynolds increases. The intensity of growth of such a difference increases with a decrease in the degree of filling the chamber. The Fr value for each of these limits increases with an increase in Re. With the relative particle size of the solid granular fill  $\psi_{dc} \approx (0.065-1.04) \cdot 10^{-3}$  and  $Re=30-500$ , the value of Fr for the boundary  $\omega_{fl}$  is 1–2.9, and for the boundary  $\omega_{dl} - Fr=0.5-1.4$ . The value of Fr for the boundary  $\omega_{dl}$  exceeds the value of 1 at  $Re=100-250$ . Exceeding the value of Fr for the boundary  $\omega_{fl}$  over the value for the boundary  $\omega_{dl}$  is 1.6–2.1 times. This is due to the steady manifestation of the established effect of the rheological hysteresis of the movement of the solid granular fill of the rotating chamber due to the quasi-liquefaction of the granular medium under the influence of solidity.

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