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A NOVEL APPROACH FOR SOLVING DECISION-MAKING PROBLEMS WITH STOCHASTIC LINEAR-FRACTIONAL MODELS

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Stochastic chance-constrained optimization has a wide range of real-world applications. In some real-world applications, the decision-maker has to formulate the problem as a fractional model where some or all of the coefficients are random variables with joint probability distribution. Therefore, these types of problems can deal with bi-objective problems and reflect system efficiency. In this paper, we present a novel approach to formulate and solve stochastic chance-constrained linear fractional programming models. This approach is an extension of the deterministic fractional model. The proposed approach, for solving these types of stochastic decision-making problems with the fractional objective function, is constructed using the following two-step procedure. In the first stage, we transform the stochastic linear fractional model into two stochastic linear models using the goal programming approach, where the first goal represents the numerator and the second goal represents the denominator for the stochastic fractional model. The resulting stochastic goal programming problem is formulated. The second stage implies solving stochastic goal programming problem, by replacing the stochastic parameters of the model with their expectations. The resulting deterministic goal programming problem is built and solved using Win QSB solver. Then, using the optimal value for the first and second goals, the optimal solution for the fractional model is obtained. An example is presented to illustrate our approach, where we assume the stochastic parameters have a uniform distribution. Hence, the proposed approach for solving the stochastic linear fractional model is efficient and easy to implement. The advantage of the proposed approach is the ability to use it for formulating and solving any decision-making problems with the stochastic linear fractional model based on transforming the stochastic linear model to a deterministic linear model, by replacing the stochastic parameters with their corresponding expectations and transforming the deterministic linear fractional model to a deterministic linear model using the goal programming approach

Keywords: Stochastic Models, Fractional Programming Problems, Goal Programming, Joint Probability Distribution

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1. Introduction

Linear programming problems have a wide range of real-world applications. The linear programming model consists of the linear objective function and linear constraints. For more details about these types of optimization problems, we refer to [1–5]. If some or all of the coefficients of decision variables are random variables with joint probability distribution, then the problem is known as stochastic linear programming (SLP). The paper [6] first presented the problem that the decision-maker must give his solution before the random variables come true, which is called stochastic chance-constrained programming (SCP). On the other hand, in some real-world applications, the decision-maker has to formulate the problem as a ratio between two linear objective functions, and then the problem is known as a fractional linear programming problem. Moreover, if some or all of the coefficients are random variables with joint probability distribution, then the problem is known as stochastic linear fractional programming (SLFP). The solution methods of this kind of optimization problems vary from one to another. The fundamental drawbacks of such solution are the difficult-to-predict uncertainty of the result as well as the complexity of transforming the stochastic linear fractional model into a stochastic linear model. Thus, to solve this optimization problem, one needs to establish a computational procedure for solving these kinds of problems.

2. Literature review and problem statement

The paper [7] presented a stochastic linear fractional programming problem, in which the parameters of the model are assumed to be mutually independent Cauchy variates. The advantage of this approach is that the resulting problem can be solved using the Charnes and Cooper method. However, the parameters of the fractional model are assumed to be mutually independent Cauchy variates. The main difficulty in solving the equations is that these equations involve infinite series.

The paper [8] introduced a genetic algorithm to solve chance-constrained programming problems. This method does not require transforming the stochastic model into a deterministic model, but it needs to use some other method to check the feasibility such as Monte Carlo simulation. The presented approach is used for solving linear problems and has not been established for fractional problems because it requires some transformation methods.

An interactive conversion technique is presented in the paper [9] to solve the linear stochastic fractional programming model. The advantage of this work is to focus on stochastic sum-of-probabilistic-fractional programming. The paper [10] introduced an application of a stochastic fractional model to address classes of assembled printed circuit boards. Using this approach, the solution of the first problem gives the upper bound of the profit as well as it helps to solve the second problem, but this approach deals with linear objective function and nonlinear objective function at the same time.

SLFP is applied to a case study of waste flow allocation within a municipal solid waste (MSW) management system in the paper [11]. This study attempts to provide a new modeling framework for solving ratio optimization problems associated with random inputs. The main difficulty in solving this model is that it requires the LFP duality theory.

The paper [12] introduced a transportation model with a stochastic fractional programming problem. This proposed model would provide a useful solution under some conditions when the company wants to optimize the ratio of profit over the cost per unit of shipment in a way to meet the stochastic demands with a clear account for variation. The presented model is a special class of stochastic linear fractional programming problems and more work is required to extend it to more general optimization problems.

An application to an assembled printed circuit board problem using the chance-constrained programming problem is presented in the paper [13]. This study requires that errors are estimable with the help of prior knowledge.

The paper [14] introduced a new method for solving chance-constrained programming problems. This algorithm is established based on the simplex method and compared with the genetic algorithm. The advantage of this method is to obtain a feasible solution for any chance-constrained programming problem. The presented method needs more work to solve the stochastic linear fractional model.

A differential evolution algorithm for solving stochastic programming problems is proposed in the paper [15]. This method has better performance in solution quality, convergence rate, and robustness, when compared to other algorithms.

The paper [16] presented a double-sided stochastic chance-constrained linear fractional programming model for managing irrigation water under uncertainty. A chance-constrained linear programming model with Weibull random coefficients is proposed in the paper [17]. This approach provides better performance for optimizing the job completion time.

The paper [18] introduced a goal programming approach to multichoice multiobjective stochastic transportation problems with extreme value distribution.

Fuzzy stochastic linear fractional programming based on fuzzy mathematical programming is suggested in the paper [19]. This method provides the solution for the stochastic linear programming with fuzzy coefficients, but different approaches need to be used for reducing the model into single-objective linear programming (LP) problem. The paper [20] presented the solution of a probabilistic fractional programming problem, where the parameters of the right-hand side constraints follow the Cauchy distribution. Using this method, the stochastic model can be transformed into a deterministic model. Type-2 fuzzy chance-constrained fractional integrated programming approach is developed for the planning of sustainable management in an electric power system under complex uncertainties in the paper [21]. The paper [22] introduced a new linear approximation technique for solving a fractional stochastic programming problem. A stochastic fractional problem involving an inequality type of constraints, where all quantities on the right side are log-normal random variables, and the objective function coefficients are fractional intervals is presented in the paper [23].

3. The aim and objectives of the study

The aim of the study is to devise a new approach for solving decision-making problems with stochastic linear fractional models.

To accomplish the aim, the following objectives have been set:

– to transform the stochastic linear fractional model into a stochastic linear model;

– to construct a computational procedure that finds the optimal solution for the stochastic linear model.

4. Materials and methods

In this section, a theoretical approach is used to establish our computational procedure for the stochastic fractional model. This is done by first transforming the stochastic fractional model into a stochastic linear model using the goal programming approach. After that, there was a need to reduce the stochastic model into a deterministic model.

4. 1. Goal programming approach

As in [24], the general goal programming model is as follows:

$$\operatorname{Min} U = \sum_{i=1}^{m} d_i^+ + d_i^-. \tag{1}$$

S.T.:

Goal constraints:

$$\sum_{j=1}^{n} a_{ij} x_j - d_i^+ + d_i^- = b_i, \text{ for } I = 1,\ldots,m.$$

System constraints:

$$\sum_{j=1}^{n} a_{ij} x_j \begin{bmatrix} \le \\ = \\ \ge \end{bmatrix} b_i, \text{ for } i = m+1,\ldots\ldots,m+p.$$

With $d_i^+, d_i^- x_j \ge 0$, for $i = 1,\ldots,m$, for $j = 1,\ldots,n$, where U – represents the objective function; a_{ij} – represents the coefficient of the decision variables; x_j – represents the decision variable; b_i – represents the right-hand side; d_i^- – represents the negative deviational variable; d_i^+ – represents the positive deviational variable.

Assumptions:
1. The decision variables are nonnegative.
2. The deviational variables are nonnegative.
3. The feasible region is bounded.
Table 1 shows the deviation cases.

Table 1

Deviation cases

Minimize	Constraint type
d_i^-	$\geq b_i$
d_i^+	$\leq b_i$
$d_i^+ + d_i^-$	$= b_i$

4. 2. Goal programming models
4. 2. 1. Lexicographic goal programming model

$$\text{Min}U = \sum_{i=1}^{m} Pi\, d_i^+ + d_i^-. \quad (2)$$

S.T.:
Goal constraints:

$$\sum_{j=1}^{n} a_{ij} x_j - d_i^+ + d_i^- = b_i, \text{ for } I = 1,\ldots,m.$$

System constraints:

$$\sum_{j=1}^{n} a_{ij} x_j \begin{bmatrix} \leq \\ = \\ \geq \end{bmatrix} b_i, \text{ for } i = m+1,\ldots,m+p.$$

With $d_i^+, d_i^- x_j \geq 0$, for $i = 1,\ldots,m$; for $j = 1,\ldots,n$, where p – represents the priority of the goal.

4. 2. 2. Weighted Goal Programming Model

In this mathematical model, the decision-maker can set up weight for each goal. The mathematical formulation for this type is given as follows:

$$\text{Min}U = \sum_{i=1}^{m} w_i^+ d_i^+ + w_i^- d_i^-. \quad (3)$$

S.T.:
Goal constraints:

$$\sum_{j=1}^{n} a_{ij} x_j - d_i^+ + d_i^- = b_i, \text{ for } I = 1,\ldots,m.$$

System constraints:

$$\sum_{j=1}^{n} a_{ij} x_j \begin{bmatrix} \leq \\ = \\ \geq \end{bmatrix} b_i, \text{ for } i = m+1,\ldots,m+p.$$

With $d_i^+, d_i^- x_j \geq 0$, for $i = 1,\ldots,m$; for $j = 1,\ldots,n$, where U – represents the objective function; a_{ij} – represents the coefficient of the decision variables; x_j – represents the decision variable; b_i – represents the right-hand side; d_i^- – represents the negative deviational variable; d_i^+ – represents the positive deviational variable; w_i – represents the weight for each goal.

5. Results of the computational procedure for the SCLFP

5. 1. Stochastic Chance-Constrained Linear Programming (SCLP) Model

As in [14, 25], SCLP can be formulated as follows:

$$\text{Max}Z(X) = C^T x.$$

S.T.:

$$P(Ax \leq b) \geq \alpha, \ x \geq 0.$$

Or

$$\text{Max}Z(x) = C^T x.$$

S.T.:

$$P(A_i x \leq b_i) \geq \alpha_i, \ i = 1,2,\ldots,m, \ x \geq 0,$$

where A is a real $m \times n$ matrix; $x \in R^n$, $b \in R^m$, $C \in R^n$, $D \in R^n$ is the *ith* confidence level of the constraint.

Assumptions:
1. $C^T x$ is nonnegative and concave.
2. The decision variables are nonnegative.
3. Underlying random variables follow a uniform distribution.

Remark 5. 1. To transform the stochastic linear model to a deterministic linear model, replace the stochastic parameters with their corresponding expectations.

5. 2. Stochastic Chance-Constrained Linear Fractional Programming (SCLFP) Model

In this section, we will present the formulation of the stochastic fractional model, which represents a ratio between two linear objective functions where some or all of the coefficients are random variables with joint probability distribution and the confidence level of the constraints.

5. 2. 1. Mathematical formulation

SCLFP can be formulated as follows:

$$\text{Max}U(x) = \frac{C^T x + \alpha}{D^T x + \beta}. \quad (4)$$

S.T.:

$$P(Ax \leq b) \geq k(SCLFP), \ x \geq 0,$$

where A is a real $m \times n$ matrix; $x \in R^n$, $b \in R^m$, $C \in R^n$, $D \in R^n$, α and β are real numbers with $\beta > 0$ and k represents the confidence level of the constraint.

Assumptions:
1. $C^T x + \alpha$ is nonnegative and concave.
2. $D^T x + \beta$ is positive and convex.
3. Underlying random variables follow a uniform distribution.

5. 2. 2. Solution Method

In this section, we will construct a computational procedure for solving (SLFP). This approach is based on Remark 5.1, which is transforming the stochastic parameters by their expectations and then make use of goal programming, which is one of the most well-known methods to solve multi-criteria decision-making problems. Now, the computational procedure for solving (SLFP) using the goal programming approach is described as follows:

Step 1. Construct two stochastic linear objective functions, which represent the numerator and the denominator of the (SCLFP) as follows:

– goal 1: $\text{Max}Z_1 = C^T x + \alpha$.

– goal 2: $\text{Min}Z_2 = D^T x + \beta$.

Step 2. Form the following (SCLP) problems:

$$\text{Max}Z_1 = C^T x + \alpha.$$

S.T.:

$$P(Ax \le b) \ge k(SCLFP),\ x \ge 0$$

and

$$\text{Min}Z_2 = D^T x + \beta.$$

S.T.:

$$P(Ax \le b) \ge k(SCLFP),\ x \ge 0.$$

Step 3. Transform (SCLP) problems in step 2 into deterministic linear programming (DLP) problems using Remark 4.1.

Step 4. Find the optimal solution of (DLP) problems in step 3 using the goal programming approach as follows:

If the reduced cost of $\text{Max}Z_1 = C^T x + \alpha \ge 0$ and $\text{Min}Z_2 = D^T x + \beta \le 0$, go to step 6.

Else go to step 5.

Step 5. Construct and update the simplex table for each objective function and go to step 4.

Step 6. Find the optimal solution for the (SCLFP) problem by dividing the optimal value of goal 1 to the optimal value of goal 2.

Numerical illustration.

We establish our computational approach by the following SCLFP model.

Let (a, b) be the random variables with uniform distribution in rectangle $\{1 \le a \le 4, 1/3 \le b \le 1\}$ and consider the following SCLFP problem (5):

$$\text{Max}U(x) = \frac{10x_1 + 3x_2}{7x_1 + x_2 + 1}.$$

S.T.:

$$ax_1 + x_2 \le 70(SCLFP),$$

$$bx_1 + x_2 \le 40.$$

Step 1. Construct two stochastic linear objective functions, which represent the numerator and the denominator of the (SCLFP) as follows:

– goal 1: Max $Z_1 = 10x_1 + 3x_2$;

– goal 2: Min $Z_2 = 7x_1 + x_2 + 1$.

Step 2. Form the following (SCLP) problems:

$$\text{Max } Z_1 = 10x_1 + 3x_2.$$

S.T.:

$$ax_1 + x_2 \le 70,$$

$$bx_1 + x_2 \le 40(SCLP1),\ x_1 \ge 0,\ x_2 \ge 0,$$

and

$$\text{Min}Z_2 = 7x_1 + x_2 + 1.$$

S.T.:

$$ax_1 + x_2 \le 70,$$

$$bx_1 + x_2 \le 40(SCLP2),\ x_1 \ge 0,\ x_2 \ge 0,$$

Step 3. Transform (SCLP) problems in step 2 into deterministic linear programming (DLP) problems as follows:

There are several approaches to transform stochastic linear programming into a deterministic linear programming problem [6]. We will use the approach that aims to replace a and b with the corresponding expectations and solve the (DLP) problems as follows:

$$\text{Max}Z_1 = 10x_1 + 3x_2.$$

S.T.:

$$\frac{5}{2}x_1 + x_2 \le 70,$$

$$\frac{2}{3}x_1 + x_2 \le 40(DLP1),\ x_1 \ge 0,\ x_2 \ge 0$$

and

$$\text{Min}Z_1 = 7x_1 + x_2 + 1.$$

S.T.:

$$\frac{5}{2}x_1 + x_2 \le 70,$$

$$\frac{2}{3}x_1 + x_2 \le 40(DLP2),\ x_1 \ge 0,\ x_2 \ge 0.$$

Step 4. Find the optimal solution of (DLP) problems in step 3 using the goal programming approach and the input data and output data are described in Table 2 and Table 3 respectively as follows:

Table 2

Win QSB Input Data

Right-Hand Solution	Direction	x_2	x_1	Variable
–	–	3	10	Max: G1
–	–	1	7	Min: G2
70	≤	1	2.5	Constraint No. 1
40	≤	1	2/3	Constraint No. 2
–	–	0	0	Lower Bound
–	–	∞	∞	Upper Bound
–	–	Continuous	Continuous	Variable Type

Table 3

Win QSB Output Data

Total Contribution	Unit Cost	Solution Value	Decision Variable	Goal Level
200	10	20	x_1	$G1$
0	3	0	x_2	$G1$
140	7	20	x_1	$G2$
0	1	0	x_2	$G2$
200	(Max)	Value	Goal	$G1$
140	(Min)	Value	Goal	$G2$
Slack or Surplus	Right-Hand Solution	Direction	Left-Hand Solution	Constraint
20	70	≤	50	$C1$
0	40	≤	40	$C2$

Step 5. Find the optimal solution for the (SCLFP) problem by dividing the optimal value of goal 1 to the optimal value of goal 2 as follows:

$$\text{Max}\,U(x)=\frac{200}{140}=1.428.$$

Now, in order to analyze the feasibility of this approach, we assume $\theta^*=(20.0)$, and consider:

$$M=\{(x_1,x_2)|ax_1+x_2\le 70, bx_1+x_2\le 40\}.$$

Then

$$P(\theta^*\in M)=P\left(\frac{18}{11}a+\frac{32}{11}\le 70,\frac{18}{11}a+\frac{32}{11}\le 40\right),$$

$$P(\theta^*\in M)=P\left(a\ge\frac{5}{2},b\ge\frac{2}{3}\right)=P\left(a\ge\frac{5}{2}\right)\cdot P\left(b\ge\frac{2}{3}\right),$$

$$P(\theta^*\in M)=\left\{1-P\left(a<\frac{5}{2}\right)\right\}\left\{1-P\left(b<\frac{2}{3}\right)\right\},$$

$$P(\theta^*\in M)=1-P\left(a<\frac{5}{2}\right)-P\left(b<\frac{2}{3}\right)+$$

$$+P\left(a<\frac{5}{2}\right)\cdot P\left(b<\frac{2}{3}\right).$$

$$P(\theta^*\in M)=1-0.5-0.5+(0.5)(0.5)=0.25.$$

This represents the probability of the feasible region.

6. Discussion of the results of solving decision-making problems with stochastic linear fractional models

To solve the problem (5), first, we have to transform it into two linear stochastic models using the goal programming approach. The two stochastic linear models represent the numerator and denominator, respectively, for the problem (5). Next, since the stochastic parameters (a and b) have uniform distribution, so using Remark 4. 1 we replace them with their expectations and obtain two deterministic linear objective functions. After that, using Win QSB solver the input of the deterministic problems as in Table 2 is interred and the optimal solution of these deterministic objective functions is obtained as in Table 3. Finally, using the optimal solution in Table 3, which represents the optimal values of the numerator and denominator respectively, we obtain the optimal solution for the linear fractional model.

Now, we compare our results of the presented example with the Charnes and Cooper method [26], denominator objective restriction method [27], and development complementary [28] method, which are well-known methods for solving linear fractional models in Table 4 as follows:

Note. We compared our new approach with well-known methods based on the value of the fractional objective function and the values of the decision-making variables.

Table 4

Comparison of the results of the presented example

Methods	Solution
Charnes and Cooper method	$X_1=20, X_2=0, \text{Max}\,U(X)=\frac{200}{140}=1.428$
Denominator objective restriction method	$X_1=20, X_2=0, \text{Max}\,U(X)=\frac{200}{140}=1.428$
Development complementary method	$X_1=20, X_2=0, \text{Max}\,U(X)=\frac{200}{140}=1.428$
Goal programming approach	$X_1=20, X_2=0, \text{Max}\,U(X)=\frac{200}{140}=1.428$

The advantages of the presented approach are that the obtained solution is exact, the transformation step is simple, and the computational procedure is efficient and easy to implement. The disadvantage of this method is that it requires replacing the parameters of the stochastic variables with their expectations.

The assumptions of our approach are that the objective function of the numerator has a finite optimal solution and the objective function of the denominator has a finite optimal solution.

The limitation of the proposed approach is that the probability of the feasible region is small. In this regard, the direction for future research is to use some new heuristic method to solve this optimization problem and the accuracy could be compromised.

7. Conclusions

1. The stochastic linear fractional model has been transformed into two linear stochastic models using the goal programming approach. Using this approach, we set the denominator and the numerator of the stochastic linear fractional model as two stochastic linear goals.

2. A computational procedure has been presented to solve the stochastic linear model. By replacing the stochastic parameters with their corresponding expectations, the stochastic model has been transformed into a deterministic model. Also, the optimal solution for the deterministic model has been found using Win QSB solver.

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