The results of mathematical modeling of stationary physical processes in the electron-hole plasma of the active region (i-region) of integrated p-i-n-structures are presented. The mathematical model is written in the framework of the hydrodynamic thermal approximation, taking into account the phenomenological data on the effect on the dynamic characteristics of charge carriers of heating of the electron-hole plasma as a result of the release of Joule heat in the volume of the i-th region and the release of recombination energy. The model is based on a nonlinear boundary value problem on a given spatial domain with curvilinear sections of the boundary for the system of equations for the continuity of the current of charge carriers, Poisson, and thermal conductivity. The statement of the problem contains a naturally formed small parameter, which made it possible to use asymptotic methods for its analytical-numerical solution. A model nonlinear boundary value problem with a small parameter is reduced to a sequence of linear boundary value problems by the methods of perturbation theory, and the physical domain of the problem with curvilinear sections of the boundary is reduced to the canonical form by the method of conformal mappings. Stationary distributions of charge carrier concentrations and the corresponding temperature field in the active region of p-i-n-structures are obtained in the form of asymptotic series in powers of a small parameter. The process of refining solutions is iterative, with the alternate fixation of unknown tasks at different stages of the iterative process. The asymptotic series describing the behavior of the plasma concentration and potential in the region under study, in contrast to the classical ones, contain boundary layer corrections. It was found that boundary functions play a key role in describing the electrostatic plasma field. The proposed approach to solving the corresponding nonlinear problem can significantly save computing resources

Keywords: asymptotic series, boundary layer correction, conformal mappings, singularity, electron-hole plasma, p-i-n-structure

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CONSTRUCTING AND ANALYZING MATHEMATICAL MODEL OF PLASMA CHARACTERISTICS IN THE ACTIVE REGION OF INTEGRATED P-I-N-STRUCTURES BY THE METHODS OF PERTURBATION THEORY AND CONFORMAL MAPPINGS

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1. Introduction

In microwave technology, p-i-n-diodes (diodes with a wide base) are widely used for switching the electromagnetic field [1, 2]. The operation of switching p-i-n-diodes is based on the possibility of creating, under the influence of the control current, a highly concentrated electron-hole plasma in the active region (n-region) of the diode. The appearance of charged particles in the n-region leads to a change in the electrodynamic characteristics of the switching system and, as a consequence, to the extinction of the electromagnetic field presented to the switch.

The basic characteristics of switches based on p-i-n-diodes – the level of the switched power of the microwave signal, the modulation depth, the speed of response – determine the concentration of the electron-hole plasma in the active region, the pulse density, energy (averaged characteristics of the plasma) and the design of the p-i-n-diodes. Complex processes occur in the active region of the diode when the electron and hole currents flow. The main ones are injection of charge carriers from highly doped bands through n-i and p-i junctions, electron-hole diffusion and drift, recombination processes in the bulk and on the surface of the n-region, energy transfer of electrons and holes to the crystal lattice, and the like. These processes are described by nonlinear mathematical models.

The development of technology of integrated circuits led to the emergence of p-i-n-diodes in integrated design: surface-oriented p-i-n-structures with deep contacts [3] (Fig. 1). A typical design of an integrated p-i-n-structure is a plate made of a semiconductor material (a region of an intrinsic semiconductor – *i*-region), on the surface of which donor (*n*) and acceptor (*p*) zones are formed with metal contacts brought to them. According to the active region of the integrated p-i-n-structure has a complex geometry.

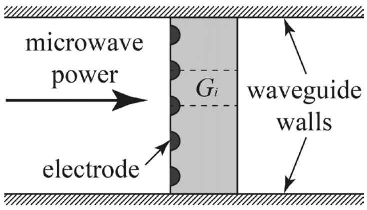

Fig. 1. Schematic representation of the p-i-n-structure in the cross section of a rectangular waveguide

The design of integrated p-i-n-structures is associated with the solution of the complex problem of the selection of semiconductor materials, the choice of the geometric dimensions of the structure elements, the characteristics of the control signals. In this case, the following conditions must be met:

– ensuring the matching of the p-i-n-structure with the signal transmission line;

– achieving optimal switching characteristics in a given frequency range;

– compliance with the required temperature regime.

Obviously, one of the stages of designing integrated p-i-n-structures is associated with solving a nonlinear mathematical problem of the dynamics of charge carriers, which is posed in a non-canonical domain. In the general case, such complex problems can be solved only by numerical methods with the involvement of significant computer resources. In this regard, the problem of increasing the efficiency of algorithms is urgent. It is especially aggravated when solving optimization problems.

To improve the design efficiency, it is proposed to decompose the original problem by methods of perturbation theory and conformal transformations. This procedure allows one to obtain a sequence of linear problems posed in the canonical domain and are solved by analytical-numerical methods.

2. Literature review and problem statement

The drift-diffusion model has become a classic in the design of p-i-n-diode structures. The properties of stationary and non-stationary mathematical models of p-i-n-structures and the corresponding experimental data were considered, for example, in the monograph [4]. In [5], the drift-diffusion model is the basis for the numerical analysis of semiconductor devices. Alternative mathematical models of electron-hole plasma are quasi-hydrodynamic model [6] and variations of the kinetic model [7, 8]. The diffusion-drift model is characterized by relative ease of use; it allows one to obtain the averaged characteristics of the plasma in the active region of electronic devices. The scope of application of this model is limited by the characteristic dimensions of semiconductor elements, which exceed the relaxation lengths of the pulse and the energy of the charge carriers. According to the diffusion-drift model, the effects associated with the imbalance and nonlocality of the electron-hole plasma are not taken into account. These limitations are significant in some cases, for example, when designing GaAs control structures. The diffusion-drift model is based on a nonlinear system of parabolic equations of current continuity with the corresponding boundary conditions, which is jointly solved by the Poisson equation for the electric field potential of charged plasma particles. Traditional approaches to the analysis of such a model are based on the use of linearization methods (for example, the ambipolar diffusion approximation [7] and the use of numerical methods [9] are used.

Let's separately note the use of asymptotic methods for splitting the corresponding problems. In [10], a one-dimensional stationary mathematical model of a p-i-n-diode in the diffusion-drift thermal approximation was proposed and analyzed by asymptotic methods. Similar methods were used to analyze the mathematical model of the p-n-junction [11]. The properties of a system of ordinary differential equations, which are the basis for mathematical models of semiconductor electronics, are considered in [12].

The advantage of linearization methods is the ability to obtain, in some cases, the simulation result in an analytical form. In this case, an acceptable level of modeling adequacy is not always achieved (as, for example, when working within the ambipolar diffusion approximation). On the contrary, numerical methods make it possible to obtain simulation results within the framework of the applied approximation methods with a relatively high accuracy. Difference schemes are most often used by themselves (for example, schemes of Samarsky, Scharfetter-Gummel, Tang, Mok, etc.). As a result of the analysis of the corresponding numerical algorithms [13], the need to keep track of the issues of monotonicity and stability of difference schemes is emphasized.

Mathematical modeling of the characteristics of an electron-hole plasma in semiconductor structures is complicated by the fact that the parameters of the models generally depend on temperature. In [13], the corresponding mathematical model is constructed to calculate the characteristics of field-effect transistors, in [14] - for diodes with a Schottky barrier. In [15], the results of modeling the characteristics of bipolar transistors are reflected. The description of the plasma state of the refined drift-diffusion model includes an additional characteristic – temperature. The system of equations of the model is supplemented by an inhomogeneous heat conduction equation, which contains a description of local heat sources. Plasma heating is caused by the Joule effects and the release/absorption of energy as a result of the process of electron-hole recombination/generation [7]. Depending on the conditions of action on a semiconductor crystal, various recombination-generation mechanisms are manifested (Shockley-Read, therefore, exciton, plasma). Depending on the selected semiconductor material, the refined mathematical models describe plasma, which can be in thermodynamic equilibrium with the crystal lattice (devices based on Si), and plasma, which has a temperature different from the crystal lattice (for example, AsGa). Along with the processes of heating semiconductor structures in temperature models, the processes of transfer of thermal energy to the environment by means of forced and natural convection and radiation are considered [16]. The exchange of energy with the environment is reflected in models, as a rule, by describing a special type of boundary conditions.

Thus, proceeding from the complexity of physical processes in the electron-hole plasma of integrated surface-oriented p-i-n-structures, let's consider the problem of modeling the characteristics of switches with active elements on p-i-n-structures to be open. One of the ways to improve the algorithms for the design of switches within the framework of the application of the traditional diffusion-drift thermal approximation is to modify the corresponding system of the mathematical model, boundary and initial conditions based on the use of phenomenological data on the relationship between the model parameters. It is also advisable to decompose the basic nonlinear model in order to reduce the level of its complexity. Let's propose to solve the corresponding problems, which contain both regularly and singularly perturbed components, by asymptotic methods with conformal mappings of the physical domain of the problem.

3. The aim and objectives of research

The aim of research is to develop tools for the design system for semiconductor switching integrated p-i-n-structures with deep contacts. This will ensure the development of the theory of p-i-n-structures and will make it possible to design the corresponding microwave switching devices with specified characteristics.

To achieve the aim, the following objectives are set:

– propose an improved stationary diffusion-drift model for predicting the state of an electron-hole plasma in the active region of p-i-n-structures, taking into account the effect of plasma heating and the peculiarities of the geometry of the physical region of the problem;

– apply the method of conformal transformations to bring the physical domain of the problem with curvilinear sections of the boundary to the canonical form;

– based on the development of methods of asymptotic corrections, develop a method for solving the corresponding perturbed nonlinear boundary value problems for the system of equations of current continuity, Poisson, heat conduction;

– carry out numerical experiments.

4. Materials and methods of research

Placement of p-i-n-diodes in the integrated structure is periodic. Therefore, it is expedient to formulate a model problem for a typical element of the active region G_i (Fig. 1, 2).

The distributions of the concentration of charge carriers p (x, y, J), n (x, y, J) in the electron-hole plasma of the active region are established under the action of a control current with a density J due to the occurrence of diffusion-drift, recombination-generation processes. Concentrations of charge carriers in the hydrodynamic approximation describe the equations of continuity of the current of holes and electrons [6]:

$$\frac{\partial p}{\partial t}=-\frac{1}{e}\nabla\cdot\vec{j}_p-R_p+G_p,$$

$$\frac{\partial n}{\partial t}=\frac{1}{e}\nabla\cdot\vec{j}_n-R_n+G_n, \quad (1)$$

where

$$\vec{j}_p=-e\mu_p p\nabla\varphi-eD_p\nabla p,$$

$$\vec{j}_n=-e\mu_n n\nabla\varphi+eD_n\nabla n, \quad (2)$$

$$\Delta\varphi=-e\left(p-n+N_d\right). \quad (3)$$

The notation is used here:

N_d – doping profile;

φ – electrical potential;

e – electron charge;

D_p, D_n – diffusion coefficients of holes and electrons, respectively;

μ_p, μ_n – mobility (is the velocity vector) of charge carriers;

R_p, R_n, G_p, G_n – rates of recombination and generation.

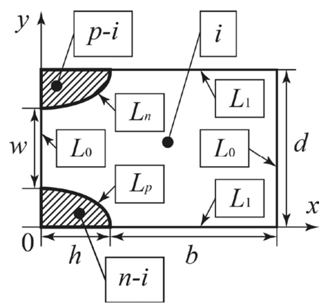

Fig. 2. 2D-model of an element of an integrated p-i-n-structure (G_i region with injection n-i-, p-i-contacts)

In semiconductor materials, the dynamics of charge carriers is affected by the heating of the crystal and electron-hole plasma as a result of the release of Joule heat in the volume of the p-i-n-diode and the release of energy as a result of the recombination of charge carriers [6, 7, 14]. To describe the temperature field, let's propose to use a modified heat conduction equation:

$$c_\rho\rho\frac{\partial T}{\partial t}-\operatorname{div}\left(\lambda_t\nabla T\right)=$$

$$=\left(\vec{j}_n+\vec{j}_p\right)\cdot\vec{E}+\left(R-G\right)\left(E_g-\frac{\alpha T^2}{T+\beta}\right), \quad (4)$$

where c_ρ – specific heat of the semiconductor material;

ρ – density of the substance;

λ_t – thermal conductivity coefficient;

E_g – semiconductor band gap;

$R=R_n+R_p$ – rate of recombination of charge carriers;

$G=G_n+G_p$ – rate of generation of electrons and holes; α, β – phenomenological steels (for Si – E_g=1.169 eV, α=$=4.9\cdot10^{-4}$ eV/K, β=655 K, for GaAs – E_g=1.519 eV, α=$=5.4\cdot10^{-4}$ eV/K, β=204 K [17]).

Plasma heating leads to a change in the diffusion coefficients and the mobility of charge carriers [18]. The mobility of electrons and holes is related to the diffusion coefficients by the following relations [7] (k_B – Boltzmann constant):

$$\mu_p k_B T/e=D_p,\ \mu_n k_B T/e=D_n. \quad (5)$$

The temperature dependences of the mobility of charge carriers are determined by phenomenological relationships, in particular, for Si in the operating temperature range of p-i-n-diodes, they take the form [18]:

$$\mu_p(T)=\mu_{0p}(T/T_0)^{-0.5},\ \mu_n(T)=\mu_{0n}(T/T_0)^{-0.5},\ T_0=300\ ^\circ\text{K}. \quad (6)$$

The rate of recombination of charge carriers is determined by processes of different types and properties of the semiconductor material. Based on the complexity of describing the combined action of recombination processes, the following phenomenological relationships are used to estimate the rate of recombination:

$$R_n = (n - n_0)/\tau_n^*, \quad R_p = (p - p_0)/\tau_p^*, \qquad (7)$$

where τ_p^*, τ_n^* – effective relaxation lifetimes of holes and electrons; n_0, p_0 – concentrations of balanced electrons and holes (in the active region of p-i-n-diodes, the concentrations of unbalanced and balanced charge carriers satisfy the conditions $n>>n_0$, $p>>p_0$).

Charge carriers appear in the active region of p-i-n-diodes also due to the processes of generation of charge carriers by the mechanisms of impact ionization, tunneling, thermal transitions of electrons to the conduction band [7]. Under typical operating modes of p-i-n structures, recombination by impact ionization and tunneling is unlikely, and thermal generation of balanced charge carriers under thermal equilibrium conditions is compensated by the reverse thermal recombination mechanism [7]. Therefore, in the future, generation processes will not be taken into account.

Equations (1)–(4) are supplemented with the following boundary conditions at the boundary of the domain $\partial G_i = L_i$ ($L_i = L_n \cup L_p \cup L_0 \cup L_1$, Fig. 2):

a) similarly to [19], let's form the conditions that determine the electric current density at the injection contacts (8), (9) and on the side surface of the diode (10):

$$\left(\vec{j}_n \cdot \vec{\text{v}}\right)\Big|_{L_n} - e\alpha_n n = J, \quad \left(\vec{j}_p \cdot \vec{\text{v}}\right)\Big|_{L_n} - e\alpha_p p = 0, \qquad (8)$$

$$\left(\vec{j}_p \cdot \vec{\text{v}}\right)\Big|_{L_p} + e\alpha_p p = J, \quad \left(\vec{j}_n \cdot \vec{\text{v}}\right)\Big|_{L_p} + e\alpha_n n = 0, \qquad (9)$$

$$\left(\vec{j}_n \cdot \vec{\text{v}}\right)\Big|_{L_0} - e\alpha_n^* n = 0, \quad \left(\vec{j}_p \cdot \vec{\text{v}}\right)\Big|_{L_0} - e\alpha_p^* p = 0, \qquad (10)$$

where J – constant that determines the density of the injection current (control current); **v** – normal vector to the region boundary; $e\alpha_p p$, $e\alpha_n n$, $e\alpha_p^* p$, $e\alpha_n^* n$ – densities of the recombinant current; $\alpha_{n,p}$, $\alpha^*_{n,p}$ – rates of surface recombination of electrons and holes, which are generally different on the contact surfaces and on the lateral surface; on the face L_1 the sought functions must satisfy the self-conjugation conditions:

$$n\big|_{L_1-0} = n\big|_{L_1+0}, \quad \frac{\partial n}{\partial \text{v}}\bigg|_{L_1-0} = \frac{\partial n}{\partial \text{v}}\bigg|_{L_1+0},$$

$$p\big|_{L_1-0} = p\big|_{L_1+0}, \quad \frac{\partial p}{\partial \text{v}}\bigg|_{L_1-0} = \frac{\partial p}{\partial \text{v}}\bigg|_{L_1+0};$$

b) the drop in the applied voltage across the diode mainly occurs in the active region:

$$\varphi\big|_{L_n} = 0, \quad \varphi\big|_{L_p} = U, \qquad (11)$$

where U – applied voltage. Boundary conditions for the potential in other parts of the surface: $\varphi'_\text{v}\big|_{L_0} = 0$, $\varphi'_\text{v}\big|_{L_1} = 0$. The conditions are written on the basis that the active region of the p-i-n-diode is high in resistance compared to the highly doped regions;

c) the heat flux removed from the active area of the p-i-n-diode through its surface is subject to the Newton-Richman law. Therefore, let's supplement the heat conduction equation with conditions of the form:

$$\frac{\partial T}{\partial \text{v}}\bigg|_{L_n} = k_1(T - T_0), \quad \frac{\partial T}{\partial \text{v}}\bigg|_{L_p} = -k_1(T - T_0),$$

$$\frac{\partial T}{\partial \text{v}}\bigg|_{L_0} = k_2(T - T_0), \qquad (12)$$

where $k_{1,2} = \sigma_{1,2}/\lambda_t$, $\sigma_{1,2}$ – heat transfer coefficients.

The nonlinear problem (1)–(12), posed on a region with curvilinear boundaries, is a complex mathematical problem, which in most such cases is solved by numerical methods [13, 14]. In this case, the effectiveness of the computational algorithm can be achieved after meeting the requirements of monotonicity, stability, convergence. The procedure for solving such problems requires the involvement of significant computer resources. To solve the problem, it is proposed to use asymptotic approaches [20–25], which allow reducing the initial nonlinear problem to a sequence of linear boundary value problems. In this case, the physical domain of the problem with curvilinear sections of the boundary of the domain is reduced to the canonical form by using the method of conformal mappings [23, 26–28].

5. Results of solving the problem of predicting the state of an electron-hole plasma in a region with curvilinear boundaries

5. 1. Stationary diffusion-drift model of the state of an electron-hole plasma. Mathematical formulation of the problem

For a given form of injection contacts ($y = g_n(x, w, h)$, $y = g_p(x,w,h)$), it is necessary to find solutions to the system of differential equations (1)–(4) in the region $G_i = G_1 \cup G_2 \cup (\partial \bar{G}_1 \cap \partial \bar{G}_2)$ (Fig. 3), $G_1 = \{(x,y)$: $0 < x < h$, $g_n(x, w, h) < y < g_p(x,w,h)\}$, $G_2 = \{(x,y)$: $h < x < h+b$, $0 < y < d\}$. The equations are supplemented by the boundary conditions (8)–(12) on a closed loop $L = L_{C^*D^*} \cup L_{D^*E^*} \cup L_{E^*E} \cup L_{ED} \cup L_{DC} \cup L_{CC^*}$, bounding the region G_i (Fig. 3). The transition from the three-dimensional physical domain of the problem to the two-dimensional one (G_i) is due to the homogeneity of the integrated p-i-n-structure along the OZ direction and the fulfillment of the condition $d << L_B$.

Let's use additional restrictions:

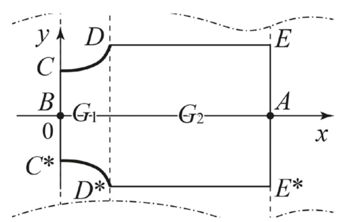

Fig. 3. The physical area of the problem

1) EMW SWITCHING is organized so that the switching time of the switch significantly exceeds the characteristic times of the dynamics of charge carriers in a semiconductor material. So, when a direct control current is applied to the contacts of the p-i-n-structure, a stationary distribution of charge carriers is established in the active region. Therefore, the distribution of the concentration of charge carriers in the region G_i will be sought at $p'_t = 0$, $n'_t = 0$;

2) the active region is weakly doped – the concentration of unbalanced electrons and holes significantly exceeds the concentration of impurities ($N_d = 0$);

3) in the investigated operating mode, the lifetime of unbalanced charge carriers in the active region and the thermal conductivity are constant values.

Let's rewrite equations (1)–(4) taking into account (5)–(7) for the stationary case. Let's apply the normalization procedure: $\check{x}=x/w$, $\check{y}=y/w$, $\check{z}=z/w$, $\check{T}=T/T_0$, $\check{\varphi}=e\varphi/(k_BT_0)$, $\check{U}=eU/(k_BT_0)$, $\check{n}=n/n_i$ $(0\leq\check{n}\leq n_{\max}/n_i)$, $\check{p}=p/n_i$ $(0\leq\check{p}\leq p_{\max}/n_i)$, where n_i – constant, determines the concentration of electrons in its own semiconductor, depends on the selected semiconductor material. After that, the system of equations (1)–(4) takes the form (the symbol "ˇ" is further omitted):

$$\begin{cases} \Delta p = -\nabla p\cdot\nabla\varphi - p\Delta\varphi + A_p(T)p,\\ \Delta n = \nabla n\cdot\nabla\varphi + n\Delta\varphi + A_n(T)n,\\ \mu\Delta\varphi = -(p-n),\\ \Delta T = -\mu\delta\left(T^{-0.5}\begin{pmatrix}\mu_{0p}(p\nabla\varphi+T\nabla p)+\\ +\mu_{0n}(n\nabla\varphi-T\nabla n)\end{pmatrix}\cdot\nabla\varphi+\right.\\ \left.+(\gamma_p p+\gamma_n n)\left(1-\alpha T_0T^2\Big/\left(E_g\left(T+\left(\frac{\beta}{T_0}\right)\right)\right)\right)\right).\end{cases} \tag{13}$$

Here the notation is used: $\mu=\varepsilon\varepsilon_0k_BT_0/(e^2w^2n_i)$ (small parameter $\mu\sim10^{-8}\div10^{-6}$),

$$A_n(T)=\frac{w^2}{D_{0n}\sqrt{T}\tau_n^*},\quad A_p(T)=\frac{w^2}{D_{0p}\sqrt{T}\tau_p^*},$$

$$\gamma_p=\frac{E_gw^2e}{\tau_p^*(k_BT_0)^2},\quad \gamma_n=\frac{E_gw^2e}{\tau_n^*(k_BT_0)^2},\quad \delta=\frac{ew^2n_i^2k_B}{\varepsilon\varepsilon_0\lambda_t}.$$

In this case, conditions (8)–(12) are reduced to the following form:

$$\left.\frac{1}{T}nE_\nu+\frac{\partial n}{\partial\nu}-\frac{\alpha_nw}{D_{0n}\sqrt{T}}n\right|_{L_n}=\frac{Jw}{eD_{0n}\sqrt{T}n_i},$$

$$\left.\frac{1}{T}pE_\nu-\frac{\partial p}{\partial\nu}-\frac{\alpha_pw}{D_{0p}\sqrt{T}}p\right|_{L_n}=0,$$

$$\left.-\frac{1}{T}pE_\nu+\frac{\partial p}{\partial\nu}-\frac{\alpha_pw}{D_{0p}\sqrt{T}}p\right|_{L_p}=-\frac{Jw}{eD_{0p}\sqrt{T}n_i},$$

$$\left.\frac{1}{T}nE_\nu+\frac{\partial n}{\partial\nu}+\frac{\alpha_nw}{D_{0n}\sqrt{T}}n\right|_{L_p}=0, \tag{14}$$

$$\left.\frac{1}{T}nE_\nu+\frac{\partial n}{\partial\nu}-\frac{\alpha_n^*w}{D_{0n}\sqrt{T}}n\right|_{L_0}=0,$$

$$\left.\frac{1}{T}pE_\nu-\frac{\partial p}{\partial\nu}-\frac{\alpha_p^*w}{D_{0p}\sqrt{T}}p\right|_{L_0}=0,$$

$$\left.\frac{\partial n}{\partial\nu}\right|_{L_1}=0,\quad \left.\frac{\partial p}{\partial\nu}\right|_{L_1}=0,$$

$$\varphi|_{L_n}=0,\quad \varphi|_{L_p}=U,\quad \left.\frac{\partial\varphi}{\partial\nu}\right|_{L_0}=0,\quad \left.\frac{\partial\varphi}{\partial\nu}\right|_{L_1}=0, \tag{15}$$

$$\left.\frac{\partial T}{\partial\nu}\right|_{L_n}=k_1(T-1)w,\quad \left.\frac{\partial T}{\partial\nu}\right|_{L_p}=-k_1(T-1)w,$$

$$\left.\frac{\partial T}{\partial\nu}\right|_{L_0}=k_2(T-1)w,\quad \left.\frac{\partial T}{\partial\nu}\right|_{L_1}=0. \tag{16}$$

The complexity of the boundary value problem (13)–(16) is due to the fact that the system of partial differential equations (13) is nonlinear and contains variable coefficients. The presence of curvilinear sections on the boundary of the region of integration further complicates the task. Therefore, let's propose to construct a conformal mapping of the physical domain of the problem G_i onto some canonical domain [26–29] G_i (Fig. 4, *b*), on the boundary of which conditions (14)–(16) must be satisfied.

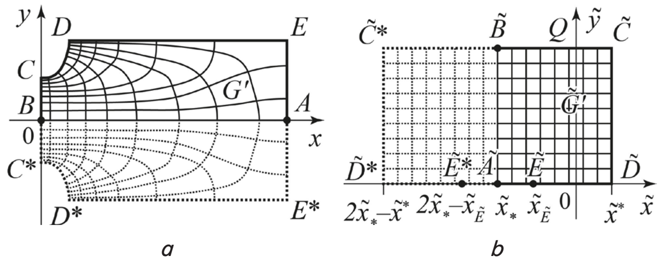

Fig. 4. Schematic images: *a* — conformal mesh in the physical region *G'*; *b* — corresponding parametric region *G'*

Since the domain G_i is symmetric ($g_n(x, w, h)=-g_p(x, w, h)$, Fig. 4, *a*), it is expedient to construct a mapping of its upper subdomain G' with the boundary $\partial G'=ABCDE$ onto the interior rectangle (Fig. 4, *b*) $\tilde{G}'=\{(\tilde{x},\tilde{y}):\tilde{x}_*<\tilde{x}<\tilde{x}^*,0<\tilde{y}<Q\}$ ($\partial\tilde{G}'=\tilde{A}\tilde{B}\tilde{C}\tilde{D}\tilde{E}$, $z=\tilde{x}+i\tilde{y}$). Moreover, apart from the sought-for functions $\tilde{x}=\tilde{x}(x, y)$, $\tilde{y}=\tilde{y}(x, y)$, the auxiliary parameters Q, $\tilde{E}$ are unknown.

The corresponding problem is to solve the system of Cauchy-Riemann equations

$$\frac{\partial\tilde{x}}{\partial x}=\frac{\partial\tilde{y}}{\partial y},\quad \frac{\partial\tilde{x}}{\partial y}=-\frac{\partial\tilde{y}}{\partial x}, \tag{17}$$

with boundary conditions:

$$\tilde{x}|_{AB}=\tilde{x}|_{0<x<h+b,\,y=0}=\tilde{x}_*,\quad \tilde{y}|_{BC}=\tilde{y}|_{x=0,\,0<y<0.5w}=Q,$$

$$\tilde{x}|_{CD}=\tilde{x}|_{0<x<h,\,y=g_n(x,w,h)}=\tilde{x}^*,$$

$$\tilde{y}|_{AED}=\tilde{y}|_{(x=h+b,\,0<y<d)\cup(h<x<h+b,\,y=d)}=0, \tag{18}$$

$$\int_{MN}-\frac{\partial\tilde{x}}{\partial y}\mathrm{d}x+\frac{\partial\tilde{x}}{\partial x}\mathrm{d}y=Q,\quad M\in BC,\quad N\in AED.$$

Let's note that the solutions of problem (17), (18) $\tilde{x}=\tilde{x}(x, y)$ and $\tilde{y}=\tilde{y}(x, y)$ from the physical point of view are, respectively, equipotential and force lines of the electrostatic field in the investigated region

5. 2. Application of the method of conformal mappings to transform the physical domain of the problem

The procedure for solving problem (17), (18) is laborious. Significant simplifications can be achieved under the condition of the implementation of the inverse conformal mapping $\tilde{x}=\tilde{x}(x, y)$ i $\tilde{y}=\tilde{y}(x, y)$ as a result of solving the equations [26, 29]:

$$\frac{\partial^2x}{\partial\tilde{x}^2}+\frac{\partial^2x}{\partial\tilde{y}^2}=0,\quad \frac{\partial^2y}{\partial\tilde{x}^2}+\frac{\partial^2y}{\partial\tilde{y}^2}=0,\quad \forall(x,y)\in G', \tag{19}$$

under the following boundary and "connecting" conditions [28, 29]:

$$y(\tilde{x}_*,\tilde{y})=0,\ \ 0\le x(\tilde{x}_*,\tilde{y})\le h+b,\ \ 0\le \tilde{y}\le Q,$$

$$x(\tilde{x},Q)=0,\ \ 0\le y(\tilde{x},Q)\le 0.5w,\ \ \tilde{x}_*\le \tilde{x}\le \tilde{x}^*,$$

$$y(\tilde{x}^*,\tilde{y})=g_n\left(x(\tilde{x}^*,\tilde{y}),w,h\right),$$

$$0\le x(\tilde{x}^*,\tilde{y})\le h,\ \ 0\le \tilde{y}\le Q,$$

$$y(\tilde{x},0)=0.5d,\ \ h\le x(\tilde{x},0)\le h+b,\ \ \tilde{x}_{\tilde{E}}\le \tilde{x}\le \tilde{x}^*,$$

$$x(\tilde{x},0)=h+b,\ \ 0\le y(\tilde{x},0)\le 0.5d,\ \ \tilde{x}_*\le \tilde{x}\le \tilde{x}_{\tilde{E}}, \tag{20}$$

$$\left.\frac{\partial x(\tilde{x},\tilde{y})}{\partial \tilde{x}}\right|_{\tilde{x}=\tilde{x}_*}=0,\ \ 0\le \tilde{y}\le Q,$$

$$\left.\frac{\partial y(\tilde{x},\tilde{y})}{\partial \tilde{y}}\right|_{\tilde{y}=Q}=0,\ \ \tilde{x}_*\le \tilde{x}\le \tilde{x}^*,$$

$$\left.\left(\begin{aligned}&\frac{\partial x(\tilde{x},\tilde{y})}{\partial \tilde{x}}+\\&+\frac{\partial g_n\left(x(\tilde{x},\tilde{y}),w,h\right)}{\partial x}\frac{\partial y(\tilde{x},\tilde{y})}{\partial \tilde{x}}\end{aligned}\right)\right|_{\tilde{x}=\tilde{x}^*}=0,\ \ 0\le \tilde{y}\le Q,$$

$$\left.\frac{\partial x(\tilde{x},\tilde{y})}{\partial \tilde{y}}\right|_{\tilde{y}=0}=0,\ \ \tilde{x}_{\tilde{E}}\le \tilde{x}\le \tilde{x}^*,$$

$$\left.\frac{\partial y(\tilde{x},\tilde{y})}{\partial \tilde{y}}\right|_{\tilde{y}=0}=0,\ \ \tilde{x}_*\le \tilde{x}\le \tilde{x}_{\tilde{E}}, \tag{21}$$

where $(\tilde{x}_{\tilde{E}}, 0)$ – point of the region G', which corresponds to the point E of the region G''.

Let's write an approximate representation of problem (19)–(21) in the uniform grid domain $\tilde{G}'^{\lambda}=\{(\tilde{x}_i,\tilde{y}_j)$: $\tilde{x}_i=\tilde{x}_*+i\Delta\tilde{x}$, $i=\overline{0,m+1}$; $\tilde{y}_j=j\Delta\tilde{y}$, $j=\overline{0,l+1}$; $\Delta\tilde{x}=(\tilde{x}^*-\tilde{x}_*)/(m+1)$, $\Delta\tilde{y}=Q/(l+1)$, $\lambda_c=\Delta\tilde{x}/\Delta\tilde{y}$, $m,l\in N\}$ using the difference representation of the corresponding expressions [26, 29]. Laplace equation (21) is approximated as follows:

$$x_{i\,j}=0.5\left(\left(x_{i\,j-}+x_{i\,j+}\right)\lambda_c^2+x_{i-\ j}+x_{i+\ j}\right)/\left(\lambda_c^2+1\right),$$

$$y_{i\,j}=0.5\left(\left(y_{i\,j-}+y_{i\,j+}\right)\lambda_c^2+y_{i-\ j}+y_{i+\ j}\right)/\left(\lambda_c^2+1\right),$$

$$\left(1\le i\le m,\ 1\le j\le l\right). \tag{22}$$

Here $x_{i,j}=x(\tilde{x}_i,\tilde{y}_j)$, $y_{i,j}=y(\tilde{x}_i,\tilde{y}_j)$, $(x_{i,j},y_{i,j})\in G'^{\lambda}$, the conformal invariant is sought by:

$$\lambda_c=\frac{1}{(m+1)(l+1)}\times$$

$$\times\sum_{i=0}^{m}\sum_{j=0}^{l}\left(\begin{aligned}&\sqrt{\left(x_{i+1,j+1}-x_{i,j+1}\right)^2+\left(y_{i+1,j+1}-y_{i,j+1}\right)^2}+\\&+\sqrt{\left(x_{i+1,j}-x_{i,j}\right)^2+\left(y_{i+1,j}-y_{i,j}\right)^2}\end{aligned}\right)/$$

$$/\left(\begin{aligned}&\sqrt{\left(x_{i,j+1}-x_{i,j}\right)^2+\left(y_{i,j+1}-y_{i,j}\right)^2}+\\&+\sqrt{\left(x_{i+1,j+1}-x_{i+1,j}\right)^2+\left(y_{i+1,j+1}-y_{i+1,j}\right)^2}\end{aligned}\right). \tag{23}$$

Boundary conditions (20), (21) can be rewritten as:

$$y_{0,j}=0,\ \ 0\le x_{0,j}\le h+b,\ \ j=\overline{0,l+1},$$

$$x_{i,l+1}=0,\ \ 0\le y_{i,l+1}\le 0.5w,\ \ i=\overline{0,m+1},$$

$$y_{m+1,j}=g_n\left(x_{m+1,j},w,h\right),\ \ 0\le x_{m+1,j}\le h,\ \ j=\overline{0,l+1},$$

$$y_{i,0}=0.5d,\ \ h\le x_{i,0}\le h+b,\ \ i=\overline{m_{\tilde{E}}+1,m+1},$$

$$x_{i,0}=h+b,\ \ 0\le y_{i,0}\le 0.5d,\ \ i=\overline{0,m_{\tilde{E}}}, \tag{24}$$

$$x_{0,j}=x_{1,j},\ \ j=\overline{1,l},$$

$$y_{i,l+1}=y_{i,l},\ \ i=\overline{1,m},$$

$$\left.\begin{aligned}&x_{m+1,j}-x_{m,j}+\\&+\left(y_{m+1,j}-y_{m,j}\right)\frac{\partial g_n(x,w,h)}{\partial x}\end{aligned}\right|_{x=x_{m+1,j}}=0,\ \ j=\overline{1,l},$$

$$x_{i,0}=x_{i,1},\ \ i=\overline{m_{\tilde{E}}+1,m},$$

$$y_{i,0}=y_{i,1},\ \ i=\overline{1,m_{\tilde{E}}}, \tag{25}$$

where $m_{\tilde{E}}-1$ – the number of nodes in the section AE (determined in the process of solving the problem).

5. 3. Technique for parallelizing the nonlinear model of diffusion-drift and thermal processes in the active region of p-i-n diodes

As a result of changing the coordinates $x=x(\tilde{x}_i,\tilde{y}_j)$, $y=y(\tilde{x}_i,\tilde{y}_j)$ and transforming the physical domain of the problem in the corresponding parametric domain G_i, the mathematical model of the stationary diffusion process (13)–(16) takes the following form:

$$\begin{cases}\Delta p=-\nabla p\cdot\nabla\varphi-p\Delta\varphi+\dfrac{A_p(T)}{\tilde{\Delta}}p,\\[2mm] \Delta n=\nabla n\cdot\nabla\varphi+n\Delta\varphi+\dfrac{A_n(T)}{\tilde{\Delta}}n,\\[2mm] \mu\Delta\varphi=-\dfrac{1}{\tilde{\Delta}}(p-n),\\[2mm] \Delta T=-\mu\delta\left(\begin{aligned}&(T)^{-0.5}\left(\begin{aligned}&\mu_{0p}\left(p\nabla\varphi+T\nabla p\right)+\\&+\mu_{0n}\left(n\nabla\varphi-T\nabla n\right)\end{aligned}\right)\cdot\nabla\varphi+\\&+\frac{\left(\gamma_p p+\gamma_n n\right)}{\tilde{\Delta}}\left(\begin{aligned}&1-\alpha T_0T^2/\\&/\left(E_g\left(T+\left(\frac{\beta}{T_0}\right)\right)\right)\end{aligned}\right)\end{aligned}\right),\end{cases} \tag{26}$$

$p=p(\tilde{x}_i,\tilde{y}_j)$, $n=n(\tilde{x}_i,\tilde{y}_j)$, $\varphi=\varphi(\tilde{x}_i,\tilde{y}_j)$, $T=T(\tilde{x}_i,\tilde{y}_j)$,

$$\tilde{\Delta}=\left(\frac{\partial\tilde{x}}{\partial x}\right)^2+\left(\frac{\partial\tilde{x}}{\partial y}\right)^2=\left(\frac{\partial\tilde{y}}{\partial x}\right)^2+\left(\frac{\partial\tilde{y}}{\partial y}\right)^2$$

– determinant of the Jacobi matrix of the transformation;

$$\left.\frac{1}{T}nE_{\tilde{x}}+\frac{\partial n}{\partial\tilde{x}}-\frac{\alpha_n w}{D_{0n}\sqrt{T}}n\right|_{L_{C^*D^*}}=\frac{J_{\tilde{x}}w}{eD_{0n}\sqrt{T}n_i},$$

$$\left.\frac{1}{T}pE_{\tilde{x}}-\frac{\partial p}{\partial\tilde{x}}-\frac{\alpha_p w}{D_{0p}\sqrt{T}}p\right|_{L_{C^*D^*}}=0,$$

$$\left.\varphi\right|_{L_{C^*D^*}}=0,\ \ \left.\frac{\partial T}{\partial\tilde{x}}\right|_{L_{C^*D^*}}=k_1(T-1)w;$$

$$-\frac{1}{T}pE_{\tilde{x}}+\frac{\partial p}{\partial \tilde{x}}-\frac{\alpha_p w}{D_{0p}\sqrt{T}}p\bigg|_{L_{CD}}=-\frac{J_{\tilde{x}}w}{eD_{0p}\sqrt{T}n_i},$$

$$\frac{1}{T}nE_{\tilde{x}}+\frac{\partial n}{\partial \tilde{x}}+\frac{\alpha_n w}{D_{0n}\sqrt{T}}n\bigg|_{L_{CD}}=0,$$

$$\varphi|_{L_{CD}}=U,\ \ \frac{\partial T}{\partial \tilde{x}}\bigg|_{L_{CD}}=-k_1(T-1)w; \tag{27}$$

$$\pm\frac{1}{T}nE_{\tilde{y}}+\frac{\partial n}{\partial \tilde{y}}-\frac{\alpha_n^* w}{D_{0n}\sqrt{T}}n\bigg|_{L_{C^*C},L_{E^*E}}=0,$$

$$\pm\frac{1}{T}pE_{\tilde{y}}-\frac{\partial p}{\partial \tilde{y}}-\frac{\alpha_p^* w}{D_{0p}\sqrt{T}}p\bigg|_{L_{C^*C},L_{E^*E}}=0,$$

$$\frac{\partial \varphi}{\partial \tilde{y}}\bigg|_{L_{C^*C},L_{E^*E}}=0,\ \ \frac{\partial T}{\partial \tilde{y}}\bigg|_{L_{C^*C},L_{E^*E}}=\pm k_2(T-1)w;$$

$$n|_{L_{DE}-0,L_{D^*E^*}-0}=n|_{L_{DE}+0,L_{D^*E^*}+0},$$

$$\frac{\partial n}{\partial \tilde{y}}\bigg|_{L_{DE}-0,L_{D^*E^*}-0}=\frac{\partial n}{\partial \tilde{y}}\bigg|_{L_{DE}+0,L_{D^*E^*}+0},$$

$$p|_{L_{DE}-0,L_{D^*E^*}-0}=p|_{L_{DE}+0,L_{D^*E^*}+0},$$

$$\frac{\partial p}{\partial \tilde{y}}\bigg|_{L_{DE}-0,L_{D^*E^*}-0}=\frac{\partial p}{\partial \tilde{y}}\bigg|_{L_{DE}+0,L_{D^*E^*}+0},$$

$$\frac{\partial \varphi}{\partial \tilde{y}}\bigg|_{L_{DE},L_{D^*E^*}}=0,\ \ \frac{\partial T}{\partial \tilde{y}}\bigg|_{L_{DE},L_{D^*E^*}}=0.$$

The presence of a small parameter μ in the formulation of problem (26), (27) makes it possible to use the asymptotic methods of perturbation theory [20–25] to solve it. In this case, the process of solving the problem consists of two stages, which alternate. If T is fixed in equations (1)–(3) of system (26), then the system of equations (1)–(3) together with the corresponding boundary conditions (27) forms singularly perturbed problems similar to the one considered in [10]. After substituting the solutions of the singularly perturbed problem (functions $\varphi(\tilde{x}_i,\tilde{y}_j)$, $n(\tilde{x}_i,\tilde{y}_j)$, $p(\tilde{x}_i,\tilde{y}_j)$) into the fourth equations of system (26) and invoking boundary conditions (27), let's obtain regularly perturbed problems for the heat equation ... In what follows, for convenience, the "~" symbol is omitted.

Based on the formulation of the singularly perturbed problem, let's propose to seek solutions in the form of the following asymptotic series [22–24]:

$$\varphi=\varphi(x,y,\mu)=\Phi_{(m)}(x,y,\mu)+\underline{\Phi}_{(m)}(\underline{\xi},\mu)+$$
$$+\overline{\Phi}_{(m)}(\overline{\xi},\mu)+R_{\varphi(m)}(x,y,\mu)=$$
$$=\sum_{i=0}^{m}\mu^i\varphi_i(x,y)+\sum_{i=0}^{m}\mu^i\underline{\Phi}_i(\underline{\xi})+$$
$$+\sum_{i=0}^{m}\mu^i\overline{\Phi}_i(\overline{\xi})+R_{\varphi(m)}(x,y,\mu),$$

$$n=n(x,y,\mu)=N_{(m)}(x,y,\mu)+\underline{N}_{(m)}(\underline{\xi},\mu)+$$
$$+\overline{N}_{(m)}(\overline{\xi},\mu)+R_{n(m)}(x,y,\mu)=$$
$$=\sum_{i=0}^{m}\mu^i n_i(x,y)+\sum_{i=0}^{m}\mu^i\underline{N}_i(\underline{\xi})+$$
$$+\sum_{i=0}^{m}\mu^i\overline{N}_i(\overline{\xi})+R_{n(m)}(x,y,\mu), \tag{28}$$

$$p=p(x,y,\mu)=P_{(m)}(x,y,\mu)+\underline{P}_{(m)}(\underline{\xi},\mu)+$$
$$+\overline{P}_{(m)}(\overline{\xi},\mu)+R_{p(m)}(x,y,\mu)=$$
$$=\sum_{i=0}^{m}\mu^i p_i(x,y)+\sum_{i=0}^{m}\mu^i\underline{P}_i(\underline{\xi})+$$
$$+\sum_{i=0}^{m}\mu^i\overline{P}_i(\overline{\xi})+R_{p(m)}(x,y,\mu),$$

where $\Phi_{(m)}(x,y,\mu)$, $N_{(m)}(x,y,\mu)$, $P_{(m)}(x,y,\mu)$ – regular parts of the asymptotics; $\underline{\Phi}_{(m)}(\xi,\mu)$, $\underline{N}_{(m)}(\xi,\mu)$, $\underline{P}_{(m)}(\xi,\mu)$, $\overline{\Phi}_{(m)}(\xi,\mu)$, $\overline{N}_{(m)}(\xi,\mu)$, $\overline{P}_{(m)}(\xi,\mu)$ – boundary corrections of the asymptotics, respectively, in the neighborhoods of the points $x=0$, $x=1$ ($\underline{\xi}=\frac{x}{\sqrt{\mu}}$, $\overline{\xi}=\frac{1-x}{\sqrt{\mu}}$ – regularizing dilations) are the remainders. Note that when constructing series (28), let's took into account the influence on the structure of solutions of the conditions on the contact sections (the neighborhood of the points $x=0$, $x=1$).

Let's seek the solution of the heat conduction equation in the following form:

$$T=T(x,y,\mu)=\sum_{i=0}^{m}\mu^i T_{(i)}(x,y)+R_{T(m)}(x,y,\mu). \tag{29}$$

At the first stage of solving (26), (27), let's obtain, similarly to [10], solutions of the following sequence of problems:

$$\begin{cases} n_0=p_0,\\ \Delta n_0-\nabla\cdot(n_0\nabla\varphi_0)-\dfrac{A_n(T)}{\tilde{\Delta}}n_0=0,\\ \Delta p_0+\nabla\cdot(p_0\nabla\varphi_0)-\dfrac{A_p(T)}{\tilde{\Delta}}p_0=0, \end{cases} \tag{30}$$

$$\begin{cases} \dfrac{\partial^2\underline{\Phi}_0}{\partial\underline{\xi}^2}=-\dfrac{1}{\tilde{\Delta}}(\underline{P}_0-\underline{N}_0),\\ \dfrac{\partial^2\underline{N}_0}{\partial\underline{\xi}^2}-\dfrac{\partial}{\partial\underline{\xi}}\left(\underline{N}_0\dfrac{\partial\underline{\Phi}_0}{\partial\underline{\xi}}\right)=0,\\ \dfrac{\partial^2\underline{P}_0}{\partial\underline{\xi}^2}+\dfrac{\partial}{\partial\underline{\xi}}\left(\underline{P}_0\dfrac{\partial\underline{\Phi}_0}{\partial\underline{\xi}}\right)=0, \end{cases} \tag{31}$$

$$\begin{cases} \dfrac{\partial^2\overline{\Phi}_0}{\partial\overline{\xi}^2}=-\dfrac{1}{\tilde{\Delta}}(\overline{P}_0-\overline{N}_0),\\ \dfrac{\partial^2\overline{N}_0}{\partial\overline{\xi}^2}-\dfrac{\partial}{\partial\overline{\xi}}\left(\overline{N}_0\dfrac{\partial\overline{\Phi}_0}{\partial\overline{\xi}}\right)=0,\\ \dfrac{\partial^2\overline{P}_0}{\partial\overline{\xi}^2}+\dfrac{\partial}{\partial\overline{\xi}}\left(\overline{P}_0\dfrac{\partial\overline{\Phi}_0}{\partial\overline{\xi}}\right)=0, \end{cases} \tag{32}$$

– $L_{C^*D^*}$:

$$\frac{dn_0}{dx}-\chi(T)n_0-\frac{\beta_n(T)\underline{N}_0}{2}+$$
$$+\frac{\beta_p(T)\underline{P}_0}{2}=\frac{J}{2eD_{0n}\sqrt{T}}\frac{w}{n_i},$$

$$\frac{d\underline{N}_0}{d\underline{\xi}}=0,\quad \frac{d\underline{P}_0}{d\underline{\xi}}=0,\quad \varphi_0+\underline{\Phi}_0=0;$$

– L_{CD}:

$$\frac{dn_0}{dx}+\chi(T)n_0-\frac{\beta_p(T)\overline{P}_0}{2}+$$
$$+\frac{\beta_n(T)\overline{N}_0}{2}=-\frac{J}{2eD_{0p}\sqrt{T}}\frac{w}{n_i},$$

$$\frac{d\overline{P}_0}{d\overline{\xi}}=0,\quad \frac{d\overline{N}_0}{d\overline{\xi}}=0,\quad \varphi_0+\overline{\Phi}_0=U; \quad (33)$$

– L_{C^*C} and L_{E^*E}:

$$\frac{\partial n_0}{\partial y}-\beta^*(T)wn_0=0,\quad \frac{\partial n_0}{\partial y}+\beta^*(T)wn_0=0,\quad \frac{\partial\varphi_0}{\partial y}=0;$$

– L_{DE} and $L_{D^*E^*}$:

$$n_0\big|_{L_1-0}=n_0\big|_{L_1+0},\quad \frac{\partial n_0}{\partial y}\bigg|_{L_1-0}=\frac{\partial n_0}{\partial y}\bigg|_{L_1+0},\quad \frac{\partial\varphi_0}{\partial y}\bigg|_{L_1}=0;$$

$$\lim_{\underline{\xi}\to\infty}\underline{\Phi}_0(\underline{\xi})=\lim_{\overline{\xi}\to\infty}\overline{\Phi}_0(\overline{\xi}),$$

$$\lim_{\underline{\xi}\to\infty}\underline{N}_0(\underline{\xi})=\lim_{\underline{\xi}\to\infty}\underline{P}_0(\underline{\xi})=\lim_{\overline{\xi}\to\infty}\overline{N}_0(\overline{\xi})=\lim_{\overline{\xi}\to\infty}\overline{P}_0(\overline{\xi})=0,$$

$$\chi(T)=\frac{1}{2}(\beta_n(T)-\beta_p(T)),$$

$$\beta_n(T)=\frac{\alpha_n w}{D_{0n}\sqrt{T}},\quad \beta_p(T)=\frac{\alpha_p w}{D_{0p}\sqrt{T}},$$

$$\beta^*(T)=\frac{1}{2}\left(\frac{\alpha_n^* w}{D_{0n}\sqrt{T}}-\frac{\alpha_p^* w}{D_{0p}\sqrt{T}}\right),$$

in the form of the principal terms of asymptotics (28), which make the main contribution to the solution of the problem.

At the second stage, let's fing an approximate solution to the nonlinear heat equation. Substitution of (29) into the corresponding equations and boundary conditions (26), (27) makes it possible to obtain a sequence of boundary value problems for linear inhomogeneous differential equations of the form:

$$\Delta T_{(0)}=0, \quad (34)$$

at

$$\frac{\partial T_{(0)}}{\partial x}\bigg|_{L_{C^*D^*}}=0,\quad \frac{\partial T_{(0)}}{\partial x}\bigg|_{L_{CD}}=0,$$

$$\frac{\partial T_{(0)}}{\partial y}\bigg|_{L_{C^*C},L_{E^*E}}=0,\quad \frac{\partial T_{(0)}}{\partial y}\bigg|_{L_{DE},L_{D^*E^*}}=0;$$

$$\Delta T_{(1)}=-\delta\left(\begin{array}{l}\left(\gamma_p\left(n_0+\overline{P}_0\right)+\gamma_n\left(n_0+\underline{N}_0\right)\right)\times\\ \times\left(1-\alpha T_0/\left(E_g\left(1+\left(\frac{\beta}{T_0}\right)\right)\right)\right)\end{array}\right)+$$
$$+\left(\begin{array}{l}\mu_{0p}\left(\left(n_0+\overline{P}_0\right)\nabla\varphi+\nabla\left(n_0+\overline{P}_0\right)\right)+\\ +\mu_{0n}\left(\left(n_0+\underline{N}_0\right)\nabla\varphi-\nabla\left(n_0+\underline{N}_0\right)\right)\end{array}\right)\nabla\varphi \quad (35)$$

at

$$\frac{\partial T_{(1)}}{\partial x}\bigg|_{L_{C^*D^*}}=k_1T_{(1)}w,$$

$$\frac{\partial T_{(1)}}{\partial x}\bigg|_{L_{CD}}=-k_1T_{(1)}w,$$

$$\frac{\partial T_{(1)}}{\partial y}\bigg|_{L_{C^*C},L_{E^*E}}=\pm k_2T_{(1)}w,$$

$$\frac{\partial T_{(1)}}{\partial y}\bigg|_{L_{DE},L_{D^*E^*}}=0.$$

The boundary corrections are obtained by solving the following system of equations with the corresponding boundary conditions (equivalent to the components of problem (31)–(33)):

$$\begin{cases}\sqrt{\mu}\dfrac{d\underline{N}_0(x,y)}{dx}=-\underline{N}_0(x,y)\Pi_N(x,y),\\ \sqrt{\mu}\dfrac{d\Pi_N(x,y)}{dx}=-\dfrac{1}{\tilde{\Delta}(x,y)}\underline{N}_0(x,y),\\ \underline{P}_0(x,y)=0,\end{cases}$$

$$\begin{cases}\sqrt{\mu}\dfrac{d\overline{P}_0(1-x,y)}{dx}=-\overline{P}_0(1-x,y)\Pi_p(1-x,y),\\ \sqrt{\mu}\dfrac{d\Pi_p(1-x,y)}{dx}=-\dfrac{1}{\tilde{\Delta}(1-x,y)}\overline{P}_0(1-x,y),\\ \overline{N}_0(1-x,y)=0,\end{cases} \quad (36)$$

where y – parameter of the function and is sequentially fixed for the corresponding chain of nodes in the parametric domain. Functions $\Pi_N(x,y)$, $\Pi_P(1-x,y)$ have the content of electric potential gradients. To determine the integration constants, along with the boundary conditions of problem (33), the following condition was used:

$$\int_0^1\left(\Pi_N(x,y)+\Pi_P(1-x,y)\right)\mathrm{d}x=U,$$

reflecting the approximate equality of the electric potential drop in the active region of the voltage U applied to the structure contacts.

5. 4. Results of numerous experiments

A series of numerical experiments was carried out, in which the following initial data were used: $e=16\cdot10^{-20}$ C,

D_n=35 cm^2/s, D_p=25 cm^2/s, α_n=2·10^5 cm/s, α_p=10^5 cm/s, n_i=10^{16} cm^3, μ=6.25·10^{-6}, U=5 V, J=2·10^5 A/cm^2, ε=11, ε_0=885××10^{-14} F/m, k_B=1.38·10^{-23} J/K, T_0=300 K, λ=130 W/(K·m), k_1=10 W/(cm^2·K), k_2=5 W/(cm^2·K). Wedge-type injection contacts were considered. According to the symmetric sections CD and C*D*, the limits of the active region were described by the functions $y=0.5(x(dw)/h+w)$ and $y=-0.5(x(d-w)/h+w)$, where w=26 μm, h=8 μm, b=72 μm, d=30 μm. The simulation results are shown in Fig. 5–7.

The algorithm for solving the difference problem (22)–(25) is based on the alternate parametrization of the limit and internal nodes of the grid region G', the conformal invariant λ_c using the ideas of the block iteration method [26, 28].

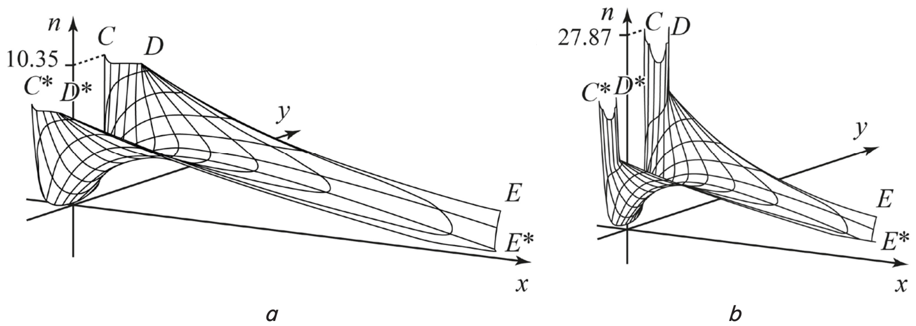

Fig. 5. Distributions of the concentration of charge carriers in the electron-hole plasma of the active region: *a* — without boundary corrections; *b* — with boundary corrections

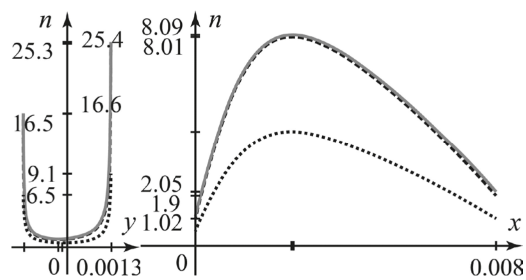

Fig. 6. Comparison of the calculated distributions of the concentration of charge carriers in cross sections without boundary corrections (....) with boundary corrections at T=300 K (▬ ▬ ▬) with boundary corrections and taking into account heating T=350 K (▬▬); *a* — cross section in the plane x=0; *b* — cross section c plane y=0)

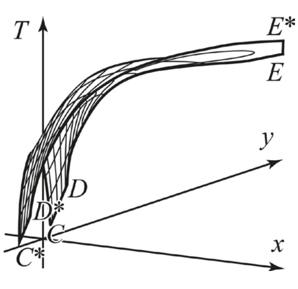

Fig. 7. Temperature field of the active region of the integrated p-i-n-structure

After executing the algorithm for reducing the physical domain of the problem to the canonical form, an iterative algorithm for solving the sequence of problems (30)–(35) is used. In this case, explicit difference schemes with a five-point pattern of the "cross" type are used to approximate the Laplace operator. The calculations were carried out on a uniform rectangular grid of nodes 100×400. The Adams algorithm was also used to solve systems of ordinary differential equations (36). It has been experimentally established that 5 iterations are sufficient to stabilize the numerical values of the computational process.

6. Discussion of the results of applying asymptotic methods and conformal mappings in modeling the characteristics of electron-hole plasma

Nonlinear problem (1)–(12), which is formed on the basis of improving the classical diffusion-drift plasma model, posed in a region with curvilinear boundaries. As a result of applying asymptotic methods and conformal transformations, it is reduced to a sequence of linear problems (22)–(25), (30)–(35), which are solved numerically. The main feature of this technique is the possibility of using, and in some cases – explicit computational schemes that do not require significant computer resources to obtain simulation results. Note that the problem is formulated in the hydrodynamic approximation.

Fig. 5, *a* illustrates the stationary distribution of plasma concentration in the active region of an integrated p-i-n-structure with wedge-shaped contacts, obtained by solving problem (1)–(3), (8)–(11) using the ambipolar diffusion approximation [7]. Let's obtain a similar result for the principal terms of the regular parts of the corresponding asymptotic series (28) at the initial stage of solving problem (26), (27) by methods of perturbation theory. Fig. 5, *b* shows the distribution of the plasma concentration with allowance for boundary corrections.

The results of modeling the distribution of plasma concentration according to the classical and improved mathematical models to a certain extent correlate with each other. Let's note significant differences in the concentration of charge carriers in the contact zones. Here, the condition of the approximate equality of the electron and hole concentrations, which is the basis for applying the ambipolar diffusion approach, is violated. The mean values of plasma concentration also differ. With the above initial data, an almost twofold increase in concentration is observed in comparison with the data of the classical model: 4.94 versus 9.14 (in relative units). Fig. 6 displays the data for comparative analysis of the results of modeling the distribution of the concentration of electrons and holes.

The temperature distribution in the active region of the integrated p-i-n-structure obtained by solving problems (30)–(35) is shown in Fig. 7. The simulation results reflect the fact that the cooling of the system is mainly due to the removal of heat through the metal contacts. Plasma heating by ~50 °C leads to a slight increase in plasma concentration (from 9.14 to 9.21 in relative units). Note that a feature of the mathematical model used in the research is the proposed description of recombination heat sources based on phenomenological data on the effect of temperature on the semiconductor band gap.

The key role in calculating the distribution of heat sources is played by the functions of the electric potential gradient, which are determined through the corresponding approximations and corrections $\Pi_N(x,y)$, $\Pi_P(1-x,y)$. Regular components of the potential gradient make an insignificant contribution to the description of the strength of the electrostatic plasma field. The characteristic power dissipated in a single p-i-n diode is 0.01–0.1 W. The classical model based on the use of the self-consistent field approximation (local gradients of the regular component of the electric field are insignificant) does not provide the indicated level of the dissipated power.

Note that at the corner points C, D, C^*, D^* of the region, the smoothness of the obtained distribution function is violated, but this circumstance does not significantly affect the description of the state of the electron-hole plasma. It is known that similar problems are solved by introducing angular corrections into consideration [20].

The accumulated charge values obtained within the framework of the ambipolar diffusion approximation and according to the refined mathematical model differ by 2–4 %. Such differences do not significantly affect the estimate of the value of the effective dielectric constant of the active region of the p-i-n-structure. And, as a consequence, on the assessment of the electrodynamic characteristics of switching systems. However, the decomposition of the nonlinear mathematical model was carried out by methods of perturbation theory and conformal mappings:

– to simplify the algorithm for solving the original problem by "splitting" it;

– to detail the physical meaning of the processes in the system under study;

– to optimize contact zones.

Let's hope that the proposed mathematical model and the proven methods of its analysis will become the basis for studying the behavior of integrated p-i-n-structures when switching powerful electromagnetic fields.

7. Conclusions

1. An improved mathematical model is proposed for predicting the stationary distribution of the concentration of electron-hole plasma in the active region of p-i-n-diodes with curvilinear boundaries under conditions of plasma heating as a result of the release of Joule heat and recombination energy of excess charge carriers in the volume of the active region. The choice of its components is reasoned. The model is represented as a nonlinear system of singularly and regularly perturbed boundary value problems for equations in accordance with the continuity of electron-hole currents, Poisson and heat conductivity with the corresponding boundary conditions.

2. The transformation of the physical domain of the problem to the canonical form is carried out by the method of conformal mappings. This ensures, to a certain extent, the universality of the developed algorithm for solving the problem.

3. The distributions of the concentration of electrons and holes in the active region of integrated p-i-n-structures with deepened contacts and the corresponding temperature field in the form of the corresponding asymptotic series in the small parameter of the problem are found, and the asymptotic series for the functions of the concentration of charge carriers and potential are proposed taking into account boundary corrections. This made it possible to reduce the nonlinear boundary value problem to a sequence of linear boundary value problems, which, in particular, include the traditional formulations. It is found that the near-edges and corrections make the main contribution to the distribution function of the electrostatic field of the electron-hole plasma. The analysis of the nonlinear mathematical model of plasma in the diffusion-drift thermal approximation makes it possible to detail the physical description of the processes in the system under study.

4. A number of numerical algorithms are built to visualize the research results. They have been tested. A computer model of an integrated p-i-n-structure with deep contacts has been developed, which allows, with an appropriate set of input data, to evaluate the basic characteristics of switches and to carry out the procedure for optimizing the shape and size of the p-i-, n-i-junction zones.

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