

The paper considers the features of the formation of an acoustic field by a spherical source with complicated properties in a regular plane-parallel waveguide, which is of practical importance in marine instrumentation and oceanographic research. The calculation algorithm is based on the use of the Helmholtz equation and the Fourier method for each partial region and the conjugation conditions on their boundaries. The presented calculation allows one to get rid of the idealized boundary conditions on the source surface, with the subsequent determination of the excitation coefficients of the waveguide modes within the framework of the Sturm-Liouville problem. In this case, the attraction of the boundary conditions on the surface and the bottom of the sea, as well as the Sommerfeld conditions, makes it possible to obtain the real distribution of the field in the vertical sections of the waveguide.

The obtained frequency dependences of the pressure and vibrational velocity components show their amplitude-phase differences, which reach 90 degrees, which partially explains the appearance of singular points in the intensity field in a regular waveguide. It has been determined that multiple reflections of sound waves from the boundaries of the working space and the space of the waveguide cause oscillations of the pressure components with a change in the amplitude level up to 6 dB. It was found that with an increase in the size of the source, a kind of resonance is formed in the working space, the frequency of which depends on the depth of the sea and corresponds to the region $kr=x=5.8$. It was found that when the acoustic field is formed in the working space, the frequency response of the impedance components is represented as a multiresonant dependence formed on the basis of the frequency characteristics of the lower modes and their combinations. Experimental studies have shown that the results of calculations of the mode composition of the acoustic field of the emitter, obtained in the conditions of the pool, correspond to the spatial characteristics of the mode components of the acoustic field with an error of up to 3 dB

Keywords: acoustic field, acoustic plane-parallel waveguide, spherical source, partial regions

UDC 534.23

DOI: 10.15587/1729-4061.2021.243161

APPLICATION OF PARTIAL AREAS METHOD IN THE PROBLEM OF SOUND RADIATION BY A SPHERE IN A WAVEGUIDE WITH SOFT ACOUSTICALLY BOUNDARIES

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Received date 27.08.2021

Accepted date 11.10.2021

Published date 29.10.2021

How to Cite: Korzhyk, O., Chaika, O., Naida, S., Zheliaskova, T., Naida, N. (2021). Application of partial areas method in the problem of sound radiation by a sphere in a waveguide with soft acoustically boundaries. *Eastern-European Journal of Enterprise Technologies*, 5 (5 (113)), 62–79. doi: <https://doi.org/10.15587/1729-4061.2021.243161>

1. Introduction

Currently, an increase in problem areas in the field of ocean research makes it necessary to search for new and improve existing formulations and solutions of problems in underwater acoustics. This is due to the need to deepen the theoretical provisions and the development of technologies for marine instrumentation and oceanographic research. At the same time, the direction of wave acoustics, which has not been fully investigated, seems to be interesting and useful in terms of the problems of the formation of acoustic fields in shallow seas. This direction implies the approximation of model concepts of traditional formulations of radiation problems to real situations of operation of complicated sound sources under conditions of multiple reflections of sound waves from the surface and bottom of a shallow sea.

In this regard, it seems promising to study the features of the formation of the acoustic field in a regular plane-parallel liquid waveguide simulating a shallow sea. The paper inves-

tigates the situation of using a complicated sound source to determine the features of the main factors of field formation. Note that taking into account the multimode nature of the source as one of the factors of changing the resistance of the emitter in the previous statements was not taken into account. At the same time, such complications should also include: the presence of sound scattering by the emitters, the phenomenon of dispersion, the actual multimode and variability of its structure, as well as the conclusion and use of functional equations to determine the partial regions, boundary conditions and conjugation conditions.

Thus, the problem of the formation of an acoustic field in a plane-parallel waveguide is subject to further research both in the formulation part and in the part of the solution. In this case, the complication of the source properties will obviously bring the calculated situation closer to the real one, and the solution algorithm will be based on the use of the methodology of partial regions for the acoustic field, the corresponding Helmholtz equations and the Fourier meth-

od for each region and the conjugation conditions at their boundaries.

The main results of the work consist in the formulation of problems, the application of the method of partial domains and the determination of the distributions of acoustic pressures in the vertical sections of the waveguide. At the same time, the boundary of the near-field region of the emitter (“working space”) is determined, starting from which the calculated pressure distributions correspond to the diagrams determined by the classical approaches. In addition, the frequency dependences of the specific impedance in the working space are considered and the changes in the operating mode of the emitter due to the change in the oscillatory mode are characterized.

The research results are important and useful for the practice of underwater communication systems, telemetry systems and hydroacoustic search facilities.

The relevance of this approach is due to the possibility of getting rid of the idealized boundary conditions on the source surface with the subsequent determination of the excitation coefficients of the waveguide modes within the framework of the Sturm-Liouville problem. At the same time, for each partial region, the attraction of the boundary conditions on the surface and the days of the sea makes it possible to obtain the real distribution of the field in the vertical sections of the waveguide, which is important for many practical problems of shallow sea acoustics.

2. Literature review and problem statement

In the problems of formation and propagation of acoustic waves in formations of the waveguide type, an essential role is played by the conditions, type, spatial and energy characteristics of spherical sources of an acoustic signal [1]. The specified characteristics in most fundamental works are presented in a generalized way [1], which does not satisfy many modern problems of hydroacoustics. Thus, the tasks of forming underwater communication channels, developing underwater communication and telemetry mean require knowledge of the fine structure of the acoustic field in the near and far zones. This approach should contain new formulations and the use of improved solution methods. However, in the traditional formulation of the problem of sound radiation in a waveguide within the framework of the Sturm-Liouville problem and the positions in terms of eigenfunctions [2], simplified situations were also used. And although the works [1, 2] thoroughly define the statements about the operation of a spherical source at an ideal boundary in the integral representation, the issues of the interference interaction of plane and spherical waves are subject to further analysis and research. In addition, the development of the formulations of wave problems should provide for the detailing of the processes of propagation of acoustic disturbances, taking into account the back effect of the reflected and scattered fields on the operation of the source.

Such tasks require the involvement of a kind of substitute that will help, when using traditional foundations, to master a higher level of knowledge about the subject of research. The method of partial regions can act as such a substitute.

The development, implementation and use of the method of partial domains in the problem of acoustics continue for a long time and is thoroughly presented in [3]. The method involves the selection of areas of the working environment close to the

canonical geometric figures and the preparation of functional equations according to the conjugation conditions and the Sommerfeld condition along the waveguide channel [1].

Nevertheless, the volume of tasks turned out to be so great, and the tasks themselves are so complex, that the slowness of mastering new results is justified.

Consequently, the expected results of using the partial domain method with the application in each partial domain of the Helmholtz equations and the Fourier method will obviously be:

- possibility of a deeper (in comparison with the data of [2–4]) study of the fine structure of the acoustic field in waveguides, which can be represented by the amplitude distribution of acoustic pressures in vertical sections of the waveguide;

- determination of the real coefficients of field excitation in the waveguide when working with a complicated source;

- determination of the frequency response of the source, taking into account the multimode.

Consequently, these expectations give reason to hope for new results and further advancement of the proposed approaches.

Undoubtedly, the current state of the problem of shallow seas requires significant detailing of the process of formation of acoustic fields when operating with both pulsed and monochromatic signals [3, 4], in the usual or end-to-end [5] setting. Nevertheless, the transition to pulsed modes should be based precisely on traditional solutions of problems of mathematical physics [6] in relation to monochromatic modes of operation and fully corresponds to the main approaches of the method of partial domains.

All this allows to assert that it is advisable to conduct a study devoted to the application of the method of partial regions to determine the features of the formation of an acoustic field by a complicated source in a plane-parallel regular waveguide, taking into account multimode.

3. The aim and objectives of research

The aim of this research is to apply the partial domain method to study the mode of sound emission in a shallow sea, represented by a regular waveguide with soft acoustically walls. This is important for the practice of underwater communication systems, telemetry systems and hydroacoustic search facilities.

To achieve the aim of research, the following objectives were set:

- to formulate the problem of sound radiation by a spherical source in a plane-parallel regular waveguide with acoustically soft boundaries;

- to select coordinate systems and determine the shape and number of partial areas;

- to compose a system of functional equations for the problem of the formation of an acoustic field;

- to solve problems and find unknown coefficients of field expansions;

- to make calculations and experimentally confirm them.

4. Materials and methods of research

Let's use a shallow sea model in the form of a regular waveguide with soft acoustically walls. In this case, a spher-

ical complicated sound source is used, acquiring important qualities of a spatial nature and multimode in order to approximate the traditional classical model and calculated situation to the real one. The algorithm for solving the problem of sound radiation by a complicated source is based on the use of the methodology of partial regions for the acoustic field, corresponding to the Helmholtz equations and the Fourier method for each region, as well as the conjugation conditions at their boundaries.

The criterion for the reliability of the proposed application of the partial area method is:

- finding by the proposed method the distributions of acoustic pressures in the sections of the working space, coinciding with the pressure (velocity) curves in the waveguide, obtained traditionally [1–3];
- determination of the distance of transformation of the field in the working space;
- determination of impedance characteristics by means of the resistivity of the medium of a spherical wave, taking into account the conditions at the boundaries;
- the spatial selectivity of the field in the implementation of a certain mode or sum of modes should be preserved when the field of the working area is transformed into a field in the waveguide.

The results of the work are the distribution of amplitudes and phases of acoustic pressures or vibrational velocities in the acoustic field of the emitter, taking into account the peculiarities of the formation of the field in the working space. Such distributions play the role of the coefficients of excitation of normal waves in the waveguide. In addition, the frequency dependences of the specific impedance were obtained, which characterize the dynamics of the mode composition of the source when the oscillatory mode of the emitting sphere changes under specified operating conditions.

5. Results of the study of the radiation problem

5.1. Statement of the problem of sound radiation by a spherical source in a plane-parallel regular waveguide with acoustically soft boundaries

The wave problem of the operation of a pulsating spherical monochromatic transducer – an emitter of sound waves in a limited elastic medium is considered. It is believed that the working space of the sphere is formed between two parallel flat acoustically soft infinite fixed boundaries (Fig. 1).

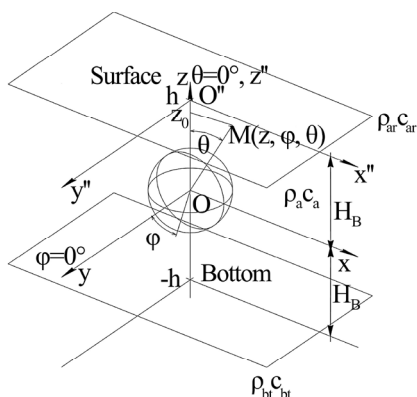


Fig. 1. Model of the waveguide channel

The space is regular and has a wave resistance $\rho_0 c_0$, which is different from the wave resistances of the outer half-spaces

$\rho_{ar} c_{ar}$ and $\rho_{bt} c_{bt}$ (air and bottom). Thus, the resulting model (Fig. 1) corresponds to a simplified waveguide channel.

5.2. Selection of coordinate systems and determination of the shape and number of partial regions

The channel is considered in rectangular O, x, y, z ; O', x', y', z' ; O'', x'', y'', z'' ; O_1, x_1, y_1, z_1 and in spherical O', r, φ, θ , in coordinate systems ($z_0=r \cos(\theta)$; $x_0=r \sin(\theta)$). The centers O and O' of rectangular coordinates O, x, y, z and O', x', y', z' are separated by a distance $H_w=H/2=h$, where H is the width of the waveguide (sea depth), and the center of spherical coordinates O' is aligned with the phase center of a spherical source having an outer radius a . When the waves reflected from the boundaries are scattered by it, the converter itself is impedance, has fully electroded surfaces, and emits a zero-order wave of a known amplitude $p_0=1$.

After the supply of electrical excitation, the oscillations of the sphere begin exclusively with pulsating movements, which is due not only to its geometric shape, but also to the type of electroding. If the working environment were unlimited, then the source should work exclusively at the zero mode and form spherical waves propagating in the form of concentric spherical surfaces.

However, already after the first reflection from the boundaries (sea surface and bottom), the wave pattern should change due to the appearance of a field scattered by the sphere and re-reflection of acoustic waves from the waveguide boundaries. Consequently, the superposition of the stray field, the direct field of the transducer and the waveguide reflected from the surfaces together form a certain total field in the working space.

Let's consider the circumstances of the formation of a complete acoustic field (Fig. 2).

The wave pattern is considered to be centrally symmetric and divided by quadrants I, II, III, IV. Thus, it will be sufficient to solve the problem for regions I and IV ($0 \leq \theta \leq \pi$) with the subsequent conjugation of the results of determining the sound potential (or pressures) along the axes O', x', O', y', O', z' , (Fig. 1). Consequently, the situation of the formation of the field presupposes the use of the method of partial regions [4] with the fulfillment of the force or kinematic conditions of conjugation.

Thus, the excitation of a waveguide and the formation of an acoustic field in it are considered from several positions:

- the first is the formation of n, m components of the near field of a spherical source $n=0, 1, 2, 3, \dots, m=0, 1, 2, 3, \dots$ at the points of its working space from the surface of the transducer (Fig. 1, region V, Fig. 2) to the conditional region of formation of the boundary of regions I, IV, V (it is from this region that the formation of normal waves with numbers n, m should begin);
- the second – at the boundaries of the regions, the conditions of conjugation of the power or kinematic types must be fulfilled.

Of course, there can be no sharp separation of the field between quadrants I, II, III, IV, and V, and the boundary of I, IV, and V only defines the boundary of the region of smooth transition from the field in regions I, IV to field V.

Let the expected acoustic field be symmetric with respect to the $\varphi=0^0$, which gives reason to go over to the plane problem and consider the situation in the above spherical coordinates only with respect to the angles θ .

It is clear that the problem of sound emission and field formation in a waveguide is reduced to determining the coefficients of excitation of normal waves.

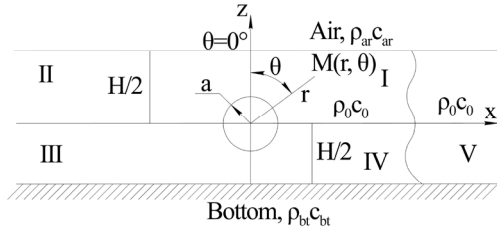


Fig. 2. Quadrants of the workspace

That is, if in a certain section of the waveguide the distribution of force, pressure or vibrational velocity of the particles of the medium is specified, then adjacent sections of the waveguide should also be characterized by the same pressure values. This follows from the conditions of conjugation by the method of partial domains (for example, the corresponding functional equations).

5.3. Drawing up a system of functional equations for the acoustic field formation problem

Thus, the source for the problem of forming the acoustic field in a plane-parallel waveguide is the field of a multimode converter $\Psi_{n,m}^{sp}(r, \varphi, \theta)$ in partial regions I–IV, both for individual modes and for their superposition:

$$\Psi_{\Sigma}^{sp}(r, \varphi, \theta) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \Psi_{nm}^{sp}(r, \varphi, \theta) = \sum_{n=0}^{\infty} \Psi_n^{sp}(r, \theta). \quad (1)$$

Therefore, the field in the waveguide can be defined as a result of the superposition of normal waves after determining the location of the conditional boundary region between the near and far fields x_0 (hereinafter referred to as the “working space of the sphere”):

$$\Psi_{\Sigma}^{st}(x, y, z) = \sum_{q=0}^{\infty} \Psi_q^{st}(x, z). \quad (2)$$

Nevertheless, let’s understand that the mode composition of the field in the working space can be different from the composition of normal waves of the waveguide. Under such conditions, the procedure for solving the problem provides for:

- application of the method of partial regions by drawing up functional equations and finding solutions to the homogeneous Helmholtz equation for each partial region;
- use of the Fourier method for solving the homogeneous Helmholtz equation with the Dirichlet and Neumann boundary conditions in the framework of the Sturm-Liouville problem [2, 7];
- use of the properties of orthogonality of wave functions on the intervals $(0, H)$, $(0, \pi)$, $(0, 2\pi)$.

Let’s consider the acoustic field in the working space (areas I, IV) relative to the pressure field $p(r, \varphi, \theta)$ represented by the homogeneous Helmholtz equation.

$$\Delta p(r, \varphi, \theta) + k^2 p(r, \varphi, \theta) = 0. \quad (3)$$

That is, the decoupling for each partial region must satisfy the Helmholtz equation (3), in which Δ – the Laplace operator, $k = \omega/c_0$ – the wave number taken for a free medium. In this case, the ratio of the speeds of sound in air c_{ar} and water c_0 corresponds to the inequality $c_{ar} < c_0$, which excludes the existence of critical angles for oblique incidence of sound from the thickness of the water-filled waveguide.

The general solution was carried out by constructing a system of functional equations for pressures, relying on the expansion (1), (2)

$$\begin{aligned} p_{\Sigma}^I &= p_{\Sigma}^I(r, \theta), \quad 0 \leq \theta \leq \pi/2, \quad a \leq r \leq R, \\ p(x, z) \Big|_{x=x_0, z=H/2} &= 0, \quad R_p^I = W_p(\theta), \\ p_{\Sigma}^{IV} &= p_{\Sigma}^{IV}(r, \theta), \quad \pi/2 \leq \theta \leq \pi, \quad a \leq r \leq R, \\ p^{IV}(x, z) \Big|_{x=x_0, z=0} &= 0, \quad R_p^{IV} = W_p(\theta), \\ p_{\Sigma}^V &= p_{\Sigma}^V(r, \theta), \quad p_{\Sigma}^{IV} = p_{\Sigma}^V(r, \theta), \quad x = x_0, \quad 0 \leq z \leq H, \\ v_{\Sigma}^{IV}(r, \theta) \Big|_{r=a} &= \frac{1}{i\omega\rho_0} \frac{\partial p_{\Sigma}^{IV}(r, \theta)}{\partial r} \Big|_{r=a}, \\ Z_{rs}^{IV} &= \frac{p_{\Sigma}^I(r, \theta)}{v_{\Sigma}^I(r, \theta)}, \quad p_{\Sigma}^{IV} = p_{\Sigma}^V(r, \theta), \quad r = R, \quad 0 \leq \theta \leq \pi, \end{aligned} \quad (4)$$

where $p_{\Sigma}^I(r, \theta)$, $p_{\Sigma}^{IV}(r, \theta)$, $p_{\Sigma}^V(r, \theta)$ – the sound pressures of the full field in workspaces I, IV, V;

$R_p^I = W_p(\theta)$ – pressure reflection coefficient at oblique incidence of a sound wave (the angle of incidence takes values in the range $0 \leq \theta \leq \pi$). Let’s note that the procedure for using the intervals of variation of the angles of incidence-reflection requires certain comments and simplifications given below;

$Z_{rs}^{IV} = \frac{p_{\Sigma}^{IV}(r, \theta)}{v_{\Sigma}^{IV}(r, \theta)}$ – resistivity of the medium of a spherical wave, $p_{\Sigma}^V(r, \theta)$ – acoustic pressure, $v_{\Sigma}^{IV}(r, \theta)$ – vibrational velocity (recorded for the outer surface near regions I, IV within the range of variation of the angle $0 \leq \theta \leq \pi$).

5.4. Solving the problem and finding the unknown coefficients of the field expansions

To solve with absolute integration (with respect to acoustic pressures and acoustic potential), the Fourier method, orthogonality properties of wave functions, the method of partial regions [5], boundary conditions for acoustically soft boundaries are used:

$$p^{IV}(x, 0, z) \Big|_{z=0} = (p^{IV}(x, 0, z) \Big|_{z=H}) = 0, \quad (5)$$

and pairing conditions of the power type

$$\begin{aligned} p^I(x', O', z') \Big|_{z=H/2} &= (p^{IV}(x', O', z') \Big|_{z=H/2}), \\ \theta &= \pi/2, \quad a_s \leq r \leq x_0. \end{aligned} \quad (6)$$

Let’s define the region I, the acoustic field in which (Fig. 2) for the n, m -th normal wave is symmetric with respect to the direction (hence, $m=0$).

In this case, the complete field $p_f = p^{00} + p + p_{rf} \pm p_{sc}$, is formed by the combination:

- 1) direct field emitted by the source when operating at zero mode

$$p^{00} = p^{00}(r, \theta) = -\omega p A_0 h_0^{(1)}(kr) P_0^0(\theta). \quad (7)$$

where $h_0^{(1)}(kr)$ – zero-order spherical Hankel function of the first kind that describes the diverging waves, $P_0^0(\theta)$ – zero-order Legendre polynomial;

- 2) the field created by higher-order modes

$$\begin{aligned}
 p &= p(r, \theta) = \\
 &= -i\omega\rho_0 \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} A_{nm} h_n^{(1)}(kr) P_n^m(\theta) \cos(m\varphi) \Big|_{n,m=0} = \\
 &= -i\omega\rho_0 \sum_{n=1}^{\infty} A_n h_n^{(1)}(kr) P_n(\theta), \tag{8}
 \end{aligned}$$

where $h_n^{(1)}(kr)$ – spherical Hankel function of the first kind of n -order for the remaining widespread waves, $P_n^m(\theta)$ – Legendre function of the first kind of order m and degree n is added, $A_0 \neq A_1, A_2, \dots$; however, seeing that in the situation of shallow seas, the “source-surface-source” path of the wave front at the beginning of radiation runs over a sufficiently short time interval (fate ms), let’s consider factor (7) for the monochromatic regime, which can be neglected. That is, use only form (8) to describe the field;

3) fields reflected from the water-air interface

$$\begin{aligned}
 p_{rf} &= p_{rf}(r, \theta) = \\
 &= -i\omega\rho_0 \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} B_{nm} y_n(kr) P_n^m(\theta) \cos(m\varphi) \Big|_{n,m=0} = \\
 &= -i\omega\rho_0 \sum_{n=0}^{\infty} B_n y_n(kr) P_n(\theta), \tag{9}
 \end{aligned}$$

where

$$B_{nm} = W_p(\theta) A_{nm} \Big|_{n,m=0} = W_p(\theta) A_{n0} = W_p(\theta) A_n,$$

$W_p(\theta)$ – pressure reflection coefficient; $y_n(kr)$ – spherical von Neumann function of the second kind n -order for converging waves;

4) the field scattered on the spherical surface of the transducer

$$\begin{aligned}
 p_{sc} &= p_{sc}(r, \theta) = \\
 &= -i\omega\rho_0 \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} C_{nm} h_n^{(1)}(kr) P_n^m(\theta) \cos(m\varphi) \Big|_{n,m=0} = \\
 &= -i\omega\rho_0 \sum_{n=0}^{\infty} C_n h_n^{(1)}(kr) P_n(\theta), \tag{10}
 \end{aligned}$$

where A_n, B_n, C_n – unknown coefficients of expansions (7)–(10).

Thus, the total pressure field in the working space is described using the following equation:

$$\begin{aligned}
 p_f(r, \theta) &= \\
 &= -i\omega\rho_0 \left[\begin{aligned} &A_0 j_0(kr) P_0(\theta) + \sum_{n=1}^{\infty} A_n j_n(kr) P_n(\theta) + \\ &+ \sum_{n=0}^{\infty} B_n y_n(kr) P_n(\theta) + \\ &+ \sum_{n=0}^{\infty} C_n h_n^{(1)}(kr) P_n(\theta) \end{aligned} \right] = \\
 &= -i\omega\rho_0 \left[\begin{aligned} &\sum_{n=0}^{\infty} [A_n j_n(kr) + B_n y_n(kr)] P_n(\theta) + \\ &+ \sum_{n=0}^{\infty} C_n h_n^{(1)}(kr) P_n(\theta) \end{aligned} \right]. \tag{11}
 \end{aligned}$$

Let’s provide comments on the determination and use of unknown expansion coefficients (6)–(11) A_n, B_n, C_n for the acoustic field in the working space, as well as for each component $p_{sur}(r, \theta)$, let’s separately present the calculations.

Falling waves.

Note that according to the problem statement, only the coefficient $A_0=1$ is known, and the other coefficients $A_n, n=1, 2, 3, \dots$ are unknown. Let’s use the results of [8] to determine the ratio of the relative amplitude values of the higher-order modes A_n and the zero mode A_0 . According to [8], with the involvement of the source [9], this occurs by introducing a dimensionless correction factor Ω_n , represented as:

$$\Omega_n = \begin{cases} 1, & n=0, \\ \frac{P_{n+1}(\cos\theta_e) - P_{n-1}(\cos\theta_e)}{(2n+1)(1+(\cos\theta_e))}, & n=1,2,3,\dots, \end{cases} \tag{12}$$

where θ_e – the angular size of the non-electroded area of the sphere surface (by opening within $\theta_e=20^\circ$, the radius of the sphere is $a=1.0$ m). Structurally, the size of such a section can determine the size of the non-electroded surface around the technological hole intended for the introduction of electrical installation elements into the cavity of the sphere.

Thus, for the first five modes, according to [8, 9], $A_{n0}=A_0\Omega_n$, whence:

$$\begin{aligned}
 n=0, A_{00}=1, n=1, A_{10}=-0.031, n=2, A_{20}=-0.026, n=3, \\
 A_{30}=-0.024, n=4, A_{40}=-0.02. \tag{13}
 \end{aligned}$$

In such conditions, the enrichment of the mode composition of the transducer is caused by the deformation of its surface due to the incident on the transducer of waves reflected from the boundaries “water-bottom”, “water-air”. The mutual influence of the formed modes is due to the connection between the mechanical vibrations of the sphere and the result of the flow of energy from the main form of vibrations to higher ones. In this case, multimode should affect the magnitude and nature of the impedance of the transducer due to the dynamics of the spatial frequency vibrational state. Note that the shape of the spatial characteristics of the converter should remain similar to the traditional ones [10, 11] when implementing certain types of electroding and the inclusion of electrodes in the excitation circle from the generator.

Reflection of a wave from plane boundaries (for example, quadrant I).

As for the coefficients B_{nm} , they characterize the reflection of a spherical pressure wave from a fixed boundary, which is acoustically soft by its properties.

Let’s suppose that the pressure reflection coefficient must be applied for each vibration mode by forming the equality:

$$B_{nm} \Big|_{n,m=0} \leftrightarrow B_{n0} = W_p A_{nm} = W_p A_{n0}. \tag{14}$$

To clarify the situation of finding the coefficient W_p , let’s use the results of [12] in accordance with the situation of operation on a long time interval and any location of the observation point of the reflection coefficient depends only on the densities of adjacent media:

$$W_p = \frac{\rho_0 - \rho_n}{\rho_0 + \rho_n}. \tag{15}$$

That is, a spherical wave is reflected from the specified boundary as a plane one. Based on the provisions of [12], as well as the fulfillment of the inequality $c_0 \gg c_{ar}$ and $c_0 \gg c_{bt}$, the use of formulas (14), (15) corresponds to the situation of propaga-

tion of homogeneous waves without limiting the values of the angles θ in the range indicated in functional equations (4)–(6).
Scattered field.

Expansions (11) and limiting condition (6) are used to determine the unknown coefficient.

$$p_{sur}(r, \theta) \Big|_{r=H/2} = -\omega \rho_0 \left[\begin{aligned} & A_0 j_0 \left(k \frac{H}{2} \right) P_0(\theta) + \\ & + \sum_{n=1}^{\infty} A_n j_n \left(k \frac{H}{2} \right) P_n(\theta) + \\ & + \sum_{n=0}^{\infty} B_n y_n \left(k \frac{H}{2} \right) P_n(\theta) + \\ & + \sum_{n=0}^{\infty} \tilde{N}_n h_n^{(1)} \left(k \frac{H}{2} \right) P_n(\theta) \end{aligned} \right] = 0, \quad (16)$$

whence, taking into account soft boundaries:

$$p_{sur}(r, \theta) \Big|_{r=H/2} = -\omega \rho_0 \left[\begin{aligned} & \sum_{n=0}^{\infty} \left[A_n j_n \left(k \frac{H}{2} \right) + \right. \\ & \left. + B_n y_n \left(k \frac{H}{2} \right) \right] + \\ & \left. + \sum_{n=0}^{\infty} C_n h_n^{(1)} \left(k \frac{H}{2} \right) \right] = 0,$$

and

$$C_n = - \left[\frac{A_n j_n \left(k \frac{H}{2} \right) + W_p A_n y_n \left(k \frac{H}{2} \right)}{h_n^{(1)} \left(k \frac{H}{2} \right)} \right]. \quad (17)$$

The impedance condition that closes the group of functional equations requires a preliminary determination of the vibrational velocity, namely, its distribution over the surface of the sphere. Therefore, it is necessary to use the condition of equality of the normal component of the speed of movement of material points of the surface of the transducer of the speed of movement along the normal of particles of the medium, which brings the situation closer to the condition of conjugation of the kinematic type.

The pressure distribution of the formula (7)–(10) is determined by the distribution of the vibrational velocity $v^l(r, \theta) = V(\varphi, \theta)$ and is the result of a complex vibrational mode installed in the working space:

$$v(r, \varphi, \theta) = \frac{1}{i \rho_0 c_0} \frac{\partial p_{sur}}{\partial r} = v(r, \varphi, \theta) = \frac{1}{i \rho_0 c_0} \times \left[\begin{aligned} & \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \left[A_{nm} j'_n(kr) + \right. \\ & \left. + B_{nm} y'_n(kr) \right] P_n^m(\theta) \cos(m\varphi) \Big|_{n,m=0} + \\ & + \frac{1}{i \rho_0 c_0} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \tilde{N}_{nm} h_n^{(1)'}(kr) P_n^m(\theta) \cos(m\varphi) \Big|_{n,m=0} = \\ & = \frac{1}{i \rho_0 c_0} \sum_{n=0}^{\infty} \left[\begin{aligned} & [A_n j'_n(kr) + B_n y'_n(kr)] P_n(\theta) + \\ & + \tilde{N}_n h_n^{(1)'}(kr) P_n(\theta) \end{aligned} \right]. \end{aligned} \right] \quad (18)$$

Based on the relationship (7)–(10) from the expression for the total pressure (11), let's pass to the vibrational velocity on the surface of the sphere:

$$V(r, \varphi, \theta) \Big|_{r=a} = \frac{1}{i \rho_0 c_0} \frac{\partial p_f}{\partial r} \Big|_{r=a} = V(\varphi, \theta) \Big|_{r=a} = \frac{1}{i \rho_0 c_0} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} A_{nm} j'_n(ka) P_n^m(\theta) \cos(m\varphi) \Big|_{n,m=0} + \frac{1}{i \rho_0 c_0} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} B_{nm} y'_n(ka) P_n^m(\theta) \cos(m\varphi) \Big|_{n,m=0} + \frac{1}{i \rho_0 c_0} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} C_{nm} h_n^{(1)'}(ka) P_n^m(\theta) \cos(m\varphi) \Big|_{n,m=0}. \quad (19)$$

Let's expand the left-hand side of equality (19) into a Fourier series in orthogonal functions $P_n^m(\theta) \cos(m\varphi)$ as follows:

$$V(\varphi, \theta) \Big|_{r=a} = \sum_{q=0}^{\infty} \sum_{s=0}^{\infty} V_{qs} P_q^s(\theta) \cos(s\varphi), \quad (20)$$

where $q, s=0, 1, 2, 3, \dots$, and V_{qs} – an unknown amplitude coefficient having a velocity content.

Let's use the orthogonality properties: equate (19) and (20),

$$\sum_{q=0}^{\infty} \sum_{s=0}^{\infty} V_{qs} P_q^s(\theta) \cos(s\varphi) = - \frac{1}{i \rho_0 c_0} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} A_{nm} j'_n(ka) P_n^m(\theta) \cos(m\varphi) \Big|_{n,m=0} - \frac{1}{i \rho_0 c_0} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} B_{nm} y'_n(ka) P_n^m(\theta) \cos(m\varphi) \Big|_{n,m=0} - \frac{1}{i \rho_0 c_0} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} C_{nm} h_n^{(1)'}(ka) P_n^m(\theta) \cos(m\varphi) \Big|_{n,m=0},$$

let's multiply the right and left sides of the resulting equality $P_n^{m'}(\theta) \cos(m'\varphi)$ and integrate on the intervals of variation of the angles φ, θ :

$$\sum_{q=0}^{\infty} \sum_{s=0}^{\infty} V_{qs} \int_0^{2\pi} \int_0^{\pi} P_q^s(\theta) P_n^{m'}(\theta) \sin(s\theta) \cos(s'\varphi) \cos(m'\varphi) \sin(\theta) d\theta d\varphi = \frac{1}{i \rho_0 c_0} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} A_{nm} j'_n(ka) \int_0^{2\pi} \cos(m'\varphi) \cos(m\varphi) d\varphi \int_0^{\pi} P_n^m(\theta) P_n^{m'}(\theta) \sin(\theta) d\theta + \frac{1}{i \rho_0 c_0} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} B_{nm} y'_n(ka) \int_0^{2\pi} \int_0^{\pi} P_n^m(\theta) P_n^{m'}(\theta) \cos(m'\varphi) \cos(m\varphi) d\theta d\varphi + \frac{1}{i \rho_0 c_0} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} C_{nm} h_n^{(1)'}(ka) \int_0^{2\pi} \int_0^{\pi} P_n^{m'}(\theta) P_n^m(\theta) \cos(m\varphi) \cos(m\varphi) \sin(\theta) d\theta d\varphi. \quad (21)$$

According to [13, 14], as a result of integration:

$$\int_0^{2\pi} \cos(m\varphi) \cos(m'\varphi) d\varphi \int_0^{\pi} P_n^m(\theta) P_n^{m'}(\theta) \sin(\theta) d\theta = 2\pi \delta_{mm'} N_{nm} \delta_{nn'},$$

$$\delta_{mm'} = \begin{cases} 1, & m = m' = 0. \\ 1/2, & m = m' \neq 0. \\ 0, & m' \neq m; \end{cases} \quad \delta_{nn'} = \begin{cases} 1, & n = n' = 0, \\ 0, & n \neq n'; \end{cases}$$

$$N_{nm} = \frac{2(n+m)!}{2n+1(n-m)!} \tag{22}$$

So, let's take into account the above assumptions about the symmetry of the field with respect to the direction $\varphi=0^\circ$ ($m=0$), the results of (21), (22) and the replacement of variable indexing of the addition operators. After carrying out these transformations, it is possible to write down the expression for the specific acoustic impedance of the medium Z_{sp}^{LIV} on the surface of the transducer for a certain mode:

$$Z_{sp}^{LIV} = i\rho_0 c_0 \frac{A_n [j_n(ka) + W_p y_n(ka)] + C_n h_n^{(1)}(ka)}{A_n [j'_n(ka) + W_p y'_n(ka)] + C_n h_n^{(1)'}(ka)} \tag{23}$$

So, the events at the boundaries of the partial regions have been clarified. After using formulas (13), (15), (17), (21) to determine the coefficients A_n, B_n, C_n, V_n and substituting them in (11), (18), (23), let's find the distribution of pressures, oscillatory velocities and impedance. To do this, let's use the ranges of variation of angles and distances based on the inequalities $0 \leq \theta \leq \pi/2, a \leq r \leq R$.

Finding the acoustic field in a regular plane parallel waveguide with acoustically soft boundaries $p_{wag}(x, z)$ is itself a well-known problem, the formulation and solution of which is presented in many literary sources, for example [2, 5, 13]. So, the general result of finding the acoustic field within the plane-parallel problem is found in the form of a series – the sum of normal waves [5]. As a basis, it is possible to take expression (2) and obtain a partial solution in the z coordinate for the field potential $\Psi(x, z)$ in a rectangular coordinate system $0, x, y, z$:

$$\Psi(x, z) = \sum_{v=0}^{\infty} \Psi_v(x, z) = \sum_{v=0}^{\infty} Z^v(z) e^{ik_v x} \tag{24}$$

where $Z^v(z) = D^v \sin\left(\frac{\pi v}{H} z\right); k^v = k \sqrt{1 - \left(\frac{\omega_{kp}^v}{\omega}\right)^2}$ – wavenumber of the v -th mode of the waveguide, and $v=0, 1, 2, 3 \dots$ is its number.

In terms of eigenfunctions, the complex amplitude of the potential or pressure can be expressed as:

$$\Psi(x, z) = \sum_{v=0}^{\infty} \delta_{v+1} u_{v+1}(z) e^{ik_v x} \tag{25}$$

where $u_{v+1}(z) = u_\mu(z), v+1=\mu, \mu \in N$ – eigenfunctions of the Sturm-Liouville problem, which for the chosen initial conditions form a complete orthogonal system of functions. The sound source, which is located in the plane $x=0$, is characterized by certain spatial properties and forms a potential $\Psi_s(z)$ so that:

$$\Psi_s(z) = \sum_{\mu=0}^{\infty} \delta_\mu u_\mu(z) = \sum_{v=0}^{\infty} \delta_{v+1} u_{v+1}(z) \tag{26}$$

where $u_{v+1}(z) = \sqrt{\frac{2}{H}} \sin\left(\frac{v\pi}{H} z\right)$.

Applying the orthogonality properties, equalities (25), (26) and the boundary conditions for acoustically soft boundaries $z=0, z=H$, let's obtain:

$$\delta_{v+1} = \sqrt{\frac{2}{H}} \int_0^H \Psi_{sr}(z) \sin\left(\frac{v\pi}{H} z\right) z dz$$

Then, for the complex amplitude (25), according to [1–3]:

$$\Psi(x, z) = \sum_{v=0}^{\infty} D_v \sin\left(\frac{v\pi}{H} z\right) e^{ik_v x}, \quad D_v = \sqrt{\frac{2}{H}} \delta_{v+1} \tag{27}$$

Thus, the process of field formation in a given waveguide consists in finding the field in the working space and then superimposing on the result (11) the properties of the waveguide itself, represented by expression (27). This makes it possible to establish the features of the process of transformation of the acoustic field in the working area of the spherical transducer and the field in the waveguide.

Therefore, let's describe the total pressure field in the working space using equation (11) and using the notation $F_s(r, \theta)$:

$$F_s(r, \theta) \Big|_{r(x)} = -\omega \rho_0 \left[\begin{aligned} & \sum_{n=0}^{\infty} A_n j_n(kr) P_n(\theta) + \\ & + \sum_{n=0}^{\infty} B_n y_n(kr) P_n(\theta) + \\ & + \sum_{n=0}^{\infty} C_n h_n^{(1)}(kr) P_n(\theta) \end{aligned} \right]_{x_0} \tag{28}$$

where $F_s(r, \theta)$ – distribution of the pressure force in the section x_0 . The transition from a spherical coordinate system $(0, r, \theta)$ to a rectangular system $(0, x, z)$ is carried out according to the standard transition formulas.

In this case, the total field in the waveguide will be written based on (7)–(11) as:

$$\begin{aligned} p_s(x, z) &= p_s(\rho, \theta) = \sum_{n=0}^{\infty} p_{sn}(\rho, \theta) = \\ &= \sum_{n=0}^{\infty} F_n(\rho, \theta) \sin\left(\frac{n\pi}{H} z\right) e^{ik_n x} \end{aligned} \tag{29}$$

It is clear that if a certain form n is considered, expressions (28), (29) acquire indexing:

$$\begin{aligned} F_n(\rho, \theta) \Big|_{x_0} &= \begin{bmatrix} A_n j_n(kr) P_n(\theta) + \\ + B_n y_n(kr) P_n(\theta) + \\ + C_n h_n^{(1)}(kr) P_n(\theta) \end{bmatrix} \\ p_{sn}(\rho, \theta) \Big|_{x_0} &= F_n(\rho, \theta) \sin\left(\frac{n\pi}{H} z\right) e^{ik_n x} \end{aligned} \tag{30}$$

Thus, the result of the solution makes it possible to determine the field in the waveguide taking into account the calculated coefficients of excitation of normal waves.

5. 5. Calculations and experimental confirmation

Calculations of the distribution of acoustic pressure amplitudes in the working space of a spherical radiator, namely, in the vertical sections of a regular plane-parallel waveguide and specific resistances of the medium to combination waves of a certain mode are performed. Note that the central symmetry of the problem allows the presentation of the results of field calculations for only one quadrant (in the given case, the first), taking into account the conditions of force-type conjugation at the boundaries of partial regions (Fig. 1, 2).

The source of a monochromatic sound signal (frequency $f=63$ Hz) is located on the horizontal axis of the waveguide at point O – the origin of rectangular coordinates O, x, y, z . Sea depth $H=100$ m. According to calculations, based on the geometry of the problem, the maximum selected number of normal waves of the waveguide was $N=4$, which does not exceed the value permissible for the given wave conditions $N = \left\lfloor \frac{2H}{\lambda} \right\rfloor = 8$.

The calculation results are presented in groups.

Group 1. Wave patterns of distribution of complex pressure amplitudes in the working space of a spherical radiator for the lower modes of sources $n=0, 1, 2, 3, 4$ and the total field (Fig. 3–9):

Group 2. Distribution of pressures in the waveguide, $x=100$ m by mode and sum of modes (Fig. 10–12).

Group 3. Bypass and plots of the distribution of pressure amplitudes pp in the working space along the coordinates x, z in the section $x=100$ m for modes 0, 1, 2, 3, 4 and for the sum of modes $0+1+2+3+4$ (Fig. 13, 16).

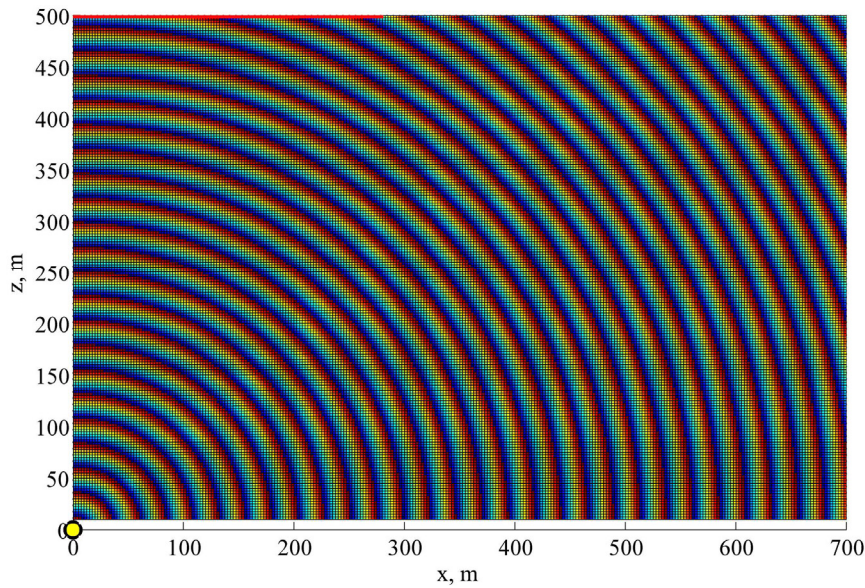


Fig. 3. Wave pattern of the field of a pulsating spherical source in an infinite ideal medium, frequency 63 Hz, $H=100$ m

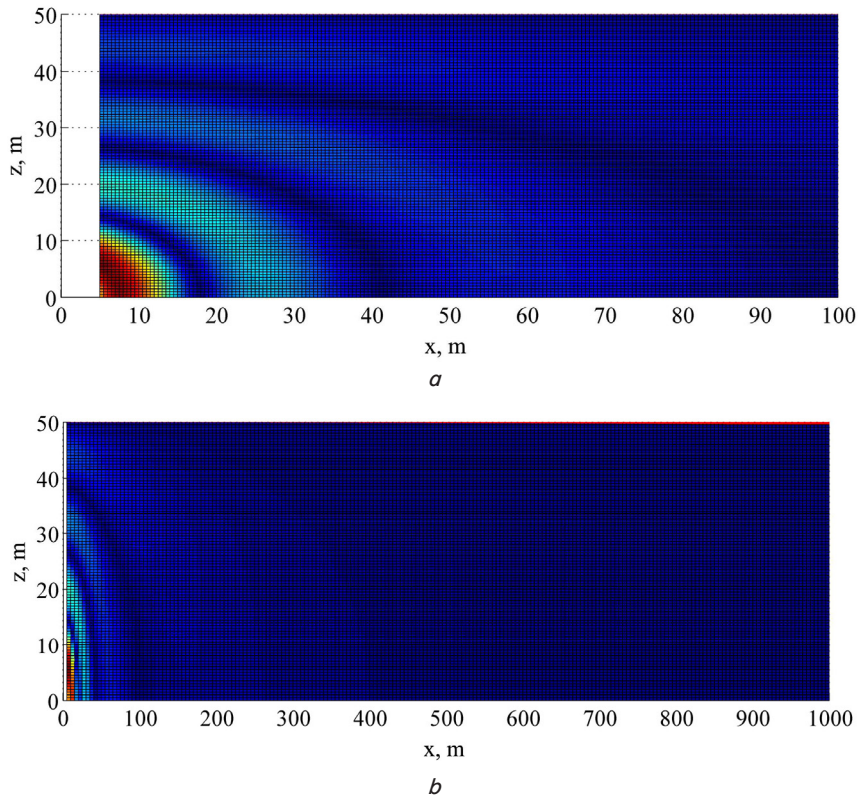


Fig. 4. Wave pattern of distribution of pressure amplitudes in the working space, mode “0”, frequency 63 Hz, $H=100$ m: $a - 0 \leq x \leq 100$ m; $b - 0 \leq x \leq 1000$ m

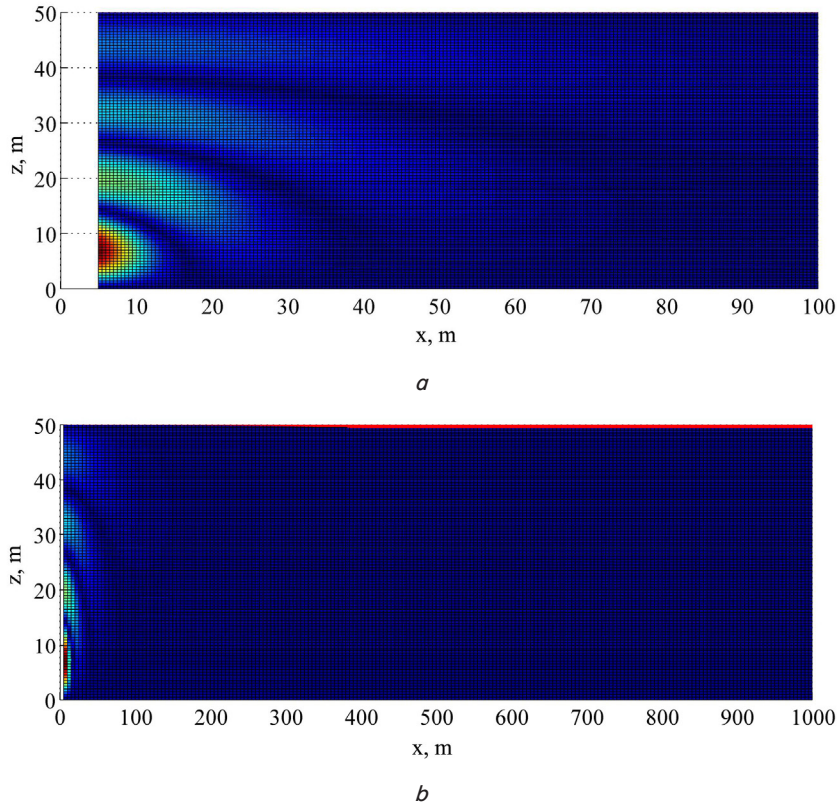


Fig. 5. Wave pattern of distribution of pressure amplitudes in the working space, mode “1”, frequency 63 Hz, $H=100$ m:
 $a - 0 \leq x \leq 100$ m; $b - 0 \leq x \leq 1000$ m

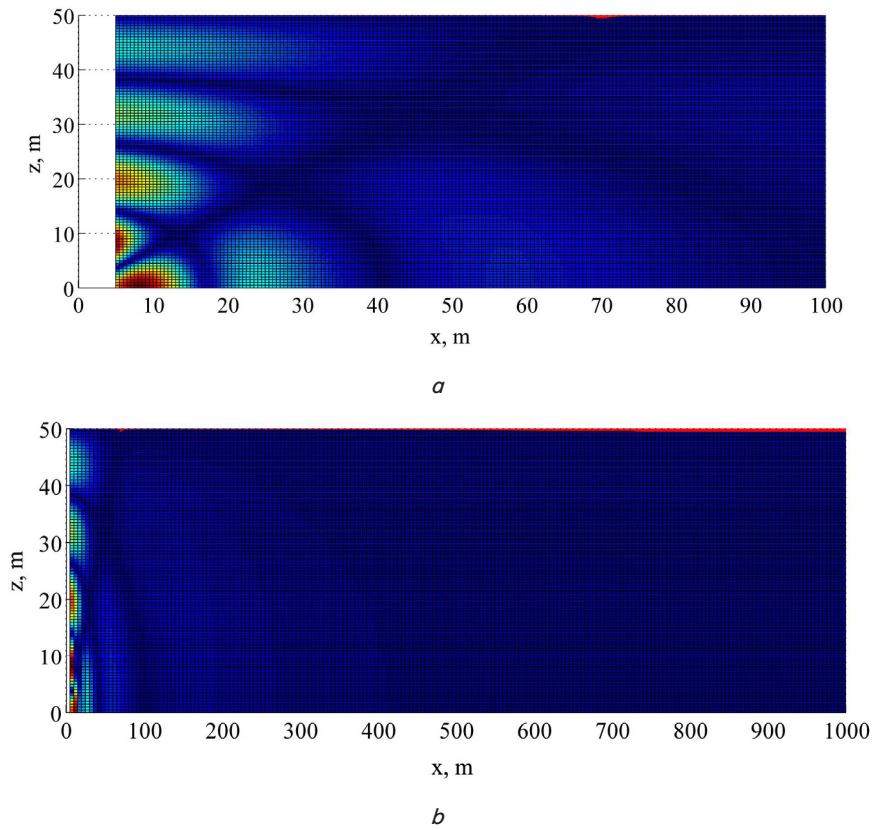


Fig. 6. Wave pattern of distribution of pressure amplitudes in the working space, mode “2”, frequency 63 Hz, $H=100$ m:
 $a - 0 \leq x \leq 100$ m; $b - 0 \leq x \leq 1000$ m

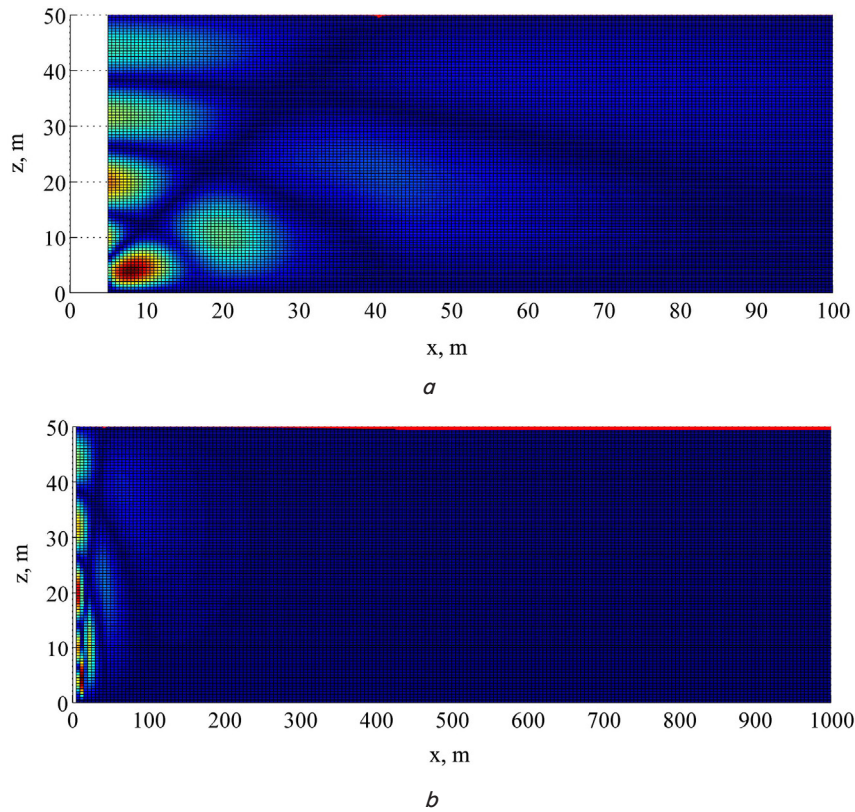


Fig. 7. Wave pattern of distribution of pressure amplitudes in the working space, mode «3», frequency 63 Hz, $H=100$ m:
 $a - 0 \leq x \leq 100$ m; $b - 0 \leq x \leq 1000$ m

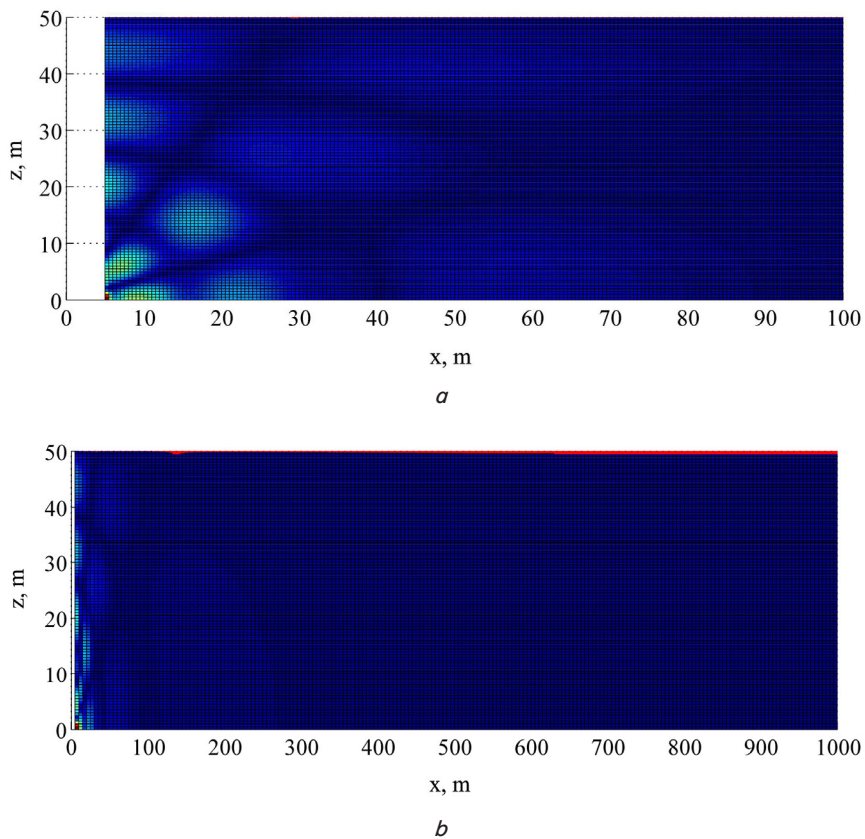


Fig. 8. Wave pattern of distribution of pressure amplitudes in the working space, mode «4», frequency 63 Hz, $H=100$ m:
 $a - 0 \leq x \leq 100$ m; $b - 0 \leq x \leq 1000$ m

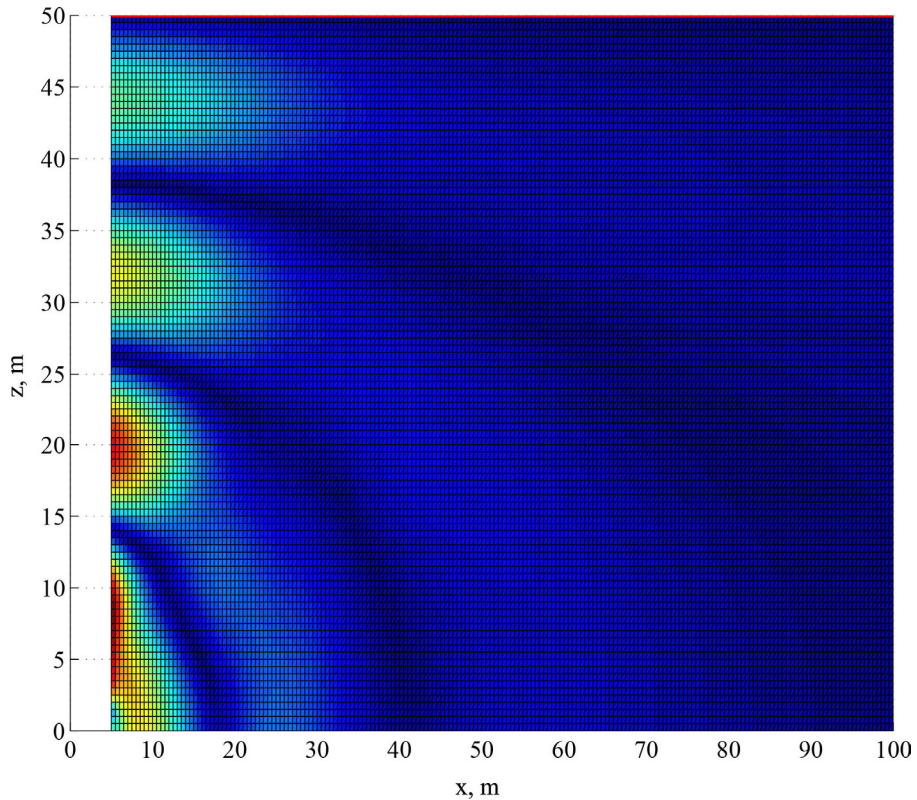


Fig. 9. Wave pattern of the distribution of pressure amplitudes in the working space, the sum of modes 0+1+2+3+4, frequency 63 Hz, $H=100$ m

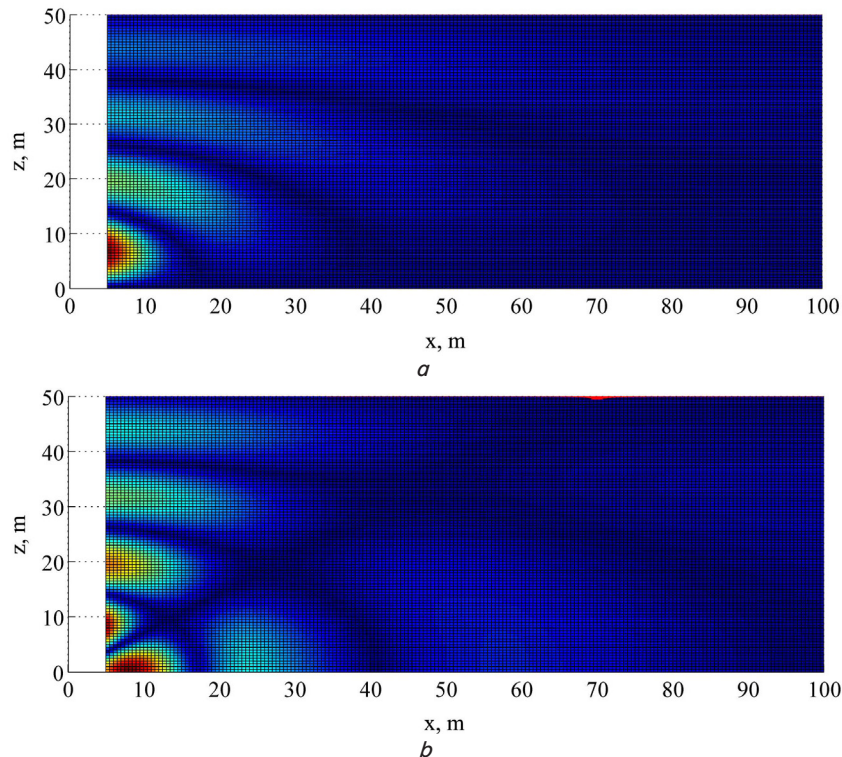


Fig. 10. Wave pattern of the distribution of pressure amplitudes in the waveguide in accordance with the mode, $x=100$ m, frequency 63 Hz, $H=100$ m: *a* – mode “1”; *b* – mode “2”

Group 4. Impedance characteristics in the form of the frequency dependence of the specific impedance of the medium reduced to the surface (Fig. 17, 18).

As for the impedance properties, the specific impedance of the medium to direct and reflected spherical waves for

sources with a radius of 1 m (Fig. 17) and 50 m (Fig. 18) was chosen as the external load of the source.

To check the correctness of the main provisions of the work and receive recommendations for determining the workspace and its boundaries, the main factor is the possi-

bility of the sphere working in a certain fashion. This leads to the need to measure the angular dependences of the distributions of the pressure amplitudes on the surface or near the sphere.

For this, the theoretical results obtained in terms of the realization of individual modes by the sphere and the forma-

tion of angular distributions of the acoustic pressure amplitudes $p_n(\theta)$, $n=0, 1, 2$ – must be confirmed experimentally.

Such an experiment was carried out in the measuring pool of the State Enterprise “Kyiv Scientific Research Institute of Hydroelectric Instruments” according to the scheme in Fig. 19.

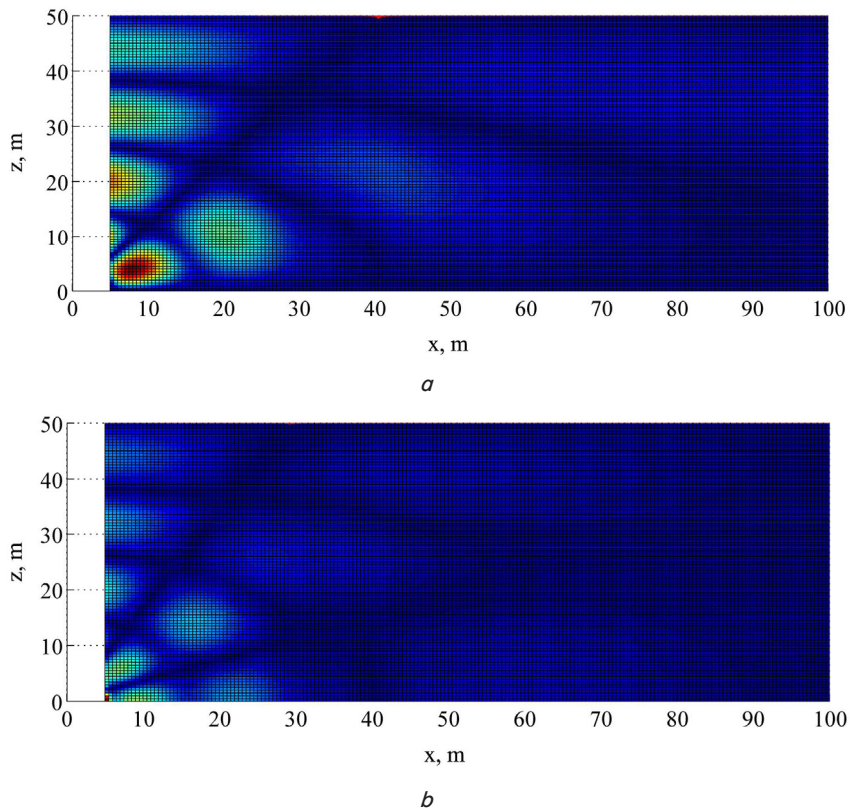


Fig. 11. Wave picture of the genealogy of pressure amplitudes in the waveguide, $x=100$ m in accordance with the mode, frequency 63 Hz, $H=100$ m: a – mode «3»; b – mode «4»

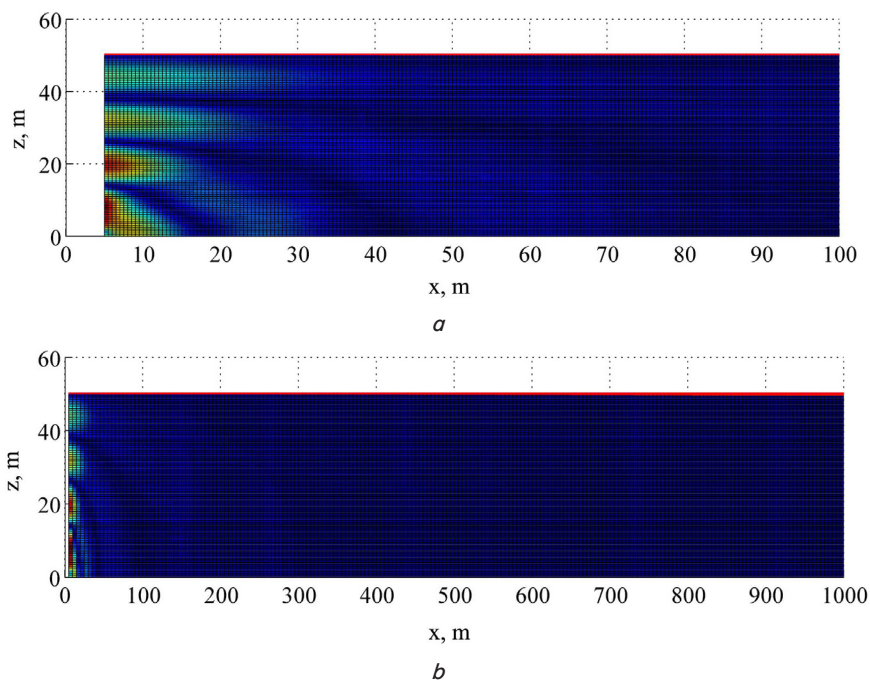


Fig. 12. Wave pattern of distribution of pressure amplitudes in the waveguide (sum of modes), frequency 63 Hz, $H=100$ m: a – $x=100$ m; b – $x=500$ m

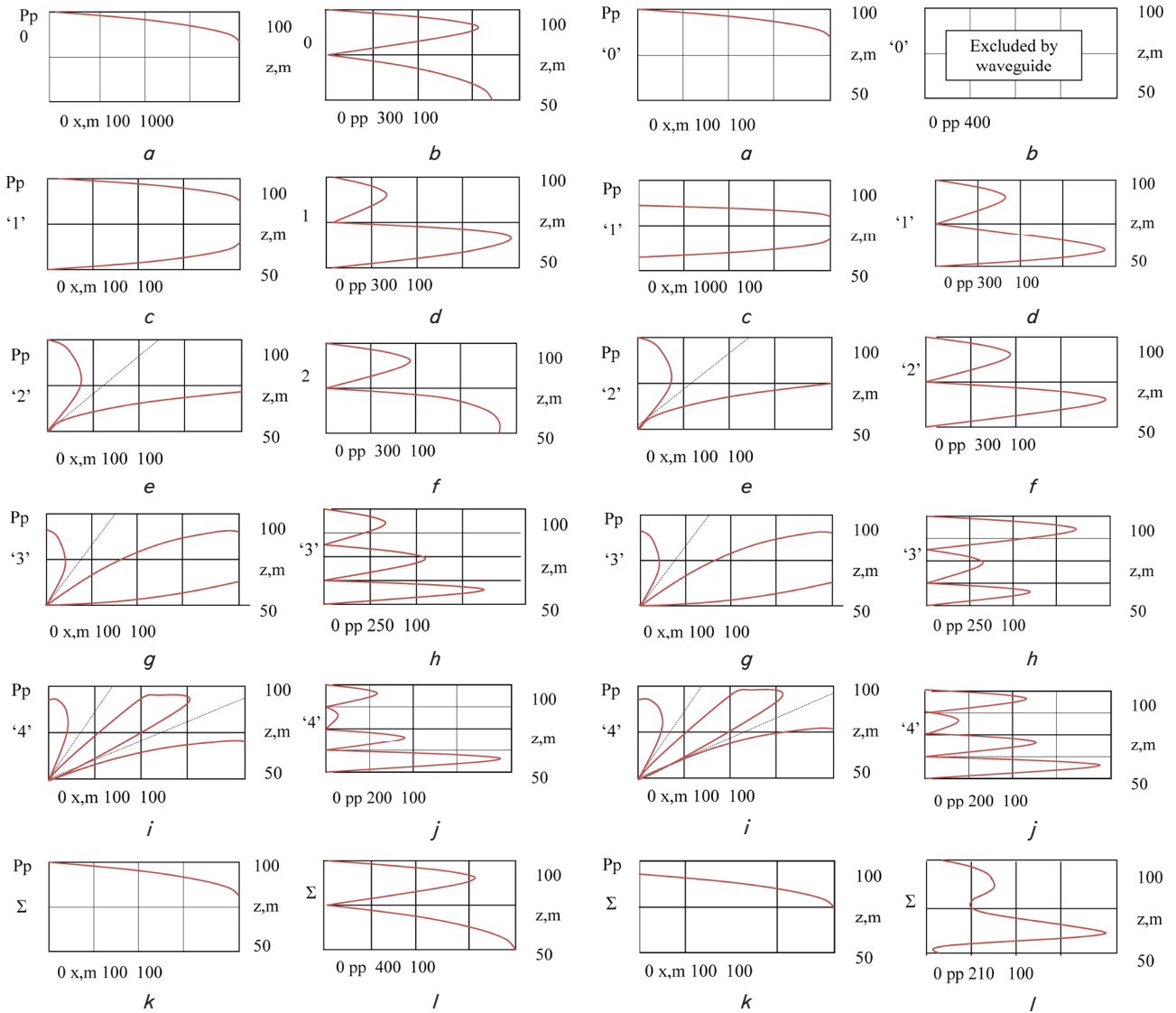


Fig. 13. Bypass distribution of pressure amplitudes pp : a, c, e, g, i, k – in the working space $pp=p(x, h), h \in [50, 100]$; b, d, f, h, j, l – in the section $x=100$ m, $pp \rightarrow p(x, h), (50 \leq z \leq 100)$ m, $(0 \leq x \leq 1000)$ m

Fig. 14. Bypass distribution of pressure amplitudes pp : a, c, e, g, i, k – in the waveguide $pp=p(x, h), h \in [50, 100]$; b, d, f, h, j, l – in the section $x=100$ m, $pp \rightarrow p(x, h), (50 \leq z \leq 100)$ m, $(50 \leq x \leq 100)$ m

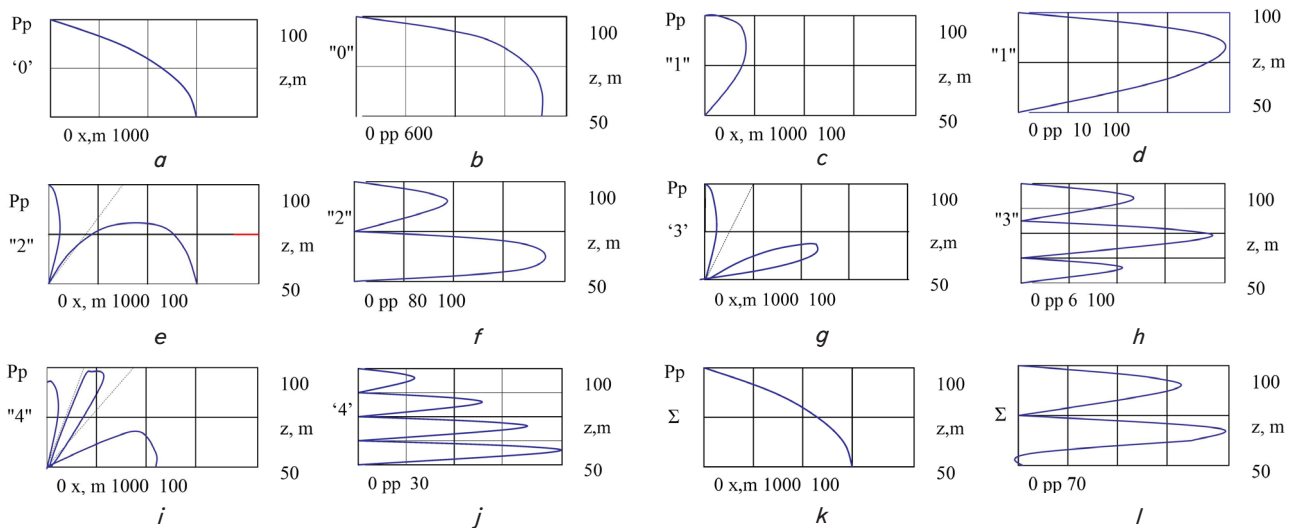


Fig. 15. Bypass distribution of pressure amplitudes pp : a, c, e, g, i, k – in the working space $pp=p(x, h), h \in [50, 100]$; b, d, f, h, j, l – in the section $x=500$ m, $pp \rightarrow p(x, h), (50 \leq z \leq 100)$ m, $(0 \leq x \leq 1000)$ m

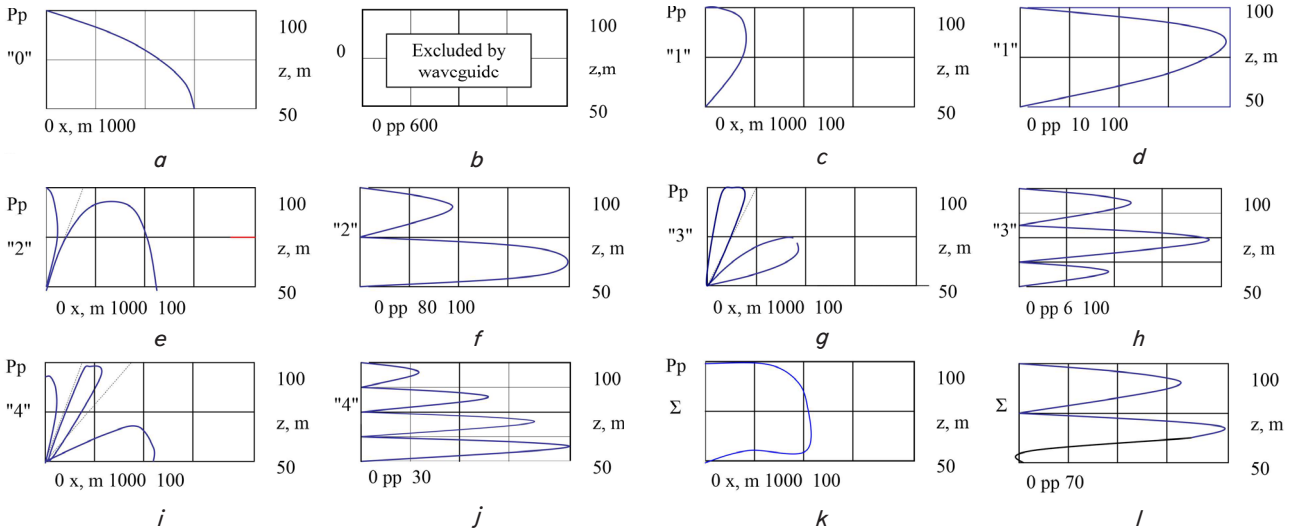


Fig. 16. Bypass distribution of pressure amplitudes pp : a, c, e, g, i, k – in the waveguide $pp = p(x, h)$, $h \in [50, 100]$; b, d, f, h, j, l – in the section $x=500$ m, $pp \rightarrow p(x, h)$, $(50 \leq z \leq 100)$ m, $(0 \leq x \leq 1000)$ m

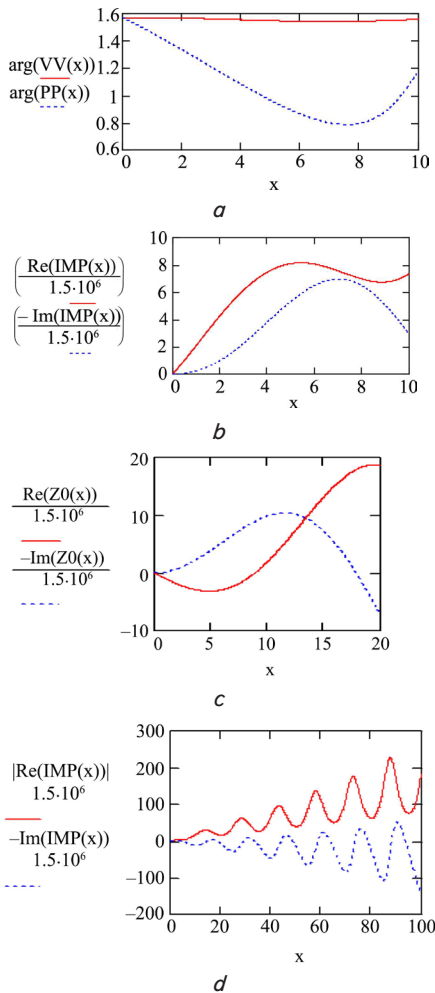


Fig. 17. Frequency dependences: a – phase displacement between pressure and vibrational velocity in a free field; c – components of the reduced impedance $\text{Re}(\text{IMP}(x))$ and $\text{Im}(\text{IMP}(x))$ in a free field; b, d – components of the reduced impedance $\text{Re}(\text{IMP}(x))$ and $\text{Im}(\text{IMP}(x))$ in the working space $a_s=1$ m

The measurement results are presented by normalized angular pressure diagrams when the sphere is operating on

vibration modes “0”, “1”, “2” at a fixed frequency, taking into account the scaling in frequency, the size of the working space and the transducer.

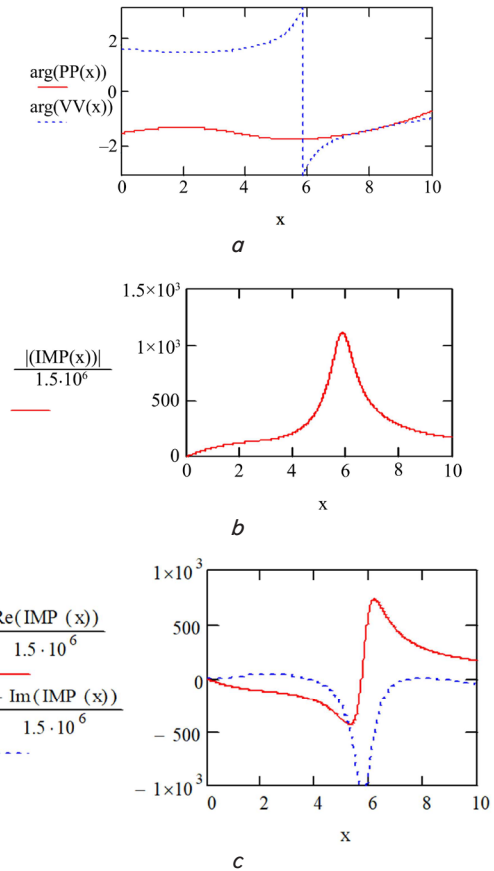


Fig. 18. Frequency dependences of phase displacement: a – between pressure and oscillatory speed; b – impedance module $|\text{IMP}(x)|$ in the working space at $a_s=50$ m; c – components of the reduced impedance $\text{Re}(\text{IMP}(x))$ and $\text{Im}(\text{IMP}(x))$ in the working space at $a_s=50$ m

During the measurements, the working space corresponded to the artificial conditions of the far field, and the method of gating the direct pulse signal in time (distance)

was implemented in the receiving path. The gating operation ensures that there is no echo in the working volume of the pool. The strobe is movable in the sweep of the indicator and registration devices (recorder and oscilloscope connected to the output of the bandpass filter unit 6 (Fig. 19)).

At the same time, the noise-signaling situation corresponded to an inequality of the form $U_s/U_i \geq 15$ dB, where U_s – electric voltage on the useful signal recorder, U_i – electric voltage of the interference. The implementation of the cases of operation of the source in a certain mode corresponded to the ideology of works [10, 14] in terms of the formation of spatial selectivity with appropriate matching of the resistances of the generator and electromechanical converter [15].

Consequently, the generator 1 generates a tone signal of a given frequency, which is amplified in power by the amplifier 2 and is fed through the caisson of block 4 to the emitter 7. The emitter electrodes are switched in such a way that the emitter can operate in a certain mode. The direct signal received by the hydrophone 8 is amplified, selection in time and frequency is performed (selection block 3) and the result is fed to the recording devices 9 and 10. Block 5 synchronizes the operation of the rotary device 4, the oscilloscope sweep, the start of the strobe and the start of the movement of the level recorder motor (Fig. 19).

As a result of the complete rotation of the emitter by the rotary device 4 (Fig. 19), the directivity characteristic is recorded in the range of 0–360 degrees (Fig. 20). In this case, the results of measuring the angular distributions of the amplitudes of the acoustic pressure at the frequency of the working range of research were analyzed (Fig. 20). As a result of the analysis of the diagrams, it was found that the spatial dependences of the pressure amplitudes measured in the working space coincide in nature with the calculated ones, and the error is no more than 3 dB.

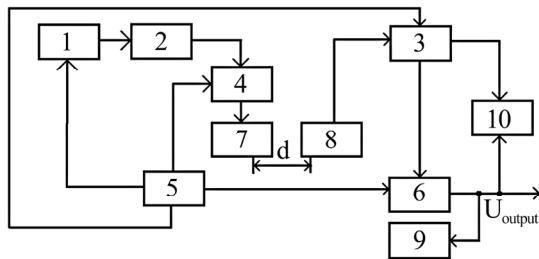


Fig. 19. Measuring circuit: 1 – master oscillator; 2 – power amplifier; 3 – selector; 4 – lifting and turning device; 5 – synchronization device; 6 – band pass filter; 7 – investigated spherical emitter; 8 – control hydrophone; 9 – level recorder; 10 – 2-channel oscilloscope

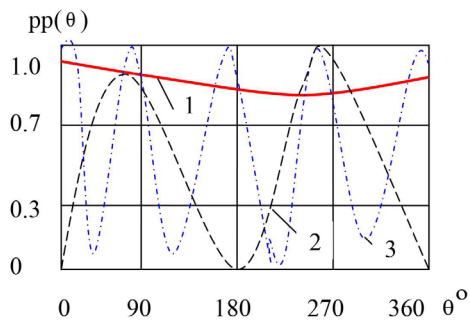


Fig. 20. Angular distribution of the pressure amplitudes of the first three modes: curve 1 – mode $n=0$; curve 2 – mode $n=1$; curve 3 – mode $n=2$

Based on the obtained angular diagrams for the three lower modes, as well as determining the distance $x=x_0 \approx 5$ m, let's consider that:

- gating of the direct signal and scaling of the experiment sufficiently ensure the approximation of the measurement conditions to the real ones;
- the measurement results determine the distance $x_0=5$ m as the limit from which the field will be formed in the waveguide and positively characterize the chosen approach to solving the given radiation problem.

6. Discussion of the results of studying the acoustic field of a spherical complicated source in a plane-parallel waveguide

The formulation and solution of the radiation problem was performed in the representation of pressure by the series (16), (27), (8), (29), (30), which was used to study the pressure distributions in the waveguide sections when replacing artificial boundary conditions with conjugation conditions, and impedance frequency dependences. As a result, two main factors of the appearance of the features of the formation of the acoustic field are determined. These are the general properties of the waveguide and a kind of transient acoustic field, as well as the field in the working space.

In this case, each of the terms in series (16)–(18) and (29), (30) can be associated with a normal traveling wave. Such a wave moves from some source in the positive direction of the Ox axis with its own phase velocity.

As is known from [16], the process of sound emission by a sphere itself presupposes the existence of a near ($kr \ll 1$) and far ($kr \gg 1$) fields. Undoubtedly, the limit, or rather, the area of separation of such fields, has no clear boundaries, and, formally, sound waves can begin to form only from a certain distance from the sphere. The used method of searching for unknown expansion coefficients of scalar and vector fields makes it possible to accurately calculate and apply the coefficients of expansions D_n (27) taking into account the mode composition of the sphere vibrations. The search for the coefficients D_n was carried out in the understanding that the sphere and the space adjacent to it are considered a complicated source of sound. Such a transition does not violate the results of the general solution, since the solution of the problem obtained in each formed partial domain satisfies the Helmholtz equation, limiting conditions, and conjugation conditions.

Consequently, the formation of the field in the waveguide will occur by using the values of the coefficients D_n from the workspace and using the eigenfunctions within the framework of the Sturm-Liouville problem. Thus, for a certain value of x_0 , in the case of coincidence of the calculated values of the pressure amplitudes in the vertical sections of the working area and the diagrams of the corresponding modes in the vertical sections of the waveguide, the coefficients D_n will uniquely correspond to the generated normal waves. In addition, it becomes possible to study the dynamics of the formation of the field when moving along the axis of the desired set Ox of normal waves (modes). Let's note that in the vicinity of the near field, the coordinate dependences of the pressures are characterized by significant sheariness, which leads to significant spatial variability of the amplitudes of the mode composition. The number and position of nodal mode lines

in this case determine the nodes and antinodes of standing waves in vertical sections of the field in the working space.

For an infinite space, the pressure distribution of the pulsating sphere is supplied by means of a system of concentric circles (Fig. 3), and the introduction of boundaries into the field should enrich the mode set of oscillations of the sphere.

In Fig. 4–9 for modes “0”–“4” and their sums 0+1+2+3+4 in the first quadrant of the working space, wave patterns of the distribution of pressure amplitudes pp at a frequency of 63 Hz are shown. Distance range ($0 \leq x \leq 100$) m, ($0 \leq x \leq 1000$) m.

Wave patterns are presented in different colors: red, orange, light blue, blue, dark blue. The colors show the change in the value of relative pressure (pp/p_0) from the highest (red) to the lowest (dark blue).

From the results of calculations (Fig. 3–9), let’s note that the pressure field in the working area is represented by a set of antinodes and alternating nodes. For a plane problem, for the number of nodal lines that determine the mode number mn , the situation is simplified. Therefore, at $m=0$, only the modes $mn \rightarrow \gg 00 \gg, \gg 01 \gg, \gg 02 \gg, \dots$ variability of pressure distribution in the working area.

Obviously, the reason for the appearance of the features of the spherical source in a confined space is the dispersion, which characterizes the decrease in the phase velocity of the n mode in frequency, asymptotically approaching the value $c=1500$ m (Fig. 3 – free space).

As can be seen from Fig. 4–9, 15, when to change the scale of the diagram, the detail of the pressure distribution shows:

1. The pressure distribution itself is the result of a complex interference-diffraction interaction of the fronts of spherical waves modulated by the known angular functional dependences (Fig. 4–9) for modes $n=0, 1, 2, 3, 4$:

$$\begin{aligned} "0" \rightarrow f(\theta) &\Rightarrow 1, \quad "1" \rightarrow f(\theta) \Rightarrow \cos(\theta), \\ "2" \rightarrow f(\theta) &\Rightarrow \cos(2\theta), \quad "3" \rightarrow f(\theta) \Rightarrow \cos(3\theta), \\ "4" \rightarrow f(\theta) &\Rightarrow \cos(4\theta). \end{aligned}$$

2. The pressure and spatial characteristics of the mode components of a spherical oscillatory system are characterized by zeros at the boundaries “sea surface” (quadrant I, II) and “bottom” (quadrant III, IV) due to the boundary conditions of the working space.

3. For any mode (sum of modes) at the point $H(x, h)$, ($x=0, h=H$), the statement about the movement of acoustic energy into the soft boundary is valid, which excludes the perturbation in the waveguide of a plane wave. In this case, the flow of energy into the limit affects the distribution components, decreasing in their amplitude and deforming in shape as the channel axis approaches the boundaries.

4. When the waveguide operates on one mode or using the total number of modes, the diagrams show a certain separation of the medium into near and far fields. In the given initial data for the “0” mode, the section passes along the coordinate $x=500$ m (Fig. 15, a, c, e, g, h, k).

To determine the pressure field in the waveguide, it is necessary for the selected n th term of series (30) to find the corresponding product, using a factor of the form of the function $\sin\left(\frac{n\pi}{H}z\right)$, of the corresponding situation of

acoustically soft waveguide boundaries. The pressure field distribution calculated for this case (Fig. 10–12) shows the appearance of additional nodal lines corresponding to zeros of the pressure dependence pp of the corresponding mode.

The considered waveguide should filter out the zero mode in accordance with the conditions at the boundary; therefore, the situation of using modes other than zero, for example, modes “1” and “2”, is of considerable interest. A separate implementation of the indicated modes by a spherical source was provided by electrical switching of separate split electrodes deposited on the surface of the sphere.

For example, according to works [10, 15]:

- “0” mode – parallel coordinated connection of all electroplated sections;
- “1” mode – by sequential counter connection of oppositely located pairs of electroplated sections;
- “2” mode – parallel-series connection, electroplated sections of orthogonal pairs of surfaces of the sphere.

The $pp \rightarrow pp(\theta), pp(z)$ dependency diagrams are arranged as follows:

- calculated envelopes of pressures pp and diagrams of pressures $pp(z)$ for the working space, modes “0”–“4”, $x=100$ m in Fig. 13, a, c, e, g, i, k and Fig. 13, b, d, f, h, j, l ;
- calculated bypass pressures pp and pressure diagrams $pp(z)$ for the waveguide, modes “0”–“4”, $x=100$ m in Fig. 14, a, c, e, g, i, k and Fig. 14, b, d, f, h, j, l ;
- calculated bypass pressures pp and pressure diagrams $pp(z)$ for the working space, modes “0”–“4”, $x=500$ m in Fig. 15, a, c, e, g, i, k and Fig. 15, b, d, f, h, j, l ;
- calculated bypass pressures pp and pressure diagrams $pp(z)$ for the waveguide, modes “0”–“4”, $x=500$ m in Fig. 16, a, c, e, g, i, k and Fig. 16, b, d, f, h, j, l .

According to Fig. 16, a, c, e, g, i, k the working space lasts along the horizontal axis of the waveguide from 1 to 700 m for lower modes. At the same time, there is a limitation of the working space by the coordinate $x=500$ m. This shows the correspondence, starting from the indicated section, to the calculated values of the amplitude distribution obtained in accordance with the provisions of the solution of the partial domains method (11), to the typical pressure distribution diagrams according to [5]. Therefore, they can be used as expansion coefficients for the field in the waveguide (27). The levels of the total field and mode components fall back towards increasing distance without an overall change in shape.

Thus, let’s choose the cross-sections $x=100$ m and $x=500$ m. For them, as can be seen from Fig. 13, 15, the bypass spatial pressure distributions in the working space are calculated. They have specific features that are manifested in the values of the levels of local maxima, zeros, and certain changes in the shapes of the petals of these diagrams. This is due to:

- differences in the selected sections of the “finest field structure” relative to the number of antiphase sections, nodes and pressure antinodes in sections $x=100$ m and $x=500$ m for each of the implemented modes;
- spatial losses for front expansion and re-reflection from the boundaries of waves formed by the source;
- differences in the phase velocities of normal waves.

Wherein (Fig. 13, a, c, e, g, i, k) the positions of the local minima of the bypass diagrams of the mode components characterize the typical directions of the angles, represented by the dotted line for modes “2”–“4”).

About the impedance features. The calculation results (Fig. 17, 18) show that in the near field (at low frequencies or small arguments $x(ka)$), there is a significant phase difference between the pressure and the vibrational velocity. Such a result, obviously, can partially explain the reason for the appearance of personal points in the intensity field in the shallow sea. In this case, the general character of the behavior of the curves (Fig. 17, *b, d*) describes the frequency dependence of the form $\text{Re}(\text{IMP}(x))$ and $\text{Im}(\text{IMP}(x))$. This coincides with the curves in Fig. 17, describing the dependence of the impedance of a monopole in a free field on frequency. Consequently, the effect of equality to zero of the average flux of acoustic energy through any small surface covering the source (monopole) can be extended to the design situation.

Also, in addition to the above, it should be noted the emergence of oscillations of the curves of active and reactive components of the impedance of a sphere of small radius, the reason for which is evidently multiple re-reflection of sound from the boundaries in the volume of the working area. The oscillation amplitudes increase with increasing frequency in the considered range.

With an increase in the size of the source in the working area, a resonance is clearly formed (Fig. 18, *b, c*), the frequency of which depends on the depth of the sea. That is, when the acoustic field is formed in the working space, the frequency response of the field is represented as a multi-resonance formation, in contrast to the emission of sound by a monopole. Thus, the impedance characteristics should be considered both for different modes and for their combination, which indicates a situation where the inertial-elastic state of the converter changes with the frequency flow. In this case, the sequence of resonances of modes or their combinations contains resonances of two origins: classical (according to modes) and additional (according to the wave characteristics of the problem under consideration).

The limitations inherent in the application of the method of partial regions are mainly in the fact that sometimes it is quite difficult to select partial regions in a shape close to the canonical. In this case, it is possible to reduce the areas and bring them, if possible, to an acceptable shape and geometric and wave sizes. In this case, the number of functional equations for boundary conditions, conjugation conditions, and partial solutions of the Helmholtz equation increases significantly, complicating the complete solution of the problem.

The disadvantages of the method include the need to compare the size of the transducer and the wavelength at the operating frequency, where the wave condition for the

smallness of the lobe region is satisfied. That is, the method is inherently limited in the high frequency range.

The development of research on the selected topic should be aimed at enriching the performances with physical factors, which should bring the results closer to the realities of field formation and the possibility of studying the fine structures of the acoustic field. One of these factors is the approach of cross-cutting problems from the field of stationary hydroelectric elasticity.

7. Conclusions

1. The problem of emission of sound waves by a spherical multimode source under the conditions of a regular waveguide is formulated. Its peculiarity lies in the fact that the influence on the source of the direct field emitted by the source, the field reflected by the boundaries of the waveguide, and the field scattered by the emitters is taken into account.

2. Based on the geometry of the emitter and the use of a plane-parallel waveguide, the coordinate systems of the problem (spherical and Cartesian) are selected. Five partial regions are identified, four of which form a symmetric system, and the fifth connects the working space of the transducer and the waveguide.

3. For the selected partial regions, a system of functional equations is compiled, which determines the coordinates of the regions, boundary conditions and conditions for conjugation of regions and make it possible to determine the unknown expansion coefficients of acoustic fields.

4. The wave problem for the full field in the working space is solved using the principle of superposition of the direct radiated field reflected from the flat boundary and the field scattered by the sphere. Due to the symmetry of regions I–IV, the problem is solved only for quadrant I. The solution of the Helmholtz equation for the indicated field components is used, followed by the application of the superposition principle for each point of the working space (quadrant I).

5. The results of calculating the pressures in the spaces of the problem determine the correspondence of the application of the obtained excitation coefficients, in the form of pressure distributions at the boundary of the working space, to the diagrams of the vertical distributions of the waveguide modes. The composition and dynamics of the mode structure of the emitter are shown during the propagation of normal waves. The boundary area of the working space was determined for the low frequency range (31.5–63 Hz) when a source with a diameter of 1 m is operated in the sea with a depth of 100 m. The distance is about 2λ .

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