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APPLIED MECHANICS

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UDC 514.18

DOI: 10.15587/1729-4061.2021.245121

DESIGNING AN OUTER TOOTHED GEAR WHOSE WHEEL TEETH ARE OUTLINED BY THE LOGARITHMIC SPIRAL ARCS

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Published date 21.12.2021

How to Cite: Pylypaka, S., Kresan, T., Volina, T., Hryshchenko, I., Pshenychna, L., Tatsenko, O. (2021). Designing an outer toothed gear whose wheel teeth are outlined by the logarithmic spiral arcs. Eastern-European Journal of Enterprise Technologies, 6 (7 (114)), 6-11. doi: https://doi.org/10.15587/1729-4061.2021.245121

1. Introduction

It is well known that toothed gears are the most common mechanical gears in machine building. Based on the shape of a tooth profile there are evolving toothed gears and non-evolving ones with a cycloidal profile. Toothed gears are characterized by high reliability and durability, a constant transfer number, and can transmit high torque. However, during toothed gear operation, the surfaces of the teeth of the geared wheels slip, which leads to the emergence of friction forces and the wear of working surfaces. To prevent this, the surfaces of the teeth need constant lubrication. There is a relevant issue related to the design of a toothed gear, which does not have friction between the surfaces of the teeth, which could minimize the wear of working surfaces. This is possible if the teeth roll over each other without slipping.

design of a gear tooth engagement, which does not have friction between the surfaces of the teeth since they roll over each other without slipping. The profile of the tooth of such a gear is outlined by congruent arcs, symmetrical relative to the line that connects the center of rotation of the toothed wheel with the top of the tooth. These symmetrical curves at the top of the tooth intersect at the predefined angle. In the depressions of the wheel, adjacent teeth also intersect at the same angle. Such a condition can be ensured by a curve that at all its points crosses the radius-vector emanating from the coordinate origin, also at a stable angle equal to half of the given one. This curve is a logarithmic spiral. If the number of teeth of the drive and driven wheels is the same, then their teeth are congruent. Otherwise, the profiles of the teeth would differ but they could be outlined by congruent arcs of the same logarithmic spiral of the same length taken from different areas of the curve.

The minimum possible angle at the top of the teeth is straight. At acute angle, the toothed gear operation is impossible. To build gear wheels with a right angle at the top of the tooth, it would suffice to set the number of teeth of the drive and driven wheels. The center-to-center distance is calculated using the derived formula. The transfer number of such a gear is variable but, with an increase in the number of teeth, the range of its change decreases. The algorithm of wheel construction is given

Keywords: toothed gear, logarithmic spiral, surface rolling, center-to-center distance, arc length

Received date 27.10.2021 Accepted date 08.12.2021 Rotational motion can be transmitted in various ways: with the help of friction, belt, chain, toothed gears, using the hydraulic and electric drives, etc. In this list, toothed gears are among the most common ones. Their design and manufacture techniques have been improving all the time and are currently high-tech. Given this, any proposals for improving gear tooth engagement deserve attention and are relevant.

2. Literature review and problem statement

Engineering practice is often associated with problems of designing geometric objects, which scientists propose to solve in quite diverse ways. For example, in [1], the authors present a vision of constructing geometric objects within a multidimensional space by approximating a solution to differential equations. In work [2], geometric modeling of multifactorial processes and phenomena is proposed to be carried out by the method of multidimensional parabolic interpolation. In [3], the use of geometric space interpolates is taken as a basis. As one can see, the methods for geometric design of objects are quite diverse. In addition, the range of tasks that require the use of geometric methods to resolve them is quite wide.

It is known that scientists pay a lot of attention to the issues of wear resistance and durability of parts. Thus, work [4] proposes a new technique to sulfide the surfaces of parts with electrical discharge machining; paper [5] outlines a way to strengthen the surfaces of thermally treated steel components, including cementation and nitriding processes. The authors of [6] devised a technique to increase the reliability and durability of metal surfaces of parts by applying a multilayer coating using electrical discharge machining. However, as noted in [7], the measures and recommendations proposed by scientists up to now are not enough. In the cited works, the wear resistance of parts is proposed to be improved by developing new ways to strengthen surfaces, which is a rather expensive and lengthy process. Our review of the above papers has shown that the search for an increase in the wear resistance of parts involves strengthening their surface layer. Instead, one can eliminate the cause of wear by eliminating friction between surfaces during their operation.

The formation of known cycloids and involutes curves whose arcs are a profile of the teeth of toothed gears is associated with rolling along a curvilinear contour. In the first case, this is the trajectory of the point of the circle, which rolls in a straight line, and in the second – on the contrary – the trajectory of the straight point, which rolls around in a circle. Rolling along a curvilinear path in a straight line is used in work [8] to construct noncircular wheels, one of which is formed by straight segments, that is, it is a polygon. The formation of congruent noncircular wheels of two symmetrical arcs of the logarithmic spiral is considered in [9]. Paper [10] proposes a geometric model for solving the problem of profiling an uncircular toothed wheel whose centroid consists of interconnected arcs. The same authors in article [11] proposed a fairly simple and accurate procedure for the automated formation of wheel teeth with an elliptical centroid. An eccentric helical toothed pair is a new type of geared pair consisting of a screw noncircular gear and an eccentric oblique curved face, which can implement the rotation of intersecting axes, as well as the movement of the output axes, proposed in [12]. It should be noted that the arcs of algebraic curves are applied when solving problems in different areas, for example, the systematization and analysis of multidimensionality [13].

Thus, there is a relevant task to design toothed wheels whose operation includes the rolling of teeth surfaces without slipping, which eliminates the friction and wear of surfaces.

3. The aim and objectives of the study

The purpose of this work is to devise a procedure for constructing an analog of the gear tooth engagement, which would ensure the absence of slipping between the surfaces of the teeth. This could improve the wear resistance and durability of the gear.

To accomplish the aim, the following tasks have been set: - to find the arcs of the curves describing the profile of the teeth, visualizing the results;

 to outline the procedure for constructing a pair of gear wheels according to the predefined output parameters;

- to consider the proposed approach using an example of constructing a pair of gear wheels with a certain ratio of the number of teeth and the predefined angle at the top of the tooth.

4. The study materials and methods

The profile of a separate tooth of gear is formed by a symmetrical representation of the arc relative to the radius-vector of the polar coordinate system, which passes through its vertex (Fig. 1, *a*). It forms the angle ε (Fig. 1, *b*) with the tangent to the profile at this point. As a result of a symmetrical representation of the profile curve, a tooth is formed with an angle of 2ε at its top (Fig. 1, *c*).



Fig. 1. Graphic illustrations for the formation of a gear tooth engagement profile: a — the shape of the teeth of the gear whose profile needs to be derived; b — the curves of the tooth profile with restrictions imposed on them; c — formation of a tooth for each wheel by the symmetrical representation of the derived arcs

This angle cannot be less than 90°, otherwise the transmission will be jammed [8]. For the same reasons, the angle between the curves in the depressions of the teeth should also be either right or larger than the right one. Consequently, the angle between the radius-vector and the curve at both the beginning and end of the arc must be at least 45° ($\varepsilon \ge 45^{\circ}$). In addition, the lengths of the arcs of the curves (in Fig. 1, *a*, marked by a thickened line) should be equal and, when rolling over each other, pass even paths. The sum of vectors ρ and ρ_1 from the poles *O* and *O*₁ to the point of contact must be constant and equal to the center-to-center distance: $\rho + \rho_1 = r$ (Fig. 1, *b*, *c*). When turning one wheel at the angle of α_0 , the second wheel must turn at angle φ_0 (Fig. 1, *a*). The values of these angles depend on the number of teeth: $\alpha_0 = \pi/n$, $\varphi_0 = \pi/n_1$, where *n* and n_1 are the number of teeth in the drive and driven wheels.

The above structural parameters of toothed gears underlie the design of the curves of the teeth profile.

5. Results of designing gears

5.1. Finding arc curves describing the profile of the teeth

The curve describing the profile of the drive wheel tooth is given by the polar equation $\rho = \rho(\alpha)$, where α is the polar angle. The parametric equations of the curve AB are:

$$x = \rho(\alpha) \cos \alpha; \quad y = \rho(\alpha) \sin \alpha.$$
 (1)

It is required to find the angle ε (Fig. 1, *b*) between the polar radius ρ and the tangent to the curve at its current point. The tangent vector can be derived by differentiating equations (1) for variable α :

$$\dot{x} = \dot{\rho}\cos\alpha - \rho\sin\alpha; \quad \dot{y} = \dot{\rho}\sin\alpha + \rho\cos\alpha.$$
 (2)

The polar radius vector is given by equations (1). A known formula can define the angle ε between vectors (1) and (2):

$$\cos\varepsilon = \frac{\dot{\rho}}{\sqrt{\rho^2 + \dot{\rho}^2}}.$$
(3)

Differential equation (3) has the following solution:

$$\rho = \rho_0 e^{\alpha \text{ctge}},\tag{4}$$

where ρ_o – the integration constant – the initial value of the polar radius on the circle of the depressions of the drive wheel teeth (Fig. 1, *a*).

Polar equation (4) describes a logarithmic spiral that crosses all polar radii at a stable angle ε . The parametric equations of the curve AB, according to (1), are recorded as:

$$x = \rho_0 e^{\alpha \operatorname{ctge}} \cos \alpha; \quad y = \rho_0 e^{\alpha \operatorname{ctge}} \sin \alpha. \tag{5}$$

In order to derive the desired arc – a tooth profile – it is necessary to change the angle α in the interval $\alpha=0...\alpha_o$ in equations (5).

Taking into consideration $\rho_1 = r - \rho = r - \rho_o^{\text{actge}}$, as well as that the radius-vector ρ_1 rotates at angle φ in the opposite direction from the angle α , the parametric equations of the desired curve on which the arc A_1B_1 is located, are recorded as:

$$x_1 = (r - \rho_0 e^{\alpha \operatorname{ctge}}) \cos \phi + r; \quad y_1 = -(r - \rho_0 e^{\alpha \operatorname{ctge}}) \sin \phi. \tag{6}$$

It is required to find expressions to determine the length of the arc curves (5) and (6). The derivative from the length of the arch of logarithmic spiral (5) is as follows:

$$\frac{ds}{d\alpha} = \sqrt{\dot{x}^2 + \dot{y}^2} = \frac{\rho_0}{\sin\varepsilon} e^{\alpha \varepsilon tg\varepsilon}.$$
(7)

Let the angle φ be dependent on the angle α , that is, $\varphi = \varphi(\alpha)$. In this case, the derivative of the length of the curve arc (6) takes the following form:

$$\frac{ds_1}{d\alpha} = \sqrt{\dot{x}_1^2 + \dot{y}_1^2} = \sqrt{\rho_0^2 \operatorname{ctg}^2 \varepsilon e^{2\alpha \operatorname{ctg} \varepsilon} + \left(r - \dot{\phi}^2 \rho_0 e^{\alpha \operatorname{ctg} \varepsilon}\right)^2}.$$
(8)

Based on the equality of the arcs of curves, their derivatives (7) and (8) are also equal. Equating expressions (7) and (8) with each other and solving with respect to $d\varphi/d\alpha$ can help derive

$$\frac{d\phi}{d\alpha} = \frac{\rho_0 e^{\alpha c tge}}{r - \rho_0 e^{\alpha c tge}}.$$
(9)

The dependence $\varphi = \varphi(\alpha)$ is obtained by integrating expression (9), which ensures equality of curve arcs when turning polar radii ρ and ρ_1 at the corresponding angles:

$$\phi = -\ln(r - \rho_0 e^{\alpha c t g \varepsilon}) t g \varepsilon + c, \qquad (10)$$

where c is the integration constant.

Based on the condition that at $\alpha = 0$, the angle $\varphi = -\varphi_o$, the expression of the constant *c* is defined, considering which renders equality (10) to take the following form:

$$\phi = \ln \left(\frac{r - \rho_0}{r - \rho_0 e^{\alpha \text{ctge}}} \right) \text{tge} - \phi_0.$$
(11)

The distance *r* can be taken arbitrarily. However, in this case, when turning the radius-vector ρ at the predefined angle α_o , the radius-vector ρ_1 would turn at an unpredictable angle φ_o , although other conditions are met. The condition is set that when turning the radius-vector ρ at the predefined angle α_o , the radius-vector ρ_1 also turns at the desired angle φ_o . It should be borne in mind that after turning the radius-vector ρ_1 takes the value $\varphi=0$ since at the beginning of rotation this angle was negative and equal to $-\varphi_o$ (Fig. 1, *a*). Substituting the resulting angle values of $\alpha=\alpha_o$, $\varphi=0$ in (11) and solving relative to *r* produces the following:

$$r = \rho_0 \frac{e^{(\alpha_0 + \phi_0) \text{ctge}} - 1}{e^{\phi_0 \text{ctge}} - 1}.$$
 (12)

Thus, the value of the center-to-center distance is not set but determined depending on the number of teeth $n=\pi/\alpha_o$, $n_1=\pi/\varphi_o$ of the drive and driven wheels and the values of ρ_o and angle ε . Substituting (12) in (11) could produce the required dependence $\varphi=\varphi(\alpha)$. If it is substituted in (6) along with constant (12), then the parametric equations of the curve could be obtained, on which it is possible to highlight the arc A_1B_1 (Fig. 1, b) when changing the angle α within $\alpha=0...\alpha_o$.

Fig. 2, *a* shows the curves constructed from equations (1) and (6) taking into consideration (12) and (11), on which we

highlighted with thickened lines the arcs AB and A_1B_1 , corresponding to the change of angle α within $\alpha=0...\alpha_o$. Visually, these arcs are congruent. This can be seen if the arc A_1B_1 is not shifted along the Ox axis by the value r. This applies to wheels with the same number of teeth.



Fig. 2. Arcs *AB* and A_1B_1 of the logarithmic spiral – the profile of a tooth of congruent wheels – constructed at $n=n_1=4$, $\rho_0=1$, $\varepsilon=45^\circ$: *a* – the initial position of the arcs outlining the profile of the teeth of the drive and driven wheels; *b* – arc A_1B_1 is turned around its pole to contact with arc *AB*; *c* – formation of the teeth of the drive and driven wheels with a symmetrical representation of the arcs relative to the straight lines that connecting the pole with the top of the tooth

This is confirmed by deriving the angle between the polar radius of curve (6) and the tangent to it. One can make sure that it is negative and equal to $-\varepsilon$. A curve that intersects all its polar radii at a constant angle can only be a logarithmic spiral. If the number of teeth of both wheels is the same, then the arcs AB and A_1B_1 are congruent; otherwise, they are not congruent but have the same length and belong to the same spiral. The difference between equations (1) and (6) of the same curve is explained by the fact that in the first case, with the growth of the polar angle α , the point when moving along the curve is removed from the pole, and in the second – on the contrary – approaching. This is shown by arrows in Fig. 2, *a*. In order for point A_1 of arc A_1B_1 (Fig. 2, *a*) to coincide with point *A* of arc *AB* (Fig. 2, *b*), one needs to additionally turn the A_1B_1 arc at angle $\pi+\varphi_0$. Thus, in equations (6), instead of the polar angle φ , there should be the angle $\varphi+\pi+\varphi_0$. The resulting expression of angle φ can be obtained after adding to it the previous value (11) of the angle $\pi+\varphi_0$:

$$\phi = \ln \left(\frac{r - \rho_0}{r - \rho_0 e^{\alpha \text{ctge}}} \right) \text{tge} + \pi, \tag{13}$$

where the center-to-center distance r is determined from formula (12).

The above considerations allow us to proceed to the design of the gear engagement with the predefined specific parameters.

5. 2. The order of construction of a pair of gear wheels according to the predefined number of teeth and angle ϵ

Our procedure for finding curve arcs describing the profile of teeth would suffice to build a gear. The center-to-center distance r is calculated while the constant ρ_o is not of fundamental importance since it plays the role of a scaling coefficient.

The order of construction is as follows:

– based on the predefined number of teeth of the drive wheel *n*, one can derive the value of angle α_o : $\alpha_o = \pi/n$. Based on equations (1), one builds the arc *AB* when changing the angle α within $\alpha = 0...\alpha_o$;

– a symmetrical arc is constructed relative to the polar radius, connecting the pole *O* with point *B* (Fig. 2, *c*);

- the resulting profile of the tooth is consistently turned around the pole *O* at angle $2i\alpha_o$, where i=1...n-1;

- the predefined number of the teeth of the driven wheel n_1 could help derive the value of angle $\varphi_o: \varphi_o = \pi/n_1$. Expression (12) is used to determine the center-to-center distance r, which must be substituted in (13). Based on equation (6), substituting with the refined expression $\varphi = \varphi(\alpha)$ (13), one can construct arc A_1B_1 when changing the angle α within $\alpha = 0...\alpha_o$;

- a symmetrical arc is constructed relative to the polar radius connecting the pole O_1 with point A_1 (Fig. 2, *c*);

– the resulting tooth profile is consecutively turned around the O_1 pole at angle $2i\varphi_0$, where $i=1...n_1-1$.

5. 3. Applying the proposed procedure to the design of a pair of gear wheels

Below is the procedure for constructing a pair of wheels with an equal number of teeth $n=n_1=4$ and different values of angle ε . The distance ρ_o is taken to equal unity: $\rho_o=1$.

The values of angles $\alpha_o = \pi/4$ and $\varphi_o = \pi/4$ are determined. The center-to-center distance *r* depends on angle ε . According to formula (12), *r*=3.19 for ε =45° and *r*=2.57 for ε =60°. In Fig. 3, the pairs of wheels are built for the two angle ε values. The radius of the circle that passes through the tops of the teeth is also different. It is defined from expression (4) when $\alpha = \alpha_o$. In the first case, it is equal to 2.19; and in the second – 1.57.

Our devised procedure can be used to build the pairs of gear wheels with any ratio of the number of teeth, including one. Fig. 4 shows the pairs of wheels constructed at a right angle at the top of the tooth (ϵ =45°) and a different number of teeth.

Worth noting is that when the angle ε approaches 90° the gear wheels deform and approach the circles passing the depressions of the teeth. At ε =90°, the radius-vector ρ , according to formula (4), is converted to a constant value ρ_0 .



Fig. 3. Pairs of congruent gear wheels at $n=n_1=4$ and $\rho_o=1$: $a - \varepsilon = 45^\circ$; $b - \varepsilon = 60^\circ$



Fig. 4. Pairs of gear wheels with different number of teeth: a - n=3, $n_1=1$; b - n=2, $n_1=5$; c - n=7, $n_1=15$

6. Discussion of results of studying the procedure for designing a pair of gear wheels

In the previous chapter, we obtained the parametric equations of the curve that describes the profile of the gear teeth (6), derived formulas for determining the angle of rotation of a radius-vector (13) and the center-to-center distance (12), which made it possible to build the arcs of the logarithmic spiral - the profile of the congruent wheel tooth (Fig. 2). A curve that crosses all its polar radii at a constant angle can only be a logarithmic spiral. Due to the use of the logarithmic spiral as an arc, which outlines the profile of the teeth of the gear wheels, the absence of slipping between the surfaces of the teeth during work of gear wheels is ensured. The condition for the absence of slipping between the surfaces of the teeth during gear wheel operation is mathematically justified. It is based on the fact that when turning the drive wheel at angle α , the driven wheel turns at angle φ ; at the same time, the profiles of the teeth during rolling pass equal paths, that is, the same lengths of the arcs of these profiles. Mathematically, this is ensured by dependence (9) between the angles of rotation of both wheels based on the equality of the lengths of the corresponding arcs. Such rolling without slipping occurs in the pairs of noncircular wheels, the design of which is widely known. In a given case, rolling the profiles within one tooth can be considered as work of a pair of noncircular wheels. The difference is that the profile of noncircular wheels is one closed curve, and, in our case, the pieces of the congruent arcs that intersect at a right angle. This makes it possible to form a wheel profile similar to a geared one.

In addition, we devised a procedure for constructing a pair of gear wheels at the predefined parameters in algorithmic form. The initial data are reduced to the number of teeth in the drive and driven wheels. Some of the possible variants of the pairs of gear wheels have been built, which could ensure that there is no slip during their operation (Fig. 3, 4). The advantage of the resulting gear is the absence of friction between the surfaces of the teeth during their work. This is achieved by establishing a dependence (9) between the angles of rotation of the drive and the driven wheels within the operation of one pair of teeth. In this case, there is no friction force, which ultimately prevents the wear of the teeth. In addition, such a gear does not require lubrication. When constructing a specific pair of gear wheels, the initial data are the number of teeth of both wheels and the angle at the top of the tooth. The remaining parameters are calculated, so the design limitations relate to the choice of a center-to-center distance, the height of the tooth, the diameters of the protrusions of both wheels.

Alternative solutions to this problem relate to gear tooth engagement not designed to transmit torque of considerable power and for which the number of full wheel rotations is important, for example, in computing devices. In a broad sense, such gear tooth engagement can be considered

as an alternative to the centroids of noncircular wheels, in contrast to which the profile of the centroid is formed not by one curve but by equal arcs of congruent curves. The evidence base of this approach is based on the mathematical derivation of the dependences whose reliability is confirmed by graphic constructions of gear wheels. The limitation of the proposed procedure is the narrowed possibilities of structural solutions for a gear transfer, in particular, the lack of choice of the center-to-center distance whose value depends on the number of teeth of the drive and driven wheels. This study can be advanced in order to design a toothed gear for the transfer of rotational motion between the intersecting axes. The scope of application of the resulting toothed gear could include computing devices for which the transfer function is insignificant, which do not transmit significant torques, and for which the number of full rotations of both wheels is important.

Given the positive point regarding the absence of friction, such a gear has drawbacks. This is an intermittent transfer function, the inability to set the predefined center-to-center distance at the specified ratio of full wheel rotations as it is calculated, as well as the high sensitivity to the center-to-center distance.

7. Conclusions

1. The use of a logarithmic spiral makes it possible to design a toothed gear in which the teeth of the wheels are outlined by the arcs of the same spiral. A characteristic feature of such a gear is the absence of slipping between the surfaces of the teeth during the operation of gear wheels. This does not cause friction forces and does not lead to the wear of working surfaces. The arcs of the spirals, which delineate the profile of the teeth, intersect at the tops and depressions of the teeth at a constant angle, which cannot be less than the right one to ensure the possibility of wheel operation.

2. The initial data for designing the wheels is the number of teeth of the drive and driven wheels, and the

value of the angle at the top of the tooth. Because the best angle is the right angle, the initial data are reduced to the number of teeth of the drive and driven wheels. The center-to-center distance, as well as other parameters, are calculated.

Our procedure could be used to build the pairs of gear wheels with any ratio of the number of teeth, including one. When the angle ε approaches 90°, the gear wheels deform and approach the circles passing the depressions of the teeth. At ε =90°, the radius-vector ρ is converted to a constant value ρ_o . Owing to the derived mathematical dependence between the angles of rotation of the drive and driven wheels, based on the equality of the arcs of the profiles of the teeth, the slip between them is eliminated.

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