This paper reports a method, built in the form of a logic function, for describing the working spaces of manipulation robots analytically. A working space is defined as a work area or reachable area by a manipulation robot. An example of describing the working space of a manipulation robot with seven rotational degrees of mobility has been considered.

Technological processes in robotic industries can be associated with the positioning of the grip, at the required points, in the predefined coordinates, or with the execution of the movement of a working body along the predefined trajectories, which can also be determined using the required points in the predefined coordinates. A necessary condition for a manipulation robot to execute a specified process is that all the required positioning points should be within a working space.

To solve this task, a method is proposed that involves the analysis of the kinematic scheme of a manipulation robot in order to acquire a graphic image of the working space to identify boundary surfaces, as well as identify additional surfaces. The working space is limited by a set of boundary surfaces where additional surfaces are needed to highlight parts of the working space. Specifying each surface as a logic function, the working space is described piece by piece. Next, the resulting parts are combined with a logical expression, which is a disjunctive normal form of logic functions, which is an analytical description of the working space.

The correspondence of the obtained analytical description to the original graphic image of working space is verified by simulating the disjunctive normal form of logic functions using MATLAB (USA)

Keywords: manipulation robot, working space boundary, elementary surface, logic function

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METHOD FOR ANALYTICAL DESCRIPTION AND MODELING OF THE WORKING SPACE OF A MANIPULATION ROBOT

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1. Introduction

The reduction of manual labor in metallurgical, chemical, and other industries is provided by robotic technological complexes consisting of individual manipulation robots (MRs).

To perform a predefined robotic operation, as part of a robotic technological system, MR must perform the positioning of the grip at the required points with known coordinates or the movement of the working body along the predefined trajectory. Assigned trajectories can also be defined by a set of positioning points with known coordinates. Then, a necessary condition for MR to perform the predefined robotic operation is the condition that the working space (WS) of a manipulation robot (MR) should include all the required positioning points.

Once an analytical model of MR WS is built, the implementation of this procedure is greatly facilitated while the presence of errors is reduced to zero. Therefore, devising an analytical description of MR WS in the form of a logic function is important and relevant.

2. Literature review and problem statement

One of the important characteristics that determine the functionality of MR is its working space, which is a set of points in a three-dimensional space bounded by a complex geometric shape. At each point within this space, it is possible to position a gripper or a working body of MR. Consider the following problem: suppose the kinematic structure of MR is given, it is required to determine the geometric structure of WS (a closed three-dimensional set of points) and build its mathematical description.

One of the directions for developing a graphic image of MR WS is the Monte Carlo approach. Using the Monte Carlo method, applying a random sample of the results of solving a direct kinematic problem, we obtain a set of coordinates for the three-dimensional space and their repeated calculation with the clarification of probabilistic characteristics; the boundary and dimensions of MR WS are determined. Using this method, the volume and configuration of MR WS are defined, as well as the optimal geometric dimensions to ensure the maximum achievability of MR

WS points [1]. In particular, it is proposed to highlight the boundary surfaces separating the subspaces that form the graphic image of MR WS. In addition, boundary surfaces can be approximated by analytical expressions. In the cited paper, MR WS is not built in the form of a geometric object; the problem of determining the optimal geometric dimensions that provide the maximum indicator of achievability is solved.

Methods and algorithms for automatically obtaining an algebraic description of MR WS boundary having *n*- rotational degrees of mobility have been devised in [2]. Algebraic equations are used to describe boundary surfaces. However, the representation of MR WS in the form of a three-dimensional geometric object is absent in the cited work. The algebraic description of MR boundaries having degrees of mobility of translational motion was built, using the Newton-Raphson method, in [3]. However, the task of analytical description of MR WS as a closed geometric spatial shape is also not considered in the cited work.

To describe the MR WS boundary, a new algorithm based on the Monte Carlo method has been proposed. For determining more precise boundary points of a two-dimensional (2D) MR WS, it is possible to apply the beta distribution to generate random values of generalized coordinates by degrees of mobility, as reported in [4]. The cited work also proposes optimal parameters for computer modeling, as well as the dimensions of 2D and 3D MR WS. This direction is interesting from the point of view of visualization of MR WS but it is not suitable for obtaining an analytical description. To improve the accuracy of the description of WS boundaries, the generation of new random values is carried out in the vicinity of the previously generated values [5]. Thus, the MR WS boundary is refined. The cited work also addresses the issues of visualization of MR WS. Obtaining an analytical description, in that case, is also impossible.

In work [6], an approach is proposed in which MR WS is considered as an arbitrary three-dimensional geometric object. In that case, the MR WS boundaries are set by known geometric shapes, a ball, a cone, a cylinder, a torus, a plane. The problem of the analytical description of MR WS, which has three rotational degrees of mobility, is considered. In the cited paper, the simplest case of MR is considered; in that case, WS is described by a fairly simple logic function; no modeling of analytical expressions was carried out.

To analyze WS, work [7] suggests using the Monte Carlo method and the matrix of transformation of the positions of the kinematic chain of MR. The Monte Carlo method is sampled in compliance with even distribution to determine each generalized coordinate of the degree of mobility of the robot. These values of the generalized coordinates are then used in the transformation matrix to obtain MR WS. In the cited paper, the problem of analyzing the type of WS depending on the kinematic structure of MR is considered. The resulting description in the form of values of generalized coordinates corresponding to MR configurations is obtained in the form of transformation matrices, the use of which for the analytical description of MR WS is not tied to the geometric dimensions and configuration of WS.

In paper [8], a method is proposed in which an inaccurate initial WS is first generated using the classical Monte Carlo method. Next, using the Gauss distribution, the original WS is transformed until the boundaries of WS are reached. The focus is on the step-by-step obtaining of the boundary surfaces of MR WS limiting the WS necessary to determine the limiting capabilities of MR. However, an analytical description using this approach is not possible.

Paper [9] reports a method for solving in real time the problem of creating a three-dimensional image of MR WS. This approach is based on the application of the Monte Carlo method, which considers randomly selected values obtained from solving a direct kinematics problem. That results in an asymptotic cover of MR WS. The cited paper deals more with the visualization of a three-dimensional image of MR WS, as well as the asymptotic cover of WS. In the case of MR having a large number of degrees of mobility, a complex configuration of WS, that cover becomes difficult to analyze.

The use of the Monte Carlo method and the voxel algorithm for the analysis of MR WS, which has 9 degrees of mobility, is proposed in [10]. First, employing the Monte Carlo method, the original image of WS is determined. Next, the resulting WS is expanded based on a normal distribution with a dynamically adjustable standard deviation. To obtain the volume of WS, a voxel algorithm is used. The application of the voxel algorithm makes it possible to build a three-dimensional visualization of MR WS, rather than an analytical description of MR WS.

In work [11], to analyze MR WS, the authors use modeling based on the Monte Carlo method followed by programming in MATLAB. In the cited paper, the analysis of the type and modeling of MR WS are carried out; the application of this approach for the analytical description of WS is not considered.

The visualization of WS of multi-stage MR, using the multi-level concept of 3D printing, is considered in [12]. The boundary formula of each layer is derived using the results of solving a direct kinematics problem. A method for highlighting the WS boundary by obtaining boundary points on each layer is proposed. The cited paper deals more with the problem of highlighting the exact boundaries of WS, which is not very suitable for the analytical description of MR WS.

A significant drawback of the proposed approaches is the difficulty of obtaining an analytical description depending on the complexity of the kinematic structure. In the case of multi-stage MRs, expressions for the analytical description become cumbersome, their applicability to solving the problem of covering the required positioning points by WS becomes impossible. Therefore, an approach in which a graphic image of WS is formed on the basis of the analysis of the kinematic chain would be promising. In this case, an increase in the number of degrees of MR mobility increases the boundary and additional surfaces that limit the working space. Accordingly, the number of Boolean variables that specify the boundary and additional surfaces would increase. That could lead to an increase in the number of parts and logic functions that form the analytical description of WS. The construction of a disjunctive normal form of logic functions describing WS, as well as its modeling in MATLAB (USA), are similar procedures that differ from each other in the number of logic variables and the increase in the number of logical operators of the same type.

In contrast to [6], it is necessary to consider a multi-stage MR to prove the applicability of a given approach for cases involving more complex shapes of WS. It is also necessary to consider the issue of modeling analytical expressions in the form of logic functions that describe MR WS. To confirm the adequacy of the obtained analytical description to the original graphic image, the analytical description of the MR WS is simulated using MATLAB (USA).

3. The aim and objectives of the study

The aim of this work is to devise a method for the analytical description of the working space of a manipulation robot as a set of all the required positioning points of the working body of the manipulation robot. This could provide for a formalized representation in the form of logic functions, as well as a simulation of the working space of a manipulation robot.

To accomplish the aim, the following tasks have been set:

– to build a graphic image of WS, based on the analysis of the kinematic chain of MR, taking into consideration the geometric dimensions of the links and the limits of change in generalized coordinates by degrees of mobility;

– to build an analytical description of WS in the form of a disjunctive normal form of logic functions obtained on the basis of splitting WS into parts, depending on which logic functions are formed that describe the parts of WS, the boundary and additional surfaces limiting the MR WS;

– to simulate in MATLAB (USA) the obtained logic function describing WS in order to determine the correspondence between the WS analytical description and its graphic image.

4. The study materials and methods

In a general form, the sequence of our study, as well as the method devised, can be represented in the form of seven steps.

Step 1. Perform a graphic representation of the kinematic chain of MR on a scale, taking into consideration the size of the links, the location of the axes of kinematic pairs, indicating the limits of movements of each kinematic pair.

Step 2. Define boundary surfaces depending on the positions of MR links that correspond to the boundary values of the generalized coordinates.

Step 3. Based on the graphic image of the kinematic chain, taking into consideration the boundary values of the generalized coordinates, build a graphic image of MR WS.

Step 4. Split the MR WS graphic image into separate parts, the boundaries of which are set by elementary rotation shapes. This also includes additional surfaces designed for the logical formation of each individual part of MR WS.

Step 5. All elementary rotational shapes that form the MR WS boundaries, as well as additional auxiliary surfaces, are given by elementary inequalities of the form $D_k(x,y,z)\geq 0$, on the basis of which Boolean variables in the form $k=\overline{1,rp},\ p=\overline{1,v}$: are built

$$L_k^p=\begin{cases}1, \text{ if } D_k(x,y,z)\geq 0,\\ 0, \text{ otherwise,}\end{cases} \tag{1}$$

where rp – approximating the p-th part of MR WS;

v – the number of individual parts of MR WS.

Step 6. For each individual part of MR WS, a logic function is constructed that forms based on the geometric image of the part of WS, as well as the boundary and auxiliary surfaces specified by Boolean variables (1):

$$L_1^pBL_2^pB\cdots BL_{rp}^p=1, \tag{2}$$

where B – a sign-logical operation (conjunction, disjunction, or negation).

Step 7. Based on the constructed logic functions (2) describing each of the parts of MR WS, we build a disjunctive normal form of logic functions describing MR WS:

$$\left(L_1^1BL_2^1B\cdots BL_{r1}^1\right)\vee\left(L_1^2BL_2^2B\cdots BL_{r2}^2\right)\vee$$
$$\vee\cdots\vee\left(L_1^vBL_2^vB\cdots BL_{rp}^v\right)\geq 0. \tag{3}$$

Thus, it is possible to describe rather complex types of MR WS as geometric objects in the form of a logical expression. The difficulty of describing MR WS is in the proper division into parts, with the identification of boundary and additional surfaces. In the case of incorrect partitioning or erroneous assignment of boundary and additional surfaces, it is possible to obtain an incorrect description of MR WS, which can be determined at the stage of modeling the resulting logical expression.

If the correct result is obtained, the analytical description in the form of a logic function can be used to determine the affiliation of the required positioning points of the robotic WS operation, for various mutual locations of MR relative to the points under consideration. Solving the problem of the feasibility of the required manipulations of MR data. The same problem occurs in the construction of layout schemes of robotic complexes and the synthesis of software trajectories of MR.

Based on the obtained graphic image, the boundary and additional surfaces are determined, which makes it possible to determine the parts of WS bounded by the boundary surfaces, the selection of which involves the introduction of additional surfaces. That makes it possible to construct a logic function describing the selected part of WS. Having thus described all the parts of WS, based on their description, a disjunctive normal form of logic functions describing the MR WS is built. To check the adequacy of the obtained logic function to the graphic image of MR WS, simulation is performed in MATLAB (USA). In the case of discrepancies between the modeling results and the graphic image of WS, possible errors may be the incorrect division of WS into parts, incorrect assignment of boundary and additional surfaces, errors in the construction of logic functions. Modeling is performed until the results coincide with the graphical image of MR WS. Application of the obtained analytical description to determine the conditions under which MR WS covers the required positioning points, in solving the problems of compliance between MR and WS for performing the predefined robotic operation, developing layout schemes of robotic systems, solving problems of synthesizing software trajectories.

We have considered MR having a kinematic structure shown in Fig. 1. Fig. 1 demonstrates that MR is an open kinematic chain consisting of 6 links interconnected by seven rotational kinematic pairs (degrees of mobility). Each degree of mobility is a fifth-class mechanism, with its own drive.

Let the axes of rotation of the kinematic pairs be mutually parallel or perpendicular. The position of each degree of mobility is given by the values of the generalized coordinates $q_i,\ i=\overline{1,7}$, ($7R$ manipulator). A working tool or MR grip is attached to the last link at point F. Let the limits of the change in the generalized coordinates be arranged symmetrically with respect to the previous link, and, for q_1, q_6, are 180°; for q_2, q_3, q_4, q_5, q_7, are equal to 90°. Link lengths are: $|0A|$=1.0, $|AB|$=0.8, $|BC|$=0.6, $|CD|$=0.6, |DE|=0.6, |EF|=0.4.

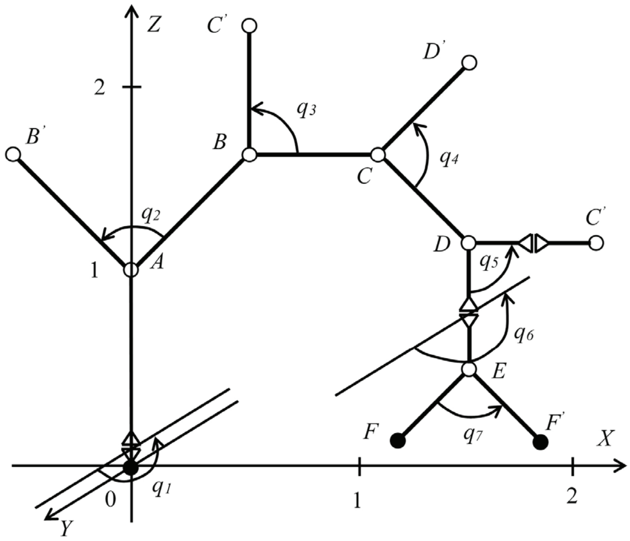

Fig. 1. Kinematic structure of manipulation robot

Based on the obtained image of the kinematic structure of MR and its further analysis, it is possible to build a graphic image of MR WS. If other permissible limits for changing generalized coordinates are set, the graphic image of MR WS would change. Consequently, the logic of splitting the WS into separate parts would change further, additional surfaces will be added. That could lead to a change in the logical expressions describing each part of the WS, therefore, the disjunctive normal form of the logic functions describing the given WS would change.

5. Results of studying the analytical description of the working spaces of manipulation robots in the form of logic functions

5. 1. Constructing a graphic image of the working space of a manipulation robot

To build WS, it is necessary to determine the boundary surfaces. Boundary surfaces depend on the position of MR links that correspond to the boundary values of the generalized coordinates $q_i,\ i=\overline{1,7}$ (Fig. 2).

Let us determine the required coordinates of the points corresponding to the articulations of MR, with an accuracy of up to the third decimal place. Point A with coordinates $x_A=0$, $y_A=0$, $z_A=|0A|=1.0$ corresponds to the first and second kinematic pairs given by the generalized coordinates q_1, q_2. The next point B, with the coordinates $x_B\approx0.565$, $y_B\approx0$, $z_B\approx1.565$, corresponds to the third kinematic pair given by the generalized coordinate q_3. Point C, with coordinates $x_C=1.165$, $y_C=0$, $z_C=1.565$, corresponds to the fourth kinematic pair given by the generalized coordinate q_4. The next point D, with the coordinates $x_D=1.589$, $y_D=0$, $z_D=1.140$, corresponds to the fifth kinematic pair given by the generalized coordinate q_5. Point E, with coordinates $x_E=1.589$, $y_E=0$, $z_E=0.540$, corresponds to the sixth and seventh kinematic pairs given by the generalized coordinates q_6, q_7.

The position of the grip or working body of MR corresponds to point F, with coordinates $x_F=1.306$, $y_F=0.257$, $z_F=0.257$. The next point G lies along the $0Z$ axis, so its coordinates would be $x_G=0$, $y_G=y_B=0$, $z_G\approx1.8$.

Let us define the coordinates of the point N, $x_N=0.742$, $y_N=0$, $z_N=2.306$. Point M is symmetric to point N relative to the axis $0Z$, so its coordinates would be $x_M=-x_N=-0.742$, $y_M=y_N=0$, $z_M=z_N=2.306$.

Similarly, based on Fig. 2, we define the required distances, which we denote: $R_1=3.0$, $R_2\approx1.502$, $R_3=2.2$, $R_4=1.6$, $R_5=1.0$, $R_6=0.4$, $R_7=R_2=1.502$.

From the analysis of the basic provisions of the links of the kinematic chain of MR, it is possible to construct the graphic image of MR WS. The MR WS takes the form shown in Fig. 3.

Based on the analysis of the graphic image of the MR WS, taking into consideration the boundary surfaces, the WS is divided into separate parts, for highlighting which additional surfaces are introduced. Let is determine those Boolean variables that define the boundary and additional surfaces, with the help of which we shall build logic functions that describe each individual part of WS. Based on the obtained logic functions that describe the individual parts of WS, a disjunctive normal form of logic functions is constructed to describe MR WS.

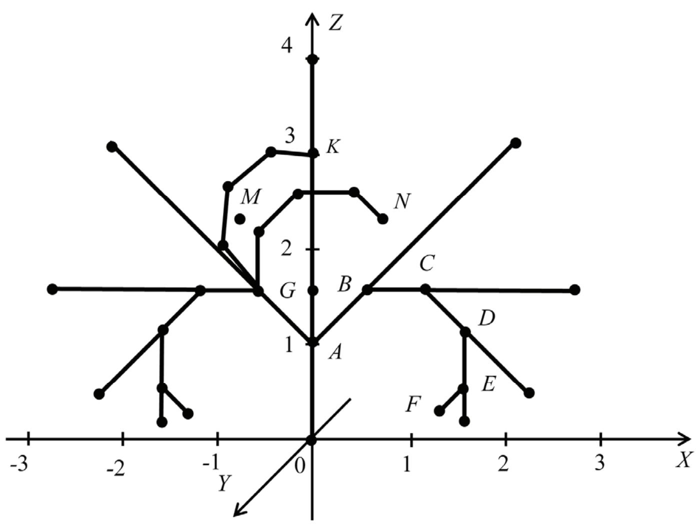

Fig. 2. Basic positions of the kinematic chain of a manipulation robot

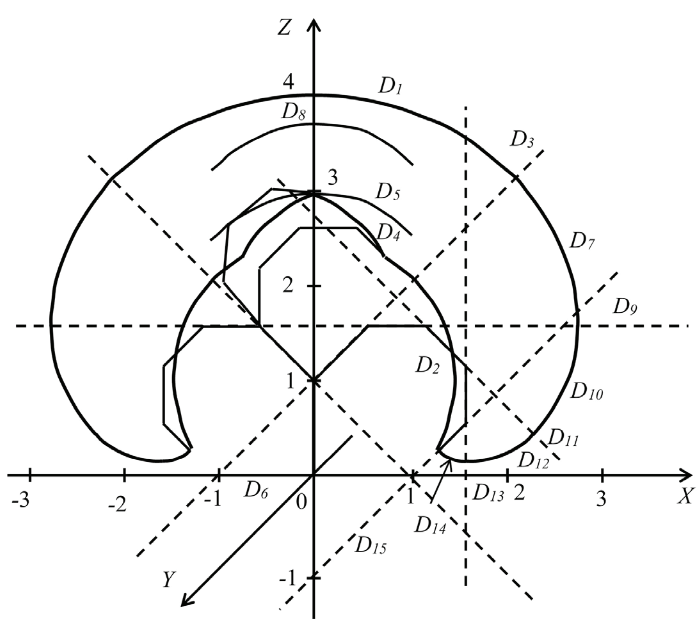

Fig. 3. The working space of a manipulation robot

5. 2. Analytical description of the working space of a manipulation robot using logic functions

We shall build an analytical MR WS having 7 rotational degrees of mobility, in the form of logic functions, based on the graphic image shown in Fig. 3, with the further goal of checking the obtained description by simulating in MATLAB (USA). To this end, it is necessary to determine the boundary surfaces that border each part of WS. Each boundary surface $D_k(x, y, z)\geq 0$, $k=\overline{1,15}$ (Fig. 3) is defined by a Boolean variable in the form (1).

The first part of MR WS is a subspace bounded by the following elementary surfaces (Fig. 3). It is the conjunction of the inner subspace of the ball D_1, the outer subspace of the ball D_2, the inner subspace of the direct circular cone D_3 (Fig. 1), the outer subspace of the torus D_4. Since the outer subspace of torus D_4 differs from the case of an ordinary torus by the presence of an inner hollow region. In this case, the torus takes the form shown in Fig. 4. If one does not take this circumstance into consideration, it would lead to the loss of a hollow area in the form of a segment and there would be a loss of part of WS.

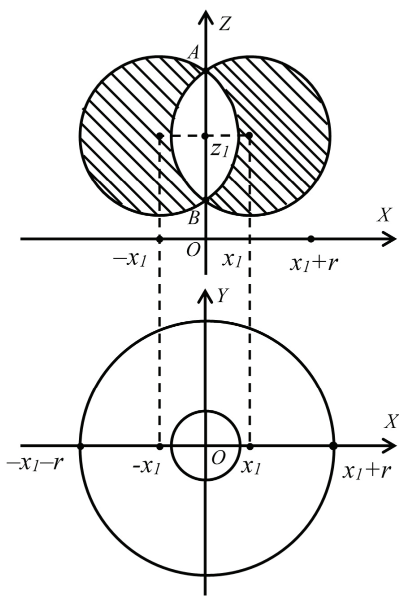

Fig. 4. Torus projections on coordinate planes at $z=z_1$

Fig. 4 shows that the torus has an inner hollow surface, in the shape of a segment, which is the outer subspace of the torus D_4 and the boundary surface at the same time. Given that the outer surface of the torus is also the outer subspace of the torus D_4 (Fig. 2), to construct this boundary surface, the external subspace of this torus and the ball is conjunct with the center at the point G (0,0,1.8), with a radius equal to $R=\sqrt{d}\approx 1.46$. Here, d is defined from expression (12), which we denote as the inner subspace of the ball D_5. Next, one must use a Boolean expression $L_{s0}=\overline{L_s}$ to obtain a subspace that is external to the inner segment of the torus. It is also necessary to add an additional logical condition, conjunction with a subspace above the plane z=0, that is, the surface D_6. For an analytical description of the first part of MR WS, we introduce the following Boolean variables into consideration.

L_1 is a Boolean variable defining the inner subspace of the ball D_1, bounded by a sphere centered at point A(0,0,1.0), having radius R_1=3

$$L_1=\begin{cases}1, \text{ if } 9-x^2-y^2-(z-1)^2\geq 0,\\ 0, \text{ otherwise.}\end{cases} \quad (4)$$

L_2 is a Boolean variable that specifies the outer subspace of the ball D_2 bounded by a sphere centered at point A(0,0,1.0) having a radius R_2=1.502

$$L_2=\begin{cases}1, \text{ if } x^2-y^2-(z-1)^2-2.256\geq 0,\\ 0, \text{ otherwise.}\end{cases} \quad (5)$$

L_3 is a Boolean variable that specifies the internal subspace D_3 of a straight circular cone obtained by rotation of a straight line z=x+1 around the 0Z axis,

$$L_3=\begin{cases}1, \text{ if } (z-1)^2-x^2-y^2\geq 0,\\ 0, \text{ otherwise.}\end{cases} \quad (6)$$

L_4 is a Boolean variable that specifies the outer subspace of the torus D_4 obtained by rotating a circle bounded by a circle with a center at a point B with coordinates (0.565,0,1.565), having a radius R_2=1.502, around the 0Z axis,

$$L_4=\begin{cases}1, \text{ if } \left(x^2+y^2+(z-1.565)^2-2.13\right)^2-\\ -1.2769\cdot\left(x^2+y^2\right)\geq 0,\\ 0, \text{ otherwise.}\end{cases} \quad (7)$$

L_5 is a Boolean variable that specifies the inner subspace of the ball D_5 bounded by a sphere centered at point G(0,0,1.8) having a radius $R=\sqrt{d}\approx 1.46$,

$$L_5=\begin{cases}1, \text{ if } 2.13-x^2-y^2-(z-1.565)^2\geq 0,\\ 0, \text{ otherwise.}\end{cases} \quad (8)$$

L_6 is a Boolean variable that specifies the subspace D_6 above the plane z=0

$$L_6=\begin{cases}1, \text{ if } z\geq 0,\\ 0, \text{ otherwise.}\end{cases} \quad (9)$$

Using the obtained Boolean variables (4) to (9), it is possible to describe the first part of MR WS in the form of the following logical expression:

$$L_1\wedge L_2\wedge L_3\wedge\overline{(L_4\wedge L_5)}\wedge L_6=1. \quad (10)$$

The second part of MR WS is a subspace bounded by the following elementary surfaces (Fig. 3). This part is the conjunction of the outer subspace of the ball D_2, the outer subspace of the direct circular cone D_3, which can be obtained by inversion of the Boolean variable $\overline{L_3}$. In this case, the boundary of the straight circular cone would belong to the first part of MR WS. Also added is the conjunction of the inner subspace of torus D_7, which has an inner hollow region in the form of a segment. In this case, the torus in question is similar to the torus D_4 (Fig. 4). In this case, the inner segment of the torus falls out of consideration since it is an external part of it. To reconstruct the inner segment, we add the inner subspace of the ball D_8 bounded by a centered sphere at point G(0,0,1.565) having a radius $R=\sqrt{d}\approx 2.126$. And a subspace located above the D_9 plane, parallel to the OXY plane that passes through a point with coordinates z=1.565. For an analytical description of the second part of MR WS, we introduce the following Boolean variables into consideration.

L_7 is a Boolean variable that specifies the inner subspace of the torus D_7 obtained by the rotation of a circle bounded by a circle centered at a point with coordinates B(0.565,0,1.565), having a radius R_3=2.2, around the 0Z axis, Fig. 2.

$$L_7=\begin{cases}1,\text{ if }\left(x^2+y^2+(z-1.565)^2-4.521\right)^2-\\-1.276\cdot\left(x^2+y^2\right)\le 0,\\0,\text{ otherwise.}\end{cases}\quad(11)$$

$L8$ is a Boolean variable that specifies the inner subspace of the ball D_8 bounded by a sphere centered at point G(0,0,1.565) having a radius $R=\sqrt{d}\approx 2.126$,

$$L_8=\begin{cases}1,\text{ if }4.521-x^2-y^2-(z-1.565)^2\ge 0,\\0,\text{ otherwise.}\end{cases}\quad(12)$$

L_9 is a Boolean variable that specifies the subspace D_9 above the plane, z=1.565

$$L_9=\begin{cases}1,\text{ if }z-1.565\ge 0,\\0,\text{ otherwise.}\end{cases}\quad(13)$$

Using the obtained Boolean variables (5), (6), (11) to (13), it is possible to describe the second part of MR WS in the form of the following logical expression:

$$L_2\wedge\bar{L}_3\wedge\left(L_7\vee L_8\right)\wedge L_9=1.\quad(14)$$

The third part of MR WS is a subspace bounded by the following elementary surfaces (Fig. 3). It is the conjunction of the outer subspace of the ball D_2, a subspace below the D_9 plane, given by inverting the Boolean variable $\bar{L}_9$. A conjunction of the inner subspace of the torus D_{10} is added, which also has an inner hollow region, but, in this case, the hollow segment is cut off by the outer subspace of the ball D_2. Therefore, this hollow segment can be ignored. It is also necessary to add conjunction of the outer subspace of the straight circular cone D_{11}. Then, for an analytical description of the third part of MR WS, we introduce the following Boolean variables into consideration.

L_{10} is a Boolean variable that specifies the inner subspace of the torus D_{10} obtained by rotating a circle bounded by a circle with a center at a point with coordinates C(1.165,0,1.565), having a radius R=1.6, around the 0Z axis

$$L_{10}=\begin{cases}1,\text{ if }\left(x^2+y^2+(z-1.565)^2-1.203\right)^2-\\-5.428\cdot\left(x^2+y^2\right)\le 0,\\0,\text{ otherwise.}\end{cases}\quad(15)$$

L_{11} is a Boolean variable that specifies the outer subspace of the straight circular cone D_{11} obtained by rotation of the straight line z=x+2.73, around the 0Z axis

$$L_{11}=\begin{cases}1,\text{ if }(z-2.73)^2-x^2-y^2\le 0,\\0,\text{ otherwise.}\end{cases}\quad(16)$$

With the help of the obtained Boolean variables (5), (13), (15), (16), it is possible to describe the third part of MR WS in the form of the following logical expression:

$$L_2\wedge\bar{L}_9\wedge L_{10}\wedge L_{11}=1.\quad(17)$$

The fourth part of MR WS is a subspace bounded by the following elementary surfaces (Fig. 3). This part is a conjunction of the outer subspace of the straight circular cone D_{11}, the inner subspace of the torus D_{12}, and the outer subspace of the straight circular cylinder D_{13}. For an analytical description of the fourth part of MR WS, we introduce the following Boolean variables into consideration.

L_{12} is a Boolean variable that specifies the inner subspace of the torus D_{12} obtained by rotating a circle bounded by a centered circle at a point with coordinates D(1.589,0,1.140), having a radius R_5=1.0, around the 0Z axis

$$L_{12}=\begin{cases}1,\text{ if }\left(x^2+y^2+(z-1.140)^2+1.525\right)^2-\\-10.1\cdot\left(x^2+y^2\right)\le 0,\\0,\text{ otherwise.}\end{cases}\quad(18)$$

L_{13} is a Boolean variable that specifies the external subspace of the straight circular cylinder D_{13} obtained by the rotation of the straight line x=1.589 around the 0Z axis

$$L_{13}=\begin{cases}1,\text{ if }x^2+y^2-2{,}525\ge 0,\\0,\text{ otherwise.}\end{cases}\quad(19)$$

With the help of the obtained Boolean variables (16), (18), (19), it is possible to describe the fourth part of MR WS in the form of the following logical expression:

$$\bar{L}_{11}\wedge L_{12}\wedge L_{13}=1.\quad(20)$$

The fifth part of MR WS is a subspace bounded by the following elementary surfaces (Fig. 3). It is a conjunction of the outer subspace of the straight circular cone D_{13}, given by the inversion $\bar{L}_{13}$, the inner space of the torus D_{14}, and the outer subspace of the straight circular cone D_{15}. For an analytical description of the fifth part of MR WS, we introduce the following Boolean variables.

L_{14} is a Boolean variable that specifies the inner subspace of the torus D_{14} obtained by rotating a circle bounded by a circle centered at a point with coordinates E(1.589,0,0.540), having a radius R_6=0.4, around the 0Z axis

$$L_{14}=\begin{cases}1,\text{ if }\left(x^2+y^2+(z-0.54)^2+2.365\right)^2-\\-10.1\cdot\left(x^2+y^2\right)\le 0,\\0,\text{ otherwise.}\end{cases}\quad(21)$$

L_{15} is a Boolean variable that specifies the external subspace of the straight circular cone D_{15} obtained by the rotation of the straight line z=x–1 around the 0Z axis

$$L_{15}=\begin{cases}1,\text{ if }(z+1)^2-x^2-y^2\le 0,\\0,\text{ otherwise.}\end{cases}\quad(22)$$

Using the obtained Boolean variables (19), (21), (22), it is possible to describe the fifth part of MR WS in the form of the following logical expression:

$$\bar{L}_{13}\wedge L_{14}\wedge L_{15}=1.\quad(23)$$

It is also necessary to add the sixth part of MR WS, which is formed from the above subspaces. It is a conjunction of the outer subspace of the ball D_2, the subspace D_6, located above the plane $z=0$. As well as the subspace below the D_9 plane given by the inversion of the Boolean variable $\bar{L}_9$, the outer subspace of the direct circular cone D_{11}, given by the inversion of $\bar{L}_{11}$. By adding the outer subspace of the direct circular cone D_{13} given by the inversion of $\bar{L}_{13}$ and the inner subspace of the direct circular cone D_{15} given by the inversion of $\bar{L}_{15}$.

Using the obtained Boolean variables (5), (9), (13), (16), (19), (22), it is possible to describe the sixth part of MR WS in the form of the following logical expression:

$$L_2 \wedge L_6 \wedge \bar{L}_9 \wedge \bar{L}_{11} \wedge \bar{L}_{13} \wedge \bar{L}_{15} = 1. \qquad (24)$$

Using the expressions (10), (14), (17), (20), (23), (24), MR WS is described as the following disjunctive normal form of the logic function.

$$\begin{aligned}&\left(L_1 \wedge L_2 \wedge L_3\overline{\left(L_4 \wedge L_5\right)} \wedge L_6\right) \vee \\ &\vee\left(L_2 \wedge \bar{L}_3 \wedge \left(L_7 \vee L_8\right) \wedge L_9\right) \vee \left(L_2 \wedge \bar{L}_9 \wedge L_{10} \wedge L_{11}\right) \vee \\ &\vee\left(\bar{L}_{11} \wedge L_{12} \wedge L_{13}\right) \vee \left(\bar{L}_{13} \wedge L_{14} \wedge L_{15}\right) \vee \\ &\vee\left(L_2 \wedge L_6 \wedge \bar{L}_9 \wedge \bar{L}_{11} \wedge \bar{L}_{13} \wedge \bar{L}_{15}\right) = 1. \qquad (25)\end{aligned}$$

The resulting expression (25) is an analytical description of MR WS having 7 rotational degrees of mobility. However, it should be noted that one rotational degree of mobility is, to a greater extent, the degree of mobility that provides orientation of the last link of the kinematic chain. It does not affect the formation of MR WS.

The adequacy of the constructed analytical description (25) to the initial graphic MR WS (Fig. 3) is to be checked by simulating the logic function (25) in MATLAB.

5. 3. Simulation of a logic function analytically describing the working space of a manipulation robot in MATLAB

A program has been developed using MATLAB for modeling a given closed spatial shape that defines MR WS described by the logic function (25), in the form of a projection onto the coordinate plane *OXZ*. Given that the graphic image of the working space is limited in the plane *OXZ* ($y=0$) by values $-3 \le x \le 3, 0 \le z \le 4$.

The algorithm for modeling the logic function (25) for compliance with the graphic image of MR WS takes the following form.

Start.

Step 1. $y=0$.

Step 2. $x=-3$.

Step 3. $z=0$.

Step 4. Based on expression (4), determine the value of the Boolean function L_1.

Step 5. Based on expression (5), determine the value of the Boolean function L_2.

Step 6. Based on expression (6), determine the value of the Boolean function L_3.

Step 7. Based on expression (7), determine the value of the Boolean function L_4.

Step 8. Based on expression (8), determine the value of the Boolean function L_5.

Step 9. Based on expression (9), determine the value of the Boolean function L_6.

Step 10. $LS_1 = L_1 \wedge L_2 \wedge L_3\overline{\left(L_4 \wedge L_5\right)} \wedge L_6$.

Step 11. Based on expression (11), determine the value of the Boolean function L_7.

Step 12. Based on expression (12), determine the value of the Boolean function L_8.

Step 13. Based on expression (13), determine the value of the Boolean function L_9.

Step 14. $LS_2 = L_2 \wedge \bar{L}_3 \wedge \left(L_7 \vee L_8\right) \wedge L_9$.

Step 15. Based on expression (15), determine the value of the Boolean function L_{10}.

Step 16. Based on expression (16), determine the value of the Boolean function L_{11}.

Step 17. $LS_3 = L_2 \wedge \bar{L}_9 \wedge L_{10} \wedge L_{11}$.

Step 18. Based on expression (18), determine the value of the Boolean function L_{12}.

Step 19. Based on expression (19), determine the value of the Boolean function L_{13}.

Step 20. $LS_4 = \bar{L}_{11} \wedge L_{12} \wedge L_{13}$.

Step 21. Based on expression (21), determine the value of the Boolean function L_{14}.

Step 22. Based on expression (22), determine the value of the Boolean function L_{15}.

Step 23. $LS_5 = \bar{L}_{13} \wedge L_{14} \wedge L_{15}$.

Step 24. $LS_6 = L_2 \wedge L_6 \wedge \bar{L}_9 \wedge \bar{L}_{11} \wedge \bar{L}_{13} \wedge \bar{L}_{15}$.

Step 25. If the condition $LS_1 \vee LS_2 \vee LS_3 \vee LS_4 \vee LS_5 \vee \vee LS_6 = 1$, is met, then the point with the coordinates *(x,y,z)* belongs to MR WS, mark it with a dot, otherwise, the point with the coordinates (x, y, z) does not belong to MR WS.

Step 26. $x=x+0.05$.

Step 27. If $x<3$, proceed to Step 3, otherwise, proceed to Step 28.

Step 28. $z=z+0.05$.

Step 29. If $z<4$, proceed to Step 4, otherwise, proceed to Step 30.

Step 30. Map the plot of MR WS projection onto the coordinate plane *OXZ*.

End.

As a result of modeling, we obtain a view of WS projection onto the *OXZ* coordinate plane (Fig. 5).

The result of MR WS simulation, in the form of a projection onto the *OXY* coordinate plane at $z=z_B \approx 1.565$ is shown in Fig. 6.

The result of MR WS simulation, in the form of a projection onto the *OXY* coordinate plane at $z=z_B \approx 1.565$ is shown in Fig. 7.

The result of MR WS simulation at $z=z_F \approx 0.257$ is shown in Fig. 8.

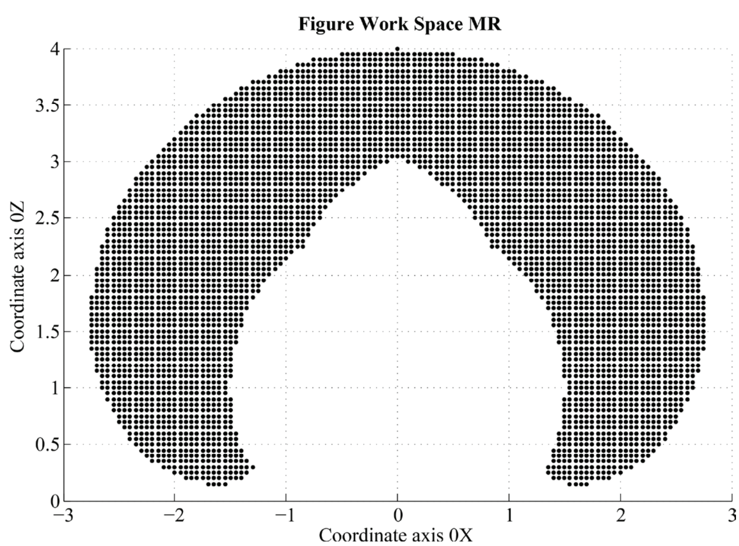

Fig. 5. Mapping the working space of a manipulation robot onto the *OXZ* coordinate plane

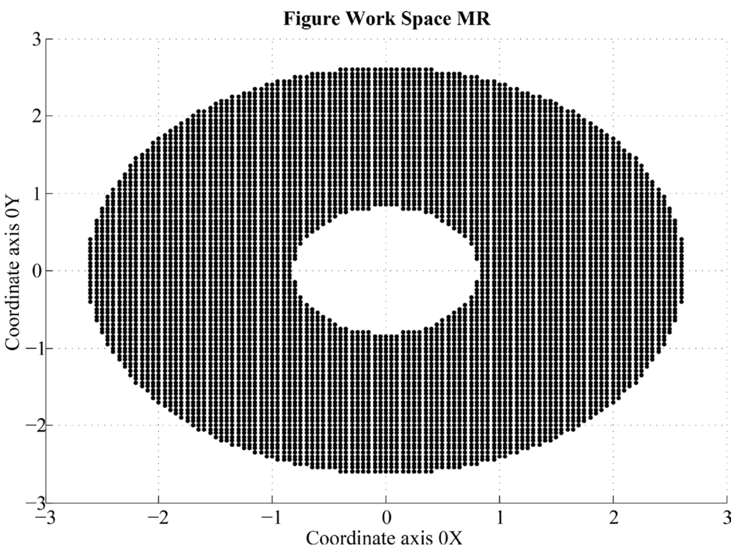

Fig. 6. Mapping the working space of a manipulation robot onto the *OXY* coordinate plane, at $z = z_N \approx 2.306$

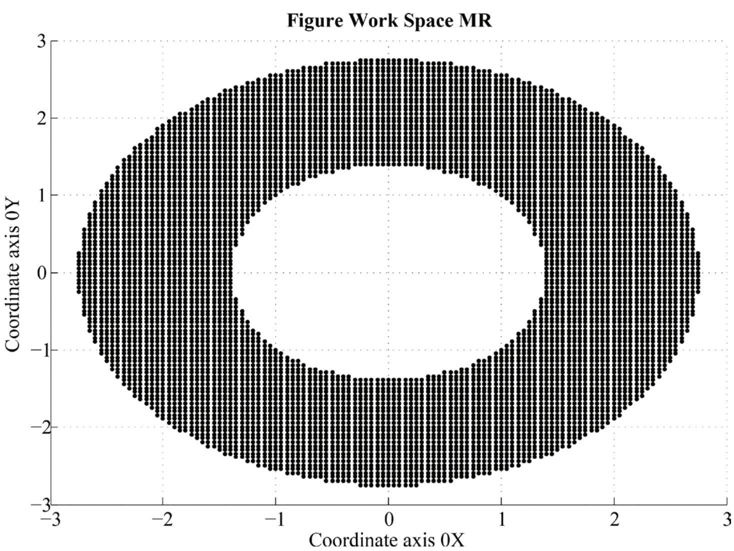

Fig. 7. Mapping the working space of a manipulation robot onto the *OXY* coordinate plane, at $z = z_B \approx 1.565$

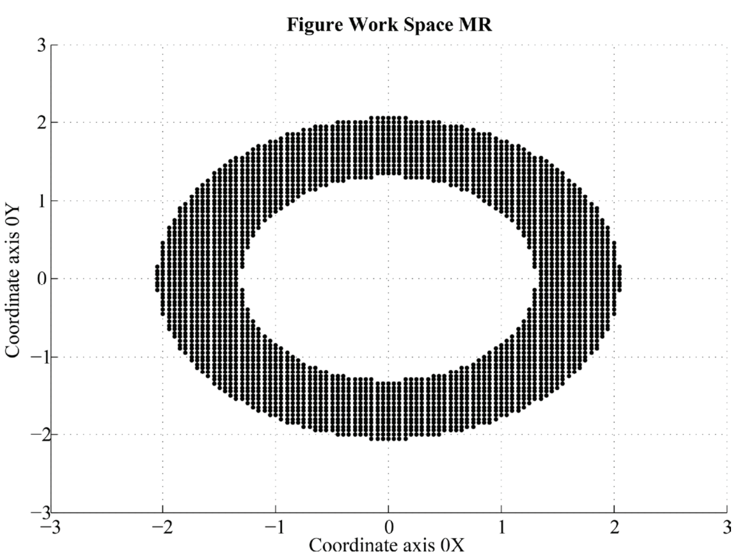

Fig. 8. Mapping the working space of a manipulation robot onto the *OXY* coordinate plane, at $z = z_F \approx 0.257$

The results of the simulation in Fig. 5 show that MR WS has the same shape as in Fig. 3, the projection onto the *OXZ* plane. Projections on the *OXY* plane (Fig. 6–8) are limited by two circles of the shape that corresponds to the original representation of the kinematic chain, the first degree of which is the degree of mobility of rotation around the *OZ* axis. Our result is a confirmation of the correspondence of the analytical description to the graphic image of MR WS.

Then it is possible to solve the analytical problem of covering the required positioning points by WS to perform the predefined robotic operation. The result is a necessary condition but not a sufficient one. Since WS is a set of all possible positioning points of the grip or working body of MR, the orientation of the grip and working body can be arbitrary. Enabling the required orientation requires an analytical description of the service areas. The partitioning of WS into service areas depending on the values of the angles of orientation of the grip is not considered in this work. This approach is applicable to determine the adequacy of the manipulation robot to perform the predefined robotic operation. When constructing the layout schemes of robotic systems, it is also necessary to meet the condition for covering all the required positioning points of WS by the MR grip. It is also applicable to determine the affiliation of the required positioning points of MR WS in the synthesis of software trajectories, as a necessary condition for solving this problem.

6. Discussion of results of studying the analytical description of working spaces of manipulation robots in the form of logic functions

The proposed method of analytical description is a sequence of steps from the analysis of the kinematic scheme to the production of a disjunctive normal form of logic functions describing WS MR. In contrast to the methods for building a graphical image of MR WS using the Monte Carlo method [1], the application of algebraic equations to describe the boundary surfaces [2], the proposed method produced an analytical description of WS of multistage MR in the form of a logic function. The method has been tested using an example of MR with 7 rotational degrees of mobility. In this case, WS takes a rather complex spatial rotational shape. However, in work [3], using the Newton-Raphson method, an algebraic description of the boundaries of MR WS with degrees of mobility of translational motion was built. That does not make it possible to obtain an analytical description of MR WS in the form of a spatial shape.

The use of logic functions makes it possible to analytically accurately describe this spatial shape. In contrast to the step-by-step procedures for refining the boundaries of MR WS, proposed in works [8, 9]. Which is a significant advantage of a given approach. It should be noted that logic functions are conveniently programmed and the analytical description of MR WS can be implemented in the form of software, for practical application when solving robotics tasks. For example, solving the issues related to choosing a PR model by type of PR, building layout schemes of robotic complexes, controlling mobile robots with a manipulator. Since the reported analytical description in the form of a logic function is implemented in the form of a software module, it can be used to implement the procedure for selecting the PR model according to WS.

The following circumstances should be borne in mind. MR is an open kinematic chain, a series connection of links by kinematic pairs of the fifth class, having parallel or perpendicular to each other axes of rotation. In a given case, it is meant that every mechanism of the fifth class has its own drive. In this case, the boundary surfaces of MR WS are given by elementary spatial surfaces that can be specified by logic functions.

If the kinematic structure consists of links connected by kinematic pairs of the fifth class, the axes of movement of which do not satisfy the condition of parallelism or orthogonality, then the boundary surfaces cannot be described by elementary rotational shapes, and the task becomes difficult to implement.

The kinematic structure of MR under consideration contains only rotational kinematic pairs, the consideration of telescopic kinematic pairs is of considerable theoretical interest. Also of interest is the problem of the analytical description of an MR WS boundary, which is a complex spatial surface, in the form of a logic function.

7. Conclusions

1. The analysis of kinematic schemes of MR, reported in this study, enables determining the geometric dimensions of links, the parameters of kinematic pairs, such as the limits of change in generalized coordinates according to the degrees of mobility of MR with the subsequent construction of a graphic image of MR WS.

2. The proposed method implements an analytical description of all the required positioning points of the MR working body and provides a formalized representation of the MR workspace in the form of logic functions, as well as its subsequent modeling. In this regard, it can be argued that the problem of formalized description of the area of coverage by the working space of the required positioning points of MR working bodies with subsequent practical application in the layout and other tasks of robotics has been analytically solved.

3. To confirm the adequacy of the derived disjunctive normal form of logic functions to the graphic image of MR WS, MATLAB-based simulation is effective. If the simulation results coincide with the original graphic image of WS, then a conclusion is drawn about the correspondence of the analytical description to the graphic image of MR WS. In the case of discrepancies between the simulation results and the graphic image of WS, the results of splitting the WS into separate parts, the boundary and additional surfaces, the built logic functions are corrected. Modeling is performed until the simulation results coincide with the graphical image of MR WS.

References

1. Rastegar, J., Fardanesh, B. (1990). Manipulation workspace analysis using the Monte Carlo Method. Mechanism and Machine Theory, 25 (2), 233–239. doi: https://doi.org/10.1016/0094-114x(90)90124-3
2. Ceccarelli, M., Liang, C. (2013). A formulation for automatic generation of workspace boundary of N-R manipulators. International Journal of Mechanisms and Robotic Systems, 1 (1), 2. doi: https://doi.org/10.1504/ijmrs.2013.051286
3. Madrid, E., Ceccarelli, M. (2014). Numerical solution for designing telescopic manipulators with prescribed workspace points. Robotics and Computer-Integrated Manufacturing, 30 (2), 201–205. doi: https://doi.org/10.1016/j.rcim.2013.09.013
4. Cao, Y., Lu, K., Li, X., Zang, Y. (2011). Accurate Numerical Methods for Computing 2D and 3D Robot Workspace. International Journal of Advanced Robotic Systems, 8 (6), 76. doi: https://doi.org/10.5772/45686
5. Liu, Z., Liu, H., Luo, Z., Zhang, X. (2013). Improvement on Monte Carlo method for robot workspace determination. Transactions of the Chinese Society for Agricultural Machinery, 44 (1), 230–235. doi: https://doi.org/10.6041/j.issn.1000-1298.2013.01.043
6. Burlibay, A. A., Beisembaev, A. A., Wójcik, W. (2014). Description of the manipulator robot's workspaces with three mobility degrees in the form of the logical expressions. PRZEGLĄD ELEKTROTECHNICZNY, 90 (8), 25–29. Available at: http://pe.org.pl/articles/2014/8/6.pdf
7. Li, J., Zhao, F., Li, X., Li, J. (2016). Analysis of robotic workspace based on Monte Carlo method and the posture matrix. 2016 IEEE International Conference on Control and Robotics Engineering (ICCRE). doi: https://doi.org/10.1109/iccre.2016.7476145
8. Peidró, A., Reinoso, Ó., Gil, A., Marín, J. M., Payá, L. (2017). An improved Monte Carlo method based on Gaussian growth to calculate the workspace of robots. Engineering Applications of Artificial Intelligence, 64, 197–207. doi: https://doi.org/10.1016/j.engappai.2017.06.009
9. Jauer, P., Kuhlemann, I., Ernst, F., Schweikard, A. (2016). GPU-based real-time 3D workspace generation of arbitrary serial manipulators. 2016 2nd International Conference on Control, Automation and Robotics (ICCAR). doi: https://doi.org/10.1109/iccar.2016.7486698
10. Zhao, Z., He, S., Zhao, Y., Xu, C., Wu, Q., Xu, Z. (2018). Workspace Analysis for a 9-DOF Hyper-redundant Manipulator Based on An Improved Monte Carlo Method and Voxel Algorithm. 2018 IEEE International Conference on Mechatronics and Automation (ICMA). doi: https://doi.org/10.1109/icma.2018.8484734
11. Zhu, J., Tian, F. (2018). Kinematics Analysis and Workspace Calculation of a 3-DOF Manipulator. IOP Conference Series: Earth and Environmental Science, 170, 042166. doi: https://doi.org/10.1088/1755-1315/170/4/042166
12. Fu, G., Tao, C., Gu, T., Lu, C., Gao, H., Deng, X. (2020). A Workspace Visualization Method for a Multijoint Industrial Robot Based on the 3D-Printing Layering Concept. Applied Sciences, 10 (15), 5241. doi: https://doi.org/10.3390/app10155241