Exceptional prospects for use in science, technology and industry are opened by highly dispersed powders (ultradispersed diamonds, nanoceramics, medicinal powders) and materials based on them. The properties of such materials depend on the particle size determined by sedimentation analysis.

An equation is proposed for processing sedimentation analysis data, which does not depend on the size distribution law of polydisperse system particles, and is used to describe the distribution functions of particles with radii for clay suspensions.

A program in the Microsoft Visual Basic for Applications (VBA) language has been created for calculating the fractional composition of suspensions according to the proposed equation; the correctness of its operation on a model system has been checked and confirmed.

Experimental research has confirmed that the use of the developed program and the "Search for a solution" add-on for the MS Excel environment makes it possible to determine the fractional composition of suspensions.

It has been proven that the proposed method can be used to analyze polydisperse systems.

It has been found that for the suspensions under consideration, as containing 25 fractions, it is possible to determine the integral curve of the distribution of the masses of the particles of the dispersed phase along the radii.

This allows to assert the possibility of using the proposed equation for processing sedimentation analysis data, which does not depend on the law of mass distribution of polydisperse system particles by size.

Thus, there is reason to assert about the possibility of a reasonable determination of the fractional composition of any polydisperse systems.

It is possible to obtain certain effects from the introduction of sedimentation analysis data processing according to the proposed equation in production, where the fractional composition of dispersed phases is regulated

Keywords: dispersed system, fractional composition, particle distribution curves, linear approximation, sedimentation

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MATHEMATICAL MODELING OF THE SEDIMENTATION PROCESS FOR DETERMINING THE FRACTIONAL COMPOSITION OF SUSPENSIONS

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1. Introduction

In real suspensions, the particles that form the dispersed phase are not the same in size – these are, as a rule, polydisperse systems. In this regard, the task of the dispersion analysis is to find the law of the size distribution of the elements of the dispersed phase.

The analysis of the particle size distribution of dispersed materials refers to such methods of monitoring the natural environment, substances, materials in which it is necessary to determine the sizes of thousands of particles in one sample.

A feature of sedimentation methods is that the determined size characterizes not only the geometric parameters of the particles, but also takes into account their interaction with the dispersion medium. Interaction with a dispersion medium is taken into account through the resistance coefficient [1–4].

Processing the results of sedimentation analysis requires the use of theoretical equations to describe the process of sedimentation of dispersed particles [5–7].

Determination of the fractional composition of suspensions using sedimentation methods requires preliminary knowledge of the law of particle size distribution of the suspension. A large number of existing dispersed materials, a variety of methods for their preparation and processing make it difficult to create a universal dependence, equally suitable for describing the law of distribution of suspension particles by size of all existing materials.

In [8, 9], a theoretical equation was proposed for processing sedimentation analysis data regardless of the size distribution law of polydisperse system particles.

Calculations of the fractional composition of dispersed systems: powders, suspensions are quite relevant and important, since the dispersion of these objects characterizes the quality indicators of various industrial products.

2. Literature review and problem statement

The description of the process of sedimentation of particles in suspensions is based on the well-known concepts: when a spherical particle falls in a gravitational field in a viscous medium, two oppositely directed forces act: the resistance force of the medium and the force of gravity [10]. But there are still unresolved issues related to the determination of the fractional composition of suspensions. The reason for this is the objective difficulties associated with the absence of ab initio equation for describing the sedimentation curve. A variant of overcoming the corresponding difficulty is possible to create an equation to describe the phenomenon of sedimentation. So, when processing sedimentation analysis data, graphical differentiation of the sediment accumulation curve is used [11]. However, the graphical definition of the derivative is rather imprecise and has a subjective component.

The logarithmically normal law is considered to be the best known of the analytical dependences used to determine the fractional composition of suspensions [12]. The application of the log-normal law to the particle size distribution always requires additional evidence for its application. Thus, in [13], the logarithmically normal law of particle size distribution is used without proving the possibility of its application. The authors of [14] use the empirical equation of the dependence of the mass of sedimentation time on time. This approach is empirical and insufficiently theoretically substantiated. Consideration of the disadvantages of methods for processing experimental data on sedimentation is presented in [15], but no algorithm is proposed by which the disadvantages can be improved. In addition to the drawbacks of methods for processing experimental data on sedimentation, there are drawbacks associated with the theoretical description of the sedimentation process [16]. However, the authors of [16] did not make changes to the model of the sedimentation process.

A variant of overcoming the corresponding difficulties can be the creation of a mathematical model of the sedimentation process, built on the consideration of the sedimentation curve as a dependence described by piecewise linear approximation.

All this allows to assert that it is advisable to conduct a study devoted to the creation of an equation for processing sedimentation analysis data, which does not depend on the law of particle size distribution of a polydisperse system.

3. The aim and objectives of research

The aim of research is to create an equation for processing sedimentation analysis data, which does not depend on the law of particle size distribution of a polydisperse system, and to solve it using the “Search for a solution” add-in for the MS Excel environment. This will make it possible to determine the fractional composition of dispersed phases of powder materials.

To achieve the set aim, the following objectives were set:

– to consider the nature and mechanism of sedimentation of particles of the dispersed phase of suspensions;

– to get an equation describing the sedimentation curve;

– to create a program in one of the programming languages that allows to determine the fractional composition of suspensions;

– to check the possibilities of the obtained equation for describing experimental data on sedimentation.

4. Materials and methods of research

During the experiments, 2 % bentonite suspension was used (quarry sample, Cherkasy deposit, Ukraine). Sedimentation analysis of the suspension by the method of continuous weighing of the sediment using a torsion balance [10].

In order to reduce the experimental errors. the experiments were repeated.

The processing of the experimental results was carried out by the created program in the VBA language.

To determine the fractional composition of the suspension with the help of the developed program, the “Search for a solution” add-on for the MS Excel environment (USA) was used.

5. Results of studies of nature and mathematical model of the sedimentation process

5. 1. The nature and mechanism of sedimentation of particles of the dispersed phase of suspensions

For a monodisperse system, the sediment mass m will increase with time proportional to the mass of particles contained in a suspension layer of unit thickness, the sedimentation rate of the particles and the sedimentation time of the particles, i. e.

$$m=\frac{m_1}{H}ut,\ \text{or}\ m=\frac{m_1}{t_1}t, \tag{1}$$

where t_1 – the time of complete sedimentation of particles of a certain radius from a height H (distance from the surface of the suspension to the scales), and m_1 – the total mass of particles. u is the sedimentation rate of particles

Since for a monodisperse system all particles are the same, they sediment at the same speed, therefore the dependence of the sediment mass m on time t is linear (Fig. 1).

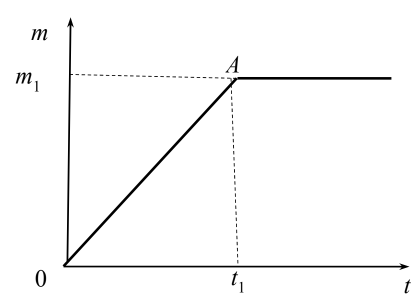

Fig. 1. Sedimentation curve of monodisperse suspension

The particle size directly from equation (1) can only be determined by a monodisperse system.

Point A corresponds to the completion of the sedimentation of particles. First, the sedimentation particles reach the pan of the torsion balance from a low height (from the lower layer of the suspension). Therefore, point A in Fig. 1 corresponds to the time during which the particles pass the path from the height H, during this time all particles will sediment. and the sediment will not accumulate.

Calculating the tangent of the angle of inclination of the straight line to the time axis $\left(\frac{m_1}{t_1}\right)$ and using the known values of t_1 and H, it is calculated for U_{sed}. Further. according to the well-known equation [10], the equivalent radius of the particles is determined with respect to the sedimentation radius $\left(r=\sqrt{\frac{9\eta U_{sed}}{2(\rho-\rho_0)g}}\right)$, of the spherical particles.

In this equation ρ – the density of the particles of the dispersion phase, kg/m^3; ρ_0 – the density of the dispersion medium, kg/m^3; g – the acceleration of gravity; V – the volume of the particle, m^3, r – the equivalent radius of the particles; η – the viscosity of the medium; U_{sed} particle sedimentation rate. It is clear that equation (3) is valid until the time $t \leq t_1$. since at $t \geq t_1$ the particles have completely sediment and the sediment mass will remain constant.

5. 2. Derivation of an equation for describing the sedimentation curve

The creation of the equation is based on the conclusions of [18] on the use of piecewise linear approximation to describe the sedimentation curve.

The equation that will describe the sedimentation curve of a monodisperse system (Fig. 1) is represented in the form:

$$m=\frac{m_1}{t_1}t\left[\left(\frac{1}{2}+\frac{t_1-t}{2|t_1-t|}\right)\right]+m_1\left[\left(\frac{1}{2}-\frac{t_1-t}{2|t_1-t|}\right)\right],$$

where $|t_1-t|$ – the modulus of the number t_1-t or in the form:

$$m=\frac{m_1}{t_1}t\left[\left(\frac{1}{2}+\frac{t_1-t}{2((t_1-t)^2)^{0,5}}\right)\right]+$$
$$+m_1\left[\left(\frac{1}{2}-\frac{t_1-t}{2((t_1-t)^2)^{0,5}}\right)\right].$$

If the suspension contains particles of only two different radii (bidisperse suspension Fig. 2), then the sedimentation in such a system can be represented as the simultaneous sedimentation of particles of two monodisperse systems (curves OBE and OCD in Fig. 2).

The equation that will describe the sedimentation curve of the bidisperse system (Fig. 2) will take the form:

$$m=\frac{m_1}{t_1}t\left[\left(\frac{1}{2}+\frac{t_1-t}{2|t_1-t|}\right)\right]+m_1\left[\left(\frac{1}{2}-\frac{t_1-t}{2|t_1-t|}\right)\right]+$$
$$+\frac{m_2}{t_2}t\left[\left(\frac{1}{2}+\frac{t_2-t}{2|t_2-t|}\right)\right]+m_2\left[\left(\frac{1}{2}-\frac{t_2-t}{2|t_2-t|}\right)\right]. \quad (2)$$

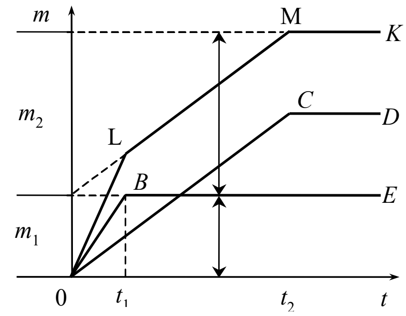

Fig. 2. Sedimentation curves: OBE — monodisperse suspensions with large particles; OCD — monodisperse suspensions with smaller particles; OLMK — bidisperse suspension

The sediment of the fraction of large-sized particles is depicted by a straight OB, smaller ones – OC; the joint subsidence of both fractions is described by the OLMK line, which is the algebraic sum of OB and OC. The breaking points on the OLMK line correspond to the time of the complete sedimentation of large and small particles, which makes it possible to calculate their sedimentation rates (H/t_1 and H/t_2) and, respectively, r_1 and r_2. Using this graph, it is also possible to determine the mass of each fraction. From the dependence, one can create that the segment m_1 on the ordinate axis corresponds to the mass of the fraction of large particles, and m_2 – to the mass of the fraction of small particles.

For a polydisperse system, the sedimentation graph is depicted by a curved line, which is a broken line with an infinitely large number of break points (Fig. 3). The graphical dependence of the mass of sedimentation particles on the sedimentation time is called the sedimentation curve. It is the initial information needed to calculate the particle size and size distribution.

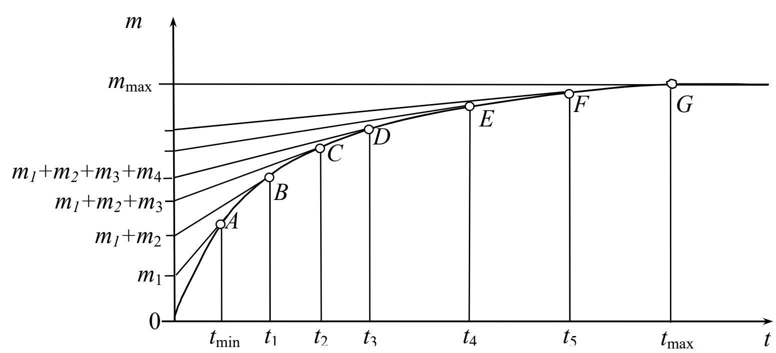

Fig. 3. Sedimentation curve of sedimentation of a polydisperse suspension

Experimental data are used to construct the sedimentation curve of subsidence, i. e. dependence of the mass m of sedimentation particles on the sedimentation time t (Fig. 3). It is usually assumed that during the sedimentation of such a polydisperse system particles of different sizes sediment independently of each other and move with a velocity $u(r)$ defined for each size. During the sedimentation of particles of polydisperse systems, the rate of sediment accumulation changes continuously and, accordingly, the dependence of the sediment mass on time has the form of a smooth curve. On this curve the initial linear section (if $t<t_{min}$) and the final section of constant sediment mass (if $t>t_{max}$) can be distinguished. The initial portion of any sedimentation curve is a straight line (Fig. 3, *OA*). This is due to the fact that at the beginning of the process particles of all sizes are sediment, and since their accumulation is proportional to the sedimentation time, the same pattern remains for the accumulation of the entire sediment.

All real dispersed systems are polydisperse, and therefore the sedimentation rate is different for different fractions: large particles have a faster sedimentation, small particles are slower. Therefore the sedimentation curve is bent towards the ordinate axis.

The purpose of sedimentation analysis is to determine the fractional composition of the dispersed system, which is necessary for the preparation of suspensions and powders. The total mass of the suspension:

$$m = m_1 + m_2 + m_3 + ...m_n,$$

where m_i – the mass of each fraction. To determine the fractional composition the mass fraction of each fraction $\left(W_i = \frac{m_i}{m}\right)$ should be calculated.

It is clear that the sum of the mass fractions of each fraction:

$$1 = W_1 + W_2 + W_3 + ...W_i,$$

where W_1 – the mass fraction of particles of the fraction that sedimented over the time interval from 0 to $t_{\min}$, i. e. fraction with particle radii from $r_{\max}$ to r_1, W_2 with particle radii from r_1 to r_2 etc. $r_{\max}>r_1>r_2>r_3>...>r_n$.

The results of sedimentation analysis of a polydisperse system for determining the fractional composition can be presented in the form of distribution curves: integral (total) and differential distribution curves ($Q=f(r)$ and $dQ/dr=f(r)$**)**. The cumulative distribution function is calculated as the fraction of the suspension fraction with particle radii from $r_{\min}$ to r_i or from $r_{\max}$ to r_i. That is, $Q_1=W_1$ there is the fraction of the suspension fraction with particle radii from $r_{\min}$ to r_1. $Q_2=W_1+W_2$, there is the fraction of the suspension fraction with particle radii from $r_{\min}$ to r_2, etc. The curve of the integral distribution function (Fig. 4) shows the content (in wt.%) of particles, of different degree of dispersion. The integral curve usually has an S-shape with a characteristic inflection point, which corresponds to the size of the particles, the mass fraction of which in the given disperse system is maximum and corresponds to r_H – the probable radius.

The integral curve can be ascending (curve *b,* Fig. 4) or descending (curve *a,* Fig. 4) depending on how the value (Q) of the particles is determined – how the fate of the suspension fraction with particle radii from $r_{\min}$ to r_i, or from $r_{\max}$ to r_i. If Q is determined by starting to sum up from the largest particles then the integral curve has a descending form (Fig. 4, curve *a*) and if from the smallest then an ascending one (Fig. 4, curve *b*).

Using the integral distribution curve of particles, the percentage of fractions is determined, that is, particles with sizes ranging from r_i to r_k: it is equal to the difference between the corresponding ordinates $Q_i–Q_k$.

A more visual representation of the particle size distribution in the system is provided by the differential distribution curve (Fig. 5). It represents the dependence of the derivative on the integral distribution function (dQ/dr) and determines the percentage in the system of the fraction of particles with radii from r to $r+dr$ in the form of an area bounded by this curve and the abscissa axis.

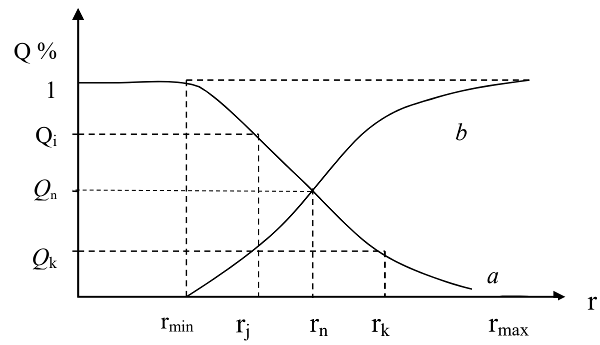

Fig. 4. Integral curves of the distribution of particles along the radii: curve *a* — descending; curve *b* — ascending

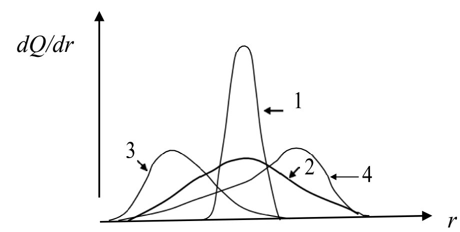

Fig. 5. Typical differential curves of distribution along the radii: 1 — the system,. which is the closest to monodisperse; 2 — polydisperse system; 3, 4 — systems containing mainly small and large particles, respectively

The fraction corresponding to the maximum of the distribution curve most likely corresponds to the radius of the polydisperse system r_n.

The narrower the interval of the radii of the differential distribution curve (from $r_{\min}$ to $r_{\max}$) and the higher its maximum the closer the system is to monodisperse. On the contrary, the more stretched the curve and the lower the maximum the more polydisperse the system is. Thus, the differential curves of the distribution of particles along the radii are an important characteristic of disperse systems, which make it possible to judge the degree of their polydispersity.

To describe the time dependence of the mass of a substance deposited during sedimentation. an equation that can be used to analyze any polydisperse systems is obtained and proposed.

So, based on the assumption that the motion of each particle is independent of others. it can be written that for an n-dispersed system (containing n particles with radii from $r_{\min}$ to $r_{\max}$), the sediment mass with time will be described by the equations:

In the time interval from $0 \le t \le t_1$

$$m = \sum_{i=1}^{n} m_i = \frac{m_1}{t_1}t + \frac{m_2}{t_2}t + \frac{m_3}{t_3}t + ...\frac{m_n}{t_n}t.$$

In the time interval from $t_1 \le t \le t_2$

$$m = m_1 + \frac{m_2}{t_2}t + \frac{m_3}{t_3}t + ...\frac{m_n}{t_n}t.$$

In the time interval from $t_2 \le t \le t_3$

$$m = m_1 + m_2 + \frac{m_3}{t_3}t + ...\frac{m_n}{t_n}t,$$

and only $m = \frac{m_i}{t_i}t$ under the conditions $t \le t_i$ and for $t > t_i$ $m = m_1$. Thus the sedimentation curve of an n-dispersed suspension (Fig. 3) can be described as a broken line consisting of n linear sections.

Thus the dependence will represent a broken curve consisting of line segments. The dependence $m=f(t)$ consisting of straight line segments can be written as follows:

$$m(t) = t\sum_{i=1}^{n}\frac{m_i}{2t_i}\left[\left(1+\frac{t_i-t}{|t_i-t|}\right)\right]+\sum_{i=1}^{n}\frac{m_i}{2}\left[\left(1-\frac{t_i-t}{|t_i-t|}\right)\right]. \quad (3)$$

In addition, to eliminate the uncertainty in the values of m at $t=t_i$, for convenience in carrying out calculations. equation (3) should be used in the form:

$$m(t) = t\sum_{i=1}^{n}\frac{m_i}{2t_i}\left[\left(1+\frac{\delta+t_i-t}{\delta+|t_i-t|}\right)\right]+$$
$$+\sum_{i=1}^{n}\frac{m_i}{2}\left[\left(1-\frac{\delta+t_i-t}{\delta+|t_i-t|}\right)\right], \quad (4)$$

where δ – some rather insignificant value for example 10^{-15} s.

Equation (4) should describe any sedimentation curve regardless of the distribution law of the particles of the polydisperse system along the radii.

Thus a real suspension can be considered as n-dispersed and, of course, the larger n is, the more accurately the above equation will describe the sedimentation curve.

Equation (4) contains n unknown values of the masses of the fractions (m_1, m_2, m_3, ... m_n) which completely sedimented in the corresponding time (t_1, t_2, t_3, ... t_n).

The number of fractions (n) for use in solving applied problems is practically no more than 10, and modern numerical methods make it possible to determine the unknown coefficients of Eq. (4) (the unknowns are the value of the masses of the suspension fractions) in the presence of experimental data.

For the practical use of equation (4), it is necessary to consider the polydisperse suspension as n-dispersed and the number of experimental measurements of the dependence of the sediment mass on time should be greater than or equal to the number of fractions. The description of the experimental data by equation (4) makes it possible to determine the fractional composition of the polydisperse suspension.

The calculation of the fractional composition (m_i values) is based on the determination of m_i values for which the theoretical sedimentation curve and the experimental one coincide in the best way. Usually, the least squares method is used to solve such problems:

$$F(t, m_1, m_2, m_3, ...m_n) =$$
$$= \sum_{j=1}^{k}\left(m(t_j) - m_{\text{ex}}(t_j)\right)^2 = \min, \quad (5)$$

where k – the number of experimental measurements of the masses of sediment particles of the suspension ($k \ge n$), $m(t_j)$ is the calculated value of the sediment mass according to equation (4), $m_{\text{ex}}(t_j)$ – the experimental value of the sediment mass, $F(t, m_1, m_2, m_3, ... m_n)$ – objective function. There are different methods for finding the minimum of the objective function [16].

5. 3. Drawing up a program in VBA language

A numerical method of data processing using the methods of computational mathematics and using the capabilities of the MS EXCEL, program which is part of the Microsoft Office suite (USA), is proposed for determining fractional suspensions. The use of numerous data processing methods makes it possible to determine the fractional composition of a polydisperse suspension regardless of the distribution law of the particles of the polydisperse system along the radii.

The methods built into MS EXCEL in the "Search for a solution" add-on are quite convenient for practical calculations of the values of m_1, m_2, m_3, ... m_n – the coefficients in equation (6). The "Search for a solution" add-in in the latest versions of MS EXCEL allows to choose a method for solving a problem depending on its type. These are the following methods: the general down-gradient method, which is used to solve smooth nonlinear problems; simplex method for solving linear problems; evolutionary method for solving nonsmooth problems.

In fact the "Search for a solution" add-in finds the minimum of the objective function and

$$F(t, m_1, m_2, m_3, ...m_n) = \sum_{j=1}^{k}\left(m(t_j) - m_{ex}(t_j)\right)^2 = \min$$

by varying the values of m_1, m_2, m_3, ... m_n. To use the "Search for a solution", it is necessary to create a Visual Basic for Applications (VBA) program to calculate the objective function.

To use the capabilities of MS EXCEL, a VBA program was developed for calculating the objective function $F(t, m_1, m_2, m_3, ... m_n)$.

The created program is shown below:

```
Function SEDIMENTATION(N. NT. A1. A2. A3. A4. A5. A6. A7. A8. A9. A10. A11. A12. A13. A14. A15. A16. A17. A18. A19. A20. A21. A22. A23. A24. A25)
Dim x As Double. i As Integer
Dim j As Integer
Dim v As Double. s As Double. z As Double. y As Double
Dim m As Double. mm As Double. mmm As Double. f As Double
x=0
v=0
s=0
For j=1 To NT
y=Cells(j. 2).Value
s=0
For i=1 To N
x=Cells(i. 1).Value
mm = Cells(i. 3).Value
a=0.00000000000001
s1=(((x–y)^2)^0.5)+a
s2=(x–y+a)
m=((mm/x)*y/2) * (1#+s2/s1)+(mm/2#)*(1#–s2/s1)
s=s+m
Next i
mmm=Cells(j. 4).Value
f=(s–mmm)^2
v=v+f
Next j
SEDIMENTATION=v
End Function
```

5. 4. Checking the reliability of the program on experimental data on sedimentation

The originally developed program was tested on simulated sedimentation data for a suspension consisting of 25 fractions with a known mass composition and complete sedimentation time for each fraction. The data are given in Table 1.

Table 1

Data for checking the created program

m_i	t_i.	m
10	308.64	162.52
15	373.46	194.54
20	451.88	230.15
25	546.78	269.03
30	661.60	310.82
35	800.54	355.10
40	968.65	401.32
45	1,172.07	448.85
50	1,418.20	496.90
55	1,716.02	544.55
60	2,076.39	590.66
65	2,512.43	633.85
60	3,040.04	672.45
55	3,678.45	706.57
50	4,450.92	736.30
45	5,385.62	761.77
40	6,516.60	783.14
35	7,885.08	800.60
30	9,540.95	814.38
25	11,544.55	824.75
20	13,968.91	832.05
15	16,902.38	836.68
10	20,451.88	839.13
5	24,746.77	840.00
0	29,943.59	840.00

	A	B	C	D	E	F	G
1	309	309		163		10	
2	373	373		195		15	
3	452	452		230		20	
4	547	547		269		25	
5	662	662		311		30	
6	801	801		355		35	
7	969	969		401		40	
8	1172	1172		449		45	
9	1418	1418		497		50	
10	1716	1716		545		55	
11	2076	2076		591		60	
12	2512	2512		634		65	
13	3040	3040		672		60	
14	3678	3678		707		55	
15	4451	4451		736		50	
16	5386	5386		762		45	
17	6517	6517		783		40	objective function
18	7885	7885		801		35	10248179,03
19	9541	9541		814		30	
20	11545	11545		825		25	
21	13969	13969		832		20	
22	16902	16902		837		15	
23	20452	20452		839		10	
24	24747	24747		840		5	
25	29944	29944		840		0	
26	t=	t_i=	Calculated m_i=	m=		Accurate m_i=	
27							

Fig. 6. MS EXCEL sheet with initial data

Fig. 6 shows an example of inputting initial data for checking the created program.

The calculation of the masses of fractions using the "Search for a solution" add-on is performed in column C (Fig. 7).

After searching for a solution. column C will contain the calculated values of the masses of fractions (m_i) (Table 2).

Comparison of the calculated values of the masses of the fractions and the exact ones (Table 2) confirms the correctness of the developed program created in the VBA language.

The developed program was used to process data on the sedimentation of a clay suspension in water. Sedimentation data are processed according to the proposed equation (4).

During the experiments, 2 % bentonite suspension was used (quarry sample, Cherkasy deposit, Ukraine).

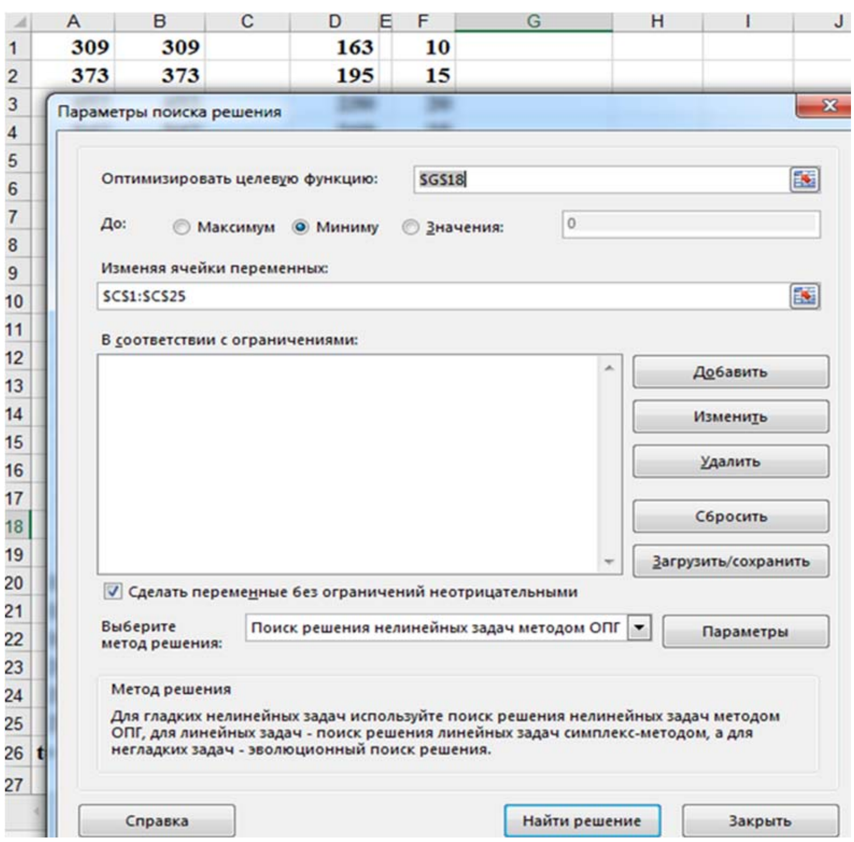

Fig. 7. MS EXCEL sheet when loading the add-in

For the sample 5, repeated experiments were performed on the sedimentation of suspensions. The measurement error did not exceed 3 %. Table 3 shows averaged experimental data.

Table 2

Calculation results after using the "Search for a solution" add-in

t	t_i	Calculated m_i	*m*	Exact m_i
309	309	10.002	163	10
373	373	14.998	195	15
452	452	20.000	230	20
547	547	25.000	269	25
662	662	30.000	311	30
801	801	35.000	355	35
969	969	40.000	401	40
1.172	1.172	45.000	449	45
1.418	1.418	50.000	497	50
1.716	1.716	55.000	545	55
2.076	2.076	60.002	591	60
2.512	2.512	64.995	634	65
3.040	3.040	60.002	672	60
3.678	3.678	55.000	707	55
4.451	4.451	50.000	736	50
5.386	5.386	45.000	762	45
6.517	6.517	40.000	783	40
7.885	7.885	35.000	801	35
9.541	9.541	30.000	814	30
11.545	11.545	25.000	825	25
13.969	13.969	20.000	832	20
16.902	16.902	15.000	837	15
20.452	20.452	10.000	839	10
24.747	24.747	5.000	840	5
29.944	29.944	0.000	840	0

Table 3

Experimental data on the increase in the mass of sediment particles of the suspension from the sedimentation time *t*

t, s	*m*, mg
30	63
60	109
120	171
180	212
240	240
300	261
360	277
420	290
480	300
600	316
720	327
840	336
960	343
1.080	348
1.200	353
1.500	361
1.800	367
2.100	372
2.400	375
2.700	378
3.300	382
4.800	387
5.200	388
5.600	389
7.000	390

The resulting sedimentation curve has a classic form (Fig. 8).

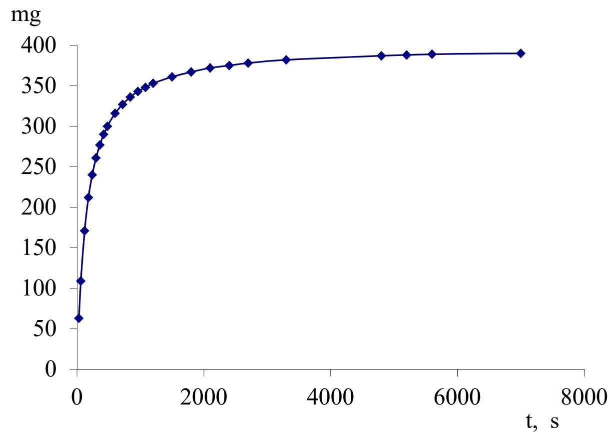

Fig. 8. Dependence of the mass of particles (m) of suspensions on sedimentation time (t)

The processing of data on the sedimentation of a suspension of clay in water was carried out according to the proposed equation (4) and the created program. The results of calculating the fractional composition of the suspension and the integral curves of the distribution of the masses of the suspension particles along the radii are shown in Fig. 9, 10.

30	30	17,001	63	
60	60	30,000	109	
120	120	42,000	171	
180	180	38,996	212	
240	240	28,009	240	
300	300	24,984	261	
360	360	18,028	277	
420	420	20,963	290	
480	480	16,033	300	
600	600	24,970	316	
720	720	12,024	327	
840	840	13,990	336	
960	960	16,003	343	
1080	1080	0,000	348	
1200	1200	17,998	353	
1500	1500	9,996	361	
1800	1800	6,071	367	
2100	2100	13,715	372	
2400	2400	0,470	375	
2700	2700	8,639	378	
3300	3300	11,426	382	objective function
4800	4800	0,000	387	0,546633429
5200	5200	0,000	388	
5600	5600	18,148	389	
7000	7000	0,006	390	
t=	t_i=	m_i=	m=	

Fig. 9. The results of calculating the fractional composition of the suspension

The form of the integral curve of the distribution of the masses of the suspension particles along the radii indicates the presence of a uniform distribution of the masses of the particles in the region of small values of the radii.

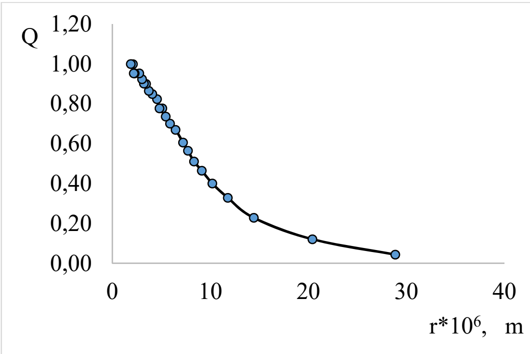

Fig. 10. Integral curves of the distribution of the masses of the particles of the suspension along the radii

6. Discussion of the results of mathematical modeling of the sedimentation process to determine the fractional composition of suspensions

As can be seen from the data in the Table 8, the calculated fractional composition for the model sedimentation data completely coincides with the initial data. This coincidence testifies to the validity of the derived equation. The obtained theoretical equation for describing the phenomenon of sedimentation makes it possible to overcome the objective difficulties associated with the absence of a non-empirical equation.

The coincidence of the calculated values with the output values confirms the absence of errors when compiling the program and confirms the ability of the "Search for a solution" add-in to calculate 25 unknown coefficients in the approximation equation. Since these coefficients are responsible for the fractional composition (equal to the masses of the fractions), there is no need for a graphical definition of the derivative, which is rather inaccurate and has a subjective component.

The results of calculations based on the experimental data made it possible to determine, using the developed program, the fractional composition of the samples (the value of the values in column 3 in Fig. 9) and the integral curve of the distribution of the mass of the suspension particles along the radii (Fig. 10).

Thus, the experiment performed and subsequent calculations show that equation (4) can be used to describe the sedimentation curve and determine the fractional composition of any polydisperse system.

Data processing by means of the proposed equation can in principle be used to determine the fractional composition of polydisperse suspensions with more than 25 fractions. Such calculations are important both from a scientific point of view and for industries where the fractional composition of dispersed phases is regulated.

A limitation in production conditions may be that the degree of influence of experimental errors of sedimentation analysis on the results of calculations of the fractional composition of polydisperse suspensions is unknown.

The main direction of further development of the study is to identify the possibility of its application to study sedimentation processes in centrifuges.

7. Conclusions

1. A new theoretical equation has been obtained to describe the phenomenon of sedimentation, which allows processing experimental data without using an empirical equation.

2. It has been proved that the method of computational mathematics can be used to determine the fractional composition of suspensions according to the proposed equation. This is confirmed by the processing of model data of sedimentation analysis using the "Search for a solution" add-in for the MS EXCEL environment. The results of such processing indicate the coincidence of the calculated and model values of the fractions.

3. Using the obtained equation and the developed program in the VBA language, the possibility of processing the data of sedimentation analysis of suspensions obeying any law of particle size distribution of the polydisperse system has been proven.

4. It has been proven that the processing of sedimentation analysis data using the "Search for a solution" add-on for the MS EXCEL environment allows one to determine the distribution of particles of a polydisperse system along radii which can be any natural suspensions.

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