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USING THE ASYMPTOTIC APPROXIMATION OF THE MAXWELL ELEMENT MODEL FOR THE ANALYSIS OF STRESS IN A CONVEYOR BELT

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The features of the propagation of dynamic stresses in a conveyor belt, the material properties of which correspond to the Maxwell element model, are considered. Analytical expressions are presented for calculating the dynamic elastic modulus, the loss modulus, and the angle of mechanical loss depending on the frequency of longitudinal oscillations in the belt of an extended transport conveyor. To analyze the dynamic stress propagation process, dimensionless parameters are introduced that characterize the specific features of the viscoelastic process in a conveyor belt, the material properties of which correspond to the Maxwell element model. The transition to the dimensionless Maxwell element model is made and the analysis of the relationship between stress and deformation of a conveyor belt element for extremely large and small values of dimensionless parameters is made. The substantiation of the scope of the Maxwell element model is given. It is shown that at sufficiently high frequencies of longitudinal stress oscillations in a conveyor belt, at which the oscillation period is much less than the characteristic oscillation decay time, the relationship between stress and deformation of the conveyor belt element corresponds to Hooke's law. A qualitative analysis of the relaxation time was carried out for a conveyor belt material, the properties of which correspond to the Maxwell element model. The analysis of the propagation of dynamic stresses in the conveyor belt for the characteristic operating modes of the transport conveyor is carried out. The conveyor operating mode with a constant deformation rate of the belt element; the mode in which a constant load is suddenly applied to the belt element; the conveyor operating mode with an instantly applied load to the belt element were investigated. It was determined that in cases where the characteristic process time significantly exceeds the stress relaxation time in the conveyor belt or the longitudinal oscillation period is much less than the stress relaxation time in the conveyor belt, the Maxwell element model can be replaced with a sufficient degree of accuracy by the Hooke element model

Keywords: viscoelastic process, Maxwell element, Hooke element, transport conveyor, dynamic elastic modulus

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1. Introduction

One of the characteristics of a transport conveyor, which determines its operational capabilities, is the strength of the conveyor belt [1, 2]. The mechanical strength of the belt is the ability to resist destruction under the action of the dynamic Maxwell element loads that occur during material transport. The tensile strength of the belt depends on the properties of the material, the rate of deformation, and the temperature of the belt material [3, 4]. For transport conveyors that operate continuously for a long time, belt failure occurs at stresses that are significantly lower than the ultimate strength of the belt material. The behavior of the belt material at the moment of destruction is determined by the relaxation and strength properties, which are interrelated [5]. The viscoelastic properties of the conveyor belt material, caused by relaxation processes, affect the destruction rate of the conveyor belt [6, 7], are closely related to the problem of saving material transport costs. One of the ways to reduce transport costs, which occupy a significant part in the cost of material extraction, is to use a conveyor belt speed control system [8]. Switching speed modes leads to acceleration or deceleration of the conveyor belt, and, accordingly, to the emergence of dynamic stresses in the belt. This imposes additional restrictions on the speed modes of the transport system. The proposed algorithms for stepwise regulation of the belt speed imply instant switching of speed modes, and algorithms for smooth speed control do not take into account the propagation of dynamic disturbances in the conveyor belt. At the same time, with a non-stationary incoming of material at the input of the transport system, the duration of the transient process takes up a fairly large part of the total time of the control process. For an in-depth analysis of these limitations, which consists in determining the dependence of the stress value on the magnitude of the belt speed and acceleration, a solution of the wave equation is required. For the Hooke element model, the solution to the wave equation is obtained in an analytical form. For more complex elastic element models, among which the Maxwell element model should be distinguished, the solution of the wave equation is associated with additional difficulties. In this regard, the problem of constructing simple analytical dependencies between the stress and deformation of the belt element is urgent, which would simplify the solution of the wave equation and make it possible to form constraints on phase coordinates when designing algorithms for optimal speed control of the belt, the material properties of which correspond to the Maxwell element model.

2. Literature review and problem statement

In [9], the results of studies of the propagation of dynamic stress disturbances in conveyor belts made of a material whose characteristics correspond to the Voigt element model are presented. It is shown that conveyor belt materials have pronounced viscoelastic properties, which imposes specific features on the propagation of disturbances [7]. For some operating modes of the conveyor, a numerical calculation of the values of the conveyor belt speed, acceleration, and stress is carried out. However, the issues of constructing an analytical solution to the problem remained open. The reason for this is the objective difficulties caused by the stress-deformation relationship in the Voigt element model. The problem of constructing an analytical solution to the problem remained open in [5, 10, 11]. In [5], for the conveyor belt of the elastic Kelvin-Voigt element model, the tension of the belt in the steady-state and transient conditions is analyzed. For the numerical calculation, the Lagrange equation system was used. The work [10] presents an analysis of the main models of elastic elements for a conveyor belt: Hooke element, Newtonian element, Maxwell element, Kelvin element, Venant element, CDI geometric element, CDI five-element. The calculation of two transport systems (conveyor length of nine kilometers) for start and stop modes with the CDI five-element composite model was performed using the finite element method. The dynamic stress in a conveyor belt, the material characteristics of which correspond to the Kelvin-Voigt element model, the combination of Hooke and Kelvin-Voigt element, was investigated in [11]. The conveyor belt segment is represented by a two-parameter rheological model.

In some cases, these difficulties can be overcome if, during the conveyor operation, it is assumed that there are small deformations in the belt, at which the relationship between stress and deformation in the element can be approximately represented by Hooke's law

$$\sigma(t,S)=E\varepsilon(t,S), \tag{1}$$

where $\sigma(t, S)$, $\varepsilon(t, S)$ are stress and deformation of the belt at time t at point S of the conveyor section of length S_d; E is the elastic modulus of the material. This approach to the analysis of the propagation of long-wave oscillations in the conveyor belt of the transport system is implemented in [12]. Also, an attempt to construct an approximate analytical solution in a particular case was made in [13] for a transport system, the belt material of which corresponds to the Winkler foundation model. All this allows us to assert the feasibility of conducting a study on the construction of approximate Maxwell element models with their subsequent use to calculate the propagation of longitudinal oscillations in a conveyor belt.

3. The aim and objectives of the study

The aim of the study is to develop asymptotic models of viscoelastic processes in a conveyor belt, the material of which corresponds to the Maxwell element model. This made it possible to synthesize belt speed control algorithms, taking into account the limitations associated with the presence of dynamic disturbances in the conveyor belt, and, accordingly, to further reduce the cost of material transport by optimizing speed modes.

To achieve the aim, the following objectives were set:

– to perform an asymptotic analysis of the Maxwell element model and justify the scope of the model;

– to analyze the characteristic operating modes of the transport conveyor, to carry out the transition for each of the considered modes from the general Maxwell element model to the asymptotic model of the viscoelastic element, justifying its use.

4. Research materials and methods

The materials of conveyor belts have both elastic and viscous properties, which leads to a rather complex relationship between the stress $\sigma(t, S)$ and deformation $\varepsilon(t, S)$ in the conveyor belt. One of the common models used to describe the viscoelastic properties of a conveyor belt material is the Maxwell element model. An analytical solution to the problem of the propagation of longitudinal dynamic stress disturbances in a conveyor belt, the material of which corresponds to the Maxwell element model, was obtained for a number of simple cases. A common way to solve this problem for more complex cases is to use numerical methods [5, 10, 13]. However, a numerical experiment is not a convenient enough means for determining the general patterns between stress and deformation in a conveyor belt element, which creates preconditions for further development of analytical methods that allow, in a particular case, studying the propagation of longitudinal dynamic stress disturbances in a conveyor belt, the material of which corresponds to the Maxwell element model. An alternative way to solve the problem is that the analytical solution can be achieved by introducing simplifications when setting the problem or in the course of solving it. The proposed simplification is based on the use of a small parameter, the presence of which reflects the essence of the formulation of the problem of determining the general patterns between stress and deformation in the Maxwell element model. In this regard, characteristic dimensionless numbers are introduced that determine the features of the viscoelastic process in the belt for studying the properties of the Maxwell element. The transition to dimensionless variables and the use of similarity criteria made it possible to generalize the research results for conveyor belts made of different materials. The next step of the study was that for the limit values of the introduced similarity criteria, asymptotic Maxwell element models were obtained, which characterize the features of the viscoelastic process in a conveyor belt. Essentially, asymptotic Maxwell element models are represented as simple relationships between stress and deformation in a conveyor belt, some of which are consistent with Hooke's Law to the required degree of accuracy. The area of application of asymptotic models is determined in accordance with the characteristic values of the introduced similarity criteria for a viscoelastic process.

5. Results of the study of the asymptotic approximation for the Maxwell element model

5. 1. Asymptotic analysis and justification for the application of the Maxwell element model

The stress $\sigma(t, S)$ and deformation $\varepsilon(t, S)$ relationship can be represented in general form

$$\sigma(t,S)=E_c\varepsilon(t,S),\quad E_c=E_1+iE_2,$$

$$|E_c|=\sqrt{E_1^2+E_2^2},\quad tg\delta=E_2/E_1, \tag{2}$$

where E_c is the complex elastic modulus of the material. The real part of the complex elastic modulus of the material $Re(E_c)=E_1$ is the dynamic elastic modulus, which characterizes the process of energy transfer through the conveyor belt element. The imaginary part of the complex elastic modulus of the material $Im(E_c)=E_2$ is the loss modulus, which characterizes the process of dissipation of oscillation energy in a viscoelastic body, in which the conveyor belt is heated. If at point S_0 of the conveyor belt there is a dynamic, periodically varying stress $\sigma(t, S_0)=\varepsilon_0\cos(\omega t)$, then taking into account the relations $E_c=|E_c|(\cos\delta+i\sin\delta)$, $\varepsilon(t, S_0)=\varepsilon_0(\cos\phi+i\sin\phi)$ and using the Moivre formula, the relationship follows:

$$\sigma(t,S_0)=\sigma_0\cos(\omega t)=|E_c|\varepsilon_0\cos(\delta+\varphi), \quad (3)$$

$$\varepsilon(t,S_0)=\sigma_0/|E_c|\cos(\omega t-\delta). \quad (4)$$

A viscoelastic element of a conveyor belt is characterized by a phase shift between stress and deformation, which is set by the tangent of the angle δ of mechanical losses. The dynamic elastic modulus and the loss modulus are the main parameters determining the propagation of longitudinal oscillations in a conveyor belt. One of the important problems in the analysis of disturbance propagation is to determine the dependence of the dynamic elastic modulus E_1 and the loss modulus E_2 on the oscillation frequency ω. Let's obtain the indicated dependencies for the models used to describe the process of propagation of longitudinal oscillations in a conveyor belt.

The equation defining the relationship between stress $\sigma(t, S)$ and deformation $\varepsilon(t, S)$ at point S_0 of the conveyor belt, the material of which corresponds to the Maxwell element model (Fig. 1), is as follows

$$\frac{1}{E}\frac{d\sigma(t,S_0)}{dt}+\frac{\sigma(t,S_0)}{\eta}=\frac{d\varepsilon(t,S_0)}{dt}, \quad (5)$$

where E is the elastic modulus of the element; η is the viscosity of the element.

Let us seek a solution to equation (5) in the form

$$\sigma(t,S_0)=\sigma_0 e^{i\omega t}. \quad (6)$$

Taking into account relations (2), after substituting expression (6) into (5), the equation is obtained

$$\frac{i\omega}{E}+\frac{1}{\eta}=\frac{i\omega}{E_c}, \quad (7)$$

which was used to determine the dynamic elastic modulus E_1 and the loss modulus E_2:

$$\begin{cases}\dfrac{\omega}{E}E_1+\dfrac{1}{\eta}E_2=\omega,\\ \dfrac{1}{\eta}E_1-\dfrac{\omega}{E}E_2=0.\end{cases} \quad (8)$$

The introduced dimensionless parameters

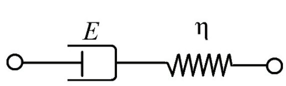

Fig. 1. Viscoelastic Maxwell element

$$\chi=\omega\eta/E,\ \ \varepsilon_1=E_1/E,\ \ \varepsilon_2=E_2/E, \quad (9)$$

are used to transform the system of equations (8) to the form

$$\begin{cases}\chi\varepsilon_1+\ \varepsilon_2=\chi,\\ -\varepsilon_1+\chi\varepsilon_2=0.\end{cases} \quad (10)$$

The solution of the system of equations (10) made it possible to obtain the dependences of the dynamic elastic modulus ε_1 and the loss modulus ε_2 on the parameter χ, which have the form (Fig. 2)

$$\varepsilon_1=\frac{\chi^2}{1+\chi^2},\ \ \varepsilon_2=\frac{\chi}{1+\chi^2},\ \ \text{tg}\delta=\frac{\varepsilon_2}{\varepsilon_1}=\frac{1}{\chi}. \quad (11)$$

The parameter χ is the dimensionless oscillation frequency

$$\chi=\omega\frac{\eta}{E}=\frac{\omega}{\omega_0},\ \ t_0=\frac{\eta}{E}=1/\omega_0. \quad (12)$$

where t_0 is the characteristic oscillation decay time. For a viscoelastic Maxwell element with parameters E=2.5e8 (Pa) and η=1875 (Nsec/m^2)), the characteristic oscillation decay time is t_0~10^{-5}(sec).

Using solution (11), the analysis of the dependence of the dynamic elastic modulus ε_1 and the loss modulus ε_2 on the dimensionless frequency χ of stress fluctuations in the conveyor belt for the Maxwell element model is carried out. At large values of oscillation frequencies (χ>>1), the solution to equation (11) can be represented in the form

$$\varepsilon_1\approx 1-\frac{1}{\chi^2}+\ldots\to 1,\ \ \varepsilon_2\approx\frac{1}{\chi}-\frac{1}{\chi^3}+\ldots\to 0,$$

$$\text{tg}\delta=\frac{\varepsilon_2}{\varepsilon_1}=\frac{1}{\chi}\to 0. \quad (13)$$

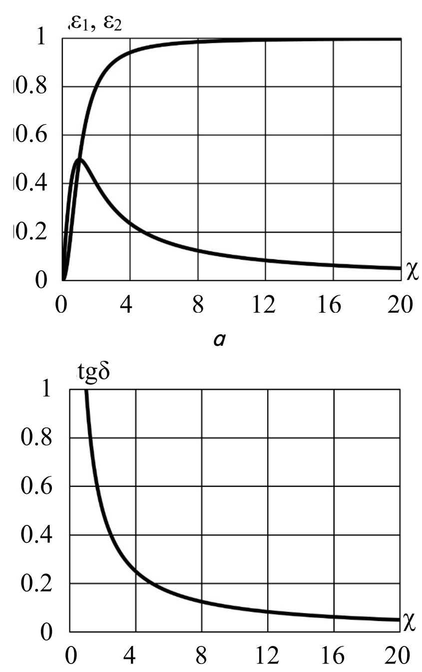

Fig. 2. Characteristics of a viscoelastic element depending on the frequency χ: *a* — dynamic elastic modulus ε_1, loss modulus ε_2; *b* — angle of mechanical losses δ

At large values of dimensionless frequencies χ>>1, the value of the dynamic elastic modulus ε_1 tends to the value of Young's modulus E. Losses that lead to the dissipation of elastic energy in the conveyor belt and are characterized by the modulus ε_2 can be neglected (Fig. 2). There is no phase shift δ between stress $\sigma(t, S)$ and deformation $\varepsilon(t, S)$ at point S_0 of the conveyor belt. Thus, solution (2) for equation (5) is presented in the form of Hooke's law (1). For high oscillation frequencies, the stress and strain relationship in the Maxwell element model follows Hooke's law.

For small values of the parameter χ<<1, the solution to equation (11) has the form

$$\varepsilon_1 \approx \chi^2 - \chi^4 + \ldots \to 0, \quad \varepsilon_2 \approx \chi - \chi^3 + \ldots \to 0,$$

$$\mathrm{tg}\delta = \varepsilon_2 / \varepsilon_1 = \chi^{-1} \to \infty. \quad (14)$$

The results obtained for the case of low oscillation frequencies showed that the dynamic elastic modulus ε_1 does not have a nonzero value. The close-to-zero value of the elastic modulus does not agree with the experimental data. Therefore, to describe the dynamic viscoelastic properties at low oscillation frequencies, the use of the model is incorrect due to the fact that the stress in the belt element tends to zero at an arbitrary deformation value of the viscoelastic element.

A qualitative assessment of the value of the characteristic decay time of oscillations t_0 in a viscoelastic element was made taking into account the results of studies of the experimental reference dependences storage $G_1(\omega)$~$E_1(\omega)/3$ and loss modules $G_2(\omega)$~$E_2(\omega)/3$ for conveyor belt materials, which were obtained in [14]. The results of the experiment [14] (Fig. 4), presented as graphical dependencies $G_1(\omega)$, $G_2(\omega)$, for the convenience of processing and perception of the results, are transformed into a tabular presentation (Table 1).

Table 1

Characteristic values of the parameters of a viscoelastic element [14]

$G_1(\omega)$, MPa	$E_1(\omega)$, MPa	$G_2(\omega)$, MPa	$E_2(\omega)$, MPa	f, Hz	ω, rad/sec
5.2	15.6	0.9	2.7	10^{-9}	$2\pi\cdot 10^{-9}$
11.2	33.6	1.4	4.2	10^{-5}	$2\pi\cdot 10^{-5}$
20.0	60.0	2.8	8.4	10^{-1}	$2\pi\cdot 10^{-1}$
22.0	66.0	3.0	9.0	$1/2\pi$	1
27.0	81.0	3.8	11.4	10^{0}	$2\pi\cdot 10^{0}$
33.0	99.0	5.0	15.0	10^{1}	$2\pi\cdot 10^{1}$
40.0	120.0	6.0	18.0	10^{2}	$2\pi\cdot 10^{2}$
170.0	510.0	90.0	270.0	10^{5}	$2\pi\cdot 10^{5}$
1700.0	5100.0	100.0	300.0	10^{10}	$2\pi\cdot 10^{10}$
1720.0	5160.0	20.0	60.0	10^{11}	$2\pi\cdot 10^{11}$

Taking into account the dimensionless expression (11), the characteristic oscillation decay time t_0 was determined by presenting the value of the dynamic elastic modulus $E_1(\omega)$ for an arbitrary frequency value ω and the dynamic elastic modulus $E_1(1)$ for the frequency ω=1 in the following form:

$$E_1(\omega) = E\frac{\omega^2 t_0^2}{1+\omega^2 t_0^2}, \quad E_1(1) = E\frac{t_0^2}{1+t_0^2}. \quad (15)$$

Divide the first equation by the second equation, we get the equation

$$\frac{E_1(\omega)}{E_1(1)} = \omega^2 \frac{1+t_0^2}{1+\omega^2 t_0^2}, \quad (16)$$

which we solve for the characteristic oscillation decay time

$$t_0 = t_0(\omega) = \frac{1}{\omega}\sqrt{\frac{E_1(\omega)/E_1(1) - \omega^2}{1 - E_1(\omega)/E_1(1)}}. \quad (17)$$

Substitution of the values $E_1(\omega)$, $E_1(1)$, ω into the last equation made it possible to obtain a qualitative dependence of the characteristic oscillation decay time $t_0(\omega)$ [sec] for the viscoelastic Maxwell element model on the frequency ω [rad/sec] (Fig. 3).

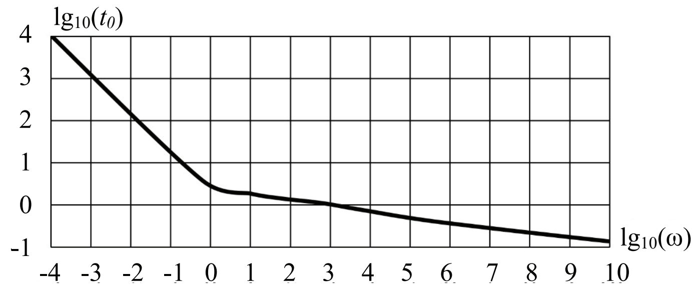

Fig. 3. Characteristic oscillation decay time $t_0(\omega)$ [sec] for the viscoelastic Maxwell element model

The time required for the transient mode of conveyor belt acceleration is several minutes [15–17], which significantly exceeds the characteristic decay time of high-frequency oscillations t_0 in a viscoelastic element. With a decrease in the oscillation frequency, the characteristic decay time of oscillations t_0 in a viscoelastic element increases significantly (11), (17).

5. 2. Construction of asymptotic models for the main operating modes of the transport conveyor

Let us consider the solution of equation (5) for some common cases of operation of a transport conveyor using conveyor belts made of a material whose characteristics correspond to the viscoelastic Maxwell element model.

Let us introduce dimensionless parameters

$$\sigma_\chi = \sigma(t, S_0)/\sigma_0, \quad \varepsilon_\chi = \varepsilon(t, S_0)/\varepsilon_0, \quad \tau = t\omega. \quad (18)$$

Since the choice of the scale factors σ_0, ε_0 is arbitrary, then setting

$$\sigma_0 = E\varepsilon_0, \quad (19)$$

we obtain equation (5) in dimensionless form

$$\frac{d\sigma_\chi}{d\tau} + \frac{1}{\chi}\sigma_\chi = \frac{d\varepsilon_\chi}{d\tau}, \quad (20)$$

which we use to analyze the characteristic operating modes of the transport conveyor. For extremely high oscillation frequencies (χ>>1), the solution to equation (20) is determined by the quasi-linear relationship between stress and deformation in a belt element (1).

5. 2. 1. The case of a constant deformation rate of a conveyor belt

This operating mode of the transport system is characteristic of the initial tension of the conveyor belt at the start of the transport conveyor. In the presence of a constant deformation rate $d\varepsilon_\chi/d\tau=\nu_\varepsilon$, equation (16) can be represented as follows:

$$\frac{d\sigma_\chi}{d\tau}+\frac{1}{\chi}\sigma_\chi=v_\varepsilon,\quad \sigma_\chi(0)=0,$$

$$\frac{d\varepsilon_\chi}{d\tau}=v_\varepsilon\cong\text{const},\quad \varepsilon_\chi(0)=0. \tag{21}$$

Let's write the solution of equations (21) in the form (Fig. 4):

$$\sigma_\chi=\chi v_\varepsilon\left(1-\exp(-\tau/\chi)\right), \tag{22}$$

$$\varepsilon_\chi=v_\varepsilon\tau,\quad \tau/\chi=t/t_0. \tag{23}$$

At $\tau/\chi>>1$, it follows that the stress σ_χ for the case of a constant deformation rate tends to the value $\chi\nu_\varepsilon$. The deformation grows indefinitely (Fig. 4).

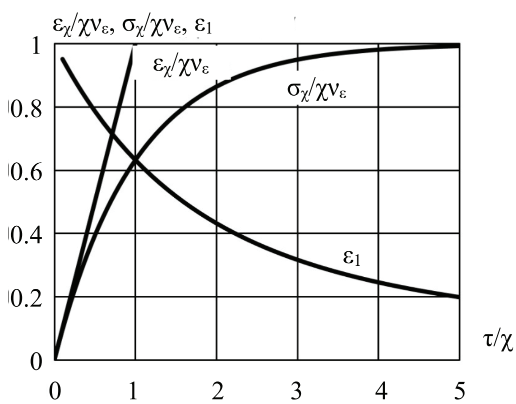

Fig. 4. Stress σ_χ, deformation ε_χ and dynamic elastic modulus ε_1 at a constant deformation rate of the conveyor belt

If the characteristic time of the deformation process t is much shorter than the relaxation time t_0 ($\tau/\chi<<1$), then

$$\lim_{\tau/\chi\to 0}\left(\sigma_\chi/\varepsilon_\chi\right)=\lim_{\tau/\chi\to 0}\varepsilon_1=\lim_{\tau/\chi\to 0}\frac{\left(1-\exp(-\tau/\chi)\right)}{\tau/\chi}=1. \tag{24}$$

At small values of the dimensionless parameter τ/χ, the behavior of the viscoelastic Maxwell element of the conveyor belt obeys Hooke's law (1).

5. 2. 2. The case of a constant rate of stress change in the conveyor belt element

With a uniform distribution of material along the conveyor belt and a constant force of primary resistance, the stress in the belt changes linearly along the length of the section. If the conveyor belt moves at a constant speed, then the rate of stress change $d\sigma_\chi/d\tau=\nu_\sigma$ in the element dS when it is moved as a result of material transportation will also be constant in magnitude. At a constant rate of stress change ν_σ, equation (16) takes the form:

$$v_\sigma+\frac{1}{\chi}\sigma_\chi=\frac{d\varepsilon_\chi}{d\tau},\quad \varepsilon_\chi(0)=\varepsilon_{\chi 0},$$

$$\frac{d\sigma_\chi}{d\tau}=v_\sigma\approx\text{const},\quad \sigma_\chi(0)=\sigma_{\chi 0}, \tag{25}$$

where the value of σ_χ is determined by the tension of the belt to eliminate its sagging during material transportation. The solution of equations (25) is presented in the form:

$$\varepsilon_\chi=\frac{v_\sigma}{2}\frac{\tau^2}{\chi}+\sigma_{\chi 0}\frac{\tau}{\chi}+v_\sigma\tau+\varepsilon_{\chi 0},\quad \sigma_\chi=v_\sigma\tau+\sigma_{\chi 0}. \tag{26}$$

At $\tau/\chi<<1$ ($t<<t_0$), the solution (26) implies a relationship between the stress and deformation of the belt element dS

$$\varepsilon_\chi\sim v_\sigma\tau+\varepsilon_{\chi 0}\sim\sigma_\chi-\sigma_{\chi 0}+\varepsilon_{\chi 0}. \tag{27}$$

The behavior of the viscoelastic Maxwell element of the conveyor belt follows Hooke's law. For $\tau/\chi>>1$, when the characteristic process time significantly exceeds the relaxation time ($t>>t_0$)

$$\varepsilon_\chi\sim\frac{v_\sigma}{2}\frac{\tau^2}{\chi}+\sigma_{\chi 0}\frac{\tau}{\chi},$$

a nonlinear increase in the value of deformation ε_χ with time is observed.

5. 2. 3. Conveyor belt element suddenly subjected to a constant load

The conveyor belt element is quite often subjected to a sudden change in the load value as a result of damage to the structural elements of the transport conveyor, which leads to a sharp increase in the primary resistance to the belt movement. The constant stress $\sigma_{\chi 0}$, suddenly applied to the element dS of the conveyor belt, can be represented in the following form

$$\sigma_\chi(\tau)=H(\tau)\sigma_{\chi 0},\quad H(\tau)=\int_{-\infty}^{\tau}\delta(\alpha)d\alpha,$$

$$H(\tau)=\begin{cases}1, & \text{if } \tau\geq 0,\\ 0, & \text{if } \tau<0,\end{cases} \tag{28}$$

where $H(\tau)$, $\delta(\tau)$ are the Heaviside function and the Dirac function, respectively. Substitution of the expression determining the value of the suddenly applied stress (28) into equation (16) makes it possible to obtain an equation that determines the deformation of the belt element

$$\delta(\tau)\sigma_{\chi 0}+\frac{1}{\chi}H(\tau)\sigma_{\chi 0}=\frac{d\varepsilon_\chi}{d\tau}. \tag{29}$$

Having integrated the last equation, let's write down the solution for the load applied at time $\tau=0$ in the form (Fig. 5):

$$\varepsilon_\chi(\tau)=H(\tau)\sigma_{\chi 0}+\int_{-\infty}^{\tau}\frac{1}{\chi}H(\alpha)\sigma_{\chi 0}d\alpha, \tag{30}$$

$$\varepsilon_\chi(\tau)=0,\quad \tau<0,\quad \varepsilon_\chi(\tau)=\sigma_{\chi 0},\quad \tau=0,$$

$$\varepsilon_\chi(\tau)=\sigma_{\chi 0}+\sigma_{\chi 0}\frac{\tau}{\chi},\quad \tau>0.$$

For an arbitrary moment of time when the load is applied $\tau=\tau_s$, the solution to equation (29) can be represented in the general form

$$\varepsilon_\chi(\tau)=H(\tau-\tau_s)\sigma_{\chi 0}+\int_{-\infty}^{\tau}\frac{1}{\chi}H(\alpha-\tau_s)\sigma_{\chi 0}\mathrm{d}\alpha.$$

With an increase in the time of load application, the deformation increases linearly.

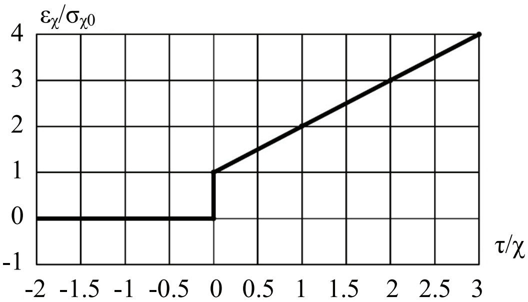

Fig. 5. Deformation of a conveyor belt element under a suddenly applied load $\sigma_{\chi 0}$

If the load is applied for a long time (even at small values), for $\tau/\chi>>1$, when the characteristic process time significantly exceeds the relaxation time ($t>>t_0$), the deformation exceeds the maximum permissible value. The deformation process becomes irreversible.

5. 2. 4. Conveyor belt element with an instantaneous force of resistance to the belt movement

Instantaneous load can be caused by instant braking of a belt element of thickness b and width h as a result of instantaneous sharp jamming of moving or rotating parts of the conveyor structure. The resulting instantaneous stress $\sigma_\chi(\tau)$, caused by the instantaneously acting force of resistance to the belt movement $P_\chi(\tau)$ is determined in the following way

$$\sigma_\chi(\tau)=\delta(\tau)\sigma_{\chi\tau},\ \ \sigma_{\chi\tau}=\text{const},\ \ P_\chi(\tau)=\sigma_\chi(\tau)bh. \qquad (31)$$

At the moment of jamming, a sharp increase in the force of resistance to the belt movement $P_\chi(\tau)$ is observed, which is instantaneous, leading to instant braking of the belt, followed by the restoration of the transport system functioning. Substitution of expression (31) into equation (16) allows one to obtain an equation for determining the change in the deformation of the element dS depending on time

$$\sigma_{\chi\tau}\frac{d\delta(\tau)}{d\tau}+\frac{1}{\chi}\delta(\tau)\sigma_{\chi\tau}=\frac{d\varepsilon_\chi}{d\tau}. \qquad (32)$$

Having integrated the last equation, let's write the solution in the form (Fig. 6)

$$\varepsilon_\chi(\tau)=\int_{-\infty}^{\tau}\frac{1}{\chi}\delta(\alpha)\sigma_{\chi\tau}\mathrm{d}\alpha+\int_{-\infty}^{\tau}\sigma_{\chi\tau}\frac{\mathrm{d}\delta(\alpha)}{\mathrm{d}\alpha}\mathrm{d}\alpha, \qquad (33)$$

$$\varepsilon_\chi(\tau)=\sigma_{\chi\tau}\delta(\tau)+\frac{1}{\chi}H(\tau)\sigma_{\chi\tau},\ \ \varepsilon_\chi(\tau)=0,$$

$$\tau<0,\ \ \varepsilon_\chi(\tau)=\frac{1}{\chi}\sigma_{\chi\tau},\ \ \tau>0, \qquad (34)$$

where

$$\int_{-\infty}^{\tau}\sigma_{\chi\tau}\frac{\mathrm{d}\delta(\alpha)}{\mathrm{d}\alpha}\mathrm{d}\alpha=\sigma_{\chi\tau}\delta(\alpha)\Big|_{-\infty}^{\tau}-\int_{-\infty}^{\tau}\delta(\alpha)\frac{\mathrm{d}\sigma_{\chi\tau}}{\mathrm{d}\alpha}d\alpha=\sigma_{\chi\tau}\delta(\tau),$$

$$\int_{-\infty}^{\tau}\frac{\delta(\alpha)}{\chi}\sigma_{\chi\tau}\mathrm{d}\alpha=\frac{H(\alpha)}{\chi}\sigma_{\chi\tau}.$$

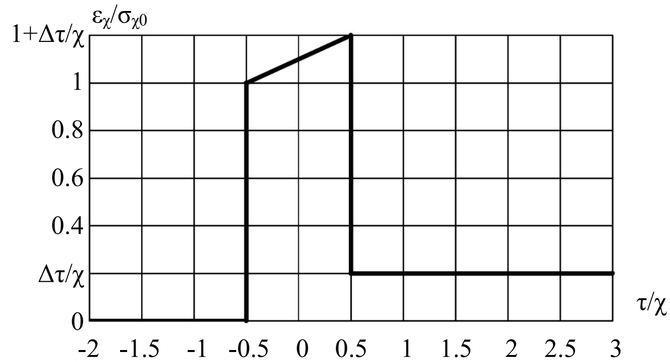

Fig. 6. Deformation of a belt element under an instantaneous applied load $\sigma_\chi(\tau)=\delta(\tau)\sigma_{\chi\tau}$

Analysis of solution (33) for the values of time τ in the vicinity of zero using the Dirac $\delta(\tau)$ and Heaviside $H(\tau)$ functions in the form

$$\delta(\tau)=\begin{cases}1/\Delta\tau, & \text{if } |\tau|\le\Delta\tau/2,\\ 0, & \text{otherwise,}\end{cases}\qquad H(\tau)=\int_{-\infty}^{\tau}\frac{1}{\Delta\tau}\mathrm{d}\alpha, \qquad (35)$$

allowed representing solution (33) in the form

$$\varepsilon_\chi(\tau)=\frac{\sigma_{\chi\tau}}{\Delta\tau}+H(\tau)\frac{1}{\chi}\sigma_{\chi\tau}=$$

$$=\frac{\sigma_{\chi\tau}}{\Delta\tau}\left(1+H(\tau)\frac{\Delta\tau}{\chi}\right)=\sigma_\chi\left(1+H(\tau)\frac{\Delta\tau}{\chi}\right),$$

$$\lim_{\Delta\tau\to 0}\varepsilon_\chi(\tau)=\lim_{\Delta\tau\to 0}\sigma_\chi\left(1+H(\tau)\frac{\Delta\tau}{\chi}\right)=\sigma_\chi. \qquad (36)$$

The behavior of the viscoelastic element for the considered case corresponds to the Hooke model.

6. Discussion of the results of the study of asymptotic Maxwell element models of the conveyor belt

The result of the study is an asymptotic analysis of the Maxwell element model. The peculiarities of the proposed method lie in the use of similarity criteria to describe viscoelastic processes in a belt, the material properties of which correspond to the Maxwell element model. Reducing the original model to a dimensionless form, the specific properties of which are characterized by the values of the similarity criteria of the process under consideration, made it possible to construct asymptotic Maxwell element models for some cases of the transport system functioning. Asymptotic models of a viscoelastic element in the form of simple analytical expressions that determine the relationship between stress and deformation of a conveyor belt element are convenient for practical use in solving specific engineering problems. The application of the proposed approach made it possible to replace the Maxwell element model with an asymptotic model close to the Hooke element model for individual operating modes of a transport conveyor.

The asymptotic analysis of the Maxwell element model clearly demonstrates that at sufficiently high frequencies of longitudinal stress fluctuations in the conveyor belt, at which the oscillation period is much less than the characteristic oscillation decay time, the Maxwell element model can be replaced with a sufficient degree of accuracy by the Hooke element model. This is due to the fact that over a period of time equal to the period of longitudinal oscillations, there is a slight change in stress associated with the component determined by the viscous properties of the Maxwell element model. In this case, the main contribution to the propagation of longitudinal oscillations along the conveyor belt is made, respectively, by the Hooke element (Fig. 1). A similar explanation can be given as a result of the analysis of the characteristic operating modes of the transport conveyor. If the characteristic time of the considered disturbance propagation process is much less than the characteristic decay time of oscillations in the conveyor belt material, then the viscous properties of the Maxwell element can be neglected. In this case, as in the case of high frequencies of longitudinal stress fluctuations in the conveyor belt, the Maxwell element can be replaced with a sufficient degree of accuracy by the Hooke element. The analysis of the propagation of longitudinal disturbances in the belt material is generally based on the numerical solution of the wave equation, the foundation of which is the model of a viscoelastic element [5, 10, 11], in particular, the Maxwell element model in general form (5). This significantly complicates the construction of an analytical solution that determines the amount of dynamic stress in the material along the conveyor belt. An analytical solution can be obtained when using fairly simple models of a viscoelastic element [12], the Hooke element model. Thus, a natural step is to build simplified models of the viscoelastic Maxwell element for the main operating modes of the transport system. The developed asymptotic models of the viscoelastic Maxwell element make it possible to construct an analytical solution of the wave equation for the corresponding operating modes of the transport conveyor, greatly simplifying the analysis of the propagation of longitudinal stresses in the conveyor belt material, which is a significant advantage of the proposed approach. However, it should be noted that the area of application of asymptotic models is limited to the range of limit values of the introduced similarity criteria for a viscoelastic process. A rather important result of the study is that the use of the asymptotic approximation made it possible to substantiate and determine the area of application of the general original Maxwell element model (5). To construct asymptotic Maxwell element models, a linear approximation is used, which limits the accuracy of the model. Future research may be aimed at eliminating this drawback by replacing linear models with nonlinear models with a given degree of accuracy to solve the problem. The possibility and expediency of using nonlinear asymptotic models require additional research. The practical significance of the study lies in the use of the results obtained for the design of optimal speed control systems of the conveyor belt, taking into account restrictions on phase coordinates and the propagation of dynamic stresses in the conveyor belt. A prospect for further research is the analysis of the propagation of stress disturbances in a conveyor belt for cases when the properties of the conveyor belt material can be represented by asymptotic Kelvin-Voigt element models.

7. Conclusions

1. The use of the asymptotic approximation to analyze the characteristic operating modes of an extended transport conveyor, the belt material properties of which correspond to the Maxwell element, made it possible to:

a) determine the stress and deformation relationship for the limit values of similarity criteria;

b) justify the scope of the Maxwell element model.

It is shown that at high frequencies $\omega >> \omega_0$, for which the oscillation period is much shorter than the relaxation time t_0, the behavior of the viscoelastic Maxwell element corresponds to Hooke's law. A qualitative analysis shows that for the oscillation frequencies $\omega = 10^1 \div 10^3$, the characteristic oscillation decay time is several seconds, $t_0 = 1 \div 10$. Thus, in the quasi-stationary mode of acceleration/deceleration of the belt during the transition period, which is several minutes, the arising disturbances quickly damp out. The most dangerous is the initial moment of acceleration/deceleration of the belt. For small frequency values $\omega << \omega_0$, the application of the Maxwell element model requires additional justification.

2. Analysis of the main operating modes of the transport conveyor made it possible to draw additional conclusions: for the case of a constant deformation rate of the conveyor belt, with a characteristic process time significantly exceeding the relaxation time $t >> t_0$, the stress in the conveyor belt tends to a constant value. In this case, an increase in the magnitude of deformation reaches the limit values. For small values of the characteristic process time compared to the relaxation time $t << t_0$, the Maxwell element model can be replaced by the Hooke element model. A similar situation is typical for the case of a constant rate of stress change in a conveyor belt element. Of practical interest is the analysis of the transport conveyor functioning for cases when a constant or instantaneous load is suddenly applied to a belt element. Of particular importance here is the analysis of the transient process with the characteristic process time $t << t_0$. In this case, the Maxwell element model can be replaced by the Hooke element model.

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