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DEVELOPMENT OF AN ESTIMATION-EXPERIMENTAL METHOD FOR ESTIMATING THE PRESERVATION INDICATORS OF SINGLE-USE ARTICLES

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To manage the operation of modern single-use products, it is necessary to evaluate their preservation indicators as uncontrolled, non-repairable, and maintenance-free objects. Data for assessing its parameters are considered as one-time censored samples with continuous monitoring, which does not correspond to the mode of storage of products during operation. Under the conditions of limited volumes of censored samples, it is problematic to identify the parametric model of persistence.

To solve this problem, a non-parametric estimation-experimental method has been devised, which is a set of models for data generation, estimation of the function of the distribution of the preservation period and preservation indicators.

The data generation model is represented by a scheme of operational tests and analytical relationships between the quantities of tested and failed articles. The model of estimating the distribution function describes the process of its construction on the generated data. Models for estimating preservation indicators are represented by ratios for their point and interval estimates, as functionals from the restored distribution function. Unlike the well-known ones, the developed method implements the assessment of indicators under the conditions of combined censorship.

The method can be used to assess the preservation indicators of single-use articles with an error of at least 7 %. At the same time, their lower confidence limits are estimated at 0.9 with an error not worse than 14 % with a censorship degree of not more than 0.23. The restored distribution function agrees well (reliability 0.9, error 0.1) with the actual persistence of articles with censorship degrees of not more than 0.73, which is acceptable for solving the problems of managing their operation

Keywords: preservation indicators, single-use articles, operation management, point assessment, interval estimate

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1. Introduction

Among modern complex technical systems, single-use objects are distinguished [1]. Single-use objects include articles that, before being used for their intended purpose, are in the mode of storage and (or) waiting for use for their intended purpose and can be used for their intended purpose only once. These include guided weapons, for example, aviation weapons, anti-aircraft guided missiles, protection devices for nuclear power plants, catapults, fire extinguishing devices, etc.

These articles are characterized by a long stay under a "storage" mode and (or) waiting for use for its intended purpose and a relatively short duration of use for its intended purpose. This highlights the problem of ensuring high reliability and persistence of such articles. As a rule, they belong to objects of increased danger. Such articles must be maintained in a high degree of readiness for use for their intended purpose and trouble-free operation during the duration of the task while ensuring the predefined level of safety. To improve the efficiency of the management of the operation of single-use articles (SUA), it is necessary to evaluate the indicators of residual preservation with the required accuracy and reliability. This is important for making timely decisions about the possibility of further operation of articles or its termination.

Reducing the cost of operating such articles while meeting the requirements for safety and efficiency of their use is possible by improving the accuracy and reliability of the assessment of preservation indicators. Resolving this issue for modern SUAs, which during operation are not controlled, non-restored, and maintenance-free, is problematic.

The application of known methods to solve these problems is not effective since they usually provide for the type of function of distributing the storage time known, which does not correspond to the actual operation of single-use articles. At the same time, the data for assessing its parameters are considered as one-time censored samples containing quantitative implementations of the storage time of articles and the duration of their trouble-free storage before censorship. The volumes of these samples depend on the size of the batches of products, the conditions and modes of their operation, the duration of trouble-free storage before censorship, and are limited values. Provided that the corresponding parametric model adequately describes the function of distributing the storage time of articles, it is possible to assess the preservation indicators quite accurately even with a censored sample. Under the conditions of limited sample sizes, especially small ones, it is problematic to identify a parametric conservation model since many models can be selected that describe the sample data equally well in terms of statistically consent criteria.

Under these conditions, it is advisable to use non-parametric methods for assessing the preservation indicators of single-use articles to obtain their estimates with acceptable accuracy and reliability.

In this regard, it is relevant to develop methods for assessing the preservation indicators of single-use articles with minimizing the cost of their operation while maintaining the predefined level of safety and efficiency of use.

2. Literature review and problem statement

Modern single-use articles belong to uncontrolled, maintenance-free, non-restorable, and non-repairable articles [2]. Paper [2] reports a method for assessing the preservation indicators of on-board equipment of anti-aircraft guided missiles (AGMs) with periodic monitoring of their technical condition. The disadvantage of the method is the impossibility of removing workable articles from tests and its applicability only to one-time censored samples. Study [3] establishes requirements for the average and gamma-percent storage time of articles. In [4], the storage period is understood as the calendar duration of storage and (or) transportation of an object, during which the object retains the ability to perform the required function. In addition, to solve the problems of managing the operation of such articles, average and gamma-percent residual retention periods are used [5]. For the experimental assessment and control over the average and gamma-percent retention periods, papers [3, 4] chose the "direct storage method" set out by regulatory documents. This method involves conducting operational tests under the conditions of long-term storage of articles with periodic monitoring of their technical condition. At the same time, implementations of pre-failure storage durations are characterized by appropriate interval uncertainty due to the frequency of control, and the uncertainty of incomplete sales of non-failed articles at the time of control.

To assess the preservation indicators, in [3, 4], test plans [NUT] and [NUr] are used, which involve continuous monitoring of the technical condition of articles and do not take into consideration these uncertainties. In [3], it is proposed to evaluate the preservation indicators using parametric methods under the assumption of continuous monitoring of the technical condition. Work [4] considers a pilot assessment of the gamma percent persistence of articles for [NUT], [NMT], and non-failure binomial test plans. At the same time, the quantitative values of the implementations of the retention periods are assumed to be known under the assumption of the exponential law of their distribution. Study [5] additionally deals with the assessment of durability indicators. The disadvantages of the cited study are the impossibility of removing articles (without failure) before the end of the tests, the continuity of monitoring the technical condition, which provides for accurate fixation of the moment of failure. Paper [6] analyzes the state of work in the field of censored samples and methods for assessing reliability, including nonparametric ones. A method of sequential transition to a new coordinate system for test plans with continuous control of articles has been developed. The issues of assessing the performance of articles with periodic monitoring are not considered. Study [7] discusses the planning of abbreviated reliability tests, parametric, and non-parametric methods for its evaluation. The disadvantages of [3–7] are not taking into consideration the uncertainties associated with the peculiarities of storing SUA during operation, which, as a rule, leads to significant errors in assessing the preservation indicators.

The preservation indicators are inherently consistent with the durability indicators and are determined by the corresponding dependences. The authors of [8] developed a non-parametric method for assessing the residual durability of a radio engineering system operated according to its technical condition. The disadvantage of the method is its inapplicability to uncontrolled, non-recoverable articles.

Paper [9] reports parametric methods for assessing reliability indicators, involving knowledge of the type of resource allocation functions (service life) that are used for engineering articles, radio electronics, etc. The disadvantage of these methods is the need to confirm the law of distribution of service life (resource), which is difficult under the conditions of censored samples.

Methods based on the use of diffusion distributions involve the use of diffusion monotone [10] for mechanical articles and diffusion non-monotone [11] for electronic products as a model for the distribution of service life (resources). The application of these methods is difficult to articles with a complex hierarchical structure, including heterogeneous elements.

Study [12] considers the issues of taking into consideration the nature of statistical information in various plans for testing technical articles for reliability, grouping data, and building empirical distribution functions by parametric and non-parametric methods. However, [12] does not address the issues of assessing the distribution function under periodic inspections and repeated censorship, which does not make it possible them to be used to assess the preservation indicators of SUA.

In [13], based on time series analysis methods, a method for estimating the residual life of mechanical articles and structures has been developed. The method makes it possible to predict the fixed values of the resource in the near future. Study [14] developed an adaptive algorithm for predicting residual resources using time series analysis methods. These methods assume a known set of resource values recorded at given points in time. Their disadvantage is the strict requirements for the initial data in the form of values of the technical resources of the product at specified points in time, which imposes significant restrictions on their use.

Paper [15] considers methods for assessing the reliability of systems with temporary redundancy. Work [16] considers an analytical method for assessing the reliability of non-recoverable articles with a hierarchical structure and various redundancy options. Such methods [15, 16] are applicable for long-term application systems with continuous monitoring. In [17], a method for predicting the reliability of technical systems using a model of the Markov degradation process is investigated. The method is applicable for assessing the reliability of non-recoverable articles of continuous long-term use. However, the cited works [15–17] do not consider the tasks of assessing the preservation indicators of SUA.

The authors of [18] examine a multi-sample hybrid censorship model of the I type for a particular system with a sequential structural scheme of reliability. This model makes it possible to obtain ratios for finding confidence intervals. This model cannot be used for repeatedly censored samples (RCSs) with interval uncertainty. Studies [19, 20] develop procedures for testing hypotheses about the compliance of product quality with the required level for progressive censored type I sampling for Gompertzian [19] and Chen [20] lifetime distributions. The disadvantage of the models reported in [18–20] is their applicability only for once censored samples of articles with continuous control.

Paper [21] shows that when analyzing values of the "lifetime" type from censored samples, nonparametric criteria for testing hypotheses lose the property of "freedom from distribution". Study [22] investigates the application of various consent criteria to test simple and complex hypotheses against unilateral censored samples. In this case, tables of upper percentages of points for the distributions of statistics of nonparametric criteria such as Kolmogorov, Kramer-Mises-Smirnov, and others are constructed by modeling methods when testing simple hypotheses. These results should be used to correctly select consent criteria in limited censored samples. However, their application for RCS with interval uncertainty is limited.

Work [23] solves the problem of estimating the unknown parameters of the predefined distribution function based on the data of a progressive hybrid censored sample of type II using the methods of maximum plausibility and Monte Carlo. The disadvantage of the cited work is the impossibility of using the obtained results for non-parametric assessment of the preservation indicators of SUA.

In [24], a hierarchical model is developed for assessing reliability indicators with a known function of distributing the MTBF of articles, random censorship of their service lives, and continuous monitoring of the technical condition. The disadvantage of the cited work is its limited applicability, due to the need to know the laws of distribution of MTBF of articles and censorship, quantitative values of implementations of MTBF.

In [25], a Bayesian method for assessing the reliability of articles from censored samples during acceptance tests is proposed. This method is designed to justify sample sizes when planning acceptance tests and meeting the requirements for specified risks. The disadvantage of the cited paper is the assumption of continuous control of articles, a known a priori distribution, and a single censored sample.

Thus, the known methods for assessing the preservation indicators are focused on the availability of information about the type of law for the distribution of the terms of preservation of articles and large sample volumes, etc. Their application to assess the preservation indicators of uncontrolled, non-recoverable SUAs is problematic.

The development of an estimation-experimental method for assessing the preservation indicators of SUAs under the conditions of uncertainties caused by ignorance of the law of distribution of the preservation period, periodic control, etc., is a relevant task.

3. The aim and objectives of the study

The purpose of the research is to develop an estimation-experimental method for assessing the preservation indicators of single-use articles to improve the efficiency of their operation management.

To accomplish the aim, the following tasks have been set:

– to develop a model for the formation of data of censored samples of single-use articles with periodic monitoring;

– to devise a model for assessing the function of distributing the storage time of single-use articles;

– to build models for assessing the preservation indicators of single-use articles according to censored samples;

– to study the characteristics of the estimation-experimental method for assessing the preservation indicators of single-use articles.

4. The study materials and methods

When conducting research, methods of probability theory, mathematical statistics, reliability theory, methods of system analysis, methods of mathematical modeling were used. When carrying out experimental studies, a computer algebra system from the class of computer-aided design systems Mathcad 15.0 M05 (USA) was employed. When validating the proposed solutions, analytical and empirical methods of comparative analysis were applied.

The following limitations and assumptions are accepted in our study:

– anti-aircraft guided missiles (AGMs) of the S-300PS type anti-aircraft missile system (the range is up to 75 km) are considered as SUAs [26];

– AGMs are considered as uncontrolled, non-recovered, maintenance-free, and non-repairable objects;

– the mode of AGM storage during operation under normal conditions is considered with the assumption of a monotonic decrease in the level of their preservation;

– the technology of AGM serial production has been worked out and provided in full on the studied articles;

– control of the performance of AGM samples of different age groups are reliable;

– the operating test conditions are the same for all articles of the AGM samples subjected to the tests.

5. Results of studying the method for assessing the preservation indicators of single-use articles

5. 1. Development of a model for the formation of data of censored samples of single-use articles with periodic control

Periodic monitoring of the technical condition (TCM) of the SUA sample leads to acquiring information about their technical condition (TC) in the form of observation intervals. The high persistence of these articles does not make it possible for a certain period of their storage to bring all articles to failure. Therefore, the assessment of SUA preservation indicators before the failure of all tested articles is a typical situation in the management of their reliability and (or) operation. Performance tests (observations) of SUA during storage will be represented by the test plan [[NU(T_1, n_1), (T_2, n_2), ..., (T_{J-1}, n_{J-1}), T_J]], according to which N articles are tested simultaneously. Articles that fail during testing are not restored or replaced. When the storage time T_1 is reached, n_1 of the non-failed articles is removed from the test (if the number of operable articles is greater than n_1, otherwise the tests are discontinued), etc. The tests shall cease at the end of the storage period of T_J. When conducting operational tests for reliability based on the results of periodic TCM, it is known that by a certain point in the duration of storage T_j, part of the articles is in a working state, and the other part is in an inoperable state. In this case, the implementation of SUA storage durations are censored samples. In a general case, a selective periodic TCM for a part of SUAs can be non-destructive, and for another part of the articles – destructive. For this and other reasons, for the j-th period of TCM [T_{j-1}, T_j], we shall distinguish between failed articles (in the amount r_j) and those that were removed from the tests at the end of this period, workable (in the amount of n_j). The data of such samples are characterized by a greater degree of uncertainty of information than with a complete sample.

When developing (choosing) a method for assessing the persistence of such articles, it is necessary to clearly understand the process of forming these censored samples and their structure.

Let us consider this process in more detail (Fig. 1). As a generator of the shelf life of articles, a “mechanism” is considered, which assumes the presence of a vector N of random variables X_1, X_2, ..., X_N (studied variables) with a distribution function (d. f.) $F(x)$ (unit 2). The generated data are their independent implementations x_1, x_2, ..., x_N. Other components of the censored sample data generation model are the censorship generation model (unit 1) and the x_1, x_2, ..., x_N (unit 3) model of interaction between censorship and data.

The combination of these three components determines the specificity of the process of generating data of operational tests (observations) of SUA according to the plan [[NU(T_j, n_j), T_J]] (Fig. 1). The model of the formation of censorship (unit 1) is deterministic and represents the moments of conducting the TCM T_j planned in accordance with the period T_c and the operational testing (observations) of the operational testing (observations) of the operational SUA in the amount of n_j. As a result of the influence of censorship (T_j, n_j) on data x_i in accordance with the interaction model (unit 3), the resulting process is formed in the form of RCS data (unit 4, Fig. 1). The RCS data are conditional and incomplete implementations. For clarity, Fig. 2 shows a diagram of the operational tests (observations) of the SUA sample (Fig. 2, *a*) and the corresponding implementation diagram (Fig. 2, *b*) for the test plan [[11U (T_j, 1), $j=\overline{1,4}$, T_5]].

The values of conditional implementations ( ) (Fig. 2, *b*) are determined by the duration of trouble-free storage of articles to the time of TCM, at which their failures are detected. Incomplete implementation values ( ) are determined by the uptime of each of the tested articles by the time the last TCM is completed. In this case, the RCS data are obtained grouped by the TCM intervals [T_{j-1}, T_j], j=1,..., 5.

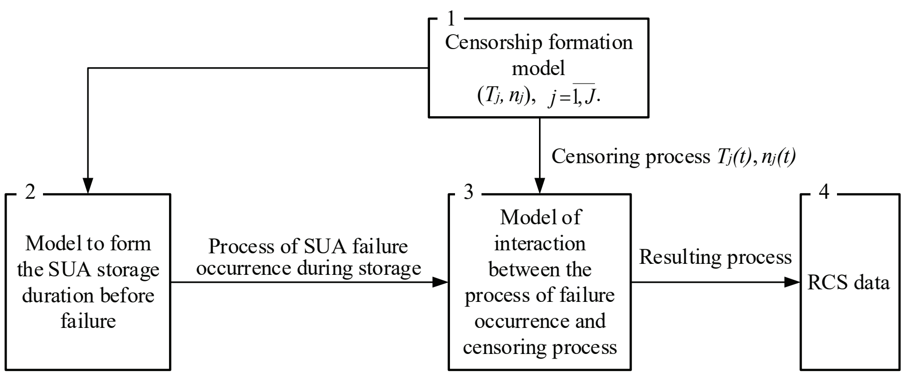


Fig. 1. RCS data generation model for single-use articles in an operational test plan [[NU(T_j, n_j), T_J]]

The implementation diagram (Fig. 2, *b*) is the resulting process of the RCS product data generation model in the test plan [[NU (T_j, 1), $j=\overline{1,4}$, T_5]]. In the classification [5, 6, 12], these data represent a progressive censored sample of type I.

Several batches of SUAs are in operation, i.e. serially manufactured according to the same technology and component elements of SUAs of different years of release. To assess the indicators of SUA preservation, samples of articles from various batches (hereinafter referred to as articles of different age groups) are formed, ordered by the calendar duration of storage during operation, starting from the moment of commissioning (Fig. 3, *a*).

Introduce the following designations (Fig. 3):

– $T_{\ell 0}$ is the moment of commissioning of articles of the ℓ-th age group;

– $T_{\ell j}$ is the moment of the j-th TCM of articles of the ℓ-th age group;

– $[T_{\ell j-1}, T_{\ell j}]$ is the j-th time interval for observing (storing) a sample of articles of the ℓ-th age group;

– J is the number of the age groups of articles;

– $T_{\ell j} - T_{\ell j-1}=T_c$ is the TCM period of the selected articles;

– $N_{\ell j}$ is the number of PCs of articles of the ℓ-th age group at the beginning of the j-th time interval or the number of articles of the ℓ-th age group observed (tested) in the j-th time interval.

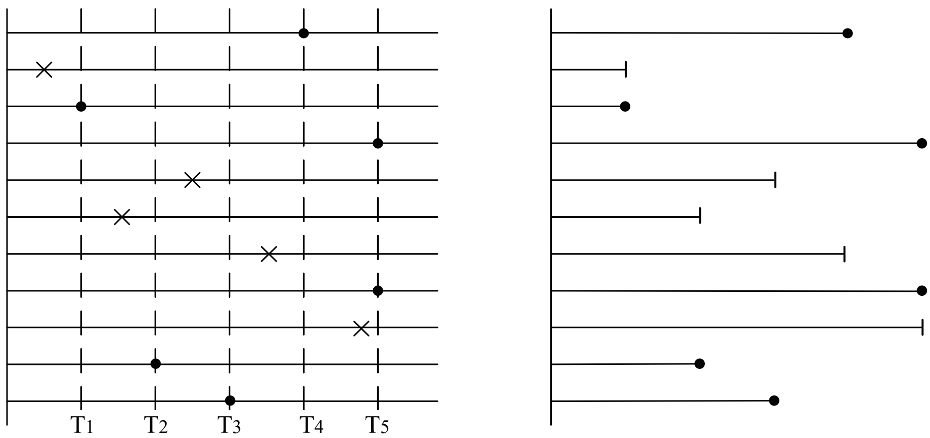


Fig. 2. Graphical representation of RCS data from a sample of 11 single-use articles in a test plan [[11U (T_j, 1), $j=\overline{1,4}$, T5]]: *a* — test process diagram; *b* — implementation diagram

Samples of articles of different age groups brought to a single point of the beginning of storage T_0 (Fig. 3, *b*) form J time intervals of observations $[T_{j-1}, T_j]$ with the number of observed articles in the j-th time interval

$$N_j = \sum_{\ell=1}^{L} N_{\ell j}, \quad j=\overline{1,J}. \tag{1}$$

The results of operational observations (tests) of samples of articles of L age groups during J time intervals at $J>L$ (Fig. 3, *b*) can be represented by two rectangular matrices $\|r_{\ell j}\|$ and $\|n_{\ell j}\|$ whose dimensionalities are $L\times J$ (Fig. 4). The $r_{\ell j}$ element of the matrix $\|r_{\ell j}\|$ characterizes the number of articles of the ℓ-th age group that failed during the j-th time interval $[T_{j-1}, T_j]$. The $n_{\ell j}$ element of the matrix $\|r_{\ell j}\|$ characterizes the number of SUAs of the ℓ-th age group censored on the right at the time of the j-th TCM T_j.

The elements of the ℓ-th line $r_{\ell j}$, $j=\overline{1, J-L+\ell}$ in the matrix $\|r_{\ell j}\|$ are a time series of the number of failed articles of the ℓ-th age group over J J–L+$\underline{\ell}$ periods of TCM. The elements of the j-th column $r_{\ell j}$, $\ell=\overline{1,L}$ of this matrix characterize the distribution of the number of failed articles by L age groups in the j-th time interval.

Elements r_{1L}, r_{2L+1}, ..., r_{LJ} of the matrix $\|r_{\ell j}\|$ characterize the number of failed articles for each age group as the results of the current (latest) TCM of articles of L age groups. The relevant elements placed to the left of the above indicate the number of failed articles for each period of the TCM preceding the current one, and the unfilled elements placed to the right – the corresponding results of SUA inspection over subsequent periods.

The elements of the ℓ-th line $n_{\ell j}$, $j=\overline{1, J-L+\ell}$ of the matrix $\|n_{\ell j}\|$ are a time series of the number of SUAs censored on the right of the ℓ-th age group over J–$\underline{L+}\ell$ TCM periods. The elements of the j-th column $n_{\ell,j}$, $\ell=\overline{1,L}$ of this matrix characterize the distribution of the number of censored articles on the right by L age groups at the time of the j-th TCM.

Elements $n_{1,L}$, $n_{2,L+1}$, ..., $n_{L,J}$ of the matrix $\|n_{\ell j}\|$ characterize the number of non-failing articles established by the results of the current (last) TCM of articles of L age groups. The corresponding elements placed to the left of the above indicate the number of non-failed articles for each TCM period preceding the current one, and the unfilled elements placed to the right – the corresponding results of SUA TCM for subsequent periods.

With $J=L$ (the number of grouping intervals is equal to the number of age groups), these matrices are converted to lower triangular ones, and the elements of their main diagonals ($r_{\ell,j}$ and $n_{\ell j}$ at $\ell=j$) characterize the results of the current TCM (latest study) of articles from L age groups.

This representation of the results of operational observations (Fig. 3, 4) makes it possible to write the following ratios for:

– the number of articles of the ℓ-th age group observed in the j-th time interval

$$N_{\ell j}=r_{\ell j}+n_{\ell j}, \tag{2}$$

– the number of articles N_j observed in the j-th time interval (1),

– the number of failed articles of L age groups observed in the j-th time interval (or the number of conditional implementations over the j-th time interval)

$$r_j = \sum_{\ell=1}^{L} r_{\ell j}, \tag{3}$$

– the number of censored on the right articles of L age groups, at the time of completion T_j of the j-th TCM (or the number of incomplete implementations over the j-th time interval)

$$n_j^* = \sum_{\ell=1}^{L} n_{\ell j}. \tag{4}$$

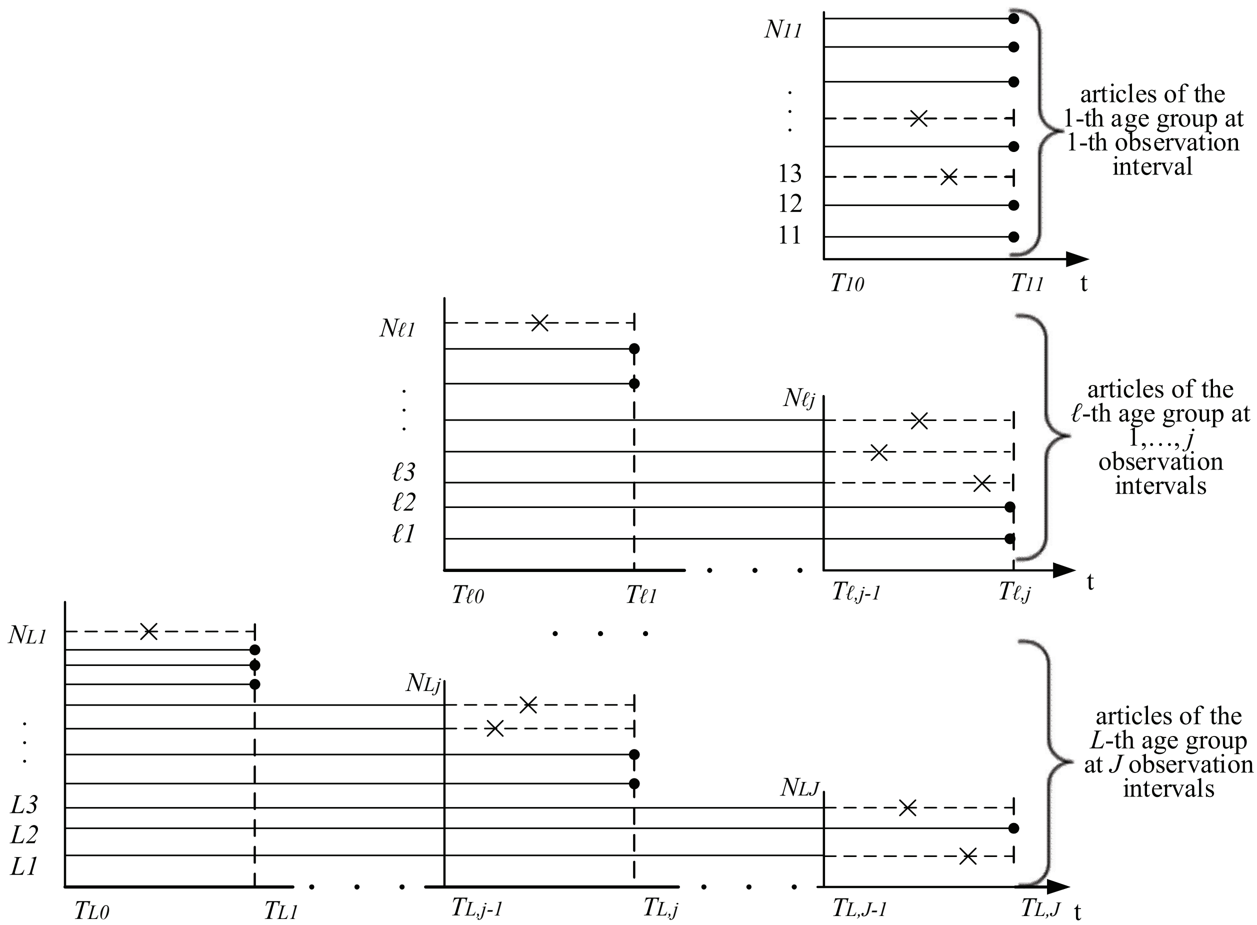
From the known values of r_{j-1} and n_{j-1}^* it is possible to determine the number of articles observed in the j-th time interval

$$N_j = N_{j-1} - \left(r_{j-1} + n_{j-1}^*\right), \tag{5}$$

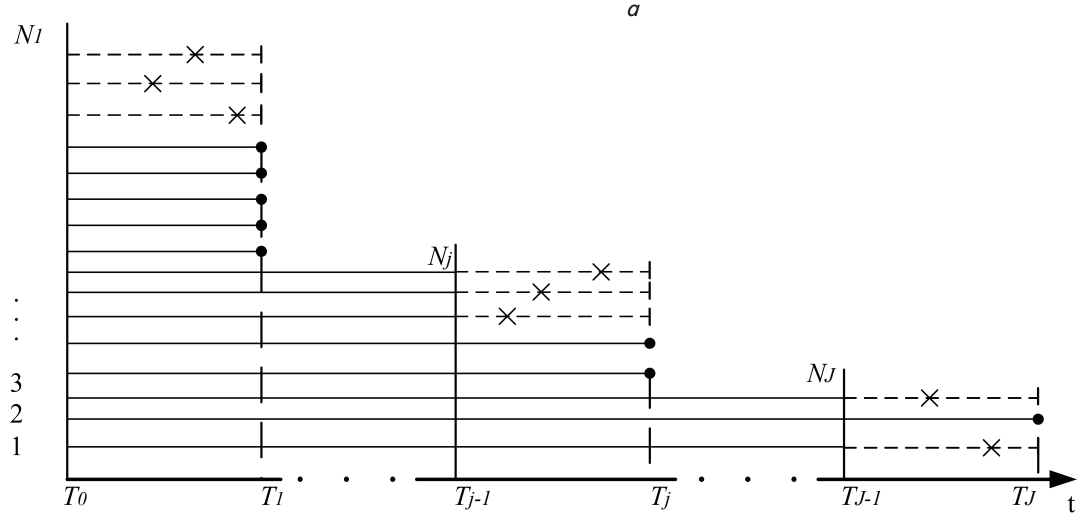
or

$$N_j = N - \sum_{i=0}^{j-1} \left(r_i + n_i^*\right). \tag{6}$$

All discussed above (Fig. 3, 4), as well as ratios (1) to (6), characterize the process of formation of RCS in regularly performed periodic TCM of articles and are used further to assess their preservation indicators.



a



b

Fig. 3. Scheme of the process of operational tests (observations) of samples of articles of L age groups in the test strategy $[[NU(T_j, n_j), T_j]]$: *a* – the initial scheme of operational tests of samples of articles of L age groups at j-th observation intervals, $j=\overline{1,J}$; *b* – a scheme of operational tests of samples of articles of L age groups grouped by J observation intervals given to a single point T_0 of the beginning of observations

When conducting one-time SUA technical surveys in different age groups, data on their technical condition for previous intervals are not available in whole or in part, and the initial volumes of $N_{\ell 0}$ samples, as a rule, are not known. This situation is also typical in the absence or irregular conduct of periodic TCM of articles. In this case, the elements of the $\|r_{\ell j}\|$ and $\|n_{\ell j}\|$ matrices located to the left of the diagonal corresponding to the one-time survey are undefined. At the same time, information about SUA failures over the intervals preceding a one-time examination is not available, the sample is censored not only on the right but also on the left. Assessment of preservation indicators based on such data is carried out with significant errors.

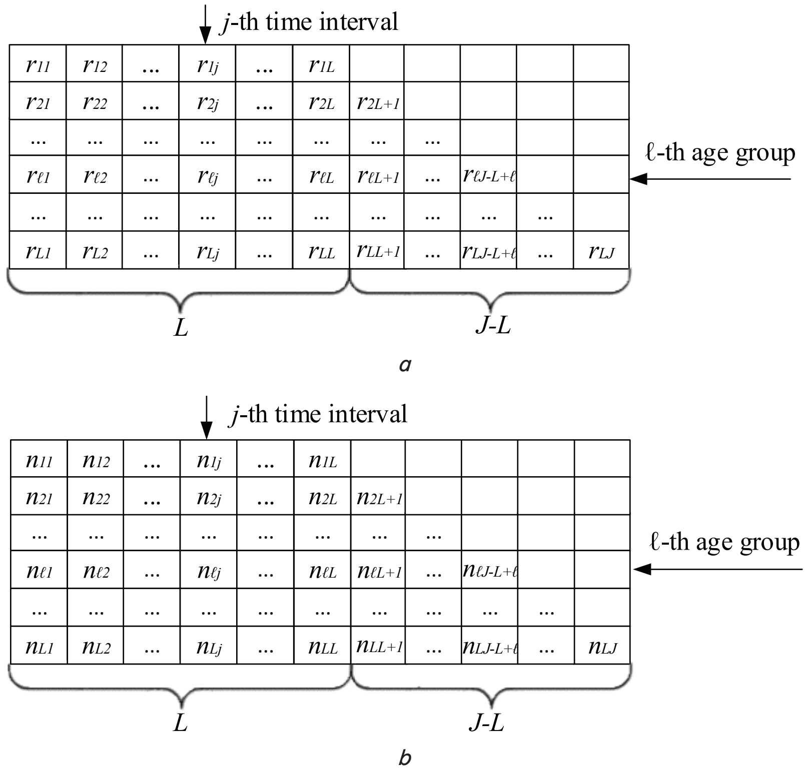


Fig. 4. Results matrices of operational observations (tests) of samples of articles of L age groups during J time intervals under the test strategy $[[NU(T_j, n_j), T_J]]$: a – matrix $\|r_{\ell j}\|$ characterizing the number of failed articles; b – matrix $\|n_{\ell j}\|$, which characterizes the number of censored articles on the right

5. 2. Development of a model for evaluating the function of distributing the shelf life of single-use articles

The implementation of the plan $[[NU(T_j, n_j), T_J]]$ of operational tests (observations) produces the RCS of type I, the statistical structure of which is shown in Fig. 3*b*. To estimate the d. f. of sample duration of storage to failure, the researcher has the following baseline data.

Test intervals (observations) $[T_{j-1}, T_j]$, $j=\overline{1,J}$, determined by the TCM plan, and their results grouped according to these intervals. At the same time, N_1 articles were observed at interval $[0, T_1]$, at the time T_1 the number of failed articles r_1 and the number of workable articles n_1 removed from the tests are known. In the interval $[T_1, T_2]$ N_2 articles were observed, at the time T_2 – respectively, r_2 of failed and n_2 removed from the tests of workable articles, etc. At the last J-th test interval, N_J articles were observed, at the time T_J – respectively, r_J failed and n_J withdrawn from the tests of workable articles. The type of function for distributing the duration of storage of articles to failure is unknown.

We shall find estimates for the values of the empirical distribution function (EDF) at certain points for its subsequent construction and use in assessing the preservation indicators.

Evaluation of EDF values from these initial data should be carried out under the conditions of combined censorship. Such censorship is due to the interval uncertainty of conditional implementations based on the results of observations of the studied continuous random variable (r. v.) only at the moments of TCM, and the uncertainty of incomplete implementations censored on the right. Incomplete implementations are due to the completion of the next control and articles that have not failed by this time. At the same time, in contrast to the well-known nonparametric methods for evaluating and analyzing the reliability of articles from censored samples, it is necessary to use rank ordinal statistics [27], rather than their quantitative values.

In this regard, a model is being developed for estimating the values of d. f. of the storage periods of SUAs at combined censorship with the use of the theory of rank ordinal statistics [27]. To this end, we shall first select the appropriate method of assessment in censored samples according to the criterion of accuracy.

In the case under study, the number of conditional implementations at the observation intervals $[T_{j-1}, T_j]$ may be small. At the same time, the number of observed articles N_j with an increase in the interval number can be significantly reduced. This leads to the need to estimate EDF values under the conditions of limited sample sizes, including small ones.

Among the nonparametric methods of estimating from censored samples, including small ones, the most preferred are the Kaplan-Mayer method (the so-called PL–assessment) and the method of sequential transition to a new coordinate system (MSTNCS) [6]. The MSTNCS assessment is more general and is represented in an additive form while a PL estimate takes a multiplicative form. The additive form of evaluation is more convenient when studying its properties and computational procedures. In this case, the estimation problem is considered sequentially at each j-th interval for N_j articles (which did not fail during the previous intervals) in the new coordinate system using the complete sampling apparatus.

The closest analog to the developed model is the EDF model of MTBF for the right of type I, obtained for the operational test plan of non-repairable articles of the $[NU(n_j, T_j), T_J]$ type with continuous TCM. In this case, complete implementations (MTBF $t_i^{(j)}$, $(i=\overline{1,r_j})$) in the j-th observation intervals $j=\overline{1,J}$ and incomplete implementations due to censorship on the right are known. In accordance with the STNCS method developed in [6], the estimate of d. f. $F(t)$ is determined from the ratios:

$$\hat{F}(t)= \\ =\begin{cases} 0, & t\le 0, \\ \hat{F}_1=\dfrac{\ell}{N_{y_1}}, & 0\le t\le T_1, \ell=0,1,\ldots,r_1, \\ \hat{F}_1+\left(1-\hat{F}_1\right)\dfrac{i}{N_{y_2}}, & T_1\le t\le T_1, i=0,1,\ldots,r_2, \\ \ldots & \ldots \\ \hat{F}_{J-1}+\left(1-\hat{F}_{J-1}\right)\dfrac{z}{N_{y_J}}, & T_{J-1}\le t\le T_J, z=0,1,\ldots,r_J. \end{cases} \quad (7)$$

In (7), the found estimates of EDF F_i at points i correspond to the value of the i-th implementation of MTBF (or the i-th number of the variation series of operation $t_i^{(j)}$). In this case, it is assumed that all incomplete operations in the j-th observation interval end at the time T_j. If this condition is met for the J-th interval, then $n_J \neq 0$ and the estimate $F_J < 1$. Otherwise, the censored sample ends with full operation, i.e. $n_J=0$ and $F_J = 1$.

In the case under study, instead of complete implementations $t_i^{(j)}$, the number of conditional implementations that fell within the observation interval under consideration $[T_{j-1}, T_j]$ is known. These implementations can be represented by a single point at each interval. The coordinates of this point should correspond to the largest implementation in the examined interval. Relations for its coordinates can be obtained by methods of the theory of ordinal statistics. Suppose that the EDF values in the interval in question are evenly distributed. In this case, in N_j observations, the event under study is implemented r_j times. Then the estimate of the value of the EDF F_j (an ordinate of the point) will be found as expectation r_j of the ordinal statistics [27]:

$$F_j = r_j \big/ \left(N_j + 1\right). \tag{8}$$

The abscissa estimate of this point will be obtained in the same way. Suppose that the conditional implementations grouped in this interval are r_j independent evenly distributed r. v. The problem of finding the moment of occurrence of the last r_j-th failure in the observation interval of duration T_c (a TCM period) is reduced to the estimation of expectation of the r_j-th ordinal statistics r_j of independent r. v. It is known [27] that expectation of the i-th ordinal statistics under these assumptions is determined from the ratio:

$$M\left[x_{r_j}^{(i)}\right] = i \big/ \left(r_j + 1\right),\ 1 \le i \le r_j. \tag{9}$$

Then the abscissa x_{rj} of the EDF point in question relative to the beginning of the interval in question can be found as

$$x_{rj} = T_c\, r_j / (r_j + 1). \tag{10}$$

From (10), it follows that at $r_1=1$ the abscissa $x_1=T_c/2$; at $r_2=2$ the abscissa $x_2=(2/3)T_c$, ..., at $r_j>>1$ $x_{rj} \approx T_c$, i.e. the middle of the j-th interval can be attributed to the EDF value for this interval only in the special case when one conditional implementation falls into this interval. With two conditional implementations, the value of the EDF should be assigned to the abscissa of the point 2/3 of its length from the beginning of the interval, and so on. The right end of the interval can only be used as a limit case if the value of the conditional implementations r_j in this interval is large enough. Determining the coordinates of EDF points at each grouping interval using ratios (8), (10) makes it possible to reduce the bias of EDF estimates.

The d. f. estimate $F(t)$ at the observation interval $[0, T_J]$ is determined from the following ratios:

$$\hat{F}(t) = \begin{cases} 0, & 0 \le t < t_1, \\ \hat{F}_1 = r_1 \big/ \left(N_1 + 1\right), & t_1 \le t < t_2, \\ \hat{F}_j = \hat{F}_{j-1} + \\ +\left(1 - \hat{F}_{j-1}\right) r_j \big/ \left(N_j + 1\right), & t_j \le t < t_{j+1}, j = \overline{2, J-1}, \\ \hat{F}_J = \hat{F}_{J-1} + \\ +\left(1 - \hat{F}_{J-1}\right) r_J \big/ \left(N_J + 1\right), & t_J < t, t_J < t \le T_J. \end{cases} \tag{11}$$

$$t_j = \begin{cases} T_{j-1} + T_c, & r_j = 0, \\ T_{j-1} + \dfrac{r_j}{\left(r_j + 1\right)} T_c, & r_j > 0, \end{cases} \quad j = \overline{1, J}. \tag{12}$$

Built in accordance with (8), (10) to (12), EDF d. f. $F(t)$ for the interval $[0, T_J]$ takes the form of a non-decreasing step function, the jumps of which occur at the points t_j. The duration of SUA storage before failure is continuous r. v., its d. f. and the corresponding function of the accumulated failure rate $\Lambda(t)$ are continuous. The relationships for such functions are $F(t)=1-\exp(1-\Lambda(t))$, $\Lambda(t)=\ln[1/(1-F(t))]$ [9].

To represent the EDF $F(t)$ as a continuous function, let us find its values at $t=T_j$, $j = \overline{0, J}$. From (11) to (12), it follows that $F(0) = 0$; $F(T_1) = r_1 / (N_1 + 1)$, etc. For each value F_j at T_j, $j = \overline{0, J}$, we shall find the corresponding value $\Lambda_j = \ln\left[1 \big/ \left(1 - F_j\right)\right]$. It is known that the persistence of SUAs is characterized by a monotonic non-decreasing function $\Lambda(t)$. Then, according to the known coordinates of the points (T_j, Λ_j), $j=0, ..., J$, we shall construct this dependence by the method of least squares in the form of a polynomial

$$\tilde{\Lambda}(t) = a_1 t + a_2 t^2 + \ldots + a_k t^k,\ t \in [0, T_J]. \tag{13}$$

In this case, it follows from the condition $\Lambda(0) = 0$ that the coefficient of the polynomial a_0 must be equal to 0. At the same time, it is assumed that the best (optimal) form of dependence is chosen in the sense of the minimum residual variance, based on a sequential search of their variants. For a known dependence $\tilde{\Lambda}(t)$, the reduced d. f. takes the form $\tilde{F}(t) = \exp\left(-\tilde{\Lambda}(t)\right)$, $t \in [0, T_J]$. If the J-th interval ends with a conditional operation rate, then $n_J=0$, $F_J = 1$, and the construction of $\tilde{F}(t)$ d. f. is completed. Otherwise, it is necessary to define this function for subsequent intervals $[T_{J+i-1}, T_{J+i}]$. Let us solve this problem under the assump-tion that the pattern of change $\tilde{\Lambda}(t)$ established at the observation intervals $[T_{j-1}, T_j]$, $j = \overline{1, J}$, continues in the future. To this end, we extrapolate the $\tilde{\Lambda}(t)$ function into intervals $[T_{J+i-1}, T_{J+i}]$, $i = \overline{1, m}$ and $[T_{J+m}, \infty)$, where the moment T_{J+m} is characterized by a magnitude $\tilde{\Lambda}_{J+m}$, which corresponds to a sufficiently small value ε probability of fail-safe storage, i.e., $\tilde{\Lambda}_{J+m} = \ln[1/\varepsilon]$. Thus, the given value $\varepsilon=0.05$ corresponds to $\tilde{\Lambda}_{J+m} = 3$, $\varepsilon = 0.01 - \tilde{\Lambda}_{J+m} = 4.605$, $\varepsilon = 0.001 - \tilde{\Lambda}_{J+m} = 6.909$.

The desired d. f. is determined from the following formula

$$\tilde{F}(t) = 1 - \exp\left[-\tilde{\Lambda}(t)\right],\ t \in [0, \infty). \tag{14}$$

When forecasting $\tilde{F}(t)$ for large forecasting intervals, errors are possible associated with a change in the pattern $\tilde{\Lambda}(t)$ at such intervals. To reduce such errors, it is advisable to calculate the dependences $\tilde{\Lambda}(t)$ by the method of "weighted deviations", attributing more weight to the points at the right end of the variation series of the sample [7], for example, the method of exponential smoothing, etc. Consider a possible application of this method at forecasting intervals.

Let the J-th observation interval for conditional implementations be followed by k censored implementations. At the same time, the J-th observation interval corresponds to a known estimate $\tilde{\Lambda}_J$. The subsequent k of potential failures at the intervals $[T_{J+i-1}, T_{J+i}]$ corresponds to k values of the accumulated failure rate Λ. The predicted values of the accumulated failure rate Λ_{J+i}, $i = \overline{1, k}$ can be found using known ratios for grouped type I data [7] characteristic of operational observations (tests) of periodically controlled SUAs. With respect to the case in question, we obtain

$$\Lambda_{J+1}=\tilde{\Lambda}_J+\frac{1}{N_J},\quad \Lambda_{J+2}=\tilde{\Lambda}_{J+1}+\frac{1}{N_{J+1}-1},\ \ldots,$$

$$\Lambda_{J+k}=\tilde{\Lambda}_{J+k-1}+1. \tag{15}$$

At certain intervals of the calendar duration of storage $[T_{J+i-1}, T_{J+i}]$, on the extrapolated dependence $\tilde{\Lambda}(t)$, we shall find the corresponding boundaries of the parameter Λ, i.e. $\Lambda_{J+i-1}, \Lambda_{J+i}$. Then, according to the k predicted values of Λ_{J+i}, $i=\overline{1,k}$, and the boundaries of the intervals $\left[\Lambda_{J+i-1},\Lambda_{J+i}\right]$, one can find the numbers of conditional implementations that fell into the corresponding intervals. Let the interval $[T_{J+i-1}, T_{J+i}]$ include k_i conditional implementations, and $\sum_{i=1}^{s}k_i=k$, where s is the number of the last forecast interval.

Then each of the s forecast intervals at the time TJ+i corresponds to the forecast value of the accumulated failure rate $\hat{\Lambda}_{J+k_i}$. If $\hat{\Lambda}_{J+k_i}<\tilde{\Lambda}_{J+i}$, then the predicted value $\tilde{\Lambda}_{J+i}$, determined by the extrapolated dependence will be pessimistic from the point of view of the probability of fail-safe storage and the $\tilde{\Lambda}_{J+i}$ values at the forecast intervals should be adjusted to the level $\hat{\Lambda}_{J+k_i}$. Otherwise, the extrapolation results do not need to be adjusted.

The reliability of the developed model is confirmed by the correct formulation of the problem for its development, the correct application of the methods of probability theory, mathematical statistics, reliability theory, methods of system analysis, mathematical modeling, analytical and empirical methods of comparative analysis. The adequacy of the model of the d. f. of the shelf life of SUAs to the real law of their persistence was checked using the Kolmogorov criterion at various values of the degree of censorship of the samples.

5. 3. Development of models for assessing the persistence of single-use articles based on censored samples

The developed method involves point and interval estimation of the following indicators of SUA preservation indicators: average (T_{av}) and gamma-percentage (T_γ) preservation periods, the average ($T_{av}(\tau)$) and gamma-percentage ($T_\gamma(\tau)$) residual storage periods, etc. The preservation indicators of SUAs are calculated as some functions from the assessment of the d. f. of their shelf life $\tilde{F}(t)$.

Determining the point estimates of these indicators is proposed on the basis of the ratios for their calculation according to the d. f. of the shelf life $F(t)$ (or the probability of fail-free storage $P(t)=1-F(t)$) and the known estimate of this function $\tilde{F}(t)$ (or $\tilde{P}(t)$). At the same time, point estimates of indicators are based on the results of SUA observations at a limited interval or on these results with forecasting for an extended operating interval $[\tau, \tau+t]$ or the entire area of determining the d. f.

The ratios for point estimates of the average and gamma-percent preservation periods take the form

$$\hat{T}_{av}=\int_0^\infty \tilde{P}(t)\mathrm{d}t, \tag{16}$$

$$\hat{T}_\gamma=\tilde{P}^{-1}(\gamma), \tag{17}$$

where $\tilde{P}^{-1}(\gamma)$ is the inverse function $\tilde{P}(t)$ or its quintile of the level γ.

The ratios for point estimates of the mean and gamma-percent persistence periods over the predefined (limited) observational interval $[0, t_{ob}]$ take the form

$$\hat{T}_{avt_{ob}}=\int_0^{t_{av}}\tilde{P}(t)\mathrm{d}t, \tag{18}$$

$$T_{\gamma t_{ob}}=\tilde{P}^{-1}(\gamma)\ \text{at}\ \tilde{P}(t_{ob})\geq\gamma. \tag{19}$$

The ratio for the point estimate of the mean residual storage period is of the form

$$\hat{T}_{av}(\tau)=\frac{1}{\tilde{P}(\tau)}\left[\hat{T}_{av}-\int_0^\tau\tilde{P}(t)\mathrm{d}t\right]. \tag{20}$$

A point estimate of the gamma-percent residual storage period $T_\gamma(\tau)$ is derived from the equation

$$\frac{\tilde{P}(\tau+T_\gamma(\tau))}{\tilde{P}(\tau)}=\gamma. \tag{21}$$

From (16) to (20), it follows that the relationship between estimates of average shelf life is:

$$T_{avt_{ob}}+T_{av}(t_{ob})\tilde{P}(t_{ob})=T_{av}. \tag{22}$$

The ratios for point estimation of truncated mean storage period over a limited observation interval $[\tau, \tau+t]$ are of the form

$$\hat{T}_{avt}(\tau)=\frac{1}{\tilde{P}(\tau)}\int_\tau^{\tau+t}\tilde{P}(t)\mathrm{d}t. \tag{23}$$

Ratios (16) to (23) demonstrate that the bias and variance of the estimates of these indicators is mainly determined by the bias and variance of the estimate $\tilde{P}(t)$. Since $\tilde{P}(t)=1-\tilde{F}(t)$, the variance $D\left[\tilde{P}(t)\right]=D\left[\tilde{F}(t)\right]$. We shall find the ratios for expectation $M\left[F_k\right]$ and variance $D\left[F_k\right]$ in the d. f. of $\tilde{F}(t)$ (11), (12). In [6, 28], it is shown that expectation of the d. f. estimate (7) can be found by applying to it the operation of conditional expectation k times. For the studied d. f. (11), (12), acting in the same way, we obtain the following ratio

$$M\left[\hat{F}(T_k;r_1,\ldots,r_k;n_1,\ldots,n_k)\right]=$$
$$=\sum_{j=1}^{k}M\left\{\left(1-\hat{F}_{j-1}\right)\frac{M\left[r_k|r_1,\ldots,r_{k-1}\right]}{N_k}\right\}. \tag{24}$$

In (24), the number of observed articles on the k-th interval N_k is calculated from formula (6), $N_1=N-n_0$, $n_0=0$, $\hat{F}_0=0$.

For the first interval (k=1), from (24), we find

$$M\left[\hat{F}_1\right]=\frac{M[r_1]}{N_1}=F_1.$$

For the second interval (k=2), we obtain

$$M\left[\hat{F}_2\right]=F_1+M\left\{\left(1-\hat{F}_1\right)\frac{M[r_2/r_1]}{N_2}\right\}. \tag{25}$$

Given that the conditional r. v. r_2 at $N_2>0$ is characterized by a binomial distribution, and, at $N_2\leq 0$, is zero, ratio (25) can be brought to the form

$$M\left[\hat{F}_2\right]=F_2-\frac{F_2-F_1}{1-F_1}\sum_{s=N_1-n_1}^{N_1}\left(1-\frac{s}{N_1}\right)P(s), \tag{26}$$

$$P(s)=C_{N_1}^{s}F_1^{s}\left(1-F_1\right)^{N_1-s}.$$

The estimate $\hat{F}_1$ is not biased. The estimate $\hat{F}_2$ is generally biased. The amount of displacement $\Delta M_2=M\left[\hat{F}_2\right]-F_2$ is determined by the values of the ratios s/N_1, $s=N_1-n_1$, ..., N_1; $(F_2-F_1)/(1-F_1)$ and the probabilities of occurrence in the second interval of observations of s failures. At $n_1<<N_1$ (by 20 times or more), the displacement is not significant. It can be shown that, at $k=3$, ..., J, the d. f. estimate $F(t)$ for (11), (12) is generally biased, and, with an increase of n_k, the amount of displacement ΔM_k increases. However, with large sample sizes N_k and small n_k samples, the estimate $\hat{F}_k$ for (11), (12) can be considered unbiased.

Variances of estimates $D\left[\hat{F}_k\right]$ can be found from known ratios [28]:

$$\begin{cases} D\left[\hat{F}_k\right]=D\left[\hat{F}_{k-1}\right]+D\left[\hat{F}_k-\hat{F}_{k-1}\right]+\\ +2\operatorname{cov}\left(\hat{F}_{k-1},\hat{F}_k-\hat{F}_{k-1}\right),\\ D\left[\hat{F}_{k-1}\right]=M\left[\hat{F}_{k-1}^2\right]-M^2\left[\hat{F}_{k-1}\right]. \end{cases} \tag{27}$$

At $k=1$, the value $n_0=0$ and the variance of the estimate $\hat{F}_1$ is determined from the ratio for complete samples [28, 29]:

$$D\left[\hat{F}_1\right]=F_1\left(1-F_1\right)/N_0. \tag{28}$$

At $k=2$, the variance of the estimate $\hat{F}_2$ is determined from (27). In this case, the variance $D\left[\hat{F}_2-\hat{F}_1\right]$, is first calculated, then, taking into consideration the binomial distribution of the value r_1 – the covariance $\operatorname{cov}\left(\hat{F}_1,\hat{F}_2-\hat{F}_1\right)$ of the quantities $\hat{F}_1$ and $\hat{F}_2-\hat{F}_1$, then the final expression is derived for $D\left[\hat{F}_2\right]$. It has a cumbersome form and is not given. From its analysis, it follows that the presence of a bias in the d. f. estimation leads, with an increase in n_1, to an increase in variance $D\left[\hat{F}_2\right]$. At $n_1<<N_1$, the ratio for the variance takes the form

$$D\left[\hat{F}_2\right]\approx F_2\left(1-F_2\right)/N_1, \tag{29}$$

which corresponds to the variance of EDF at point T_2 for a complete sample.

For $k=3$, ..., J, doing similarly, one can use (27) to obtain a ratio for the variance of the estimate $D\left[\hat{F}_k(t)\right]$, which is cumbersome and is not given. Its analysis confirms the conclusion obtained at $k=2$.

Thus, the point estimates of the preservation indicators obtained from formulas (16) to (23) are generally biased and ineffective. With large sample sizes N_k and $n_k<<N_k$, the point estimation of EDF (11), (12) can be considered unbiased and effective.

Consider the interval estimation of preservation indicators on the example of calculating their lower confidence boundaries (LCB). To this end, we shall first find an estimate of the LCB probability of fail-safe storage $\tilde{P}(t)$ using its approximation by the normal distribution, i.e.

$$\underline{P}_q(t)=\tilde{P}(t)-u_q\sqrt{D\left(\tilde{P}(t)\right)}, \tag{30}$$

where u_q is the quantile of the normal distribution of the level q;

$D\left(\tilde{P}(t)\right)$ is the variance of the estimate $\tilde{P}(t)$.

With a known function (30), the LCB assessment of the level q, the preservation indicators can be found from the following ratios.

The lower confidence boundaries of the mean and gamma-percent storage periods of level q

$$\underline{T}_{avq}=\int_0^\infty \underline{P}_q(t)\mathrm{d}t, \tag{31}$$

$$\underline{T}_\gamma=\underline{P}_q^{-1}(\gamma). \tag{32}$$

The lower confidence bounds of the mean and gamma-percent residual storage periods of level q

$$\underline{T}_{avq}(\tau)=\frac{1}{P(\tau)}\int_\tau^\infty \underline{P}_q(t)\mathrm{d}t, \tag{33}$$

$$\underline{T}_{\gamma q}(\tau)=\underline{P}_{\tau q}^{-1}(\gamma), \tag{34}$$

where $\underline{P}_{\tau q}(t)=\underline{P}_q(t)/\tilde{P}(\tau)$.

The lower confidence bound of the truncated mean storage period of level q

$$\underline{T}_{avtq}(\tau)=\frac{1}{\hat{P}(\tau)}\int_\tau^{\tau+t} \underline{P}_q(t)\mathrm{d}t. \tag{35}$$

The construction of dependence (30) for $\underline{P}_q(t)$ is reduced to finding the dependence $D\left(\tilde{P}(t)\right)$. For this, we shall first find variances of estimates of the probability of fail-safe storage of SUAs at the end of the intervals $[T_{k-1}, T_k]$, $k=1$, ..., $J+1$. According to the totality of these points and the known field of determining the function $\tilde{P}(t)$, the method of least squares makes it possible to construct a monotonously decreasing dependence $D\left(\tilde{P}(t)\right)$ in the form of a polynomial.

To find the variance of the estimates of the probability of fail-safe storage at the end of the intervals $[T_{k-1}, T_k]$, $k=2$, ..., $J+1$, instead of the cumbersome formulas obtained from (27), it is proposed to use the approximate Greenwood formula [6]. This formula is obtained for RCS under the assumption that there is no bias in the estimate $\hat{P}(T_k)$ and a large sample size. Then, with large sample volumes N_j and the inequality $n_j<<N_j$, the estimate of this variance will be found using Greenwood's formula in the following form

$$D\left(\tilde{P}(T_k)\right)=\tilde{P}^2(T_k)\sum_{j=1}^{k}\frac{1-\hat{P}_j}{N_j\hat{P}_j}, \tag{36}$$

where $\hat{P}_j$ is the estimation of the conditional probability of fail-safe storage at the point T_j. In (36), the value N_j is calculated from (5) or (6), the probability value $\tilde{P}(T_k)$ is calculated from the previously constructed dependence $\tilde{F}(t)$ and the ratio $\tilde{P}(T_k)=1-\tilde{F}(T_k)$. The values of conditional probabilities $\hat{P}_j$ are estimated from formula $\hat{P}_j=1-r_j/N_j$. Note that the values N_j and r_j are known only for observation intervals $[T_{j-1}, T_j]$, $j=1$, ..., J. For intervals $[T_{J+i-1}, T_{J+i}]$, $i=1$, ..., m, these values must be predicted.

To predict the number of SUAs N_{J+i} that are expected to be observed at the intervals $[T_{J+i-1}, T_{J+i}]$, $i=1$, ..., m, and the number of potential failures r_{J+i} at these intervals, we shall use the extrapolated data of the recovered d. f. $\tilde{F}(t)$. At the same time, the extrapolation of d. f. at the points $t=T_{J+i}$, $i=1, ..., m$ will be carried out in accordance with the dependence established according to the data of operational tests at the observation intervals $[T_{j-1}, T_j]$, $j=1, ..., J$.

An estimate of the expected number of SUA failures r_{J+i} at subsequent intervals $[T_{J+i-1}, T_{J+i}]$, $i=2, ..., m$ can be found from known estimates of the restored d. f. $\tilde{F}_{J+i-1}$, $\tilde{F}_{J+i}$ at points T_{J+i-1}, T_{J+i}, respectively, and the number of articles N_{J+i} that are expected to be observed in this interval.

From the ratio $\tilde{F}_{J+i}=1-\exp\left[-\Lambda_{J+i}\right]$, it follows that $\Lambda_{J+i}=\ln\left(1-\tilde{F}_{J+i}\right)^{-1}$ and

$$\Lambda_{J+i}-\Lambda_{J+i-1}=\ln\left(1-\tilde{F}_{J+i-1}/1-\tilde{F}_{J+i}\right).$$

The expectation of the expected number of failures for the considered observation interval at N_{J+i} tested SUAs will be found from the ratio

$$r_{J+i}=N_{J+i}\ln\left(\frac{1-\tilde{F}_{J+i-1}}{1-\tilde{F}_{J+i}}\right). \quad (37)$$

Then the values r_{J+i} и N_{J+i}, $i=\overline{1,m}$, can be found in the following order:

1. Find the number N_{J+i} of articles tested in the interval $[T_J, T_{J+1}]$, according to the known N_J, r_J and n_J over the last observation interval $[T_{J-1}, T_J]$ and (5):

$$N_{J+1}=N_J-\left(r_J+n_J\right).$$

2. Using the known N_{J+1}, $\tilde{F}_J$ and $\tilde{F}_{J+1}$ and ratio (37), we obtain:

$$r_{J+1}=N_{J+1}\ln\left(\frac{1-\tilde{F}_J}{1-\tilde{F}_{J+1}}\right).$$

3. According to the known data N_{J+1}, r_{J+1}, n_{J+1} and ratio (5), we shall find N_{J+2}, then, by using ratio (37), we shall find r_{J+2}, etc.

As a result, we shall get all the initial data for estimating the variance $D\left(\tilde{P}(t)\right)$ by (36).

Calculations of the LCB preservation indicators from ratios (31) to (35) are obtained under the assumption of the validity of the assumptions taken for the application of formulas (30), (36). In this regard, we shall consider the assessment of the preservation indicators of SUAs using bootstrap modeling from the restored d. f.

The restored d. f. $\tilde{F}(t)$ is used as a true number to generate pseudo-random numbers – the durations of storage of SUAs before failure. In this case, the specified number M of independent copies of data arrays is obtained. To obtain each copy, the process of observing implementations is simulated, then, based on implementations, an array of data corresponding to the plan of operational tests (observations) is formed. Then an ordinal table is formed and, on its basis, estimates of the preservation indicators are calculated. Baseline data shall be generated in relation to the operational tests (observations) of SUAs at a limited interval in accordance with the test plan $[NUz]$ or $[NUT]$ over the entire area of determining the d. f. in accordance with the $[NUN]$ plan. In this case, the original RCS with progressive censorship is converted to a single censored sample corresponding to the plan $[NUz]$ and $[NUT]$ with full and incomplete implementations or to a full sample ($[NUN]$).

The results of modeling the process of observing implementations will be shown in Fig. 5 in the form convenient for assessing the preservation indicators of the nomenclature under consideration. This makes it possible to estimate the preservation indicators using known ratios [5] given below, namely to calculate for the $[NUT]$ plan:

a) the point estimates of the average $\hat{T}(\tau)$, truncated average $\hat{T}_t(\tau)$, and gamma-percent $\hat{T}_\gamma(\tau)$ residual periods of article preservation:

$$\hat{\hat{T}}_{av}(\tau)=\sum_{j=k+1}^{N} z_j \Big/ K_N(\tau)(N-k), \quad (38)$$

where $z_j=t_j-\tau$ is the residual (after τ) preservation period of the j-th article;

$K_N(\tau)=1-\left[1-\hat{P}(\tau)\right]^N$ is the shift coefficient;

k is the number of failed articles in the interval $(0, \tau)$.

$$T_{avt}(\tau)=\frac{1}{r}\left[\sum_{j=1}^{\rho} z_j+(r-\rho)t\right], \quad (39)$$

where r, ρ is the number of failed and observed SUAs in the interval $(\tau, \tau+t)$, respectively.

$$\hat{T}_\gamma(\tau)=z_{m-1}+\frac{\left(z_{(m)}-z_{(m-1)}\right)\left[\hat{P}_\tau\left(z_{(m-1)}\right)-\gamma\right]}{\hat{P}_\tau\left(z_{(m-1)}\right)-\hat{P}_\tau\left(z_{(m)}\right)},$$

where $z_{(m-1)}$, $z_{(m)}$ are the residual storage periods of the $(m-1)$ and m-th SUA in their variation series, $z_{(1)}<z_{(2)}<...<z_{(m-1)}<z_{(m)}$, for which the following inequality holds:

$$\hat{P}_\tau\left(z_{(m)}\right)\le\gamma<\hat{P}_\tau\left(z_{(m-1)}\right), \quad (40)$$

$$\hat{P}_\tau(t)=\frac{\hat{P}(\tau+t)}{\hat{P}(\tau)};$$

b) the lower confidence limits (LCB) of the truncated average $\underline{T}_{tq}(\tau)$ and gamma-percent $\underline{T}_{\gamma q}(\tau)$ residual preservation periods of articles:

$$\underline{T}_{avtq}(\tau)=\hat{T}_t(\tau)\Big/\left(1+u_q\big/\sqrt{r}\right). \quad (41)$$

$$\underline{T}_{\gamma q}(\tau)=\frac{\hat{T}_\gamma(\tau)}{1+u_q\phi(\gamma,r)}, \quad (42)$$

$$\phi(\gamma,r)=\frac{1}{\ln(1/\gamma)}\sqrt{\frac{1/\gamma-1}{r}}. \quad (43)$$

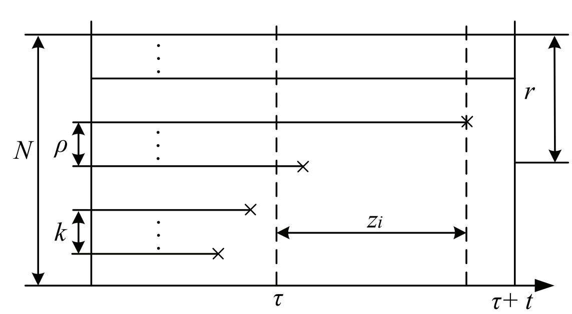


Fig. 5. Representation of the results of modeling the terms of preservation of articles using the restored d. f. $\tilde{F}(t)$

Each copy of the data set corresponds to estimates $\tilde{F}_k(t)$, $\tilde{P}_k(t)$, $\hat{T}_{avk}$, $\hat{T}_{\gamma k}$, etc. The value of operation rate t is put in correspondence to the variation series formed by the values of the estimates under consideration ordered by magnitude. For example, when evaluating the d. f., a variation series is obtained

$$\tilde{F}_1(t) \le \tilde{F}_2(t) \le \ldots \le \tilde{F}_{M}(t). \qquad (44)$$

Denote $M^{-} = \left[M \frac{1-\gamma}{2} \right]$, $M^{+} = \left[M \frac{1+\gamma}{2} \right]$, where $[x]$ is the integer part of the number x. To obtain an analog of γ – the confidence interval for $F(t)$, one needs to take as the lower (upper) boundary M^{-} (M^{+})) the values of the variation series (44), that is, $\tilde{F}_{(M^{-})}(t)$ $\left(\tilde{F}_{(M^{+})}(t)\right)$. In accordance with the bootstrap method, the value of the difference $\tilde{F}_{(M^{+})}(t) - \tilde{F}_{(M^{-})}(t)$ roughly corresponds to the width γ of the confidence interval for $F(t)$.

5. 4. Study of the characteristics of the estimation-experimental method for assessing the preservation indicators of single-use articles

The estimation-experimental method for assessing the SUA indicators of preservation is represented in the form of a sequence of operations in Fig. 6.

As characteristics of the estimation-experimental method, the accuracy and reliability of assessing the preservation indicators of SUA of the predefined nomenclature, which are used in managing their operation, are considered. These characteristics are determined by the quality indicators of the developed models for the formation of data of censored samples, the assessment of the function of distributing the storage periods of SUAs on the censored sample and the assessment of their preservation indicators from these data.

The study of characteristics was conducted using a computer algebra system from the class of computer-aided design systems Mathcad 15.0 M05. The adopted limitations and assumptions are formulated above.

The following parameters are used as inputs for modeling. At the considered storage intervals $[0, T_J]$, performance monitoring with a period of T_c=4 years is carried out on a sample of AGMs. Based on the results of this control, the fact of operability (inoperability) of AGM at the time of control is established. Inoperable articles are decommissioned, workable ones continue to be operated. At the same time, n_j launches of AGMs are carried out from among workable articles in accordance with the test plan.

Three options for setting observation intervals $[0, T_J]$, the duration of which are 20, 28, and 36 years, are considered. The results of operational tests for each interval under consideration are censored samples, the structure of which is shown in Fig. 3.

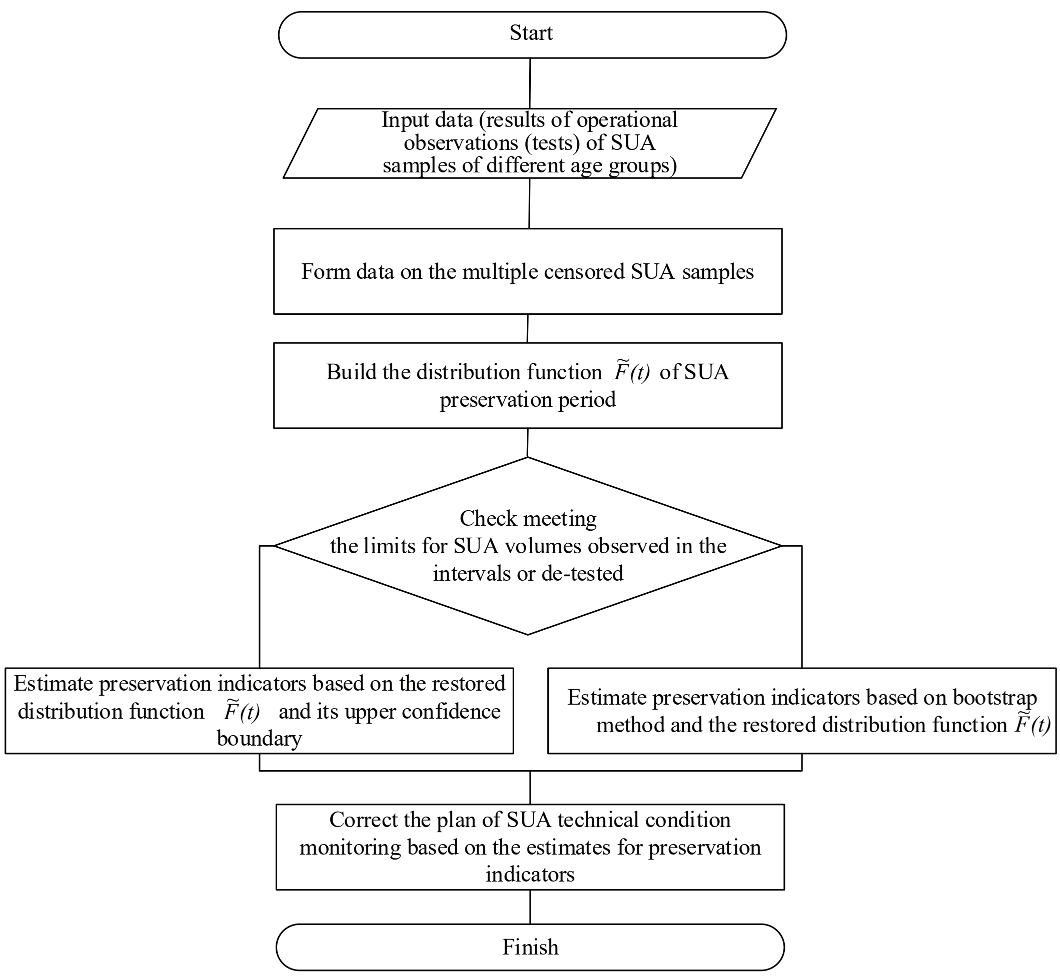


Fig. 6. The sequence of operations in the estimation-experimental method for assessing the indicators of preservation of single-use articles

To assess the accuracy and reliability of the developed method, implementations of the duration of AGM preservation periods were generated in accordance with the theoretical distribution function of the form

$$F(t)=1-\exp(-t/a)^{b}, \tag{45}$$

where the scale parameter a=30 years, the shape parameter b=3.

The collected initial data of the results of periodic control of AGM samples in accordance with the model for the formation of censored sample data are represented in the form of implementations (Fig. 3, *b*) corresponding to the test plan [[100U(T_j, n_j), T_J]].

As reference estimates of the indicators of AGM preservation, the values of these indicators found from the theoretical (reference) function, i.e. the average and gamma-percentage conservation periods, were considered:

$$T_{av}=a\Gamma(1+1/b), \tag{46}$$

$$T_{\gamma}=a(\ln 1/\gamma)^{1/b}, \tag{47}$$

where $\Gamma(x)$is the known gamma function [9, 29]

the mean and gamma-percentage duration of residual preservation:

$$T_{av}(\tau)=\frac{1}{P(\tau)}\left[T_{av}-\int_{0}^{\tau}P(t)\mathrm{d}t\right], \tag{48}$$

$$T_{\gamma}(\tau)=a(\ln 1/\gamma_1)^{1/b}-\tau,\ \ \gamma_1=\gamma\exp\left(-(\tau/a)^{b}\right). \tag{49}$$

To assess the variance in the preservation periods and the residual periods of AGM preservation, their variances were additionally calculated:

$$D=a^{2}\left[\Gamma(1+2/b)-\left(\Gamma(1+1/b)\right)^{2}\right], \tag{50}$$

$$D(\tau)=\frac{2a^{2}}{b}\exp\left(\frac{\tau}{a}\right)^{b}\Gamma\left[\left(\frac{2}{b},\left(\frac{\tau}{a}\right)^{b}\right)\right]-$$
$$-2\tau T_{av}(\tau)-\left(T_{av}(\tau)\right)^{2}, \tag{51}$$

where

$$\Gamma\left[(a,z)\right]=\int_{z}^{\infty}y^{a-1}\exp(-y)\mathrm{d}y$$

is the incomplete gamma-function.

The errors in the calculation of point estimates of AGM preservation indicators were found relative to the benchmark estimates, and the errors in the calculation of their interval estimates were found relative to the indicators calculated by the parametric method. The estimation of the preservation indicators by the parametric method was carried out under the assumption about a known type of preservation period distribution function (45) and the test plan [100UT_J]. Point estimates of the parameters *a* and *b* of this distribution were found according to a known methodology in accordance with [5, 11]. The point estimates of preservation indicators (residual persistence) were found from ratios (45) to (49) with the replacement of the parameters *a* and *b* with their point estimates $\hat{a}$ and $\hat{b}$. Thus, the ratio for the point assessment of the d. f. of the preservation period (45) takes the form:

$$\hat{F}(t)=1-\exp(-t/\hat{a})^{\hat{b}}. \tag{52}$$

The interval estimates of the mean and gamma-percent storage periods of the level *q* were based on the following ratios from [5, 11]:

$$\underline{T}_{avq}=\hat{T}_{av}\left(1+\varepsilon_{davq}\right), \tag{53}$$

$$\underline{T}_{\gamma q}=\hat{T}_{\gamma}\left(1+\varepsilon_{d\gamma q}\right), \tag{54}$$

where the ε_{davq}, $\varepsilon_{d\gamma q}$ values are determined from plots given in [5, 11].

The interval estimates of the mean and gamma-percent residual storage periods of the level q were derived from the following ratios:

$$\underline{T}_{avq}(\tau)\cong\hat{T}_{av}(\tau)-u_q\sqrt{D\left(\hat{T}_{av}(\tau)\right)}, \tag{55}$$

$$\underline{T}_{\gamma q}(\tau)\cong\hat{T}_{\gamma}(\tau)-u_q\sqrt{D\left(\hat{T}_{\gamma}(\tau)\right)}, \tag{56}$$

wherein the variance $D\left(\hat{T}_{\gamma}(\tau)\right)$ is calculated from the formula for variance $D\left(\hat{T}_{\gamma}\right)$, given in [5, 11], with the substitution of the parameter γ by γ_1, which is found from (49).

In accordance with the developed method for implementations corresponding to the test plan [[100U(T_j, 1), T_J]], EDF is found from (9) to (12), and, on its basis, the reconstructed d. f. from (13) to (15). At the same time, data on the number of conditional and complete implementations in the intervals under consideration are obtained from the results of their modeling using the theoretical d. f. and the model of formation of the RCS. Fig. 7–9 show plots of the theoretical, parametric, and reconstructed d. f. of the SUA preservation period. The parametric and reconstructed f d. f. are built for three intervals of observations. At the same time, the values *a* of the degree of censorship of the sample were 0.73 for the interval (0, 20], 0.43 for the interval (0, 28], and 0.24 for the interval (0, 36].

To characterize the accuracy and reliability of the description of the reconstructed d. f. of the actual preservation of SUAs, let us determine the boundaries within which the theoretical d. f. of the shelf life is located with the predefined probability. To this end, we shall use Kolmogorov's theorem [29] and the results of research [22]. In the case of a complete sample (a=0) for the predefined confidence probability *q*, according to the tabulated values of Kolmogorov's d. f. [30], we shall find the quantile t_q.

Hence, it follows from the Kolmogorov's theorem [29]:

$$P\left\{\tilde{F}(t)-t_q/\sqrt{N}\le F(t)\le\tilde{F}(t)+t_q/\sqrt{N}\right\}\approx q. \tag{57}$$

Since $0\le F(t)\le 1$, inequality (57) must be refined:

$$\max\left(0;\tilde{F}(t)-t_q/\sqrt{N}\right)\le F(t)\le\min\left(1;\tilde{F}(t)-t_q/\sqrt{N}\right). \tag{58}$$

With a confidence probability q=0.9 according to the tabulated values of the Kolmogorov d. f. [30], we find $t_{0.9}$=1.22. At the same time, the value $t_{0.9}/\sqrt{100}$ in (58) is 0.122, that is, the greatest discrepancy between the theoretical and reconstructed d. f. in the case of a complete sample should not exceed 0.122. For the correct application of Kolmogorov's criterion for censored sampling (Fig. 8, 9) in accordance with [22], we shall determine the required minimum sample sizes. For the type of censorship under consideration and the above values of the degree of censorship of the sample, these volumes are: 20 at a=0.24; 30 – at a=0.43; and 76 – at a=0.73. The sample size requirements can be considered fulfilled. Then the quantile values $t_{0,9}$(a) of the Kolmogorov criterion statistics for the established values a are determined in accordance with [22]: $t_{0.9}(0.24)=1.209$, $t_{0.9}(0.43)=1.167$, $t_{0.9}(0.73)=0.927$. In this case, the expression $t_{0.9}(a)/\sqrt{100}$ in (58) will take the values of 0.121, 1.117, and 0.093, respectively. Fig. 7 shows that the greatest discrepancy between the theoretical (curve 1) and the reconstructed d. f. (curve 3) is observed at t=38 years and is 0.082. This allows us to assert that the reconstructed d. f. agrees well with the theoretical one at $a \le 0.73$. Or, from the point of view of the model being developed, the reconstructed d. f. with a reliability of 0.9 and an error of not more than 0.1 describes the actual preservation of SUAs, which is acceptable for solving the problems of managing the operation of AGMs. In addition, from Fig. 7, it follows that the discrepancy between the theoretical (curve 1) and parametric d. f. (curve 2) is 0.052, which satisfies (57) at q=0.9. The accuracy and reliability of estimating the d. f. according to the parametric model (Fig. 8) with the accepted assumptions is higher than that according to the reconstructed d. f. (Fig. 9). This makes it possible to use it for the subsequent assessment of the accuracy and reliability of the preservation indicators of AGMs by the developed method.

Dependence analysis in Fig. 9 shows that with an increase in the observation interval (a decrease in the magnitude of the degree of censorship), the accuracy and reliability of estimating the d. f. of the preservation period by the developed method increases. In addition, the plots of these d. f.'s make it possible to assess the sensitivity of the developed model for estimating the d. f. to a change in the magnitude of the observation interval.

The results of assessing the AGM preservation indicators based on the parametric and reconstructed d. f. and on the reconstructed d. f. using bootstrap modeling are given in Tables 1–4.

The accuracy of estimating the i-th preservation indicator by the j-th method was calculated from the magnitude of the relative error Δ_{ij} according to the ratios:

– for point estimates

$$\Delta_{ij} = \left|T_{ri} - \hat{T}_{ij}\right| \cdot 100\% \big/ T_{ri}, \quad i=1, 2, 3, 4, 5; j=1, 2, 3, \tag{59}$$

– for interval estimates

$$\Delta_{ij} = \left|\underline{T}_{i1q} - \underline{T}_{ijq}\right| \cdot 100\% \big/ \underline{T}_{i1q}, \quad i=6, 7, 8, 9, 10; j=2, 3, \tag{60}$$

where T_{ri}, $\hat{T}_{ij}$ is the reference value of the i-th preservation indicator and its point estimate obtained by the j-th method;

$\underline{T}_{i1q}$ and $\underline{T}_{ijq}$ is the LCB of the assessment of the i-th preservation indicator of level q, obtained by the parametric and examined j-th method, respectively.

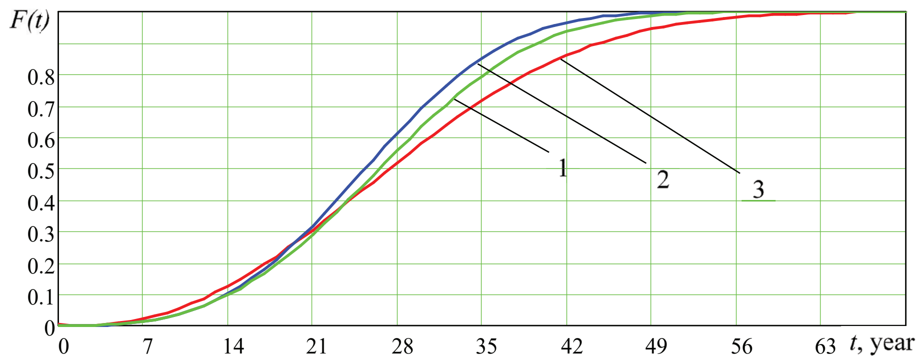


Fig. 7. Plots of the distribution function of single-use articles preservation: 1 — theoretical; 2 — parametric (test plan [100U20]); 3 — reconstituted (test plan [[100U(T_j, 1), 20]])

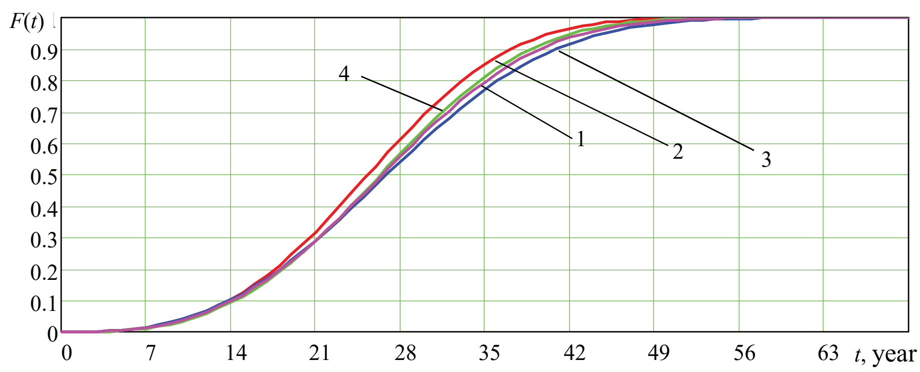


Fig. 8. Plots of the theoretical and parametric distribution function of the preservation of articles in different intervals of observations: 1 – theoretical; 2, 3, 4 – parametric in the intervals of observations (0, 20], (0, 28], and (0, 36], respectively

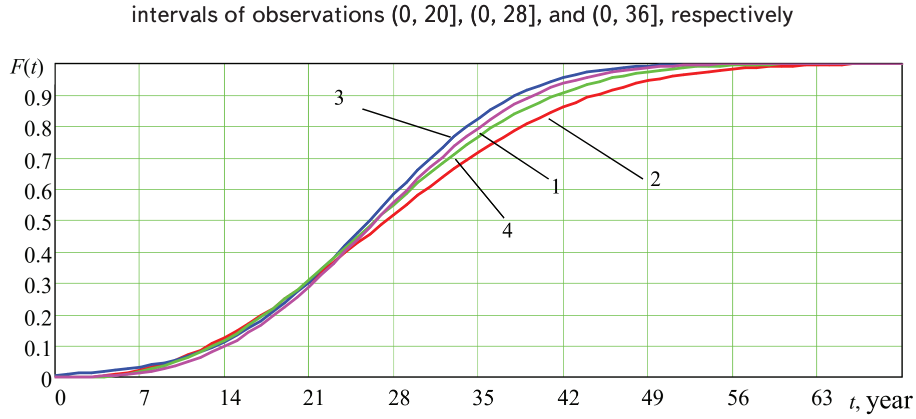


Fig. 9. Plots of the theoretical and reconstructed distribution function of the preservation of articles in different intervals of observations: 1 – theoretical; 2, 3, 4 – reconstructed in the observation intervals (0, 20], (0, 28], (0, 36], respectively

Table 1

Point estimates of SUA preservation indicators in the test plan [[100U(T_j, n_j), 20]] and their errors Δ

Number and designation of the point estimate of the indicator		The reference value of the indicator	Parametric estimation for plan [100U20]		Non-parametric estimation			
					for $\tilde{F}(t)$		for $\tilde{F}(t)$ with bootstrap modeling	
			Indicator score	Δ_{i1}, %	Indicator score	Δ_{i2}, %	Indicator score	Δ_{i3}, %
1	$\hat{T}_{av}$	26.79	25.50	4.82	28.29	5.60	28.50	6.38
2	$\hat{T}_{90\%}$	14.17	13.89	1.98	13.00	8.26	13.41	5.36
3	$\hat{T}_{av}(12)$	15.97	14.63	8.39	17.54	9.83	17.40	8.95
4	$\hat{T}_{av8}(12)$	7.27	7.15	1.65	7.05	3.03	6.90	5.09
5	$\hat{T}_{90\%}(12)$	4.60	4.24	7.83	4.98	8.26	3.94	14.35

Table 2

Interval estimates of SUA preservation indicators at the level q=0.9 in the test plan [[100U(T_j, n_j), 20]] and their errors Δ

Number and designation of the point estimate of the indicator		Parametric estimation for plan [100U20]	Non-parametric estimation			
			for $\tilde{F}(t)$		for $\tilde{F}(t)$ with bootstrap modeling	
		Indicator score	Indicator score	Δ_{i2}, %	Indicator score	Δ_{i3}, %
6	$\underline{T}_{avq}$	19.13	25.66	34.13	26.5	38.53
7	$\underline{T}_{90\%q}$	10.42	11.3	8.45	11.72	12.48
8	$\underline{T}_{avq}(12)$	11.71	15.52	32.54	15.84	35.27
9	$\underline{T}_{av8q}(12)$	6.6	6.9	4.55	6.05	8.33
10	$\underline{T}_{90\%q}(12)$	3.18	2.3	27.67	2.7	15.09

Table 3

Point estimates of SUA preservation indicators in the test plan [[100U(T_j, n_j), 36]] and their errors Δ

Number and designation of the point estimate of the indicator		The reference value of the indicator	Parametric estimation for plan [100U36]		Non-parametric estimation			
					for $\tilde{F}(t)$		for $\tilde{F}(t)$ with bootstrap modeling	
			Indicator score	Δ_{i1}, %	Indicator score	Δ_{i2}, %	Indicator score	Δ_{i3}, %
1	$\hat{T}_{av}$	26.79	26.55	0.90	27.02	0.86	27.2	1.53
2	$\hat{T}_{90\%}$	14.17	14.32	1.06	13.20	6.85	13.5	4.73
3	$\hat{T}_{av}(12)$	15.97	15.65	2.00	16.58	3.82	15.4	3.57
4	$\hat{T}_{av8}(12)$	14.94	14.75	1.27	15.25	2.07	14.32	4.15
5	$\hat{T}_{90\%}(12)$	4.60	4.55	1.09	4.30	6.52	4.28	6.95

Table 4

Interval estimates of SUA preservation indicators at the level q=0.9 in the test plan [[100U(T_j, n_j), 36]] and their errors Δ

Number and designation of the point estimate of the indicator		Parametric estimation for plan [100U36]	Non-parametric estimation			
			for $\tilde{F}(t)$		for $\tilde{F}(t)$ with bootstrap modeling	
		Indicator score	Indicator score	Δ_{i2}, %	Indicator score	Δ_{i3}, %
6	$\underline{T}_{avq}$	21.77	24.68	13.37	25.03	14.97
7	$\underline{T}_{90\%q}$	11.45	10.8	5.68	10.3	10.04
8	$\underline{T}_{avq}(12)$	13.3	14.29	7.44	14.8	11.28
9	$\underline{T}_{av24q}(12)$	14.1	13.6	3.55	13.5	4.26
10	$\underline{T}_{90\%q}(12)$	3.68	3.14	14.67	3.3	10.33

Tables 1, 3 show that the point estimates of preservation indicators at the observed part of the sample of 27 % (T_J=20 years) are characterized by errors of up to 10 % when they are estimated by the reconstructed d. f., and up to 14 % by the reconstructed d. f. with the bootstrap method. An increase in the observed part of the sample to 76 % (T_J=36 years) leads to a decrease in these errors by about 1.5 times. With a known type of d. f. and the period of monitoring the performance of AGMs, much less than their average shelf life, these errors are significantly smaller, and, with the observed part of the sample of 76 %, do not exceed 2 %.

Tables 2, 4 demonstrate that the LCB of the estimated AGM preservation indicators at the level of 0.9 by the developed method with the observed part of the sample of 27 % are characterized by errors of the order of 7–39 %. Increasing the proportion of the observed part of the sample to 76 % leads to a decrease in these errors by about 2 times, which are about 4–15 %.

Note that the errors of the interval estimation of the preservation indicators were calculated relative to the parametric estimates. Their greater value in comparison with the errors of point estimates is also due to the errors of formulas (36), (55), (56), large errors in estimating the d. f. in the non-observed part of the sample, and the sensitivity of interval estimates to errors.

The results of our studies (Tables 1–4) show that the accuracy of estimating the preservation indicators by the developed method is largely determined by the accuracy of estimating the $\Lambda(t)$ dependence in the observation interval. In addition, it is determined by the results of forecasting $\Lambda(t)$ in the non-observed part of the field of determining the d. f. The quality of this dependence is determined by the initial values of $\Lambda(t)$ at points $t=T_j$, $j=1,...,J$, the choice of the form of this dependence, usually nonlinear, by the method of least squares. Almost any form of the nonlinear dependence $\Lambda(t)$ can be constructed by the method of least squares. However, this method restores only functions linear in parameters, or variables reduced to them by means of linearizing transformations, for example, power, exponential, etc. The optimal form of the monotonously non-decreasing dependence $\Lambda(t)$ is proposed to be chosen according to the criterion of minimal residual variance.

The results of the simulation show that the point and interval estimates of AGM preservation indicators obtained by the developed method provide acceptable accuracy and reliability. This, in turn, ensures the adoption of informed decisions on the choice of moments for periodic monitoring of the technical condition on the samples of AGMs, their volumes, and other measures to manage their operation.

Unlike the well-known ones, the developed nonparametric method implements the assessment of AGM preservation indicators under the conditions of combined censorship, due to the interval uncertainty of conditional implementations and the uncertainty of incomplete implementations of product storage periods censored on the right. At the same time, the ranks of conditional implementations of the AGM preservation periods and the quantitative values of their incomplete implementations at observation intervals and the predicted number of implementations in the non-observed part of the field of determining the d. f. are used. They are focused on the parametric assessment of preservation indicators with continuous monitoring of their technical condition and one-time censorship. Therefore, no comparison of the developed and known methods for assessing the SUA preservation indicators was carried out due to different test plans and conditions.

6. Discussion of results of studying the characteristics of the developed estimation-experimental method for assessing the preservation indicators of a single-use product

The operation of modern SUAs requires solving the problems of assessing the preservation indicators with acceptable accuracy and reliability, for example, the error is not worse than 15 % and the reliability is not worse than 0.9 (Fig. 7, 9; Tables 1, 3, 4; ratios (58) to (60).

The developed estimation-experimental method makes it possible to ensure that the accuracy and reliability of the assessment of SUA preservation indicators are acceptable for managing their operation when fulfilling the requirements for the degree of censorship of samples (the region of observation) (Fig. 7–9). This is achieved by reconstructing the d. f. of SUA preservation period throughout the field of determining by converting the initial RCS within a limited interval into an equivalent full one using bootstrap modeling if necessary.

The developed estimation-experimental method for assessing SUA preservation indicators (Fig. 8) is based on the following:

– formation of operational test data (observations) of AGM samples of different ages in the form of RCS (Fig. 3, *b*), grouped by observation time intervals and represented in the matrix form (Fig. 4);

– the reconstruction of the d. f. of SUA preservation period (ratios (8) to (15)) under the conditions of combined censorship with an accuracy not worse than 0.1 and a reliability not lower than 0.9 with a value of the degree of censorship not exceeding 0.73;

– the calculation, based on the reconstructed d. f., of the point estimates (ratios (16) to (23)) of SUA preservation indicators, their interval estimates (ratios (30) to (35)), or according to the simulated implementations with bootstrap modeling and reconstructed d. f. (ratios (38) to (44)). The choice of the calculation option is determined by the volumes of SUAs observed at observation intervals and removed from the tests.

The results of modeling the processes of assessing the preservation indicators (residual preservation) of SUAs are obtained according to the data of modeling the d. f. of the terms of preservation of AGMs, selective periodic monitoring of their technical condition. These results are represented in the form of d. f. plots (Fig. 7–9), the tables of point and interval estimates of preservation indicators and their errors (Tables 1–4). At the same time, the d. f. parameters and the technical condition control correspond to the conditions and storage modes of the S-300PS AGM system during operation.

The developed method is a development of well-known non-parametric methods for assessing reliability indicators according to RCS in relation to the assessment of the preservation indicators of SUAs during their selective periodic monitoring. At the same time, the development of models that form the basis of the method is aimed at correctly taking into consideration the uncertainties of implementations of the storage periods of articles under the conditions of combined censorship, checking the accuracy and reliability of the reconstructed d. f., and choosing an evaluation model. The totality of these provisions confirms the theoretical validity of the developed method.

The reliability of the developed method is confirmed by the fact that the results of estimating the d. f. of the preservation period (Fig. 9) and the indicators of preservation (Tables 1, 3) with an increase in the observation intervals converge to their theoretical values. The adequacy of the model for estimating the d. f. is confirmed by checking its agreement with the theoretical d. f. of the preservation period using the Kolmogorov criterion at different values of the degree of censorship of samples. The adequacy of the developed models for assessing preservation indicators is confirmed by the results of modeling (Tables 1–4, Fig. 7–9) and their convergence in particular cases to known results. Thus, the known d. f. (7) for the type I RCS, found under the condition of continuous monitoring of the technical condition of articles [6], is reduced to d. f. (11), (12) for periodic control.

The method should be used to assess the preservation indicators (residual preservation) of various types of SUAs, for example, anti-aircraft guided missiles, protection devices for nuclear power plants, catapults, fire extinguishing devices.

It should be noted that the developed method does not take into consideration the a priori information on the preservation of articles available at the time of the beginning of their serial operation. This imposes certain restrictions on the scope of the developed method, which can be attributed to its shortcomings. Further research related to their elimination should be directed to improving the calculation and experimental method, taking into consideration a priori information about the preservation of articles. Such a method could have a wider scope of application for solving the problems of assessing the preservation indicators of SUAs while minimizing the cost of managing their operation.

7. Conclusions

1. The developed model for the formation of data of censored samples of SUAs is represented by a structural scheme, a graphic scheme of the process of their operational tests, analytical ratios between the quantities of articles tested, failed, and removed from the tests in time intervals. It makes it possible to form initial data for constructing the d. f. of preservation time of articles in the form of matrices of the number of failures and the number of SUAs censored on the right at observation intervals grouped by time intervals.

2. The developed model for estimating the d. f. of the shelf life of articles according to RCS describes the process of constructing an empirical d. f. according to the initial information on the number of observed and failed articles at the grouping intervals under consideration. According to the empirical d. f., which takes the form of a stepped function, the continuous d. f. of the shelf life over the observation interval is restored by the method of least squares. The proposed procedure for reconstructing d. f. is completed if the observation interval ends with a conditional operation, otherwise the d. f. is extrapolated to subsequent (future) intervals. It involves the choice of the best (optimal) form of dependence in the sense of a minimum residual variance based on a sequential search of variants of dependence forms.

3. Models for assessing the preservation indicators of single-use articles are represented by analytical ratios for their point and interval estimates, as a function of the reconstructed d. f. of the preservation period. The nomenclature of SUA preservation indicators includes average and gamma-percent preservation periods, including residual ones. With large volumes of articles observed at grouping intervals and a small number of workable articles removed from testing, the preservation indicators are calculated directly from the reconstructed d. f. and its lower confidence boundary. Otherwise, estimates of the preservation indicators of SUAs are calculated from the implementations obtained by bootstrap modeling and the reconstructed d. f.

The developed estimation-experimental method is a set of models for data generation, estimation of the d. f. of the preservation period, and assessment of preservation indicators. The method provides acceptable accuracy and reliability of estimates of the preservation indicators of SUAs for making informed decisions on the management of their operation.

4. Unlike the well-known ones, the developed nonparametric method implements the assessment of the preservation indicators of SUAs under the conditions of combined censorship, due to the interval uncertainty of conditional implementations and the uncertainty of incomplete implementations of product storage periods censored on the right. At the same time, in addition to the implementations formed in the observed part of the incomplete sample, the predicted number of implementations in the non-observed part of the field of determining the d. f. is used.

The method makes it possible to estimate the average and gamma-percent preservation periods of SUAs, including residual ones, with an accuracy of not less than 7 %, and their lower confidence limits with an acceptable confidence of 0.9 and an error of not less than 14 %. At the same time, the value of the degree of censorship of the sample is not more than 0.23. The reconstructed d. f. of the shelf life with a reliability of 0.9 and an error of not more than 0.1 describes the actual preservation of SUAs with a censorship degree of not more than 0.73, which is acceptable for managing their operation.

It is recommended to use the developed calculation-experimental method to assess and control preservation indicators of uncontrolled, maintenance-free, non-restorable SUAs in solving the problems of managing their operation, including for solving the problems of extending their operation.

References

1. Lanetskii, B., Lukyanchuk, V., Khudov, H., Fisun, M., Zvieriev, O., Terebuha, I. (2020). Developing the model of reliability of a complex technical system of repeated use with a complex operating mode. Eastern-European Journal of Enterprise Technologies, 5 (4 (107)), 55–65. doi: https://doi.org/10.15587/1729-4061.2020.214995
2. Lanetskii, B., Koval, I., Popov, V., Grynevych, V. (2020). Method for evaluating the shelf-life indicators for on-board equipment of surface-to-air missiles during periodic inspections of their operability. Weapons and Military Equipment, 2 (26), 59–64.
3. Otsenka pokazateley sohranyaemosti tsifrovyh ustroystv releynoy zaschity. Zaharov O.G. Vsyo o releynoy zaschite. Available at: https://rza.org.ua/article/read/Otsenka-pokazateley-sohranyaemosti---tsifrovih-ustroystv-releynoy-zashchiti--Zaharov-O-G-_119.html
4. Mikhailov, V. S. (2019). Estimation of the gamma-percentile life for the binomial test plan. Dependability, 19 (2), 18–21. doi: https://doi.org/10.21683/1729-2646-2019-19-2-18-21
5. Sudakova, R. S., Teskina, O. I. (Eds.) (1989). Nadezhnost' i effektivnost' v tekhnike. Vol. 6: Eksperimental'naya otrabotka i ispytaniya. Moscow: Mashinostroenie, 376.
6. Skripnik, V. M., Nazin, A. E.., Prihod'ko, Yu. G., Blagoveschenskiy, Yu. N. (1988). Analiz nadezhnosti tekhnicheskih sistem po tsenzurirovannym vyborkam. Moscow: Radio i svyaz', 184.
7. Aronov, I. Z., Burdasov, E. I. (1987). Otsenka nadezhnosti po rezul'tatam sokraschennyh ispytaniy. Moscow: Izd-vo standartov, 184.
8. Lukianchuk, V., Lanetskii, B., Khudov, H., Terebuha, I., Zvieriev, O., Shknai, O. et. al. (2021). Development of an experimental-estimation method for estimating indices of residual life of a radio technical complex. Eastern-European Journal of Enterprise Technologies, 3 (9 (111)), 27–39. doi: https://doi.org/10.15587/1729-4061.2021.233538
9. Gnedenko, B. V., Belyaev, Yu. K., Solov'ev, A. D. (2017). Matematicheskie metody v teorii nadezhnosti. Osnovnye harakteristiki nadezhnosti i ih statisticheskiy analiz. Moscow: KD Librokom, 582.
10. Strel'nikov, V. P. (2000). Opredelenie ozhidaemoy ostatochnoy narabotki pri DM-raspredelenii. Matematychni mashyny i systemy, 1, 94–100.

11. Beichelt, F., Franken, P. (1984). Zuverlässigkeit und instandhaltung. Mathematische methoden. Berlin: Verlag Technik, 392.
12. Belyaev, Yu. K. et. al.; Ushakov, I. A. (Ed.) (1985) Nadezhnost' tekhnicheskih sistem. Moscow: Radio i svyaz', 608.
13. Chopra, S., Meindl, P. (2004). Supply Chain Management: Strategy, Planning and Operation. Prentice-Hall, 2, 40–44.
14. Chen, H. M., Vidakovic, B., Mavris, N. D. (2004). Multiscale forecasting method using ARMAX models. Technological Forecasting and Social Change, 1, 34–39.
15. Kredentser, B. P. (2019). Raschyot pokazateley nadyozhnosti tekhnicheskih sistem s izbytochnost'yu. Kyiv: Feniks, 520.
16. Guo, J., Wang, X., Liang, J., Pang, H., Goncalves, J. (2018). Reliability Modeling and Evaluation of MMCs Under Different Redundancy Schemes. IEEE Transactions on Power Delivery, 33 (5), 2087–2096. doi: https://doi.org/10.1109/tpwrd.2017.2715664
17. Ding, F., Sheng, L., Ao, Z. et. al. (2017). Research on reliability prediction method for traction power supply equipment based on continuous time Markov degradation process. Proc CSEE, 37, 1937–1945.
18. Górny, J., Cramer, E. (2020). Type-I hybrid censoring of multiple samples. Journal of Computational and Applied Mathematics, 366, 112404. doi: https://doi.org/10.1016/j.cam.2019.112404
19. Wu, S.-F., Hsieh, Y.-T. (2019). The assessment on the lifetime performance index of products with Gompertz distribution based on the progressive type I interval censored sample. Journal of Computational and Applied Mathematics, 351, 66–76. doi: https://doi.org/10.1016/j.cam.2018.10.044
20. Wu, S.-F. (2018). The performance assessment on the lifetime performance index of products following Chen lifetime distribution based on the progressive type I interval censored sample. Journal of Computational and Applied Mathematics, 334, 27–38. doi: https://doi.org/10.1016/j.cam.2017.11.022
21. Lemeshko, B. Y., Lemeshko, S. B., Postovalov, S. N. (2010). Statistic Distribution Models for Some Nonparametric Goodness-of-Fit Tests in Testing Composite Hypotheses. Communications in Statistics - Theory and Methods, 39 (3), 460–471. doi: https://doi.org/10.1080/03610920903140148
22. Lemeshko, B. Yu., Chimitova, E. V., Pleshkova, T. A. (2010). Proverka prostyh i slozhnyh gipotez o soglasii po tsenzurirovannym vyborkam. Nauchniy vestnik NGTU, 4 (41), 13–28.
23. Panahi, H., Moradi, N. (2020). Estimation of the inverted exponentiated Rayleigh Distribution Based on Adaptive Type II Progressive Hybrid Censored Sample. Journal of Computational and Applied Mathematics, 364, 112345. doi: https://doi.org/10.1016/j.cam.2019.112345
24. Zhu, T. (2020). Reliability estimation for two-parameter Weibull distribution under block censoring. Reliability Engineering & System Safety, 203, 107071. doi: https://doi.org/10.1016/j.ress.2020.107071
25. Miao, Y., Wang, G., Rui, X., Tu, T. (2019). An innovative Bayesian sequential censored sampling inspection method and application to test design. Applied Mathematical Modelling, 76, 867–882. doi: https://doi.org/10.1016/j.apm.2019.07.019
26. S-300 PS SA-10B Grumble B Surface-to-Air missile (2020). Available at: https://www.armyrecognition.com/s-300ps_sa-10b_grumble_b_systems_vehicles_uk/s-300_ps_s-300ps_sa-10b_grumble_b_long_range_surface-to-air_missile_technical_data_sheet_information.html
27. Deyvid, G. (1979). Poryadkovye statistiki. Moscow: Nauka, glavnaya redaktsiya fiziko-matematicheskoy literatury, 336.
28. Borovkov, A. A. (1986). Teoriya veroyatnostey. Moscow: Nauka, 432.
29. Ivchenko, G. I., Medvedev, Yu. I. (1984). Matematicheskaya statistika. Moscow: Vyssh. shkola, 248.
30. Bol'shev, L. N., Smirnov, N. V. (1983). Tablitsy matematicheskoy statistiki. Moscow: Nauka, 416.