

There is a tendency of intensive development of a new scientific area aimed at optimizing the processes of comprehensive ensuring the life of society and industrial processes of countries, specifically logistics, and its more important aspect, military logistics. This paper considers typical contradictions between the need and opportunities for additional development of the theory of processes involving this system. On the one hand, the military has important, dynamic, multifaceted processes for the comprehensive provision of their combat operations to analyze, which requires significant intensification of the development of methods and models for quantitative analysis of the effectiveness of the functioning of military logistics systems. On the other hand, there is now limited availability of theoretical developments and the practical application of the necessary, convenient, effective mathematical tools aimed at computerization of solving the problems of providing military scientific and technical problems in real time.

Matrix technology for forecasting the dynamics of functioning of closed systems of military logistics of various military purposes is proposed. Matrix calculus makes it possible to obtain intermediate and ultimate results in a compact form and carry out complex and cumbersome calculations using effective algorithms. A method to precisely solve the system of linear differential equations describing processes of arbitrary type has been proposed. The method is based on the use of the operational calculus by Laplace. The possibilities of the method and procedures of forecasting are illustrated by solving practical military tasks that arise during the functioning of military logistics systems of varying complexity. These tasks differ in configuration, different numbers of possible states, and state transitions

Keywords: military logistics, dynamics of functioning, matrix forecasting technique, support system, system states, operational calculus

UDC 656.073

DOI: 10.15587/1729-4061.2021.249270

DEVISING MATRIX TECHNOLOGY FOR FORECASTING THE DYNAMICS IN THE OPERATION OF A CLOSED MILITARY LOGISTICS SYSTEM

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Received date 02.11.2021

Accepted date 15.12.2021

Published date 29.12.2021

How to Cite: Ugol'nikov, A., Diachenko, V., Kliat, Y., Kosenko, A., Shelukhin, S. (2021). Devising matrix technology for forecasting the dynamics in the operation of a closed military logistics system. *Eastern-European Journal of Enterprise Technologies*, 6 (3 (114)), 36–46. doi: <https://doi.org/10.15587/1729-4061.2021.249270>

1. Introduction

Improvement of management of logistics processes for the training and combat use of military units and subunits occurs under the conditions of constant growth of the dynamics of this application and the shortage of material resources. This causes the need to develop scientific methods of prompt substantiation of management decisions made. This problem should be solved in theory and in practice by a new structure, which includes command, planning, and executive bodies and which is called the military logistics system.

Logistics refers to the science of planning and optimizing material flows, service flows, as well as related flows, in a certain system or subsystem to achieve the goal due to these flows.

Military logistics is a scientifically based, optimized comprehensive equipment of troops, units, and subunits of all types of the Armed Forces:

- armament, transport, and means of management;
- missiles and ammunition;

- repair equipment and military property;
- fuel and lubricants;
- property, food, and medical supplies.

The equipment takes place in peacetime, during preparation for combat operations and their implementation during the movement of troops, their activities in defense, and in the offensive. Optimization of processes in military logistics is carried out according to the criterion of minimum time spent on achieving the purpose of the functioning process and is organized in content, volume, time, and place.

The novelty and complexity of the tasks facing the military logistics system require the development of new approaches to solve them. Therefore, it is necessary to use scientific methods in the analysis, planning, and optimization of the processes of the provision in the military logistics system. The following means are used to scientifically substantiate decisions on optimizing the processes of equipping troops, units, and subunits:

- the scientific and methodological apparatus of inventory management;
- software products based on predictive software models;

- the methods of linear, dynamic programming to optimize the processes of comprehensive support;
- an apparatus of computer calculations using MATLAB software;
- staff dialog and information models implemented on a computer that works in real time.

The existence of a military logistics system requires the development of adequate models of the process of its functioning in the implementation of various practical tasks. This is due to the increasing complexity of the tasks that logistics must perform. The volume of information flows is increasing, the time for the implementation of various types of support is decreasing, the range of supplies is expanding, new vehicles are used for deliveries, for example, unmanned aerial vehicles, and so on. Modeling makes it possible to give an analytical description of the processes of functioning of the military logistics system in order to determine and improve its effectiveness. These factors determine the relevance of work in the area of forecasting the dynamics of the functioning of the closed military logistics system.

2. Literature review and problem statement

In work [1], the tasks of provision are considered in terms of minimizing the cost of storage and supply of industrial products. From a mathematical point of view, the problem is reduced to finding a minimum of the objective function, depending on many variables.

Article [2] investigates a complicated situation where the objective function is not continuous and has non-sliding gaps of the first kind. In [3], to optimize the operation of the motor transport enterprise of the military logistics base, linear programming methods are used in the multi-criteria scorecard. The problem of optimal placement of goods (the problem of three-dimensional packaging) is solved in work [4] provided that cars are additionally loaded at specialized points.

Work [5] is methodological. It examines the existing approaches to modeling logistics systems – deterministic-optimal, probabilistic, and knowledge-oriented, assessing the possibility of their use to describe processes in the auto-technical units of military units. The analysis of the advantages and disadvantages of each of these approaches and models based on them leads to the conclusion that the introduction of a knowledge-oriented approach is the most effective in modern military logistics. There are no calculated algorithms in the cited work, which reduces its applied value.

Paper [6] reports the development of a methodology for building a model of the system of ensuring the work of the enterprise engaged in production and sales activities. The substantiation of the importance of the development of general theoretic research methods is given. A universal method of developing a support system, starting with the simplest tank systems, is suggested. The model is used to solve optimization problems of building systems for ensuring the operation of an enterprise. Optimization is carried out using numerical methods.

Study [7] considers the task of choosing a safe cargo supply route in war zones. Unlike the classic setting of the task about the salesman, a parameter is introduced into consideration, which characterizes the safety of the selected route (the probability of delivery of goods to its intended destination). The criterion for the optimality of the route is the maximum probability of delivery of goods to all desti-

nations. It is noted that the formulated problem is the task of integer programming and it is proposed to solve it by the method of branches and bounds.

Article [8] proposes solving a narrow problem of building a statistical model for assessing and repairing combat damage to weapons and equipment. On this basis, it is proposed to deploy the forces of the military logistics system and the formation of its resources.

Work [9] builds on the research started in [8], and considers the development of a simulation model to study the factors influencing combat damage to equipment. Modeling results are planned to be used as input information to build a more complex model of logistics of the military industry.

Another work by the same team of authors [10] proposes the construction of a model of optimal solutions based on statistical material and simulation modeling. The result of the integrated system is a built model of optimal solutions for the elimination of combat damage to equipment.

Work [11] considers the issue of improving the reliability of logistics by the forces of the military logistics system. Two deception strategies are considered: the inclusion of empty convoys on routes and the spread of misinformation about the route. It is assumed that convoys can be controlled on routes by the enemy. A two-step integer model of linear programming is proposed, which determines the quantitative parameters of both options for deceiving opponents. In addition, the model takes into consideration several other tasks of the military logistics system. Among them is the presence of restrictions on the range at points of supply, compliance with time restrictions, the choice of the best route option.

Article [12] describes the peculiarities of logistical support in the implementation of military expeditionary operations and humanitarian missions. The model of a non-stationary mass service network is used, which makes it possible to take into consideration uncertainties and risk assessment. To verify the model, the data of logistical support of combat operations during the operation in Iraq were used.

A common feature of those studies is the focus on the interaction of the military logistics system with consumers of its services. At the same time, not enough attention is paid to the internal problems of the functioning of the military logistics system itself.

The problem of increasing (in publication) the adequacy of the model, as well as reliable risk assessment, is not solved. The functioning of the military logistics system is always carried out under conditions of uncertainty of both random and antagonistic nature. Under these conditions, the application of the specified mass service model requires the use of pre-known parameters of the law of the flow of random orders of consumers, which during practice are difficult to determine. In this situation, predicting the dynamics of the functioning of the military logistics system can make unacceptable mistakes in determining risks.

Under these conditions, it seems more reliable to use models that are not based on the laws of a probability distribution of the flow of applications and refusals.

The use of more adequate (in such a situation) models of the dynamics of mean, that is, the apparatus of discrete Markov processes, seems more acceptable. And overcoming the uncertainty of the antagonistic type, for example, with the help of the minimax principle, that is, taking into consideration the expected options for the enemy's actions and options for counteracting the logistics system, does not create technical difficulties.

That is why the use of more adequate models, which are based on the widespread use of matrix calculation technology in real time, is promising. It should be noted that the input data, in this case, are the data of experience surveillance and use under the conditions of high dynamics of changes in the situation of the investigated support processes.

3. The aim and objectives of the study

The purpose of this work is to devise an informative and convenient (in use) method for describing and predicting the dynamics of the functioning of the military logistics system. This technique makes it possible to find the probabilities of the process of functioning of the system in its main states, which contributes to the adoption of informed decisions when ensuring the implementation of combat missions.

To accomplish the aim, the following tasks have been set:

- to build a system of differential equations describing within the model the process of functioning of the military logistics system when performing various tasks;
- to derive a solution to the system of differential equations in the form of expressions to calculate the probability of the functioning of the logistics system in its main states;
- to illustrate the performance of the model, using it to describe the process of functioning of the military logistics system in the performance of specific support tasks, for example, forecasting the survivability of the transport logistics subsystem during the march of the military unit.

4. The study materials and methods

One of the effective methods of researching complex phenomena and processes is the method of mathematical modeling. The functioning of the military logistics system is influenced by a significant number of poorly defined factors of random and antagonistic nature. Simplification of solving the problem can be achieved by introducing into consideration some generalized performance indicator [13]. In work [14], as such a generalized indicator, it is proposed to use the probability of the military logistics system being in its working condition when performing specific tasks. This way of assessing the effectiveness of the system makes it possible to take into consideration the uncertainties that quantify the change in time, that is, the dynamics of the functioning of the logistics system in real conditions. Therefore, it was chosen during the construction of the model in this work.

Analysis of the dynamics of the functioning of the military logistics system makes it possible to present its functioning as a discrete Markov process [14]. This process is characterized by a set of typical states that the system can enter, as well as state transitions. The intensity and probability of these transitions are determined by specific conditions of application of the logistics system for its intended purpose. The possibility of using the Markov model is based on the fact that the next state of the military logistics system depends only on its current state and does not depend on how the system entered that state. Transitions of the Markov system between states occur many times, and this reflects the real transitions of the military logistics system between states in the process of its functioning.

The dynamic model of the functioning of the military logistics system, which can be in $n+1$ different states

S_i ($i=0, 1, 2, \dots, n$) with the probabilities $P_i(t)$ and make transitions from any state to any other, is considered. The system can be depicted using the state and transition graph. Figure 1 shows an example of such a graph for a system that can be in four states and execute all possible transitions between them.

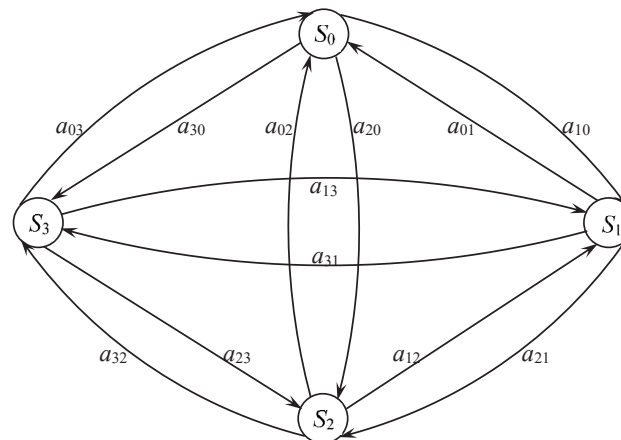


Fig. 1. Graph of states and transitions of the functioning of the full-time military logistics system, which can be in four main states

The transition from S_j state to S_i state is characterized by the intensity h_{ij} and the probability H_{ij} . Then the coefficients $a_{ij}=h_{ij} \cdot H_{ij}$ ($i \neq j$) characterizes the impact of the probabilities $P_j(t)$ of the system staying in the S_j state on the probability $P_i(t)$ of its being in the state S_i . The coefficients a_{ii} with the same indexes are equal to $a_{ii} = -\sum_{\substack{j=0 \\ j \neq i}}^n a_{ji}$, reflecting the fact that

all objects that are out of the S_i state have moved to one of the other states S_j .

5. Results of studying the model of the functioning of the military logistics system

5.1. A system of differential equations describing the dynamics of the process model

These are some practical examples of building a system of differential equations that are inherent only in the military logistics system and worthy in solving the typical problems of this system.

We have proposed an effective matrix technology for solving applied problems of military logistics. Unlike known ones, this computation technique is based on the use of the well-known apparatus of discrete Marking processes and Laplace transformations. This could contribute to a significant acceleration of the calculation of forecast estimates of the probability levels of the system being in a state of effective functioning, which is important information for the command staff when planning the processes of comprehensive provision of actions and planning maneuvers by material means.

The probability P of the system being in the S_i state increases due to the transition of the entire system from other states S_j and decreases due to transitions in the opposite direction. Then the differential equations describing the time dependence of the probabilities $P_i(t)$ of the presence of the studied system in states S_i , take the form:

$$\begin{cases} \dot{P}_0(t) = a_{00}P_0(t) + a_{01}P_1(t) + \dots + a_{0n}P_n(t), \\ \dot{P}_1(t) = a_{10}P_0(t) + a_{11}P_1(t) + \dots + a_{1n}P_n(t), \\ \dot{P}_2(t) = a_{20}P_0(t) + a_{21}P_1(t) + \dots + a_{2n}P_n(t), \\ \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \\ \dot{P}_n(t) = a_{n0}P_0(t) + a_{n1}P_1(t) + \dots + a_{nn}P_n(t), \end{cases} \quad (1)$$

where $\dot{P}_i(t) = \frac{dP_i}{dt}$ is the probability derivative by time. The properties of the system of equations (1) are fully determined by the matrix **A** of the coefficients of the system:

$$\mathbf{A} = \begin{pmatrix} a_{00} & a_{01} & \dots & a_{0n} \\ a_{10} & a_{11} & \dots & a_{1n} \\ \dots & \dots & \dots & \dots \\ a_{n0} & a_{n1} & \dots & a_{nn} \end{pmatrix}.$$

Unknown $P_i(t)$ functions are subject to additional restrictions. These are, first, the initial conditions that specify the probability values at the initial point in time ($t=0$):

$$P_i(0) = P_i^0 \quad (i = 0, 1, 2, \dots, n),$$

and, second, the following normalization condition

$$\sum_{i=0}^n P_i(t) = 1,$$

that is, a set of all possible system states forms a complete group of events. Obviously, the presence of the system in one of these states is a reliable event, the probability of which is equal to one.

When applying the proposed model to the description and forecasting of the functioning of the military logistics system in the performance of typical provision tasks, it is necessary to build a graph of states and transitions and determine the quantitative characteristics of all transitions. This can be done using regulatory documents and an expert evaluation method.

5.2. Solving a system of model equations that describes the process of functioning

Solving the system of linear differential equations with constant coefficients is advisable to perform using the method of operating calculus, which is based on the Laplace transformation. Next, for a more compact notation, the argument t in parentheses after the sign of the function $P_i(t)$ or its time derivative $\dot{P}_i(t)$ is omitted.

Since the system equations (1) are bound by a condition of normalization, they are not independent. This makes it possible to exclude one of the equations and consider a system containing n independent equations. The exclusion of the first equation (with index "0") and the substitution of the corresponding expression to all other equations produce the following system

$$\begin{cases} P_0 = 1 - P_1 - P_2 - \dots - P_n, \\ \dot{P}_1 = a_{10}(1 - P_1 - P_2 - \dots - P_n) + \\ + a_{11}P_1 + a_{12}P_2 + \dots + a_{1n}P_n, \\ \dot{P}_2 = a_{20}(1 - P_1 - P_2 - \dots - P_n) + \\ + a_{21}P_1 + a_{22}P_2 + \dots + a_{2n}P_n, \\ \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \\ \dot{P}_n = a_{n0}(1 - P_1 - P_2 - \dots - P_n) + \\ + a_{n1}P_1 + a_{n2}P_2 + \dots + a_{nm}P_n. \end{cases}$$

The first equation is removed, the terms of other equations are regrouped, and the designation $a'_{ij} = a_{ij} - a_{i0}$ ($i, j = 1, 2, \dots, n$) is entered to bring the so-called shortened system:

$$\begin{cases} \dot{P}_1 = a'_{11}P_1 + a'_{12}P_2 + \dots + a'_{1n}P_n + a_{10}, \\ \dot{P}_2 = a'_{21}P_1 + a'_{22}P_2 + \dots + a'_{2n}P_n + a_{20}, \\ \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \\ \dot{P}_n = a'_{n1}P_1 + a'_{n2}P_2 + \dots + a'_{nn}P_n + a_{n0}. \end{cases} \quad (2)$$

The next step is to apply the Laplace transformation to system (2) using the conversion table and the original differentiation formula:

- $P_i(t) \rightarrow X_i(p)$ is the mapping of the unknown function $P_i(t)$, where p is the complex variable;
- $\dot{P}_i(t) \rightarrow pX_i(p) - P_i^0$ is the mapping of the derivative;
- $a_{i0} \rightarrow a_{i0}/p$ is the mapping of the constant value.

As a result, the system of linear differential equations relative to probabilities $P_i(t)$ is replaced by a system of linear algebraic equations relative to their mapping $X_i(p)$:

$$\begin{cases} pX_1(p) - P_1^0 = a'_{11}X_1 + a'_{12}X_2 + \dots + a'_{1n}X_n + \frac{a_{10}}{p}, \\ pX_2(p) - P_2^0 = a'_{21}X_1 + a'_{22}X_2 + \dots + a'_{2n}X_n + \frac{a_{20}}{p}, \\ \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \\ pX_n(p) - P_n^0 = a'_{n1}X_1 + a'_{n2}X_2 + \dots + a'_{nm}X_n + \frac{a_{n0}}{p}, \end{cases}$$

or, in a standard form:

$$\begin{cases} (a'_{11} - p)X_1 + a'_{12}X_2 + \dots + a'_{1n}X_n = -\frac{pP_1^0 + a_{10}}{p}, \\ a'_{21}X_1 + (a'_{22} - p)X_2 + \dots + a'_{2n}X_n = -\frac{pP_2^0 + a_{20}}{p}, \\ \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \\ a'_{n1}X_1 + a'_{n2}X_2 + \dots + (a'_{nn} - p)X_n = -\frac{pP_n^0 + a_{n0}}{p}. \end{cases} \quad (3)$$

Introducing into consideration the matrix of the shortened system **A'**, the matrix-column of free terms **B(p)**, and the matrix-column of unknown **X(p)**:

$$\mathbf{A}' = \begin{pmatrix} a'_{11} & a'_{12} & \dots & a'_{1n} \\ a'_{21} & a'_{22} & \dots & a'_{2n} \\ \dots & \dots & \dots & \dots \\ a'_{n1} & a'_{n2} & \dots & a'_{nm} \end{pmatrix}, \quad \mathbf{B}(p) = -\frac{1}{p} \begin{pmatrix} pP_1^0 + a_{10} \\ pP_2^0 + a_{20} \\ \dots \dots \dots \\ pP_n^0 + a_{n0} \end{pmatrix},$$

$$\mathbf{X}(p) = \begin{pmatrix} X_1(p) \\ X_2(p) \\ \dots \dots \\ X_n(p) \end{pmatrix},$$

allows the system (3) to be written as a single matrix equation

$$(\mathbf{A}' - p\mathbf{E}) \cdot \mathbf{X} = \mathbf{B},$$

where **E** is the unit matrix of the n -th order. The methods of solving systems of linear algebraic equations are well known.

A solution to the matrix equation is convenient to present in the form

$$\mathbf{X}(p) = \frac{1}{p \cdot \det(\mathbf{A}' - p\mathbf{E})} \mathbf{R}(p). \tag{4}$$

where $\det(\mathbf{A}' - p\mathbf{E})$ is the main determinant of system (3) (characteristic polynomial of the matrix \mathbf{A}'), and the elements $R_i(p)$ of the matrix-column $\mathbf{R}(p)$ with accuracy to the multiplier p are the auxiliary determinants of this system:

$$R_i(p) = - \begin{vmatrix} a'_{11} - p & \dots & a'_{1,i-1} & pP_1^0 + a_{10} & a'_{1,i+1} & \dots & a'_{1n} \\ a'_{21} & \dots & a'_{2,i-1} & pP_2^0 + a_{20} & a'_{2,i+1} & \dots & a'_{2n} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ a'_{n1} & \dots & a'_{n,i-1} & pP_n^0 + a_{n0} & a'_{n,i+1} & \dots & a'_{nn} - p \end{vmatrix}. \tag{5}$$

To obtain the solution $\mathbf{P} = \mathbf{P}(t)$, one must inversely convert the mapping of $\mathbf{X}(p)$ by Laplace. $X_i(p)$ elements of the matrix $\mathbf{X}(p)$ are correct rational fractions, so they decompose into the sum of elementary fractions by difference $(p - p_j)^{-1}$ ($j = 0, 1, \dots, n$), where $p_0 = 0, p_j \neq 0$ are the proper numbers of matrix \mathbf{A}' (the roots of its characteristic equation $\det(\mathbf{A}' - p\mathbf{E}) = 0$). From the content of the problem, it follows that the roots must be valid, and, in real conditions, they are all simple. As a result, the decomposition of $X_i(p)$ ($i = 1, 2, \dots, n$) elements is:

$$\left\{ \begin{aligned} X_1(p) &= \frac{R_1(p)}{p \cdot \det(\mathbf{A}' - p\mathbf{E})} = \\ &= \frac{C_{10}}{p} + \frac{C_{11}}{p - p_1} + \frac{C_{12}}{p - p_2} + \dots + \frac{C_{1n}}{p - p_n}, \\ X_2(p) &= \frac{R_2(p)}{p \cdot \det(\mathbf{A}' - p\mathbf{E})} = \\ &= \frac{C_{20}}{p} + \frac{C_{21}}{p - p_1} + \frac{C_{22}}{p - p_2} + \dots + \frac{C_{2n}}{p - p_n}, \\ &\dots \\ X_n(p) &= \frac{R_n(p)}{p \cdot \det(\mathbf{A}' - p\mathbf{E})} = \\ &= \frac{C_{n0}}{p} + \frac{C_{n1}}{p - p_1} + \frac{C_{n2}}{p - p_2} + \dots + \frac{C_{nn}}{p - p_n}. \end{aligned} \right. \tag{6}$$

In matrix form, we obtain the equation $\mathbf{X}(p) = \mathbf{C} \cdot \mathbf{D}(p)$. The symbols \mathbf{C} and \mathbf{D} indicate the matrix of decomposition coefficients and the matrix of decomposition parameters, respectively:

$$\mathbf{C} = \begin{pmatrix} C_{10} & C_{11} & C_{12} & \dots & C_{1n} \\ C_{20} & C_{21} & C_{22} & \dots & C_{2n} \\ \dots & \dots & \dots & \dots & \dots \\ C_{n0} & C_{n1} & C_{n2} & \dots & C_{nn} \end{pmatrix}, \quad \mathbf{D} = \begin{pmatrix} p^{-1} \\ (p - p_1)^{-1} \\ (p - p_2)^{-1} \\ \dots \\ (p - p_n)^{-1} \end{pmatrix}.$$

Calculation of unknown elements of matrix \mathbf{C} , in the case of simple real roots, is performed by substitution of zeros of the denominator p_j ($j = 0, 1, 2, \dots, n$). The algorithm is that fractions in the right parts of expressions in each line of system (6) lead to a common denominator, and then equate the numerator of the resulting fraction to the corresponding numerator $R_i(p)$ on the left side. These equalities

must hold at all values of the complex variable p , including at the values $p = p_j$ ($j = 0, 1, 2, \dots, n$), which are zeros of the denominator. The substitution of these values gives the following expressions for calculating the C_{ij} elements of matrix \mathbf{C} :

$$C_{ij} = \frac{R_i(p_j)}{(-1)^n \prod_{k=0, k \neq j}^n (p_j - p_k)} \quad (i = 1, 2, \dots, n; j = 0, 1, \dots, n). \tag{7}$$

Determining C_{ij} coefficients and using a tabular formula

$$\frac{C_{ij}}{p - p_j} \rightarrow C_{ij} e^{p_j t},$$

and the linear properties of the Laplace transformation makes it possible to find the originals $P_i(t)$ in the form

$$\left\{ \begin{aligned} P_1(t) &= C_{10} + C_{11} e^{p_1 t} + C_{12} e^{p_2 t} + \dots + C_{1n} e^{p_n t}, \\ P_2(t) &= C_{20} + C_{21} e^{p_1 t} + C_{22} e^{p_2 t} + \dots + C_{2n} e^{p_n t}, \\ &\dots \\ P_n(t) &= C_{n0} + C_{n1} e^{p_1 t} + C_{n2} e^{p_2 t} + \dots + C_{nn} e^{p_n t}. \end{aligned} \right. \tag{8}$$

In matrix form, we obtain $\mathbf{P}(t) = \mathbf{C} \cdot \mathbf{F}(t)$, where $\mathbf{F}(t)$ is the matrix of the inverse transformation of Laplace of the matrix $\mathbf{D}(p)$:

$$\mathbf{F}(t) = \begin{pmatrix} 1 & e^{p_1 t} & e^{p_2 t} & \dots & e^{p_n t} \end{pmatrix}^T.$$

The solution (8) is derived without any additional assumptions, that is, it is precise and fully analytical. This makes it possible to use this result to solve specific tasks of ensuring the actions of the military logistics system, that is, to evaluate the quality of its functioning.

5. 3. Illustrating the feasibility of the forecasting model for the dynamics of a military logistics system

The calculation algorithm developed using operational and matrix calculus is quite generalized.

We are talking about a set of consistent actions, rules that make it possible to achieve the goal of the operation to determine the effectiveness of the functioning of military logistics systems under typical difficult conditions for the implementation of military tasks.

It can be applied to the description of the processes of functioning of the military logistics system in different conditions. The specificity of particular tasks is determined by the list of the main possible states of the system, the list of possible transitions, and the numerical values of input parameters of transitions. To determine them, it is advisable to use normative documentation and practical experience (in the form of an expert evaluation method). Features of the model are illustrated in several examples.

The subsystem of technical support of the military logistics system, which ensures the functioning of the transport flow, is considered. Analysis of the dynamics of the process shows that the system can be in three main states and make transitions between them. A graph of the main states and transitions of the military logistics system, which provides the transport flow, is shown in Fig. 2.

In this practical example, the content and typical nature of each state are defined. The following state notations are used:

- S_0 – the system is in a working, non-busy state, that is, it is not involved before the march and at rest, if there is no damaged equipment that needs to be restored;
- S_1 – the system is in a working, busy state, that is, the movement of the column occurs, and there is no damaged equipment that needs to be restored;
- S_2 – the system is involved in the elimination of damages caused by equipment during movement or at rest.

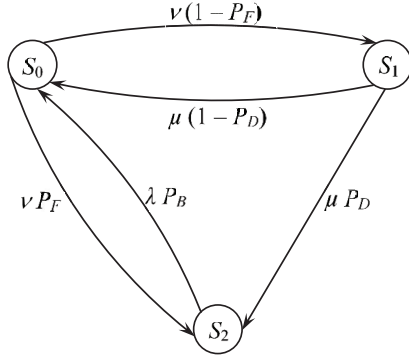


Fig. 2. Graph of three states and transitions of the functioning of the military logistics system through the implementation of the transport flow during the provision of troops' activities

Suppose, for example, the march of the battalion tactical group to the area of concentration to perform a combat mission at a distance of 240 km from the original boundary. The purpose of the march is to concentrate troops for combat operations. The column contains:

- armored weapons – 31 units;
- rocket and artillery weapons – 6 units;
- automotive communication and control means – 3 units;
- trucks – 17 units;
- technical means of ensuring the march – 3 units.

The organization of the march includes decision-making, planning of the march, setting tasks for parts (units), organization of interaction, air defense, comprehensive support, curfew service, and management.

The commander makes the decision on the march personally based on clarification of the task received, assessment of the situation, the route of nomination and tactical calculations carried out by the headquarters. The march purpose includes:

- determining the structure of marching order and the distribution of forces and means in columns;
- composition, tasks, and remote protection;
- routes and average speed;
- output frontier (point) and frontiers (points) of regulation;
- the number and duration of halts over the first daily transition;
- organization of air defense on the march.

When organizing a long-distance march, the number and size of daily transitions is also determined; the number of routes at each transition areas of the day (night) daily rest of units and time of stay in them.

It is planned to carry out a daily march. After moving for six hours, it is assumed to rest for an hour and a half, four hours is the time of restoration of weapons and military equipment that failed due to operational reasons, as well as due to damage by the enemy. According to the established standards, the daily operational failures of weapons and mil-

itary equipment (WME) are equal to 5 %, daily damage to these means at rest – 6 %, daily damage during movement – 7 % [15]. Recovery rates during the march of WME column for vehicles that failed due to operational reasons are 95 %, and the means that failed as a result of combat losses – 65 %. From this data, one can calculate values for the input parameters of the model (the intensity and probability of each state transition).

The intensity of transition from any state to another is defined as the value inverse to the time of the system's stay in this state. The WME provision system is during the march in a non-busy state S_0 only at rest. Therefore, the intensity ν of the system transitions from this state to states S_1 or S_2 is equal to

$$\nu = \frac{1}{1.5} \approx 0.667 \text{ hour}^{-1}.$$

Accordingly, the intensity μ of transition of the supply system from state S_1 to states S_0 and S_2 is equal to

$$\mu = \frac{1}{6} \approx 0.167 \text{ hour}^{-1}.$$

The intensity λ of transition of the support system from state S_2 (restoration of WME failed during the march or at rest) to the state S_0 , is equal to

$$\lambda = \frac{1}{4} = 0.25 \text{ hour}^{-1}.$$

According to the above-mentioned standards, the probability P_F of WME failure at rest should be accepted equal to $P_F=0.06$.

The probability P_D of WME damage during movement consists of the amount of probabilities of operational losses and losses as a result of the enemy's actions and is equal to $P_D=0.05+0.07=0.12$.

Finally, the probability P_B of WME recovery over the planned time consists of three additions. This is the probability of restoration of equipment that failed during operation, and the probability of restoration of equipment that was damaged due to the enemy's actions during movement or at rest. Hence

$$P_B = 0.05 \cdot 0.95 + 0.06 \cdot 0.65 + 0.07 \cdot 0.65 = 0.132.$$

It is advisable to assume that at the time $t=0$ the march support system was in state S_0 (the system is working, non-busy). This assumption corresponds to the initial conditions $P_0^0 = 1, P_1^0 = 0, P_2^0 = 0$.

Calculating the a_{ij} elements of matrix A that characterize the transitions between the states of the system produces:

$$a_{01} = \mu(1 - P_D) \approx 0.147, \quad a_{02} = \lambda P_B \approx 0.033,$$

$$a_{10} = \nu(1 - P_F) \approx 0.627, \quad a_{12} = 0,$$

$$a_{20} = \nu P_F \approx 0.040, \quad a_{21} = \mu P_D \approx 0.020,$$

$$a_{00} = -(a_{10} + a_{20}) \approx -0.667,$$

$$a_{11} = -(a_{01} + a_{21}) \approx -0.167,$$

$$a_{22} = -(a_{02} + a_{12}) \approx -0.033.$$

Then the matrix A of the system of differential equations (1) is equal to

$$A = \begin{pmatrix} -0.667 & 0.147 & 0.033 \\ 0.627 & -0.167 & 0 \\ 0.040 & 0.020 & -0.033 \end{pmatrix}$$

The calculations performed according to the above algorithm give the roots of the characteristic equation $p^2 - \alpha_1 p + \alpha_2 = 0$

$$p_1 = -0.810, \quad p_2 = -0.056.$$

and the C matrix elements:

$$C_{10} = 0.456, \quad C_{11} = -0.797, \quad C_{12} = 0.341,$$

$$C_{20} = 0.423, \quad C_{21} = 0.031, \quad C_{22} = -0.455.$$

Then, taking into consideration the condition of probability normalization, the solution to the system takes the form:

$$\begin{cases} P_0(t) = 0.121 + 0.766e^{-0.810t} + 0.113e^{-0.056t}, \\ P_1(t) = 0.456 - 0.797e^{-0.810t} + 0.341e^{-0.056t}, \\ P_2(t) = 0.423 + 0.031e^{-0.810t} - 0.454e^{-0.056t}. \end{cases}$$

The plot in Fig. 3 demonstrates that during the march of the column, the technical support system is highly likely (~0.70) in the state S_1 . In this case, the movement of the column occurs and there is no damaged equipment that needs to be restored. This fully corresponds to a real situation and indicates the adequacy of the proposed model and the realism of the selected values of the input parameters.

To assess the quality of the technical support system, an efficiency indicator is used. It is introduced, for example, as the ratio of the total probability of the system staying in a state of working capacity to the probability of its stay in a state of disability:

$$E(t) = \frac{P_0(t) + P_1(t)}{P_2(t)}.$$

The plot of dependence of this indicator on time for the case in question is shown in Fig. 4.

The proposed method of modeling the technical support system of military logistics can be applied to more complex cases, with a greater number of possible system states and transitions between them.

As a second example, a generalized model of the process of technical support for the restoration of military equipment that is damaged in battle is considered. This system, according to our analysis, can enter one of the four main states:

- S_0 is the state of combat use of equipment;
- S_1 is the state of preparation of equipment for use;
- S_2 is the state of restoration of equipment after its damage;
- S_3 is the state of maintenance of equipment before or after the end of hostilities.

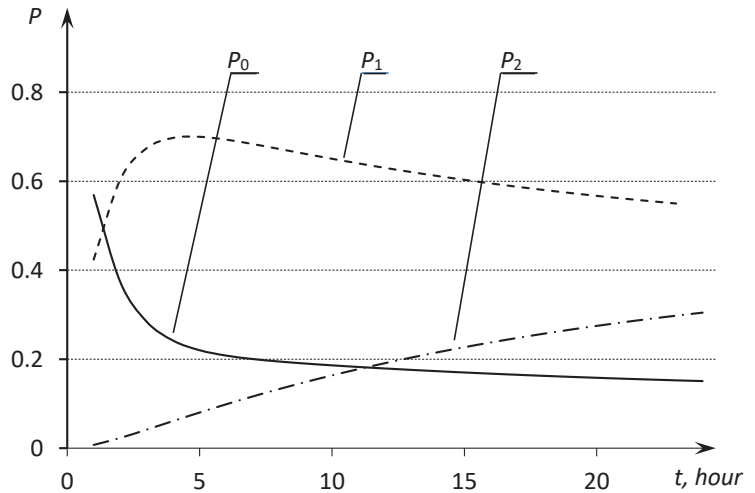


Fig. 3. Probability of the process of functioning of the military logistics system in its main states during the provision of troops' actions through the transport flow of material means

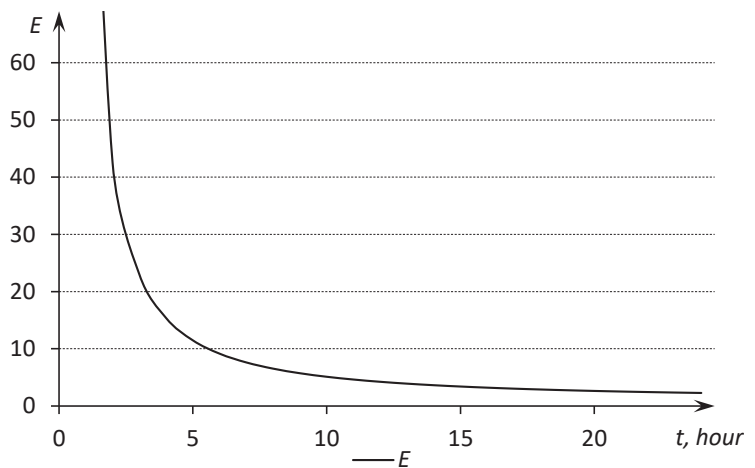


Fig. 4. Performance indicator E of the functioning of the transport flow of the military logistics system during the provision of troops' actions

The graph of states and transitions of the system is shown in Fig. 5.

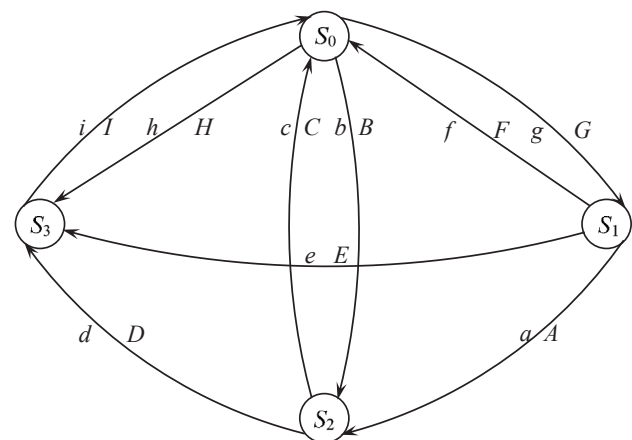


Fig. 5. Graph of states and transitions of the process of functioning of the disabled typical system of military logistics during the provision of actions of troops in the implementation of a more complex task

System transitions between states are characterized by the following parameters:

– *a*, *A* is the intensity and probability of transitions of the military-logistic support system of the weapons and military equipment from the state of preparation of equipment to the state of its maintenance;

– *b*, *B* is the intensity and probability of transitions from the state of maintenance to the state of combat use of automotive equipment;

– *c*, *C* is the intensity and probability of transitions from the state of combat use to the state of maintenance of equipment;

– *d*, *D* is the intensity and probability of transitions from the maintenance state to the state of recovery of equipment after damage;

– *e*, *E* is the intensity and probability of transitions from the state of preparation to the state of recovery of equipment after damage;

– *f*, *F* is the intensity and probability of transitions from the state of preparation to the state of combat use of equipment;

– *g*, *G* is the intensity and probability of transitions from the state of combat use to the state of training of equipment for the purpose of its use;

– *h*, *H* is the intensity and probability of transitions from the state of recovery of equipment after damage to the state of its combat use;

– *i*, *I* is the intensity and probability of transitions from the state of combat use of equipment to the state of recovery after its damage.

The a_{ij} elements of matrix **A** of system (1) are equal to:

$$\begin{aligned} a_{01} &= fF, \quad a_{02} = hH, \quad a_{03} = bB; \quad a_{10} = gG; \\ a_{20} &= iI, \quad a_{21} = eE, \quad a_{23} = dD; \quad a_{30} = cC, \quad a_{31} = aA; \\ a_{00} &= -(a_{10} + a_{20} + a_{30}), \quad a_{11} = -(a_{01} + a_{21} + a_{31}), \\ a_{22} &= -a_{32}, \quad a_{33} = -(a_{03} + a_{23}). \end{aligned}$$

The values of the intensity and probability of each of the transitions of the technical support system, in this case (as in the following example), are applied without justification. They are illustrative.

$$a = b = c = d = e = f = g = h = i = \frac{1}{2},$$

$$A = B = C = D = E = F = G = H = I = \frac{1}{9},$$

Therefore, we obtain the matrix of the system of differential equations in the form

$$\mathbf{A} = \begin{pmatrix} -\frac{1}{6} & \frac{1}{18} & \frac{1}{18} & \frac{1}{18} \\ \frac{1}{18} & -\frac{1}{6} & 0 & 0 \\ \frac{1}{18} & \frac{1}{18} & -\frac{1}{18} & \frac{1}{18} \\ \frac{1}{18} & \frac{1}{18} & 0 & -\frac{1}{9} \end{pmatrix}.$$

The probability values of the system states at the initial point in time (initial conditions) are accepted equal to

$$P_0^0 = 1, \quad P_1^0 = P_2^0 = P_3^0 = 0.$$

The roots of the characteristic equation $p^3 - \alpha_1 p^2 + \alpha_2 p - \alpha_3 = 0$ are equal to

$$p_1 = -\frac{1}{9}, \quad p_2 = -\frac{2}{9}, \quad p_3 = -\frac{1}{6}.$$

Calculations produce a solution to the system in the form:

$$\begin{cases} P_0(t) = \frac{1}{4} + \frac{3}{4} e^{-\frac{2}{9}t}, \\ P_1(t) = \frac{1}{12} - \frac{3}{4} e^{-\frac{2}{9}t} + \frac{2}{3} e^{-\frac{1}{6}t}, \\ P_2(t) = \frac{1}{2} - \frac{1}{2} e^{-\frac{1}{9}t}, \\ P_3(t) = \frac{1}{6} + \frac{1}{2} e^{-\frac{1}{9}t} - \frac{2}{3} e^{-\frac{1}{6}t}. \end{cases}$$

The curves of dependence of probabilities of staying of the system of ensuring hostilities in its possible states on time are shown in Fig. 6.

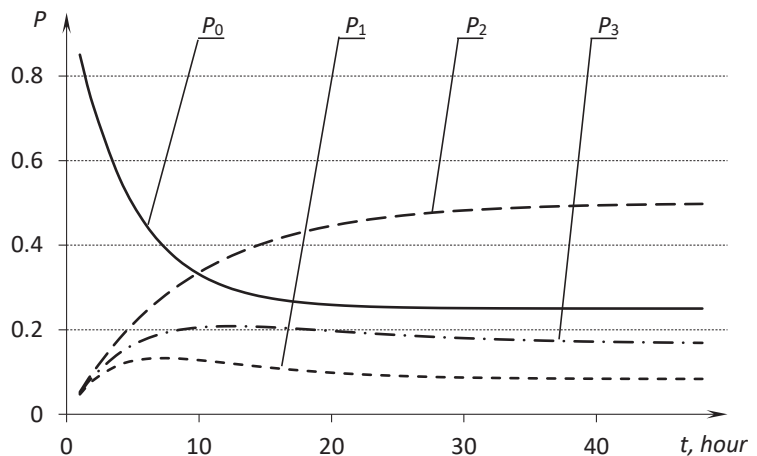


Fig. 6. Probability of staying in the process of functioning of the disabled typical system of military logistics in its main states during the restoration of military equipment, which is damaged in battle

The indicator of the effectiveness of the combat operations support system is defined as the ratio of the probability of its stay in a state of combat use to the probability of its stay in the state of recovery, takes the form

$$E(t) = \frac{P_0(t)}{P_2(t)}.$$

As an example of a system that can be in five different states, the system of military logistics of comprehensive support of combat operations is considered with the dominance of the importance of the functioning of the military logistics system of comprehensive support of troops' actions in full readiness. According to the analysis, the system can be in the following main states:

– S_0 is the state of stay of the functioning of the military logistics system of comprehensive support of troops' actions in full readiness;

– S_1 is the state of stay of functioning of the combat (operational) subsystem in unpreparedness;

- S_2 is the state of stay of functioning of the subsystem of technical support in unpreparedness;
- S_3 is the state of stay of functioning of the medical support subsystem in unpreparedness;
- S_4 is the state of stay of functioning of the subsystem of the rear support in unpreparedness.

Obviously, the main task is to maintain the state of operation of the comprehensive readiness of the military logistics system for use. This is clearly demonstrated by the graph of states and transitions of this system, shown in Fig. 7, where the S_0 state occupies a special place.

The numeric intensity and probability values of all possible state transitions have been selected as arbitrary but realistic. The result is the following matrix **A** for this system:

$$A = \begin{pmatrix} -1.3 & 0.9 & 0.8 & 0.7 & 0.6 \\ 0.5 & -0.9 & 0 & 0 & 0 \\ 0.4 & 0 & -0.8 & 0 & 0 \\ 0.3 & 0 & 0 & -0.7 & 0 \\ 0.2 & 0 & 0 & 0 & -0.6 \end{pmatrix}$$

The initial conditions for this system are as follows:

$$P_0^0 = 1, \quad P_1^0 = P_2^0 = P_3^0 = P_4^0 = 0.$$

The solution to the characteristic equation (the fourth power) produces four real roots:

$$p_1 = -2.193, \quad p_2 = -0.735, \\ p_3 = -0.849, \quad p_4 = -0.622.$$

Their substitution to (5), (7) produces the ultimate result of solving it in the form:

$$\begin{cases} P_0(t) = 0.355 + 0.635e^{-2.193t} + 0.004e^{-0.735t} + \\ + 0.003e^{-0.849t} + 0.003e^{-0.622t}, \\ P_1(t) = 0.197 - 0.245e^{-2.193t} + 0.011e^{-0.735t} + \\ + 0.031e^{-0.849t} + 0.006e^{-0.622t}, \\ P_2(t) = 0.177 - 0.182e^{-2.193t} + 0.022e^{-0.735t} - \\ - 0.025e^{-0.849t} + 0.008e^{-0.622t}, \\ P_3(t) = 0.152 - 0.128e^{-2.193t} - 0.032e^{-0.735t} - \\ - 0.006e^{-0.849t} + 0.013e^{-0.622t}, \\ P_4(t) = 0.118 - 0.080e^{-2.193t} - 0.010e^{-0.735t} - \\ - 0.003e^{-0.849t} - 0.031e^{-0.622t}. \end{cases}$$

The dependences of probabilities of staying by a subsystem of comprehensive provision of combat operations in its possible states and performance indicator on time are shown in Fig. 8.

An indicator of the effectiveness of the system of comprehensive provision is the ratio of the probability of its stay in a state of comprehensive readiness to the total probability of being in a state of unpreparedness

$$E(t) = \frac{P_0(t)}{P_1(t) + P_2(t) + P_3(t) + P_4(t)}$$

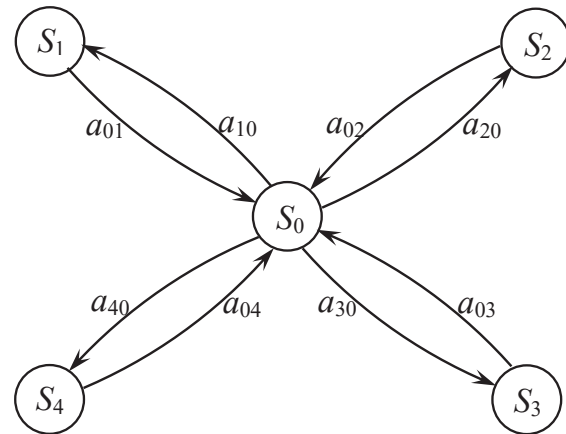


Fig. 7. Graph of states and transitions of the functioning of the partially closed military logistics system for comprehensive support of troops' actions

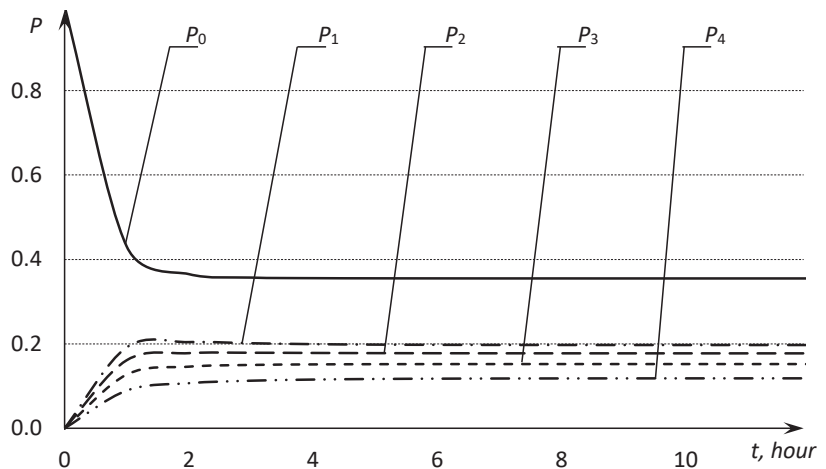


Fig. 8. Probabilities of staying for a partially closed military logistics system with the predominant dominance of the state of comprehensive readiness S_0

The efficiency indicator of this support system is always reduced during the operation and its limit value, as a rule, decreases with an increase in the complexity of the system, that is, with an increase in the number of types of provision.

6. Discussion of results of studying the model of functioning of a military logistics system

In the Markov model, the process of functioning a military logistics system is depicted as a graph of states and transitions (Fig. 1). This graph corresponds to the system of linear differential equations of the first order (1), which describes the probability of the functioning of the logistics system in its main states. The use of such a simple mathematical apparatus favorably distinguishes the proposed model from those reported in [1–7] (methods of minimizing the functions of many variables or methods of integer programming).

The application of the Laplace transformation and matrix calculus when solving the system of equations (1) led to obtaining a precise analytical solution in the form of ratios (8). As a result, a simple and easy-to-use tool for predicting the dynamics of the functioning of the military logistics system in ensuring the actions of troops was con-

structured. All calculations can easily be implemented on personal computers or tablets using effective numerical algorithms of linear algebra.

The proposed model and calculation technique, developed on its basis, were applied to the forecasting of the dynamics of the functioning of the military logistics system in the execution of various typical tasks for ensuring the actions of troops. For the first example (Fig. 2), the algorithm for determining the input parameters of the model has been described in detail. In all cases, qualitative coordination with the real situation was obtained (Fig. 6, 8), and in the first example – quantitative (Fig. 4, 5).

All this allows us to assert that a reliable and convenient technique for predicting the dynamics of the functioning of the military logistics system has been devised.

Both the model under consideration and its mathematical implementation are not free from restrictions, some of which are purely technical in nature, and some are fundamental. The first of the technical limitations is that a fully analytical solution can be obtained only with a number of possible system states of no more than five. With a larger number of states to find their own numbers of a shortened matrix, it is necessary to use numerical algorithms. The second constraint related to the fact that only the case when all the proper numbers of the shortened matrix are different was considered. If some of them coincide, then, to determine the coefficients in probability expressions, it is necessary to use a different method than the method used in this work. It should be noted that this restriction is not significant since such a situation is somewhat artificial and unlikely under real conditions.

The fundamental limitation of the model is that the coefficients of the initial system of differential equations themselves may depend on time. The nature of such dependence is currently unknown. In addition, even in the case of the simplest linear dependence, the system would be described by nonlinear differential equations, solving which is a complex mathematical problem. Obviously, the development and improvement of the proposed model should tackle these areas.

The toolset proposed to predict the dynamics of the process and the effectiveness of the functioning of the military logistics system does not require, unlike models reported, for example, in [12], the use of differential probability distribution laws for input parameters during

calculations. In the study, the use of experience data on input parameters is enough. At the same time, due to errors in the case of non-compliance of the parameters of these laws with real parameters in practice, this could cause [12] a repeated increase in errors in the results of solving an actual task.

This statement does not contradict the recommendations of E. S. Wenzel about the expediency to pay more attention not to sophisticated calculations but to the search for reliable input data.

This study may be advanced by improving the toolset offered for its application under conditions where the parameters of the state and transition graph are the functions of time.

7. Conclusions

1. To describe and predict the functioning of a military logistics system, a model of discrete Markov processes was used since this system can be considered a system without memory. For a process with an arbitrary number of possible states, a system of differential equations has been built, describing a change in the probability of the process of functioning in its possible states. This result makes it possible to adequately take into consideration the influence of the factor of the uncertainty of operating conditions on the nature of the process.

2. The exact solution to the system of differential equations was derived by using the method of operational calculus. The application of matrix calculus has made it possible to obtain ultimate results in the form of simple analytical expressions for the probability of the functioning of the military logistics system in its main states. This also allows the use of effective numerical algorithms of linear algebra during calculations.

3. The performance of the proposed model has been illustrated by several examples. They consider the processes of functioning of the military logistics system and its subsystems in solving various tasks of ensuring the operation of troops. The sequence of calculations describes in detail the procedure for determining the input parameters of the model. The results reported in this work can be used to plan the processes of functioning of a military logistics system and its operational adjustment in real time.

References

1. Kolomytseva, A. O., Yakovenko, V. S. (2012). Modeliuvannia protsesiv optymalnoho upravlinnia lohistychnymy rozpodilchymy systemamy. *Biznes Inform*, 7, 18–21. Available at: http://nbuv.gov.ua/UJRN/binf_2012_7_5
2. Vorobeva, O. (2019). Methodology for searching optimal solutions of operational planning by cargo transportation in dynamically changing economic conditions. *Transportnoe delo Rossii*, 5, 188–192. Available at: <https://www.elibrary.ru/item.asp?id=41578629>
3. Yusupova, N. I., Valeev, R. S. (2020). Operational level problems in transport logistics. *Modern high technologies*, 3, 107–111. doi: <https://doi.org/10.17513/snt.37950>
4. Androschuk, A. S., Melenchuk, V. N. (2014). Logistic model autotechnical software military parts. *Systemy ozbroyennia i viyskova tekhnika*, 3(39), 3–7. Available at: http://nbuv.gov.ua/UJRN/soivt_2014_3_3
5. Sherstennykov, Y. V. (2019). The Methodology for Modeling Logistics Systems: Implementation Principles and Examples. *The Problems of Economy*, 4 (42), 306–314. doi: <https://doi.org/10.32983/2222-0712-2019-4-306-314>
6. Bayramov, A. A., Talibov, A., Pashaev, A., Sabziev, E. (2019). The mathematical model of technical supply logistics in the war activity zones. *Modern Information Technologies in the Sphere of Security and Defence*, 2 (35), 77–80. doi: <https://doi.org/10.33099/2311-7249/2019-35-2-77-80>

7. Li, X., Zhao, X., Pu, W., Chen, P., Liu, F., He, Z. (2019). Optimal decisions for operations management of BDAR: A military industrial logistics data analytics perspective. *Computers & Industrial Engineering*, 137, 106100. doi: <https://doi.org/10.1016/j.cie.2019.106100>
8. Li, X., Zhao, X., Pu, W. (2020). Knowledge-oriented modeling for influencing factors of battle damage in military industrial logistics: An integrated method. *Defence Technology*, 16 (3), 571–587. doi: <https://doi.org/10.1016/j.dt.2019.09.001>
9. Li, X., Zhang, W., Zhao, X., Pu, W., Chen, P., Liu, F. (2021). Wartime industrial logistics information integration: Framework and application in optimizing deployment and formation of military logistics platforms. *Journal of Industrial Information Integration*, 22, 100201. doi: <https://doi.org/10.1016/j.jii.2021.100201>
10. Ausseil, R., Gedik, R., Bednar, A., Cowan, M. (2020). Identifying sufficient deception in military logistics. *Expert Systems with Applications*, 141, 112974. doi: <https://doi.org/10.1016/j.eswa.2019.112974>
11. McConnell, B. M., Hodgson, T. J., Kay, M. G., King, R. E., Liu, Y., Parlier, G. H. et.al.(2019). Assessing uncertainty and risk in an expeditionary military logistics network. *The Journal of Defense Modeling and Simulation: Applications, Methodology, Technology*, 18 (2), 135–156. doi: <https://doi.org/10.1177/1548512919860595>
12. Horodnov, V. P. et. al. (2004). *Modeliuvannia boiovykh diy viysk (syl) protypovitrianoi oborony ta informatsiynе zabezpechennia protsesiv upravlinnia nymy (teoriya, praktyka, istoriya rozvytku)*. Kharkivskiy viyskoviy universytet.
13. Sukhin, O. V., Demianchuk, B. O., Kosenko, A. V. (2019). Model protsesiv systemy tekhnichnoho zabezpechennia boiovoho zastosuvannia zrazkiv ozbroiennia. *Systemy ozbroiennia i viiskova tekhnika*, 4 (60), 94–101. doi: <https://doi.org/10.30748/soivt.2019.60.13>
14. Boiovyi statut mekhanizovanykh i tankovykh viysk Sukhoputnykh viysk Zbroinykh Syl Ukrainy. Ch. II (2016). *Komanduvannia Sukhoputnykh viysk Zbroinykh Syl Ukrainy*, 135–136.