

*It is shown that due to the growth of waste generated by the metropolis, the processes of their removal and disposal must be more accurately accounted and controlled. If it is impossible to introduce "smart" control systems, it is proposed to search for reserves to increase the efficiency of the processes in their structure. A structural model of operations has been developed that can reduce time costs. The use of incomplete information on the accumulation and removal of garbage leads to unplanned mileage of trucks. In order to avoid unforeseen costs, it is proposed to use early garbage collection, which reduces the frequency of emptying containers. This leads to an increase in the number of truck arrivals to load, but eliminates unforeseen mileage due to inconsistencies in the loading forecast. It is shown that to effectively organize the work of garbage trucks on the transport network of the city, an active, shortest schedule of operations is required, which must be made for several periods. To develop an optimal cyclic schedule of garbage trucks, a method based on the ordering of mixed graphs is proposed. The mixed graph shows the set of garbage collection operations and the time relationships between their execution times. In order to develop an optimal schedule from such a graph, cycles must be removed from the graph. To do this, the "divide and conquer" method was used. The proposed algorithm for graph ordering is used to study the current garbage collection system. As a result of research, higher productivity of garbage trucks and timely removal of organic waste were achieved. The reduction of the weekly working time of 6 garbage trucks with the use of the 70 % container filling level reached 42 hours*

**Keywords:** organization of transportation, household waste, schedule of operations, frequency of processes, mixed graphs

# OPTIMIZATION OF SCHEDULES FOR EARLY GARBAGE COLLECTION AND DISPOSAL IN THE MEGAPOLIS

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## 1. Introduction

The organization of collection, removal and utilization of cities garbage is the most difficult part of the municipal administration. The main reason is that the logistics of garbage collection do not meet the objective needs of cities for their disposal often. The process of waste accumulation is stochastic. Its intensity is only partially known to public utilities. It is also known that the generation of solid waste is characterized by seasonality, as well as weekly and daily irregularities. Therefore, it is extremely difficult to plan the operation of motor vehicles (MV) in such conditions.

There is also a contradiction in the methodology of organizing the removal of solid waste in large cities. Public

utilities are still guided by the norms of garbage generation depending on the population. Road hauliers try to shorten the planned prefabricated routes of MV in order not to run too much in the absence of reliable information about the actual accumulation of waste. However, the frequency of garbage collection is usually greater than the actual accumulation in garbage containers of fixed volume. It is important for garbage trucks not only to develop the shortest route, but also to schedule it. Due to the lack of clear schedules of MV on the routes, some containers remain overflowing with waste, which violates the conditions of sanitary norms.

The current level of technology allows public utilities to use means of remote control over the process of filling garbage containers. However, the use of automated accounting

tools does not fully solve the problem. Current and retrospective information clarifies the forecast of the volume of garbage transportation. The available forecasts do not correspond to the method of compiling the travel routes of garbage trucks. Scheduling of garbage trucks on the territory of the metropolis in accordance with the filling of containers is not practiced. In most cases, the transition of the municipal waste disposal system to an automated level is constrained by economic factors. Territorial communities of megacities are often financially unable to equip all garbage containers of the metropolis by telemetry. The problem is getting worse by the growing population of cities and, consequently, the intensity of waste generation. Therefore, research related to solving the problem of garbage collection is relevant and growing.

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## 2. Literature review and problem statement

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The paper [1] confirms that the organization and planning of solid waste collection are complex and the degree of their complexity is growing. The demands of the world community to reduce pollution and garbage sorting are growing due to the increasing number of citizens of megacities. On the other hand, the paper noted that efficient computer decision-making tools are not used yet to route garbage trucks. The authors proposed to solve the MV routing problem for a separate collection of solid waste using a large neighborhood search algorithm. A specific example of a metropolis shows that a problem is the one that contains a large amount of input data, making it difficult and sometimes impossible to solve. However, the studies did not take into account the actual intensity of garbage accumulation in garbage cans, which generally does not solve the problem.

The authors acknowledge that the management of solid waste collection and disposal is often ineffective due to the high uncertainty associated with the actual filling levels of garbage cans in [2]. Therefore, one of the possible solutions is the use of sensors for filling garbage cans. However, the use of data from sensors must be accompanied by appropriate algorithms for optimizing the routes of garbage trucks. Such special algorithms are still unknown.

Modern surveillance and monitoring devices, such as volume sensors, radio frequency identification, GPRS (General Packet Radio Service) and GPS (Global Positioning System) technology, allow receiving data on the accumulation of solid waste remotely, in real time. This is the basis for improving waste management. Well-known publications on “smart” waste management technologies can be divided into three groups. The difference lies in the basic approaches used to organize the collection of solid waste:

- 1) methodology of standards and limits;
- 2) “smart” waste collection routing (SWCR);
- 3) “smart” periods of garbage collection [3].

All algorithms of the limited approach do not have a justification of the established **minimum** level of filling of garbage containers (limits)  $k_d$ . Thus, all garbage containers with a filling level of  $k_i$ ;  $k_d$  are considered at the routing stage. The second significant disadvantage of the limited approach is that routing problems are solved cyclically, with a fixed period, preferably one day. This does not take into account the impact of past results on the newly built plan for garbage collection by trucks. Therefore, there may be problems at some stages of route planning with the accumulation of garbage to

the level of  $k_d$  in many containers that are not included in the previous plan for solid waste collection. The work of garbage trucks in this case is uneven every day, so it is inefficient.

The impact of uneven loading of garbage trucks in the SWCR approach is somewhat reduced, in which the choice of containers to be visited daily and the justification of the sequence of their collection are carried out simultaneously [4]. Criteria for maximum profit from solid waste transportation are used. The model used by the researchers represents the problem of MV routing, provided that the current level of container filling is known to the decision-maker. However, the model has no restrictions on the speed of container filling at the end of the planning period. As a result, the model will have a suboptimal solution when the filling rates approach the capacity of the container in the last days of the horizon. In addition, this problem is solved only for one MV.

The approach of “smart” periods of garbage collection model is slightly modified by researchers in [5]. The model is combined with a heuristic procedure that determines when (on what day) route planning/rescheduling should begin in order to maximize profits over a period of time. This approach explores the **minimization** of resource inefficiencies and takes into account a predetermined level of service. However, the model used by the authors dynamically improves the obtained schedule of MV only in relation to overcrowded containers. The results of using MV fleets for garbage collection are unknown.

The growing complexity of methods and algorithms for planning the collection of solid waste is due to the fact that the number of points in the network of garbage containers in the metropolis is growing. Nevertheless, the authors of the review [6] claim that quite satisfactory methods have been developed for routing tasks. However, these tasks remain in the focus of research in the management of the processes of collection and delivery of goods.

Garbage collection and delivery processes that are equipped with online parameters control tools are considered as a part of “smart” cities. However, the paper [7] based on a review of known research indicated that the possibility of future work of “smart” cities is based on the support and involvement of all stakeholders, especially citizens, in decision-making processes. Thus, the volume of data on future solid waste collection systems will increase with much greater intensity, which will complicate the application of modern algorithms.

The paper [8] investigates the stochastic variant of the MV routing problem, where both the location of customers and the volume of traffic are uncertain. A model of a decentralized decision-making system has been created, where vehicles independently establish their routes in accordance with the observed state of the system. However, researchers have provided formulations based on Markov’s decision-making processes for this task, which is ineffective in the development of “smart” systems of the metropolis.

The problem of planning and routing of waste collection without establishing the frequency of routing is considered in [9]. However, this choice increases the complexity of the model and eliminates the possibility of using dynamic routing to provide the shortest routes for the early filling of containers.

Mathematical programming problems are the main tools for the mathematical formulation of the problem of garbage collection and transportation. Despite their great diversity, the number of formulations still grows and the question of

their adaptation to specific formulations remains relevant. In particular, the paper [10] presents a recently developed model of mixed-integer programming (MIP) for the problem of selective collection of solid waste with time windows, limited heterogeneous fleet of vehicles and different varieties of solid waste that are collected separately. However, the problems of mathematical programming have a significant disadvantage, which can not be applied to cyclical problems. They do not reflect the physical nature of the processes they control.

An important value of the paper [11] is the description of the mathematical model of traditional and cyber-physical waste management system, which describes the impact of Industry 4.0 technologies such as RFID, cloud and fog computing, big data analysis. The scientific contribution of this work is mathematical modeling and optimization of waste collection processes based on binary optimization. However, such intelligent technology is still at an early stage, but its potential needs further study.

One can draw the following general conclusion from the analysis of literature sources [1–11]. The problem of organizing the collection and disposal of solid waste on the road network of megacities is relevant and, at the same time, has become more complex. Garbage truck routing tasks without information about the intensity of container filling are meaningless. When using intelligent data collection systems for garbage collection, the conditions of the task of transportation organizing are complicated by the fact that the optimal routes must be established, taking into account the filling level of containers. For effective planning, it is necessary to evenly distribute routes by days of the calendar planning period. This means that the planning process is cyclical. For cyclical processes and for a growing metropolis, the amount of input data on waste accumulation grows rapidly, on the other hand, information is accumulated in the form of experience of transportation. There is also reason to believe that future waste collection systems should take into account the exchange of information between all subjects of the collection and disposal process: from citizens to recycling companies. Due to the increasing intensity of waste generation, road transport, which is used in the process of their collection and transportation as the main one, is becoming another threat of environmental pollution. Therefore, all the following tasks should be solved by the criterion of minimizing their impact in the form, first of all, of the duration of transport cycles, as well as the use of electric vehicles and other measures to reduce harmful emissions.

### 3. The aim and objectives of the study

The aim of the study is to form an optimal cyclical schedule of garbage trucks. The optimal schedule will allow a movement on the shortest collection routes of the network of garbage container sites, taking into account the actual intensity of their filling. This will reduce transportation costs for the collection and disposal of solid waste in large cities.

To achieve this aim, the following objectives are accomplished:

- to develop a mathematical model of cargo flows and time relations of transport operations on garbage collection and transportation;
- to develop an algorithm for finding the shortest active schedule of garbage collection operations, which depends on the actual filling intensity of garbage cans;

- to study the impact of the time fund in the early collection of garbage on an active optimal schedule for the duration of the process and the efficiency of the fleet of garbage trucks.

## 4. Research materials and methods

The process of garbage accumulation is a discrete and stochastic one. The amount of garbage contained at moment  $t$  in the  $i$ -th container of the garbage collection zone is a piecewise monotonic function with a continuous argument  $k_i(t)$  (Fig. 1).

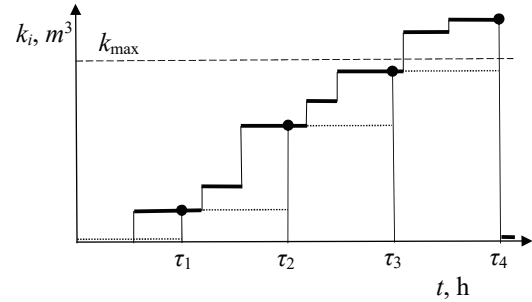


Fig. 1. Schedule of accumulation, control and emptying of the garbage container with time modulation of the input data flow:  $k_{\max}$  – maximum volume of the container;  $\tau_1 \dots \tau_4$  – moments of control of the container filling level

The garbage container filling schedule is typical for both collection systems equipped with telemetry and filling control. What unites them is that the tanks are emptied at fixed points in time, which are separated by a period of  $\tau$  called tact. The volume of emptying is carried out at times when  $k_i \approx k_{\max}$  in the presence of sensors. A certain tolerance for the maximum value of  $k_{\max}$  is sometimes set, but it is difficult to ensure due to the fixed moments of garbage collection. The graph shows that in “smart” solid waste collection systems, the moments of emptying can occur at  $\tau_4$ , as well as at  $\tau_3$  in the presence of tolerance. If the conventional collection system is used, then road hauliers use the experience of previous periods, compiling a schedule of garbage collection. However, the information on actual volumes is underestimated (dotted lines) as shown in Fig. 1. In this case, there are forced deviations from the planned routes, the mileage of garbage trucks becomes unreasonably high, sanitary norms of garbage removal are violated.

Correspondence of a priori information to actual data may exist if the control rate is small enough. But such a flow of data is difficult to provide in almost both “smart” and conventional collection systems. Therefore, we consider the system of control of incoming solid waste flows at landfills when using modulation of input data by the value of  $k_i$  (Fig. 2). As can be seen from Fig. 2, the process of garbage collection in this case is subject to the flow of control data on the volume of filling.

It is necessary to clearly establish the tolerance for the maximum filling of containers  $\Delta k$ , which is a measure of the uncertainty of the filling process. Another difference between this approach is that containers can be emptied earlier than  $\tau_4$ . The choice of the moment of emptying each particular container depends on the planned route of MV and the constructed optimum schedule of garbage collection on the territory of the metropolis. Thus, the time links between the operations of emptying garbage containers are set as more rigidly, by “linking” the schedule to the moment of reaching the planned filling level of containers.

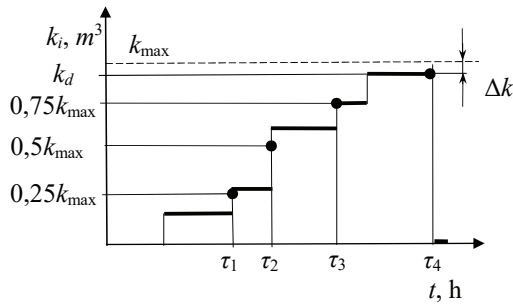


Fig. 2. Schedule of accumulation, control and emptying of the garbage container with the modulation of the process on the volume of filling:  $k_d$  – admissible actual volume of the container;  $\Delta k$  – filling level

**5. Results of optimizing the work of the fleet of garbage trucks with early garbage collection**

**5.1. Mathematical model of material flows based on time connections of the process**

The time relationships of the process operations can be displayed using the following dependencies. Let the average intensity of garbage accumulation in the  $i$ -th garbage container  $\mu_i$  be a constant value. With sufficient accuracy, this value is a priori known from the experience of previous operations to collect garbage on the  $i$ -th container. It is also likely that this value was read from telemetry filling sensors. Given the discrete nature of the process of garbage accumulation, the filling intensity of the container can be recorded as follows:

$$\mu_i = \frac{k_{d,i}}{\tau_i}, \tag{1}$$

where  $k_{d,i}$  – allowable actual volume of the container to be unloaded,  $m^3$ ;  $\tau_i$  – tact of the data received about the  $i$ -th container filling. Given that the operation of unloading the container must coincide with the time when the container is filled, the tact  $\tau_i$  is the value of duration between the start and end of the  $i$ -th operation, which is repeated periodically.

Garbage collection from container sites is carried out cyclically. The garbage collection cycle is a set of operations for unloading containers into one garbage truck. The collection cycle takes time between the time of sending an empty MV on the route until the MV is unloaded at a landfill or recycling plant. Each  $j$ -th operation, which concerns the emptying of the  $j$ -th container, can be performed from cycle to cycle with a variable sequence relative to other operations (Fig. 3).

Fig. 3 indicates the starting points of the corresponding operations –  $t_{b,i}, t_{b,j}, t_{b,j+1}, \dots, t_{b,i+\xi}, t_{b,i+\varphi}$ . The duration of any  $j$ -th operation depends on which operation was previously performed by the same MV. After all, the operation consists of such techniques as the entrance and positioning of the garbage truck near the containers, preparatory receptions, loading, trip with cargoes to the next container site. The duration of each operation is a random variable from cycle to cycle. Its guaranteed numerical value does not exceed with a probability of 0.995 some maximum value of  $a_{i,j}$ . Thus, the moment of completion of the  $i$ -th operation is defined as

$$t_{e,i} = t_{b,i} + a_{i,j}, \tag{2}$$

Fig. 3 shows that the operations  $j$  in the first cycle become  $a_{i,j}$ , and in the second cycle –  $a_{x,j}$ , which are

different values. The fact that cycles of the same operation may differ means that the condition  $\tau_{i,z} \neq \tau_{i,z+1} \neq \tau_{i,z+w}$  exists. However, we take into account the assumption that the filling level of the same container from cycle to cycle should be constant, i.e., based on the expression (1),  $\tau_{i,z} = \text{const}$ , where  $z$  is the number of cycle. This also means that the schedule of operations can be compiled for one cycle of MV and it will be designed for several consecutive cycles to ensure clarity. Given that the fleet of garbage trucks of one company is associated with a common production task, a clear schedule should be made for the entire fleet of MV. An unambiguous schedule is set in one of the possible ways:

- a) by a set of moments of the beginning of operations  $\{t_{b,1}, t_{b,2}, \dots, t_{b,N}\}$  and valid values  $\{a_{i,j}\}$ , where  $i \neq j, i, j = 1..N, N$  is the total number of containers in the service area;
- b) by a set of moments  $\{t_{b,1}, t_{b,2}, \dots, t_{b,N}\}$  and  $\{t_{e,1}, t_{e,2}, \dots, t_{e,N}\}$ , at  $t_{b,z} \neq t_{e,z}$ ;
- c) by a set of moments of completion of operations  $\{t_{e,1}, t_{e,2}, \dots, t_{e,N}\}$  and a set of values  $\{a_{i,j}\}$ .

It is accepted that no garbage truck will have a time delay with operations and the transport cycle in particular. Therefore, condition (2) must be strictly observed:

$$t_{e,j} - t_{e,i} = e_{i,j}, \quad e_{i,j} \neq a_{i,j}, \tag{3}$$

where  $e_{i,j}$  – relationship of the time of completion of two consecutive operations.

According to the schedule theory [12], the requirement for an active optimal garbage collection schedule must be met, where any two operations  $i, j$  can be performed in any sequence:

$$t_{b,i} = \max_{i \leq j \leq N} (0, v_{j,i}), \tag{4}$$

where  $v_{j,i}$  – chain of operations of the longest duration from operation  $i$  to operation  $j$ .

The requirement of an unambiguous schedule is also known [13]. If, under the conditions of the problem, operation  $i$  is to be preceded by operation  $j$ , and these operations are performed by the same MV in one cycle, then the following equations must be satisfied:

$$t_{b,j} - t_{b,i} = a_{i,j} = -(t_{b,i} - t_{b,j}) = -a_{j,i},$$

whence:

$$a_{i,j} = -a_{j,i}. \tag{5}$$

This condition can be demonstrated by the example (Fig. 4).

Fig. 4 shows the model of the process in the form of an oriented graph, the vertices of which are the moments of the beginning of operations 1, 2 – actual, 0, F – fictitious. Fictitious operations are used only to define the time limits of the process. The arcs of the graph  $a_{0,1}, a_{1,2}, a_{2,F}$  have a weight that corresponds to the duration of operations. The arcs of the graph  $a_{F,1}, a_{F,2}$  according to expression (5) mean the time constraints  $a_{1,2}, a_{2,F}$ , i.e.:

$$a_{1,2} + a_{2,F} = -(a_{F,1}), \quad a_{2,F} = -a_{F,2}. \tag{6}$$

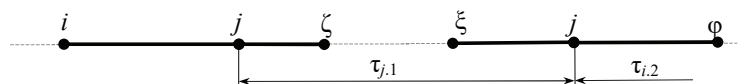


Fig. 3. Scheme of the cycle of the  $j$ -th operation of the garbage collection process:  $\tau_{j,1}, \tau_{j,2}$  – cycles of the  $j$ -th operation

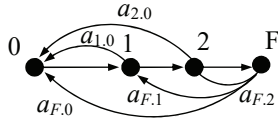


Fig. 4. Time model with feedback: 0,  $F$  – fictitious operations

Restrictions (6) mean that the duration of operations 1, 2 should not exceed the specified value, respectively  $a_{F,1}$ ,  $a_{F,2}$ .

The arcs of the graph  $a_{1,0}$ ,  $a_{2,0}$  similarly, according to expression (5), mean the restriction that operations 1, 2 must begin no earlier than the corresponding time values  $a_{1,0}$  and  $a_{2,0}$ . Taking into account the definition of the operation cycle, one can write expressions that relate the operation cycle to the structure of the process:

$$\tau_i = -(a_{i,0} + a_{F,i}). \quad (7)$$

The total length of run of garbage trucks on routes must be kept to a minimum in order for them to be used as efficiently as possible. At the same time, there is a condition that the entire amount of accumulated garbage was taken out for disposal in a time that does not exceed sanitary norms. This means that it is necessary to build a schedule of solid waste collection operations, in which the duration of the process will meet the condition [14]:

$$T_p = \min \left\{ \max_{0 \leq i \leq N} (t_{b,i} + a_{i,F}) \right\}, \quad (8)$$

where  $i=0$  – fictitious operation, which formally means the start of the whole process;  $F$  – fictitious operation, which formally means the completion of the whole process;  $a_{i,F}$  – time relations with the final operational processes, which are important for the duration of delivery of collected solid waste by MV to landfills or processing plants.

Limitations on the completeness of the waste collection process mean that the intensity of garbage collection should be determined by expression, taking into account (1):

$$\mu_{out} = \sum_{i=1}^N \mu_i = \sum_{i=1}^N \frac{k}{\tau_i}, \quad (9)$$

where  $\mu_i$  is the intensity of solid waste accumulation in the  $i$ -th container;  $k$  is the filling level of garbage containers accepted in the problem (in % from  $k_{max}$ ).

## 5. 2. Method and algorithm for constructing the optimal schedule of trucks

The ordering of the mixed graph is taken as the basis of the decomposition optimization method [14]. The formulation of the problem is as follows. The transport network of the city is given in the form of a graph  $G(Q, V)$ , the vertices  $G$  of which are container sites of solid waste collection, and the ribs  $V$  are the shortest paths between each pair of vertices. We assume that  $a_{i,j}=a_{j,i}$ , i.e. the incidence matrix of the graph  $G$  is symmetric about the diagonal. It is accepted that there is a path between any two points on the city's road network (the network is strongly connected). So the distance between them is known, but for convenience it is estimated indirectly by the travel time  $a_{i,j}$  at a known constant average operating speed.

Each container site of the metropolis places a set of containers, which are divided by sort of waste. The number of sorts is determined by laws and at the level of the territorial community of the city. Each sort of waste has to be

transported for disposal by individual MV according to the technologies used. Accordingly, separate routes are made. If the sorting of garbage is taken as the norm in the settlement, then each variety and each container site are characterized by the intensity of accumulation  $\mu_i$ , which is a random variable and is characterized by the average value of  $\mu_{m,i}$ . Among all types of garbage, there is one that is characterized by the highest intensity of accumulation. This mainly applies to organic waste and plastic [3]. Sorted waste is also characterized by acceptable disposal times, in accordance with locally accepted sanitary norms. Thus, a maximum allowable storage time is set for each container, even if the container is not full. Based on this, the duration of the planning of transportation  $T$  is set, which must be observed taking into account the restrictions. The peculiarities of this problem are that all points of the transport network, except the last point of all routes, are loading points, i.e. the required routes are prefabricated. Also, not all container sites need to be visited by garbage trucks in one cycle, as the approach of garbage removal by filling level has been adopted. In addition, two different garbage trucks cannot drive to the same site in one cycle, as all garbage of the same type is unloaded at the MV in one arrival. Given these conditions, the problem can be attributed to the compilation of cyclic unitary schedules for the flow of process operations by several devices [12]. Therefore, it is necessary to develop the shortest route for each MV that will be involved in the process, planning the implementation of transportation. On the other hand, we need the best schedule for the collection of garbage trucks in the presence of time restrictions on the removal of goods. Given the structure and properties of a typical transport process in medium and large transport systems, this task should be attributed as the optimization one [12]. According to the complexity of the algorithm for finding the optimal solution, it belongs to the problems of complexity  $O(n^3)$ . There is an indeterminate algorithm for finding a successful exact or approximate solution in a reasonable time [13]. In contrast to the classical method of ordering mixed graphs, the whole set of container points is represented by the non-oriented graph  $G(Q, V)$ . The vertex  $q_0$  of the graph is fictitious, representing the formal moment of the beginning of the whole process. The vertex  $q_F$  is fictitious, symbolizing the end of the planned cycle of duration  $T$ .  $V$  is a set of ribs, each of which reflects the time relationships  $a_{i,j}$  and  $a_{j,i}$  between the start of the  $i$ -th and  $j$ -th operation of the same MV. If there is a rib between two vertices  $q_i$ ,  $q_j$  of the graph  $G$ , it means their sequence independence and the corresponding maintenance operations of the  $i$ -th and  $j$ -th container will be performed simultaneously or with a partial overlap in time by different MV.  $R$  vehicles can be involved in the transportation process. They must work synchronously, performing several downloads in succession. This means that it is needed to find  $R$  paths in the graph  $G$  that start at the vertex  $q_0$ , pass through some vertices of the graph, which relate to existing containers and end at the vertex  $q_F$ . We search the minimum duration to perform all operations with existing MV in this version of the problem. Therefore, the required chains of vertices must pass through those vertices for which  $a_{i,j} > 0$ . If  $a_{i,j} = \infty$ , it means that the  $j$ -th operation cannot be performed after the  $i$ -th one. If the chain reaches the vertex  $i$  and there is no other path in the graph  $G$  then with non-negative or non-zero weight, the chain goes to the vertex  $q_F$ . The transport cycle for this truck will be considered complete, although there is a reserve of time to fulfill other, not yet fulfilled orders. The problem in this for-

mulation is similar to the typical multisalesmen problem [7] with differences:

- 1) planning period  $T$  is a predetermined value;
- 2) the length of links of any chain is a variable and depends on the set of operations;
- 3) each operation has restrictions on when to start and end.

The algorithm for solving the problem is as follows. The set of ready-to-perform operations is formed during the period  $T$ . There is a rib between each pair of vertices symbolizing the operations with the corresponding weights  $a_{ij}, a_{ji}$ . The number of MV  $R$  is limited. It is necessary to design a schedule for the specified garbage collection operations, i.e. for each operation specify the time of its beginning  $t_{b,i}$  and the time relationship with the next operation  $a_{ij}$ , as well as the number of MV that performs this operation. The optimal schedule is the one for which criterion (8) is fulfilled.

The search part of the proposed algorithm is based on the “divide and conquer” method. For this purpose, the known algorithm for constructing the optimal schedule [12] is adapted to the formulated conditions of the problem. Let’s mark some used terms. The path in the graph  $G(Q,U,V)$  is called a sequence of arcs  $U_k=\{(q_1, q_i), (q_i, q_j), \dots (q_x, q_p)\}$ , where all the vertices  $q_i\dots q_p$  are different, and the initial and final may coincide. A contour is a circular path in the graph  $G$ . The weight of the path is the sum of the weights of the arcs included in. The weight of the path is expressed numerically within the interval  $(-\infty, +\infty)$ , i.e. is a real number. Therefore, the term contour or path of positive (or negative) weight is used. The path of the largest positive weight in the graph  $G$  connecting  $i, j$  vertices is denoted by  $\vartheta_{ij}$ . If there is no path from the vertex  $i$  to the vertex  $j$  in the graph  $G$  due to the elimination of some edges, then  $\vartheta_{ij}=-\infty$ . Condition (3) must be met in order for the requested schedule to be unambiguous (there was no time incoordination). The moment of the beginning of any  $i$ -th operation is looked for from relation (4).

There is a one-to-one correspondence between the set of all active schedules constructed from the graph  $G$  and the set  $P(G)$  of all graphs that do not contain contours of positive weight. Therefore, we consider unambiguous the decomposition generated by the graph  $G'$  and does not contain contours of positive weight, and ribs  $V$  hence. A sequential analysis of variants with search of all graphs from the set  $P'$  and search for the optimal one by criterion (8). We used the procedure of successive division of  $P'$  into subsets on the basis of the current record by criterion (8) to organize such a search, in order to avoid unproductive search of suboptimal options, [13].

The content of operations with the graph  $G(Q,U,V)$  is as follows. If the two operations  $i, j$  are designed to be performed independently by different vehicles, then there should be no time relationship between them, and the rib of the disordered model  $G$  is eliminated. If these operations are performed sequentially, regardless of their volume, the edge  $[i, j]$  is replaced by an arc  $(i, j)$  of weight  $a_{ij}$ , or an arc  $(j, i)$  of weight  $a_{ji}$ .

We create and use an auxiliary undirected graph  $Hr(Q_r, V_r)$ , the vertices of which are operations of the subset  $Q_r$ , where  $r \leq R$ , in addition to the operation with the main graph  $G(Q, U, V)$ . Each subset of  $Q_r$  consists of vertices that symbolize the operations performed by one truck. At zero simulation step  $Q_r=Q, V_r=V, Hr(\alpha_0)=Hr(Q, \emptyset)$ . The number of such graphs at the beginning of the simulation, respectively, can be  $r > R$ . Then the graph  $Hr$  is transformed as follows. If the operation  $\alpha_i$  with the main graph  $G$  is the elimi-

nation of the rib  $[i, j]$ , then the graph  $Hr(\alpha_i)$  is obtained from the graph  $Hr(\alpha_0)$  as a result of adding the edge  $[i, j]$ . If the operation  $\alpha_i$  is the replacement of the edge  $[i, j]$  by one of the arcs  $(i, j)$ , or  $(j, i)$  in the graph  $G$ , then the graph  $Hr(\alpha_i)$  is obtained from the graph  $Hr(\alpha_0)$ , identifying the vertices  $i, j$  with one vertex. At each  $n$ -th step of the algorithm, we calculate the chromatic number of the graph  $\chi(Hr(\alpha_n))$  according to the method [14]. As a result, it is possible to determine the constraint imposed by the current number of MV on the current schedule:

$$\chi(H_r(\alpha_n)) \leq R. \tag{10}$$

This condition means limited capabilities of the process: to perform several operations simultaneously due to lack of the required number of MV.

The notion of a conflicting rib is used, i.e. the one for which condition (5), (6) is not fulfilled, in order to conduct a purposeful search for expedient transformations in the graph  $G$ . The most conflicting ribs of the graph  $G$  can be found among the conflicting ribs, i.e. those whose transformation leads to a more effective search effect. To do this, one needs to find a value for each conflicting rib.

$$h_{ij} = t_{e,j}(G,U) + \vartheta_j(G,U) + a_{i,j} - \vartheta(G,U), \tag{11}$$

where  $\vartheta_j(G, U)$  – the maximum weight of the path in the graph  $G(Q, U)$ , starting at the vertices  $q_j$ ;  $\vartheta(G, U)$  – the longest (critical) path in the graph  $G(Q, U)$ .

Choosing the most conflicting rib of all conflicting sets  $V(\alpha_n)$ , we are guided by the value  $\min(h_{ij}, h_{ji})$ . We will call that rib the most conflicting, the value of which will be the largest.

The designed algorithm consists of eleven steps:

1. Check whether the graph  $G(\alpha_n)=G(Q, U)$  contains a contour of positive weight, where  $\alpha_n$  is the  $n$ -th cycle of the algorithm,  $n=0, 1, \dots$ . If there is such a contour, then go to step 10.
2. Find the early start of each order after (4). Find the latest completion of orders after (2). If the values found do not comply with the directives, go to step 10.
3. Find the set of conflicting edges of the graph  $G(\alpha_n)$  and the most conflicting among them by criterion (11). If the set is empty, go to step 11.
4. Replace the conflicting edge  $[i, j]$  in the graph  $G(\alpha_{n,1})$  with an arc  $(i, j)$ , in the graph  $G(\alpha_{n,2})$  with an arc  $(j, i)$ ; in the graph  $G(\alpha_{n,3})$  – eliminate the rib  $[i, j]$ .
5. Create the corresponding auxiliary graphs  $Hr(\alpha_{n,1}), Hr(\alpha_{n,2}), Hr(\alpha_{n,3})$ . Calculate the chromatic number of each of the auxiliary graphs  $Hr$ .
6. If inequality (10) does not hold for any graph  $Hr$ ; then go to step 10.
7. If there are loops in the graph  $Hr$ , i.e.  $Hr(i, i)=1$ , then go to step 10.
8. For the graphs  $G(\alpha_{n,1}), G(\alpha_{n,2}), G(\alpha_{n,3})$  perform steps 2, 3, 9 alternately.
9. Calculate the lower estimate of the optimality of the desired schedule among the entered graphs after (8). If  $Tp(G(\alpha_n)) \leq Tp(G(\alpha_{n-1}))$ , then go to step 11, if not – to step 4.
10. The graph  $G$  is considered as degenerate (there is an inadmissible error in its construction), or restrictions and conditions are not met. It is impossible to construct an unambiguous active schedule according to the graph  $G$ .
11. The required schedule is  $\{t_{b,1}, t_{b,2}, \dots, t_{b,N}\}$ . Look for the goal function by (8).

The developed algorithm was translated into the Delphi programming language and applied both to the test model and to the array of actual initial data. The test model consisted of five garbage collection operations from five different container sites, which are represented as five vertices of the graph  $G(Q, V)$ . It is accepted that the average intensity of garbage accumulation on sites is a known and constant value:  $\mu_1=0.4$  m<sup>3</sup>/hour,  $\mu_2=0.35$  m<sup>3</sup>/hour,  $\mu_3=0.4$  m<sup>3</sup>/hour,  $\mu_4=0.38$  m<sup>3</sup>/hour,  $\mu_5=0.32$  m<sup>3</sup>/hour. The total filling intensity of containers is 1.85 m<sup>3</sup>/hour. The permissible level of container filling – 1.1 m<sup>3</sup>. The specified amount of garbage must be removed by  $R=3$  garbage trucks, each of which has a volume of 5.5 m<sup>3</sup>. Therefore, if one truck will be used, the garbage must be removed no later than in 180 minutes. An example of the initial disordered order model is shown in Fig. 5. Between each pair of vertices of the graph, there are ribs the weights of which are estimated in minutes, respectively:  $a_{0,1}=15$ ,  $a_{0,2}=25$ ,  $a_{0,3}=15$ ,  $a_{0,4}=10$ ,  $a_{0,5}=28$ ,  $a_{1,2}=18$ ,  $a_{1,3}=23$ ,  $a_{1,4}=40$ ,  $a_{1,5}=56$ ,  $a_{2,3}=12$ ,  $a_{2,4}=20$ ,  $a_{2,5}=32$ ,  $a_{3,4}=10$ ,  $a_{3,5}=21$ ,  $a_{4,5}=10$ .

It was assumed that the duration of garbage collection operations when moving in both directions of the route is approximately the same, i.e.  $a_{i,j}=a_{j,i}$ .

The “net” duration of operations 1–5 is the one that includes only the time spent loading garbage from the  $i$ -th container and transporting it to the landfill. Parameters of the net duration of operations are:  $a_{1,F}=25$ ,  $a_{2,F}=15$ ,  $a_{3,F}=23$ ,  $a_{4,F}=20$ ,  $a_{5,F}=35$ .

Time limits on the moments of emptying containers are formed on the basis of (7). One can specify the following restrictions, given that the allowable volume of containers is set at known intensities:

$$a_{1,0}+a_{F,1}=-150 \text{ min.}; a_{2,0}+a_{F,2}=-170 \text{ min.};$$

$$a_{3,0}+a_{F,3}=-148 \text{ min.};$$

$$a_{4,0}+a_{F,4}=-160 \text{ min.}; a_{5,0}+a_{F,5}=-170 \text{ min.}$$

There are no restrictions on the start of each of the five operations, given that all containers were empty at the beginning of the simulation.

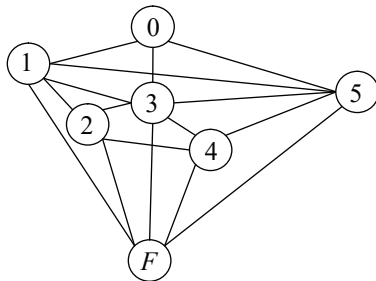


Fig. 5. Initial graph is a test model for schedules building

To get closer to the real situation, we assume that such restrictions exist and accept:  $a_{1,0}=-30$  min.,  $a_{2,0}=-50$  min.,  $a_{3,0}=-28$  min.,  $a_{4,0}=-40$  min.,  $a_{5,0}=-70$  min.:

1. We check whether the graph  $G(Q, U)$  contains contours of positive weight (CPW), removing the edges between the vertices 1–5 of the graph. Only paths remain in the column:

$$0 - 1 - F; 0 - 2 - F; 0 - 3 - F; 0 - 4 - F; 0 - 5 - F.$$

There are no contours of positive weight.

2. Calculate the early beginnings of operations:  $t_{b,1}=15$  min.,  $t_{b,2}=25$  min.,  $t_{b,3}=10$  min.,  $t_{b,4}=28$  min.,  $t_{b,5}=22$  min.

3. Looking for conflicting ribs. The following edges for which the pair inequality holds will be in conflict:

$$-a_{j,i} \leq (t_{b,j} - t_{b,i}) \leq a_{i,j}. \quad (12)$$

So, rib 1–2:  $-18 \leq 5 - 25 \leq 18$  is conflicting.

Rib 1–3:  $-23 \leq 15 - 10 \leq 23$  is conflicting.

Rib 1–4:  $-40 \leq 15 - 28 \leq 40$  is conflicting.

Rib 1–5:  $-56 \leq 15 - 22 \leq 56$  is conflicting.

Rib 2–3:  $-12 \leq 25 - 10 \leq 12$  is not conflicting.

Rib 2–4:  $-20 \leq 25 - 28 \leq 20$  is conflicting.

Rib 2–5:  $-32 \leq 25 - 22 \leq 32$  is conflicting.

Rib 3–4:  $-10 \leq 10 - 28 \leq 10$  is not conflicting.

Rib 3–5:  $-21 \leq 10 - 22 \leq 21$  is conflicting.

Rib 4–5:  $-10 \leq 28 - 22 \leq 10$  is conflicting.

Of all the values of  $h_{i,j}$  calculated by (11), the maximum is found for the rib 1–5:  $h_{1,5}=22+35+56-57=56$ .

We perform operations with the most conflicting rib:

1. First, change the rib [1, 5] to an arc (1, 5). As a result, early operations will change:  $t_{b,1}=15$  min.,  $t_{b,2}=25$  min.,  $t_{b,3}=10$  min.,  $t_{b,4}=28$  min.,  $t_{b,5}=37$  min. The critical path of the modified graph:  $0 - 1 - 5 - F$ . Its weight – 106 min. Restrictions on the timing of the process are met.

2. Change the rib [1, 5] to an arc (5, 1). Early operations will change as follows:  $t_{b,1}=78$  min.,  $t_{b,5}=22$  min. The weight of the critical path will become 93 min.

3. Eliminate the rib [1, 5]. This means that operations 1 and 5 will be performed independently. Early start of these operations will be:  $t_{b,1}=15$  min.,  $t_{b,5}=22$  min. Critical path:  $0 - 5 - F$ , weight – 57 min.

4. Create auxiliary graphs  $Hr$ .

5. The chromatic number of auxiliary graphs is  $\chi_1=2$ ,  $\chi_2=2$ ,  $\chi_3=3$ .

6. The chromatic numbers of each of the graphs do not exceed the value  $R=3$ .

7. Auxiliary graphs do not contain loops, so all three solutions are valid.

8. The graphs in which the rib [1, 5] was changed to arcs and the edge was removed do not contain contours of positive weight, so they can be the basis for constructing an acceptable schedule. However, not all ribs of the initial graph are revised. So, we determine the lower current estimate of the criterion.

9. According to the weight of the critical path, we choose the graph in which the rib [1, 5] is eliminated.

The algorithm is repeated until all edges are selected in the initial graph and steps 2–9 are performed with them. As a result of the application of the algorithm, the final graph is obtained, which meets all the restrictions and conditions of transportation and according to which the shortest schedule can be made (Fig. 6).

Fig. 6 with thickened arrows shows the arcs of the flows of trucks. From the 0-th vertex to the final  $F$ , there are three such flows. So all three trucks are involved. According to the graph, one can make a schedule according to which the maximum duration of execution is in the 5th operation. Let us calculate the cycles of operations with (7):  $\tau_1=150$  min.,  $\tau_2=170$  min.,  $\tau_3=148$  min.,  $\tau_4=160$  min.,  $\tau_5=180$  min. Thus, all restrictions are observed. The first truck is busy for 48 min. (operations 1–2), the second truck is busy for 40 min. (operations 3–4), the third truck is busy for 57 min., of three trucks (operation 5) for 180 min.

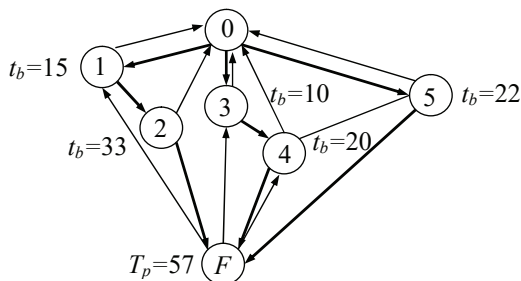


Fig. 6. The final ordered graph without cycles

In order to increase the efficiency of trucks exploitation, two more options were considered, in which the predetermined number of trucks is, respectively,  $R=2$  and  $R=1$ . An option of ordering the initial graph in the presence of two vehicles is shown in Fig. 7.

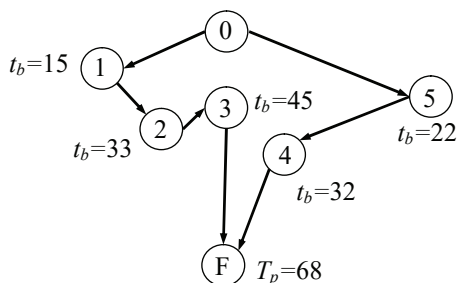


Fig. 7. Option to organize the graph for a given number of trucks  $R=2$

Fig. 7 shows that the involvement of two trucks increases the duration of the process, but not significantly. Conditions and time limits of performance of transportations at such schedule are observed. The first truck is busy for 52 min. The second is busy for 68 min. Thus, the efficiency of using two trucks is higher.

The option of ordering the initial graph in the presence of one truck is shown in Fig. 8.

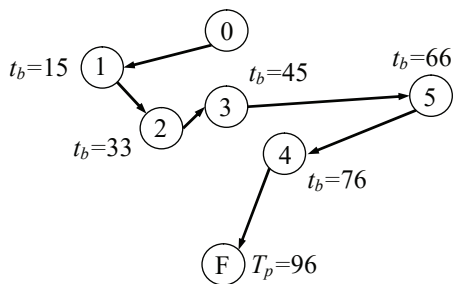


Fig. 8. Option of ordering the graph for one truck,  $R=1$

The duration of garbage collection according to the obtained schedule for one truck is 96 minutes, which is much higher than the previous results. However, the garbage truck is used for more than 50 % of its time fund.

The tact of operations is maintained so that the volume of containers filled with garbage does not exceed a predetermined level in each of the three results obtained. However, it is necessary to set such a level to ensure higher efficiency of MV use, and to adhere to the permissible terms of garbage removal. Therefore, it is necessary to study the impact of the filling level of containers on the process performance.

### 5. 3. Use of time funds of the garbage truck fleet and duration of early garbage collection

The developed algorithm and computer program were used to determine the efficiency of the fleet depending on the degree of filling and the corresponding frequency of garbage collection. To do this, we used data on the garbage collection system in the metropolis (Fig. 9).

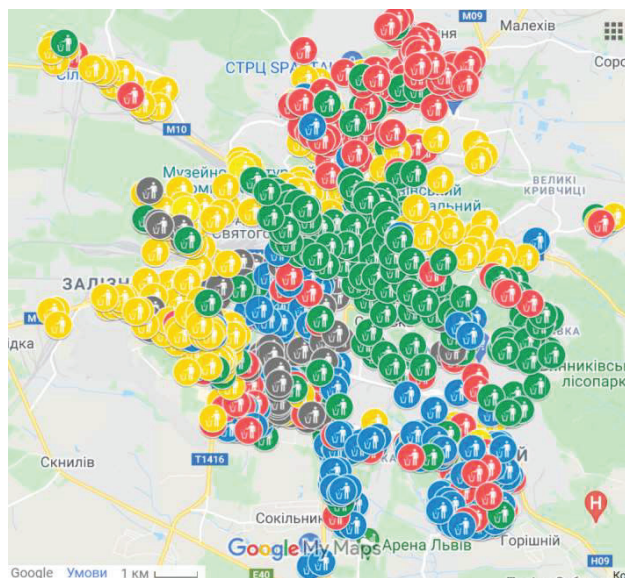


Fig. 9. Map of landfills with containers for organic waste [15]

Landfills in the city include several containers (1–4) that correspond to a particular type of garbage. Organic waste is considered to be the most urgent. In addition, the intensity of organic waste accumulation is almost 30 % higher than all other solid waste by volume. In this regard, the main focus is on the collection and export of organic matter. The waste disposal system in the metropolis is set up in such a way that garbage collection and transportation are carried out mainly by several private and state utility companies (in Fig. 9, the sites assigned to them are marked with 5 different colors). On the one hand, this satisfies the competitive environment and services are provided with quality. On the other hand, the network of sites divided in this way does not contribute to successful routing and MV enterprises carry out large runs. That is why the task of organizing the work of garbage trucks in a decentralized system is very important.

The initial data for the construction of optimal schedules were incomplete in this case. The work of the state utility company, which maintains garbage sites, which are shown in yellow on the map, was considered. On the map of the metropolis, the locations of garbage containers are displayed with satisfactory accuracy so that using the Google Map router, one can determine the shortest distances between any pair of geolocations (Fig. 10).

If the router finds two or more route options, then the longer one is selected from them. It is taken into account that in the conditions of the metropolis, traffic jams are possible and the time of movement between landfills is longer. The utility company's fleet is located within the city limits, and the organic waste processing plant, where garbage of the appropriate grade is transported, is located in the suburban area.

Transportation is carried out by specialized MAZ-5907S2-310 garbage trucks, each of which technologically holds 97 m<sup>3</sup> of solid waste. The average operational speed



of MV within the city is 22.5 km/h. Speed data were taken from tachograph data. Organic waste is not compacted in the garbage truck, so its volume of accumulation corresponds to the volume of removal. The number of garbage trucks in the enterprise fleet is 6.

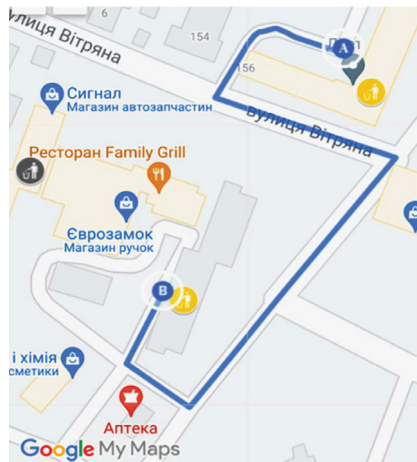


Fig. 10. Example of determining the shortest distance between two given landfills A and B

Accurate information on the intensity of accumulation and removal of organic waste in the metropolis is not available. The garbage collection system is equipped only with means of telemetric control over the location and movement of MV on routes. There are no devices to control the amount of garbage before disposal. So far, the work of garbage trucks is planned on the basis of garbage removal standards, which are often underestimated and do not correspond to reality. Therefore, the actual volumes of garbage removed for the past periods were taken into account in the problem, on the basis of which the average indicator of accumulation intensity was formed (Table 1). Garbage removal is planned at the enterprise on a weekly basis. Therefore, the data presented in the table relate to the results of modeling the weekly production task. To calculate the hourly intensity, it was assumed that garbage is accumulated for 16 hours during the day: from 7–00 to 23–00. The intensity of accumulation is uneven throughout the year. In the summer months, the amount of organic waste accumulates more.

Table 1

Average annual data on the amount of organic waste removed by the utility

Indicator	Year			
	2017	2018	2019	2020
Average total amount of garbage removed per week, m <sup>3</sup>	840	1,180	930	1,108
Number of containers	192	216	232	244
Annual coefficient of unevenness of volumes	1.46	1.62	1.66	1.87
Average intensity of garbage accumulation in one container, m <sup>3</sup> /h	0.039	0.0489	0.0358	0.0405

The coefficient of unevenness thus shows the ratio of maximum intensity to average. This ratio increases, as the total amount of waste does. It should be noted that the pandemic periods distorted the statistics for 2019 and 2020. However, the calculation can be based on the intensity of

the past 2020, 0.04 m<sup>3</sup> per hour, taking into account that the maximum value of intensity can reach 0.075 m<sup>3</sup>/h. Container sites use 1.1 m<sup>3</sup> containers. This means that on average the container is filled in 27 hours. However, the average values for planning the routes of garbage trucks have deviations, mostly to a greater extent. Therefore, for modeling and construction of optimal schedules, the allowable filling of garbage cans is used, which is, respectively 0.55, 0.66, 0.77, 0.88, 1.0 m<sup>3</sup>. Therefore, the frequency of garbage collection from one tank will vary within  $\tau=13.5...24.5$  hours.

The following additional conditions and restrictions are applied to model the waste collection process:

- planned period of garbage removal – 7 days;
- maximum duration of the garbage truck on duty – 7.5 hours;
- working days: Monday – Saturday;
- duration of zero run – no more than 20 min.

The results obtained during the simulation are the optimal performance schedules planned for the week. The parameters of the obtained schedules are given in Table 2.

Table 2

Results of modeling and optimization of schedules of trucks of the garbage removal enterprise

Schedule index, units of measurement	Maximum load of the container allowed, %					
	50	60	70	80	90	100
Average emptying rate of the container, hours	13.5	16.0	19.0	21.5	24.0	27
6 trucks						
Total duration of all MV for a week, hours.	222	192	174	189	207	216
Average duration of MV per day, hours	7.4	6.4	5.8	6.3	6.9	7.2
5 trucks						
Total duration of all MV for a week, hours.	190	188	174	165	178	185
Average duration of MV per day, hours	7.5	7.3	7.0	6.58	7.2	7.4
4 trucks						
Total duration of all MV for a week, hours.	–	152	150	146	150	152
Average duration of MV per day, hours	–	7.5	7.5	7.4	7.4	7.5

Using such initial data, positive simulation results were obtained for 6, 5, 4 trucks that can remove garbage at the same time. With fewer trucks, it was impossible to ensure the allowable time of drivers on duty. 50 % of the solutions were not found with 4 trucks and a permissible level of container filling, due to exceeding the limit of the weekly drivers' time fund.

## 6. Discussion of the results of optimization of garbage removal schedules

The results of the study of the time sequence of garbage collection operations in the metropolis suggest that since the operations form a process that is periodic, its model can be shown using a mixed graph. In this case, one needs to use feedback with negative weights. The negative feedback model reflects the time constraints that the periodicity of the process imposes on the duration of its execution. Given the

content of the constraints, the optimization of the schedule can be done according to the lower estimate of the global optimum –  $\inf(Tp)$ , which greatly simplifies the achievement of a guaranteed solution.

The algorithm for organizing a mixed graph, which is used to optimize the work schedules of garbage trucks, differs from the known ones in the possibility of achieving a guaranteed accurate solution. The complexity of the algorithm does not exceed the value of  $O(n^3)$ , where  $n$  is the number of operations on the graph (in this case, the number of edges of the mixed graph). Testing the algorithm on a test model made it possible to determine its operability. There is no time limit for using the algorithm for the existing initial conditions.

The obtained results of compiling optimal schedules of MV for solid waste removal in the metropolis network show that the application of the optimization algorithm reduces the duration of runs on routes with a constant amount of waste accumulation. The random nature of the filling of containers in landfills is a significant obstacle to the use of the shortest schedule. However, the application of tolerances on actually filled containers has a positive effect, as can be seen from Table 2, which is the increased productivity of MV on the city routes. Thus, the reduction in the weekly duration of 6 garbage trucks with the use of 70 % level of containers filling reaches 42 hours, i.e. the actual release of one crew. Early emptying of containers leads to the lower intensity of the garbage collection process, so the process can be carried out on a more favorable schedule. This effect can be explained by the fact that at a lower required frequency (lower container emptying rate according to (1)), the choice of configuration of daily MV routes is more favorable for reducing the total mileage. Although the frequency, and therefore the weekly number of empty containers, is increasing, the time spent on unloading containers is compensated by the use of more efficient routes.

However, such a positive effect is not typical of any number of vehicles running garbage collection. With the decrease in the number of active fleets of MV, the opportunity to build a shorter active schedule decreases due to the limited time of drivers and the impossibility of simultaneous operations. With a critical number of trucks, the proposed algorithm is not able to build a unique active schedule.

The proposed method of building the shortest schedules of MV was first used to organize the collection and removal of solid waste in the metropolis. The use of optimal schedules has an advantage over the routing of cars that periodically perform the same transport work. The advantage is that the use of schedules allows you to apply dynamic routing depending on the filling intensity of containers. Applying tolerances to the actual capacity of containers prevents excessive mileage of cars and ensures the timely removal of garbage, especially of organic origin.

It should also be noted that the known methods of routing garbage trucks in cities are all mainly based on mathematical programming methods, which are limited in efficiency and ability to find an exact solution due to the dimension. If the planning period for scheduling is extended to one week, most of the known methods fail. The applied algorithm, which is based on the methodology of operations on graphs, can be applied to the planning of processes that are periodically repeated and covers more than 300 operations of the planning process. However, the main limitation of the algorithm is the maximum achievable accuracy of achieving the absolute optimum. This is due to the peculiarities of the

“divide and conquer” method. This method is a compromise in the transition of the solid waste collection system from compromises to “smart”. Given the tolerances for filling containers, method errors are eliminated.

The proposed method of optimizing the schedules of solid waste removal is based on the reservation of garbage in containers. The amount of reservation depends on the available information on the intensity of garbage accumulation. If a system of “smart” waste collection planning is implemented in the metropolis, using fill level sensors and other telemetry devices, then the accuracy of the obtained schedules will be much higher.

We compare the results achieved in reducing the duration of collection and transportation of solid waste in cities with systems where “smart” planning has not yet been introduced. From the comparison, it can be argued that early collection is a more economical transitional state of the garbage collection system, provided the optimal schedules of MV on the routes. Further research should address the methodology for implementing “smart” garbage collection and disposal systems with minimal financial investment.

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## 7. Conclusions

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1. It is necessary to plan and control the processes of garbage collection and transportation with greater accuracy and on a larger scale with the growth of waste generated by the population of megacities. The applied time model of garbage collection and transportation processes is a mixed graph with negative time feedback. This model reflects the transport cycles that correspond to the allowable filling of garbage containers. The model allows building not only the shortest route for one, or for a set of MV, but also to coordinate cyclic operations of garbage collection in time. The proposed time models are also the basis for building the shortest active schedules of MV on routes.

2. The proposed algorithm for organizing time models of garbage collection and transportation processes is meta-heuristic, but it provides a guaranteed solution to the problem of scheduling optimization with satisfactory accuracy. The algorithm can be used for processes consisting of more than 300 repetitive operations, which removes its limitations on application compared to known algorithms. The algorithm uses negative inverse time connections, which set limits on the duration of the process. Due to such limitations, the algorithm has a satisfactory complexity, in contrast to the known algorithms for ordering mixed graphs.

3. Applying tolerances for filling garbage containers allows you to reduce the mileage of garbage trucks on the routes, as in this case the route changes dynamically relative to the filled containers. The effectiveness of the proposed changes to the schedule of the solid waste collection process depends on the number of garbage trucks involved and the established tolerances for filling containers. Thus, for 6 MV and a network of 244 containers at the sites, the duration of garbage collection and transportation is reduced by 16 % (42 hours of weekly time). Higher results are possible with the use of container filling sensors. In the transition of the system of garbage collection and removal in cities from normative to “smart”, which is equipped with means of telemetric measurement of process parameters, the system of tolerances may be the only beneficial for such a transition. However, the effectiveness of the tolerance system signifi-

cantly depends on the number of MV fleets involved in development and application of the shortest early garbage transportation. With a small number of garbage trucks, the collection are impossible.

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