This paper reports a solution to the inverse problem when using indicators in several calculation functions. Such problems arise during the formation of a multi-level scorecard; solving them makes it possible to determine the value of arguments in order to achieve the specified value of the resulting indicator of each level. Thus, the characteristics of an economic object can be defined in order to achieve the specified indicators of its functioning. Optimization models are given in the presence of various types of conditions for achieving the result. In contrast to existing methods, the approach based on building nonlinear programming models makes it possible to solve the inverse problem for the case where several indicators are used in different calculation functions. Algorithms for solving the inverse problem have been constructed, involving the transformation of constraints and the use of an iterative procedure based on inverse calculations. For the case of using coefficients of relative importance, two techniques of solving the problem have been considered: the formation of a single model for subtasks and the adjustment of the solution to subtasks while minimizing the sum of squares of argument changes. In comparison with the existing method, the proposed algorithms have made it possible to derive a solution with a greater correspondence of the changes in the arguments to the coefficients of relative importance. A solution to the inverse problem has been considered related to the formation of marginal profit of an enterprise in the presence of two points of sale and three types of products, as well as the joint formation of revenue and cost. The results of this study could prove useful to specialists in the field of decision-making in the economy and to developers of software decision support systems that include functions for solving inverse and optimization problems

Keywords: inverse problem, economic analysis, inverse calculations, optimization problem, iterative algorithm

CONSTRUCTION OF ALGORITHMS FOR SOLVING THE INVERSE PROBLEM WHEN USING INDICATORS IN SEVERAL CALCULATION FUNCTIONS

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1. Introduction

When assessing the activities of an economic entity and choosing ways to achieve a goal set, specialists are faced with the need to solve problems that, in terms of causation, can be divided into direct and inverse.

Inverse problems have become widespread in various fields such as physics [1], astronomy, image processing [2], economics [3, 4]. Solving such problems in economics makes it possible to receive information that is useful for specialists carrying out activities in the field of management. As a result of solving the problem, a specialist acquires information about how the predefined state of an entity can be achieved. Thus, based on this information, measures can be taken to change the controlled characteristics (price, cost, etc.) to achieve the target indicators of an entity. Thus, the literature considers solving inverse problems for managing the efficiency of a company [5], for the formation of the cost of an industrial enterprise, for improving the structure of the credit and deposit base of commercial banks, for managing an educational institution using a rating system.

Solving inverse problems makes it possible to define the characteristics of an object required to achieve the specified values of strategic and operational indicators, which, in turn, provides for an increase in the quality of management decisions and the efficiency of the functioning of an entity. At the same time, there is a need to analyze and process a large amount of information, which is difficult to implement without the appropriate mathematical apparatus and software tools. In this regard, it is a relevant task to investigate the development of mathematical and algorithmic toolsets for solving inverse problems and increasing the speed of decision-making by automating data processing.

2. Literature review and problem statement

There is a conditional classification of inverse problems in mathematical physics (retrospective, coefficient, boundary, geometric) [6], which is based on conditions not specified in the model (initial values, coefficients, etc.). A given classification is also reflected in the tasks related to economics. Thus, if it is necessary to determine the arguments of the function of calculating an economic indicator, the problem of forming a characteristic is considered. Once the input values x and the operation $h(x)$ to process them (Fig. 1) are defined, then solving a direct problem is to determine the output value y. Solving inverse problems makes it possible

to define the set of input data x or their changes Δx at the specified initial values that provide for the predefined value of the indicator y^*.
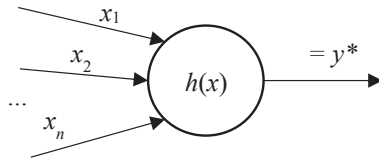

Fig. 1. Building an indicator

The problem to build an indicator can be solved by using regularization [7] and reverse calculations (Fig. 2) [8].
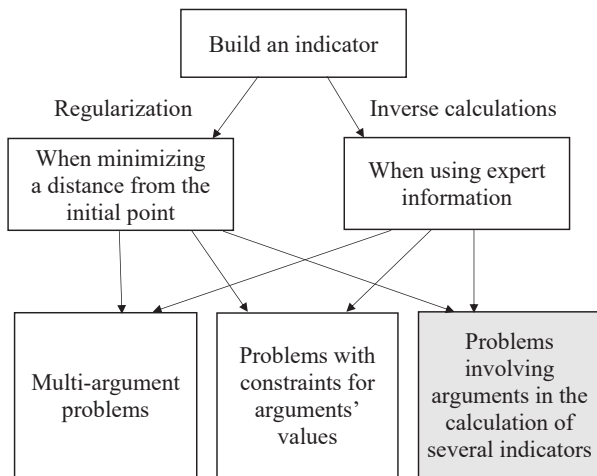

Fig. 2. Classification of problems on building an indicator

Regularization involves the introduction of an additional component of regularization into the functionality and is used to solve problems while minimizing deviation from the initial values [9]. The problem in this statement when using Tikhonov's regularization is considered, for example, in work [10]. However, when using regularization, the task of determining the regularization parameter arises, the solution to which can be found in various ways. Thus, one of these techniques is given in [11] and requires determining the value of a user constant. As a variant of overcoming this difficulty, the authors propose iterative algorithms for solving the inverse problem. Such an algorithm is the Landweber algorithm, which is given in [12] and is based on a sequential change in the argument in accordance with the value of the elements of the gradient vector of the constraint function. However, a given approach makes it possible to find a solution only taking into consideration the distance from the initial values and does not allow one to change the arguments in accordance with the specified settings in the form of expert information. In addition, a given algorithm does not take into consideration the presence of additional limitations-inequalities.

To solve problems using expert information, an inverse computing apparatus has been developed [8]. At the same time, in the case of an increase in dimensionality, the use of a classical apparatus is difficult. Thus, it is necessary to determine the agreed expert information (the directions of change in indicators and coefficients of relative importance must correspond to the goal), which is difficult to implement with a large number of variables, and, otherwise, a solution could not be found. In addition, in the classical version of the apparatus, the use of indicators in different problems is not considered.

The use of arguments in the calculation of several indicators occurs when considering multi-level problems, which are solved step by step along the direct and inverse path of the goal tree. The hierarchical structure of such a problem is reported, for example, in work [5], which considers the formation of cost-effectiveness and the level of competence of personnel. However, the cited work does not give a method for solving such problems.

A method for solving problems in which the same argument is used in the calculation of two indicators is considered in [13]. The essence of the proposed method for solving such a problem is to solve two inverse problems sequentially in order to determine a compromise option. To this end, there is a change in the indicator participating in the two problems, in proportion to the resulting values. The search for compromise values of the arguments continues until the analyst decides to stop the calculations and select the obtained values as a solution to the problem. Thus, a given procedure requires the involvement of additional expert information in terms of finding a compromise option. In addition, it is much more complicated in the case when the number of problems is greater than two, and the variables participating simultaneously in different problems exceed unity.

To eliminate the shortcomings inherent in the known methods, an approach to solving the problem of building an indicator based on the representation of the problem in the form of a conditional optimization problem was devised [14]. Paper [15] discusses the solution to inverse problems represented as a nonlinear programming problem using the variable substitution method, the Lagrange multiplier method. Classical methods for solving the conditional optimization problem are laborious: in the method of Lagrange multipliers [16], additional parameters are determined, which increases the dimensionality of the problem; in the penalty method, multiple optimization is required with a sequential change in the parameter; in the simplex method, a multiple transition from one basic solution to the constraint system of the linear programming problem to another is required. Given such difficulties, algorithms have been developed to solve such problems, including iterative ones based on inverse calculations. Work [14] deals with the algorithm for solving the problem of nonlinear programming; paper [17] considers the inverse problem while minimizing the sum of models of argument changes. These algorithms demonstrated efficiency in solving inverse problems with a single constraint. Therefore, it is advisable to conduct a study aimed at devising conditional optimization models and iterative algorithms for solving the inverse problem when using arguments in the calculation of several indicators.

3. The aim and objectives of the study

The aim of this work is to study the possibility of using iterative algorithms to solve inverse problems when using indicators in several calculation functions. This would make it possible to derive a solution to the problem with lower computational resources and with greater correspondence of the specified expert information in comparison with existing methods.

To accomplish the aim, the following tasks have been set:
– to construct algorithms for solving the problem when using indicators in several calculation functions for three

options: when minimizing the sum of squares of argument change, minimizing the sum of modules for changing arguments, using coefficients of relative importance;

– to solve the inverse problems using the algorithms built and compare the results with the solutions to the problems in the Mathcad mathematical software.

4. The study materials and methods

To conduct the study, the apparatus of inverse calculations, optimization methods, and the theory of economic analysis were used. Previously developed algorithms for solving inverse problems and nonlinear programming problems were also employed. The VBA programming language was applied to implement the algorithms. To check the adequacy of solving optimization problems, standard functions from the mathematical software Mathcad were used.

Optimization models were built to solve the tasks set.

An optimization model for solving the inverse problem when using arguments in the calculation of r indicators can be represented in the form:

$$f(\Delta x) \to \min,$$

$$h_i(\Delta x) = y_i + \Delta y_i, \ i = 1..r. \tag{1}$$

Here, the objective function is the sum of the squares of argument changes $\left(f(\Delta x) = \sum_{i=1}^{n} \Delta x_i^2 \right)$, or the sum of the modules of changes in the arguments $\left(f(\Delta x) = \sum_{i=1}^{n} |\Delta x_i| \right)$, $h_i(\Delta x)$ is the function to construct the i-th indicator.

Minimizing the sum of the squares of argument changes makes it possible to define the solution in such a way that the changes in the input variables are as close to zero as possible, and to ensure a minimal change in the characteristics of an entity. Minimizing argument change modules would make it possible to achieve the desired state by changing individual variables that are selected as the best.

In the case of using coefficients of relative importance, two ways of solving the problem can be considered.

The first technique is to build the objective function f, which characterizes the degree of deviation of the ratio of the received changes in the arguments from the ratio of coefficients of relative importance established by the expert:

$$f(\Delta x) = \sum_{k=1}^{r} \sum_{i=2}^{n_{r-1}} \left(\Delta x_q \pm \Delta x_i \frac{\alpha_{kq}}{\alpha_{ki}} \right)^2 \to \min,$$

$$h_i(\Delta x) = y_i + \Delta y_i, \ i = 1..r, \tag{2}$$

where n_r is the number of arguments involved in the construction of the indicator r; q is the number of the argument selected as the base argument.

Thus, a single nonlinear programming problem is formed for all subtasks. The expression in parentheses is derived from the equations of the inverse calculation system. The sign in parentheses indicates the direction of change of the arguments: the minus sign indicates that the change should occur in the same direction, the plus sign – in different directions [18]. Each constraint equation corresponds to the indicator being built.

The second technique is to solve each subtask separately:

$$f_u(\Delta x) = \sum_{i=2}^{n_{r-1}} \left(\Delta x_q \pm \Delta x_i \frac{\alpha_{kq}}{\alpha_{ki}} \right)^2 \to \min,$$

$$h_u(\Delta x) = y_u + \Delta y_u,$$

$$u = 1..r. \tag{3}$$

Next, the arguments are subsequently adjusted while minimizing the sum of the squares of the argument changes.

5. Results of studying and constructing algorithms for solving the inverse problem when using indicators in several calculation functions

5. 1. Construction of algorithms for solving the problem when using indicators in several calculation functions

To solve the problem when minimizing the sum of the squares of changes in arguments (1) and using the algorithm developed earlier for solving the inverse problem [14], it is necessary to reduce problem (1) to the problem with one constraint. Consider also the case where there is one inequality constraint $g(\Delta x) \geq / \leq y_j + \Delta y_j$ in the problem.

The algorithm for solving the problem when minimizing the sum of the squares of argument changes includes the following steps:

Step 1. Convert constraints-equalities h to one constraint $h^*(x)$:

$$h^*(\Delta x) = \sum_{i=1}^{r} \left(h_i(\Delta x) - y_i - \Delta y_i \right)^2. \tag{4}$$

Step 2. Solve a problem with one constraint (4):

$$f(\Delta x) \to \min,$$

$$h^*(\Delta x) = \sum_{i=1}^{r} \left(h_i(\Delta x) - y_i - \Delta y_i \right)^2 = 0.$$

To solve the problem, iterative formulas are used (β is a small number that provides for a gradual change, t is an indicator of the direction of change, taking the values of -1 or 1):

$$\Delta x_i = \Delta x_i + t \cdot \beta \cdot \frac{\partial h^*(\Delta x_i)}{\partial \Delta x_i}. \tag{5}$$

Step 3. If the resulting solution Δx satisfies the constraint-inequality g, then the algorithm is terminated.

Step 4. Replace the inequality sign with an equal sign and build one constraint-equality h^*:

$$h^*(\Delta x) = \sum_{i=1}^{r} \left(h_i(\Delta x) - y_i - \Delta y_i \right)^2 + \left(g(\Delta x) - y_j - \Delta y_j \right)^2.$$

Solving a problem with one constraint:

$$f(\Delta x) \to \min,$$

$$h^*(\Delta x) = \sum_{i=1}^{r} \left(h_i(\Delta x) - y_i - \Delta y_i \right)^2 + \left(g(\Delta x) - y_j - \Delta y_j \right)^2.$$

The solution to the problem is the obtained values Δx.

When minimizing the sum of the modules of argument changes, a method is used based on the selection of elements with the largest value of the private derivative of a single constraint function [17]. In this case, the number of selected items is equal to the number of restrictions. Next, the system of equations is solved with respect to the selected elements.

The algorithm for solving a problem when using importance coefficients (2) and building a single objective function also includes its transformation into a problem with one constraint and the use of iterative formulas. In this case, there is a change in Δx_q at a certain step; for each of the variants, they are solved according to iterative formulas. In this case, the initial values of Δx used are the values obtained by unconditional optimization of the objective function, which can be calculated from the formulas:

$$\Delta x_i = \mp \Delta x_q \cdot \frac{\alpha_{ki}}{\alpha_{kq}}.$$

In iterative formulas (5), it is necessary to take into consideration the effect of changing the argument on the change in the objective function [14, 19]:

$$\Delta x_i = \Delta x_i + t \cdot \beta \cdot \frac{\dfrac{\partial h(\Delta x_i)}{\partial \Delta x_i}}{\dfrac{\partial f^2(\Delta x_i)}{\partial \Delta x_i^2}}.$$

The algorithm for solving the problem when using coefficients of relative importance and while separately building each indicator (3) includes the following steps:

Step 1. Solve the inverse problem for each indicator built to determine x^* (a problem with one constraint) (3).

Step 2. Solve the problem of determining argument changes:

$$f(\Delta x^*) = \sum_{j=1}^{n} (\Delta x_j^*)^2 \rightarrow \min,$$

$$h_i(\Delta x^*) = y_i^*, \; i = 1..r,$$

where n is the total number of characteristics involved in the construction of indicators.

The initial values of x are the values obtained in step 1; for the values that were calculated in several subtasks, the average value is determined.

5. 2. Solving inverse problems by using the built algorithms

As an example of the use of the built algorithm, solving the problem of forming a marginal profit, which is the most important indicator of an enterprise's activity, is considered [20]:

$$L = \sum_{j=1}^{m} p_j,$$

where L is the total marginal profit; p is the marginal profit of the j-th point of sale.

The marginal profit of an individual point of sale is calculated from the formula:

$$p_j = \sum_{i=1}^{n} s_{ji} \cdot w_i \cdot r_i,$$

where s_{ji} is the share of the distribution of the i-th product for the j-th point of sale; w_i is the volume of purchase of the i-th product; r_i is the marginal profit from the sale of a unit of the i-th product.

The problem is to determine the volume w_i of the purchase of each type of product to achieve the predefined value of the total marginal profit. At the same time, there is a limitation on the procurement budget Q:

$$Q = \sum_{i=1}^{n} w_i \cdot q_i,$$

where q_i is the cost of purchasing a product of the i-th type.

Consider solving a problem for two points of sale and three types of products. The goal tree is shown in Fig. 3.
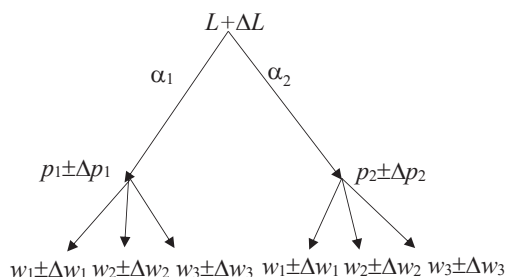

Fig. 3. Goal tree of the profit margin formation problem

Thus, solving the problem includes two stages:

1. Determine a change in profit Δp_1, Δp_2 at two points of sale to achieve the predefined value of the total profit $L+\Delta L$.

2. Determine a change in the volume of purchases Δw_1, Δw_2, Δw_3 of products of the first, second, and third types in order to achieve the profit values of the first and second points of sale obtained at the previous stage.

Initial data on products are given in Tables 1, 2; the initial value of the marginal profit is 665 monetary units; the set value of the marginal profit is 720 monetary units; the limit on the procurement budget is 1,600 monetary units.

Table 1

Initial data on products

Indicator	Product 1	Product 2	Product 3
Marginal profit, r	2	3	2.5
Purchase amount, w	100	80	90
Purchase cost, q	6	5	5.5

Table 2

Initial data on the scheme of distribution of products among points of sale

Point of sale No.	Distribution share of product 1	Distribution share of product 2	Distribution share of product 3
1	0.3	0.1	0.5
2	0.7	0.9	0.5

The initial profit margin values for the first and second points of sale are, respectively, 196.5 monetary units, and 468.5 monetary units:

$$100 \cdot 0.3 \cdot 2 + 80 \cdot 0.1 \cdot 3 + 90 \cdot 0.5 \cdot 2.5 = 196.5,$$

$$100 \cdot 0.7 \cdot 2 + 80 \cdot 0.9 \cdot 3 + 90 \cdot 0.5 \cdot 2.5 = 468.5.$$

The problem of the first stage is to be solved with coefficients of the relative importance of 0.427 and 0.573 and a positive direction of change in indicators. For the first and second points of sale, respectively, the values of changes in profit Δp would equal $(720-665)\cdot0.427=23.5$ and $(720-665)\cdot0.573=31.5$. Accordingly, the new profit margin values for the first and second points of sale are 220 monetary units and 500 monetary units.

At the second stage, the problem is represented in the form of an optimization one:

$$f(\Delta x)=\sum_{j=1}^{n}\Delta w_j^2 \to \min,$$

$$h_1(\Delta w)=(100+\Delta w_1)\cdot0.3\cdot2+$$
$$+(80+\Delta w_2)\cdot0.1\cdot3+(90+\Delta w_3)\cdot0.5\cdot2.5=220,$$

$$h_2(\Delta w)=(100+\Delta w_1)\cdot0.7\cdot2+$$
$$+(80+\Delta w_2)\cdot0.9\cdot3+(90+\Delta w_3)\cdot0.5\cdot2.5=500,$$

$$g(\Delta w)=(100+\Delta w_1)\cdot6+$$
$$+(80+\Delta w_2)\cdot5+(90+\Delta w_3)\cdot5.5\le1,600.$$

Consider the use of the above algorithm to solve the problem. At the initial stage, a problem with one limitation is built:

$$f(\Delta x)=\sum_{j=1}^{n}\Delta w_j^2 \to \min,$$

$$h(\Delta w)=\left(\begin{array}{c}(100+\Delta w_1)\cdot0.3\cdot2+(80+\Delta w_2)\times\\ \times 0.1\cdot3+(90+\Delta w_3)\cdot0.5\cdot2.5-220\end{array}\right)^2+$$

$$+\left(\begin{array}{c}(100+\Delta w_1)\cdot0.7\cdot2+(80+\Delta w_2)\times\\ \times 0.9\cdot3+(90+\Delta w_3)\cdot0.5\cdot2.5-500\end{array}\right)^2.$$

The problem is solved using an iterative algorithm. Thus, the iterative formula for changing arguments takes the form:

$$x_1=x_1-\beta\frac{\dfrac{\partial h(x)}{\partial x_1}}{2}=$$
$$=x_1-\beta\frac{4.64x_1+7.92x_2+5x_3-116.4}{2},$$

$$x_2=x_2-\beta\frac{7.92x_1+14.76x_2+7.5x_3-184.2}{2},$$

$$x_3=x_3-\beta\frac{5x_1+7.5x_2+6.25x_3-137.5}{2}.$$

A solution to the problem $(\beta=10^{-8})$: $\Delta w_1=6.538$, $\Delta w_2=1.154$, $\Delta w_3=15.385$.

The constraint g is not met, so a new constraint function is constructed:

$$h(\Delta w)=\left(\begin{array}{c}(100+\Delta w_1)\cdot0.3\cdot2+(80+\Delta w_2)\times\\ \times 0.1\cdot3+(90+\Delta w_3)\cdot0.5\cdot2.5-220\end{array}\right)^2+$$

$$+\left(\begin{array}{c}(100+\Delta w_1)\cdot0.7\cdot2+(80+\Delta w_2)\times\\ \times 0.9\cdot3+(90+\Delta w_3)\cdot0.5\cdot2.5-500\end{array}\right)^2+$$

$$+\left(\begin{array}{c}(100+\Delta w_1)\cdot6+(80+\Delta w_2)\times\\ \times 5+(90+\Delta w_3)\cdot5.5-1,600\end{array}\right)^2.$$

A solution to the problem using an iterative algorithm: $\Delta w_1=-5$, $\Delta w_2=5$, $\Delta w_3=20$.

The value of the objective function f is 449.9994. The value of the objective function when solving the problem using the Mathcad mathematical software was 450.002.

Next, consider the case of profit formation taking into consideration the coefficients of relative importance. The initial data for the problem are given in Table 3. The direction of change of arguments: positive for the volume of output; negative for the cost and price.

Table 3

Initial data for the problem of profit formation using importance coefficients

Indicator	Initial value	Formed value
Product unit cost, monetary units, x_2	2	–
Output volume, arbitrary units, x_1	10	–
Price, monetary units, x_3	5	–
Sale cost, monetary units	20	15
Revenue, monetary units	50	47
Importance coefficient β_{12}	0.2	–
Importance coefficient β_{11}	0.8	–
Importance coefficient β_{21}	0.7	–
Importance coefficient β_{23}	0.3	–

To solve the problem with the first technique, it is necessary to build an objective function; the optimization problem takes the form:

$$f(\Delta x)=\left(\Delta x_1+\Delta x_2\frac{0.8}{0.2}\right)^2+\left(\Delta x_1+\Delta x_3\frac{0.7}{0.3}\right)^2\to \min,$$

$$(x_1+\Delta x_1)\cdot(x_2+\Delta x_2)=15,$$

$$(x_1+\Delta x_1)\cdot(x_3+\Delta x_3)=47. \qquad (6)$$

A solution to the problem by transforming constraints and changing the argument Δx_1 in increments of 0.1: $\Delta x_1=2.6$; $\Delta x_2=-0.81$; $\Delta x_3=-1.26$.

Fig. 4 shows a plot of change in the objective function when increasing Δx_1.

When using the second technique, the following values are obtained in the first step:

– the problem of cost formation: $\Delta x_{1,1}=3.583$; $\Delta x_2=-0.896$;

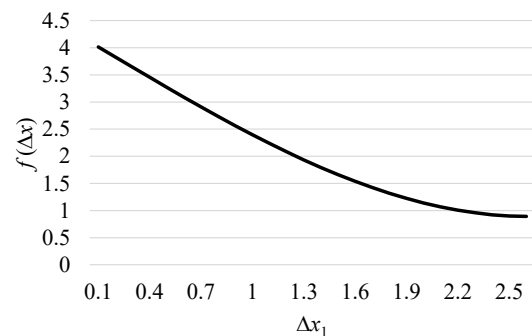– the problem of revenue generation: $\Delta x_{1,2}=1.212$; $\Delta x_3=-0.81$.

Fig. 4. Change in the objective function

Next, the optimization problem is solved while minimizing the sum of argument changes:

$$f(\Delta x) = \left(\Delta x_1^*\right)^2 + \left(\Delta x_2^*\right)^2 + \left(\Delta x_3^*\right)^2 \to \min,$$

$$\left(x_1 + \frac{\Delta x_{1,1} + \Delta x_{1,2}}{2} + \Delta x_1^*\right) \cdot \left(x_2 + \Delta x_2 + \Delta x_2^*\right) = 15,$$

$$\left(x_1 + \frac{\Delta x_{1,1} + \Delta x_{1,2}}{2} + \Delta x_1^*\right) \cdot \left(x_3 + \Delta x_3 + \Delta x_3^*\right) = 47. \tag{7}$$

A solution to the problem: $\Delta x_1^* = -0.103$; $\Delta x_2^* = -0.116$; $\Delta x_3^* = -0.36$.

Table 4 gives a solution to the problem when using two algorithms and when applying the mathematical software Mathcad. A solution obtained using a simulation algorithm reproducing the actions of a specialist according to the algorithm given in [13] is also given.

Table 4

Results of solving the problem of profit generation

Algorithm	x_1	x_2	x_3
Building a single problem for two subtasks, $\beta = 10^{-7}$	12.6	1.19	3.73
Minimizing the deviation from the solution to each problem, $\beta = 10^{-7}$	12.29	1.22	3.82
Simulation algorithm	11.21	1.1	4.19
Using Mathcad to solve problem (6)	12.61	1.19	3.73
Using Mathcad to solve problem (7)	12.29	1.22	3.82

Fig. 5 shows the value of the Euclidean norm for deviations of the obtained ratio of argument changes from the established coefficients of relative importance for the three algorithms.
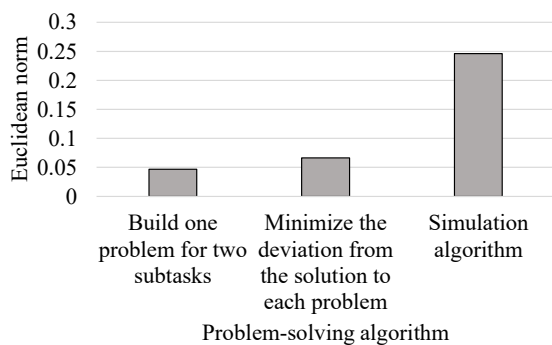

Fig. 5. Value of the Euclidean norm for problems

According to the values shown in Fig. 5, the algorithm based on the construction of a single problem provides the best correspondence to the predefined coefficients of relative importance.

6. Discussion of results of building algorithms for solving inverse problems

The proposed algorithms for solving inverse problems of economic analysis when using indicators in several calculation functions provided for a solution corresponding to the solution derived when using mathematical software (Table 4). This is explained by the used rule of movement to the final solution based on the values of the elements of the gradient vector of the constraint function and the second partial derivatives of the objective function.

When using coefficients of relative importance, two ways to solve the problem have been considered: the construction of a single optimization problem for subtasks and the adjustment of the solution to the subtasks while minimizing the sum of the squares of argument changes. The method, based on the construction of a single optimization problem, produced a lower value of the Euclidean metric characterizing the deviation of argument changes from the established values of importance coefficients (Fig. 5). This is due to the expression of the objective function, minimizing the deviation from the specified expert information. The algorithm for solving the problem by correcting the solution to subtasks does not require building an objective function based on these subtasks and, therefore, is easier to implement.

The proposed algorithms provided a lower value of the Euclidean metric of deviations of argument changes from the established values of importance coefficients compared to the existing method of solving the problem (Fig. 5). The Euclidean norm when using the method reported in [13], based on the sequential solution of two inverse problems in order to determine the compromise option, was 0.25. This value is 5.25 and 3.7 times greater than the Euclidean metric when using an algorithm based on the construction of a single problem and an algorithm based on an isolated solution to subtasks, respectively. In addition, the proposed algorithms do not require movement through the network from one indicator to another and the repeated solving of the inverse problem based on the information received from the specialist. In addition, unlike the existing method, the algorithms make it possible to define a solution to the problem when using more than one argument in the calculation of several indicators.

The disadvantages of algorithms include the need to calculate the gradient and second partial derivatives, which increases the complexity of the implementation. The limitations of the algorithms are associated with the inability to solve problems in which there are several limitations-inequalities.

Further research could involve studying those inverse problems, in which there are several constraints-inequalities, and both expert information and minimization of deviation from the initial values of the arguments are used simultaneously. In addition, further work would aim at devising iterative algorithms and their application for solving linear programming problems.

7. Conclusions

1. Algorithms for solving the inverse problem when using indicators in several calculation functions have been proposed. A feature of the proposed approach is the representation of the inverse problem in the form of an optimization problem with one limitation using iterative algorithms for the case of minimizing the sum of the squares of changes in arguments and using coefficients of relative importance. The approach used ensures a greater correspondence of the obtained solution to the coefficients of relative importance: the Euclidean norm in the example considered is more than three

times less than in the existing method of solving the problem. In addition, the proposed algorithms can define the solution when several indicators participate in different subtasks and do not require the involvement of additional expert information. The use of iterative algorithms in comparison with classical methods of nonlinear optimization makes it possible to determine a solution to the problem without the need to build and optimize the modified function.

2. The solution to two inverse problems of profit formation was derived with the help of the developed algorithms: when minimizing the sum of the squares of changes in arguments and when using coefficients of relative importance. The results of numerical problem solving are consistent with the results of using the standard function from the Mathcad mathematical software; the absolute difference in the values of the objective function ranged from $2.6 \cdot 10^{-3}$ to $2.8 \cdot 10^{-5}$.

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