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In a variety of engineering, scientific challenges, mathematics, chemistry, physics, biology, machine learning, deep learning, regression classification, computer science, programming, artificial intelligence, in the military, medical and engineering industries, robotics and smart cars, fuzzy nonlinear equations play a critical role. As a result, in this paper, an Optimization Algorithm based on the Euler Method approach for solving fuzzy nonlinear equations is proposed. In mathematics and computer science, the Euler approach (sometimes called the forward Euler method) is a first-order numerical strategy for solving ordinary differential equations (ODEs) with a specified initial value. The local error is proportional to the square of the step size, while the general error is proportional to the step size, according to the Euler technique. The Euler method is frequently used to create more complicated algorithms. The Optimization Algorithm Based on the Euler Method (OBE) uses the logic of slope differences, which is computed by the Euler approach for global optimizations as a search mechanism for promising logic. Furthermore, the mechanism of the proposed work takes advantage of two active phases: exploration and exploitation to find the most important promising areas within the distinct space and the best solutions globally based on a positive movement towards it. In order to avoid the solution of local optimal and increase the rate of convergence, we use the ESQ mechanism. The optimization algorithm based on the Euler method (OBE) is very efficient in solving fuzzy nonlinear equations and approaches the global minimum and avoids the local minimum. In comparison with the GWO algorithm, we notice a clear superiority of the OBE algorithm in reaching the solution with higher accuracy. We note from the numerical results that the new algorithm is 50 % superior to the GWO algorithm in Example 1, 51 % in Example 2 and 55 % in Example 3

Keywords: algorithms, approach, fuzzy, global, Euler method, intelligent techniques, nonlinear equations, numerical optimization, swarms

UDC 4861:2007
DOI: 10.15587/1729-4061.2022.252014

OPTIMIZATION ALGORITHM BASED ON THE EULER METHOD FOR SOLVING FUZZY NONLINEAR EQUATIONS

Kais I. Ibraheem
Corresponding author
Associate Professor, Dean of College
Department of Computer Science
College of Education for Pure Science*
E-mail: kaisismail@uomosul.edu.iq
Hisham M. Khudhur
Lecturer
Department of Mathematics
College of Computer Science
and Mathematics*
*University of Mosul
Al Majmoaa str., Mosul, Iraq, 41002

Received date 10.12.2021
Accepted date 25.01.2022
Published date 25.02.2022

How to Cite: Ibraheem, K. I., Khudhur, H. M. (2022). Optimization algorithm based on the Euler method for solving fuzzy nonlinear equations. Eastern-European Journal of Enterprise Technologies, 1 (4 (115)), 13–19. doi: <https://doi.org/10.15587/1729-4061.2022.252014>

1. Introduction

The paper [1–3] developed the concept of fuzzy numbers and arithmetic operations on them, which was expanded in [4]. Later, the work [5] contributed significantly by developing the main thought of LR fuzzy numbers and then presented a computational formula toward fuzzy number operations. The solution of the mentioned equations, the main parameters of which are fuzzy numbers has emerged as one of the key areas for the application of fuzzy numbers as the theory of fuzzy numbers has progressed. The solution of

fuzzy equations is necessary in diverse fields such as chemistry, economics, physics, and others.

Take a look at the set of j nonlinear equations:

$$g_d(\chi_1, \chi_2, \dots, \chi_j) = 0, \quad d = 1, 2, \dots, j.$$

The general form of the nonlinear equation for $j=1$ can be stated simply according to a value for the variable x , which is computed as follows:

$$G(\chi) = 0, \tag{1}$$

where G denotes any nonlinear function of χ and g_i denotes a mapping from R^n to R^n . The value of X is then referred to the root or solution of this equation, which could be one of several. The above-mentioned equation appears frequently in engineering, natural and social sciences. Several approaches have been proposed and used to gain its solution. However, fuzzy numbers are used to represent the parameters of these structures of nonlinear equations instead of clear numbers. Thus, the final results are dependent on the roots of the fuzzy equation [6]. Some standard analytical techniques [7–9] are not suitable for solving systems of nonlinear equations of the form:

$$h\chi^5 + k\chi^4 + f\chi^3 + r\chi - u = z,$$

$$\chi - \csc(\chi) = w,$$

where χ , h , k , f , r , u , and w are fuzzy numbers. Therefore, we need to suggest the metaheuristic methods to find the solution to this type of equation.

Therefore, studies that are devoted to solving fuzzy nonlinear equations and finding the optimal solution to the equations with the fewest number of iterations and in record time compared to the GWO method are of scientific relevance.

2. Literature review and problem statement

Actually, the Euler method calculates the penchant and solves ordinary differential equations (ODE) using a special Euler formulation [10]. OBE's fundamental notion is grounded on the Euler method's suggested calculated penchant concept. The OBE employs the calculated penchant as searching reasoning to find the most convenient area in the area of search and construct a group of statutes for the development of a population group based on the logic of the swarm-based optimization technique. The next subsections go over the mathematical formula for OBE. Some researchers have solved fuzzy nonlinear equations by some methods. In 2004, the work [6] proposed Newton's method. The disadvantage of the method is that it requires the calculation of the Hessian matrix. In 2006, the steepest descent method was proposed [11]. One of the disadvantages of the method is that it is slow. In 2008, the Harmonic Newton method was proposed [12], which is an ineffective way to get to the roots. In 2010, they used the Broyden's method [13]. Broyden's method requires calculating the inverse of the Hessian matrix. Also in 2010, the work [14] used the general iterative method for an ineffective method with all kinds of equations. In 2011, the work [15] used the iterative secant method. One of the disadvantages of the method is that it sometimes does not reach the solution. In 2016, some iterative methods were used to solve fuzzy equations such as the Bisection method [16], the False Position method [17], and the secant method modified in [18]. One of the disadvantages of these methods is that they are very slow to reach a solution. In 2018, a new class of paired gradient method [19] and Barzilai-Borwein gradient method were used [20]. These methods need to calculate the gradient vector at each iteration. In the same year, some numerical methods were proposed [21]. One of the disadvantages of these methods is that they are old, traditional and need an initial point. Stirling's-like method [22] does not guarantee reaching a solution. In 2019, the Chord Newton method was used [23], which needs to compute a Hessian matrix for each iteration. In 2020, the accelerated method

was proposed [24]. This method does not guarantee reaching a solution. The quasi-Newton method was proposed to solve nonlinear fuzzy equations [25]. One of the disadvantages of the method is that it requires calculating the inverse Hessian matrix for each iteration. In 2021, they proposed four numerical methods for solving fuzzy equations [26]. Traditional numerical methods need an initial point. A Special Iterative Algorithm [27], the Free Levenberg-Marquardt method [28] were proposed. One of the disadvantages of the method is that it requires calculating the inverse Hessian matrix for each iteration. The Shamanskii method was used in [29]. One of the disadvantages of the method is that it does not guarantee reaching a solution. Spectral CG algorithm for solving fuzzy nonlinear equations was developed in [30]. This method does not reach a global solution.

All this allows us to assert that it is expedient to conduct a study on metaheuristic methods for solving fuzzy nonlinear equations, which is an effective optimization algorithm method that depends on the Euler method.

3. The aim and objectives of the study

The aim of the study is to develop metaheuristic methods for solving nonlinear fuzzy equations by using numerical methods such as Euler iterative method. The OBE technique is easy to use and does not require many complications, as is the case of traditional numerical methods.

To achieve this aim, the following objectives are accomplished:

- to find the minimum of fuzzy equations;
- to get the best value for variables;
- to get the best value for functions.

4. Materials and methods

In this paper, fuzzy nonlinear equations are solved using a stochastic swarm-based model. The proposed OBE approach is expressed in a non-metaphorical manner, focusing on the mathematical essence as a set of active rules that are applied at the right time. It is not recommended to use metaphors with the population-based models because the only advantage is that it hides the basic structure of the equations used by the optimizers. As a result, OBE accounts for the Euler method's core logic as well as a throng of the agent's population-based evolution.

This stage's logic is to construct an initial swarm that will evolve throughout the specified number of iterations. For a population of size N , N sites are generated at random in OBE. Each individual in the population, X_n ($n=1,2,\dots,N$), is a D dimensional optimization problem solution. The initial positions are created at random in general using the following concept:

$$x_{n,l} = L_l + rand \cdot (U_l - L_l).$$

The problem's l th variable $l=1,2,\dots,D$ has lower and upper limits L_l and U_l , respectively, and is a random number range $[0, 1]$. Only a small number of solutions are generated by this rule.

The proposed work uses the Euler method to explore the solution space and construct an appropriate global and local search. Furthermore, it is used to determine the proposed OBE search mechanism. The SM formula is defined as follows:

$$k = \frac{1}{2\Delta\chi}(\text{rand} \times \chi_w - u \times \chi_b),$$

$$u = \text{round}(1 + \text{rand}) \times (1 - \text{rand}),$$

$$SM = (\chi_{RK})\Delta\chi,$$

$$\chi_{RK} = uk,$$

where rand_1 and rand_2 are two random values ranging from 0 to 1. Delta X is defined by the following formula:

$$\Delta\chi = 2 \times \text{rand} \times |\text{Stp}|,$$

$$\text{Stp} = \text{rand} \times ((\chi_b - \text{rand} \times \chi_{\text{avg}}) + \gamma),$$

$$\gamma = \text{rand} \times (\chi_n - \text{rand} \times (u - l)) \times \exp\left(-4 \times \frac{i}{\text{max}i}\right),$$

where X_w and X_b are determined by the following:

$$\text{if } g(\chi_n) < g(\chi_{bi})$$

$$\chi_b = \chi_n$$

$$\chi_w = \chi_{bi}$$

$$\text{else}$$

$$\chi_b = \chi_{bi}$$

$$\chi_w = \chi_n$$

$$\text{end.}$$

In the next step, we explain the solution update for the above technique.

The OBE algorithm uses a search mechanism (SM) based on the Euler method to update the current solution location at each iteration,

$$\text{if } \text{rand} < 0.5$$

$$\text{(exploration phase)}$$

$$\chi_{n+1} = (\chi_c + r \times SF \times g \times \chi_c) +$$

$$+ SF \times SM + \mu \times \text{rand}n \times (\chi_m - \chi_c)$$

$$\text{else}$$

$$\text{(exploitation phase)}$$

$$\chi_{n+1} = (\chi_m + r \times SF \times g \times \chi_m) +$$

$$+ SF \times SM + \mu \times \text{rand}n \times (\chi_{r1} - \chi_{r2}) \quad (4)$$

$$\text{end}$$

where r is an integer that can be either 1 or -1 , g is a random number between 0 and 2, SF is a coping mechanism, where μ is a number chosen at random.

The formula of SF is as follows:

$$SF = 2 \times (0.5 - \text{rand}) \times g, \quad (5)$$

$$g = \alpha \times \exp\left(-\beta \times \text{rand} \times \left(\frac{i}{\text{max}i}\right)\right), \quad (6)$$

where $\text{max}i$ stands for the largest number of iterations. The formulas of X_c and X_m are as follows:

$$\chi_c = \vartheta \times \chi_n + (1 - \vartheta) \times \chi_{r1}, \quad (7)$$

$$\chi_m = \vartheta \times \chi_{\text{best}} + (1 - \vartheta) \times \chi_{\text{best}}, \quad (8)$$

where ϑ is a random number between 0 and 1. X_{best} has shown to be the most effective solution so far. At each iteration, X_{best} represents the best position.

Enhanced Solution Quality (ESQ) is used by the OBE algorithm to improve the quality of the solutions and then override the local improvement on each iteration. ESQ is used to generate the answer ($X_{\text{new}2}$) using the following technique:

$$\text{if } \text{rand}m < 0.5$$

$$\text{if } \varpi < 1$$

$$\chi_{\text{new}2} = \chi_{\text{new}1} + r \times \varpi \times |(\chi_{\text{new}1} - \chi_{\text{avg}}) + \text{rand}m|$$

$$\text{else}$$

$$\chi_{\text{new}2} = (\chi_{\text{new}1} - \chi_{\text{avg}}) + r \times \varpi \times$$

$$\times |(\chi_{\text{new}1} - \chi_{\text{avg}}) + \text{rand}m| \quad (9)$$

$$\text{end}$$

$$\text{end}$$

$$\varpi = \text{rand}(0.2) \cdot \exp\left(-c \left(\frac{i}{\text{max}i}\right)\right), \quad (10)$$

$$\chi_{\text{avg}} = \frac{\chi_{r1} + \chi_{r2} + \chi_{r3}}{3}, \quad (11)$$

$$\chi_{\text{new}1} = \kappa \times \chi_{\text{avg}} + (1 - \kappa) \times \chi_{\text{best}}, \quad (12)$$

where k is a random value between 0 and 1, and in this paper, c is a random number that equals $5 \times \text{rand}$, x_{best} is the better solution found so far, r is an integer number that can be 1, 0, or -1 . The present solution (i.e., $f(X_{\text{new}2}) > f(X_n)$) may not have the best fitness than the solution determined in this section ($X_{\text{new}2}$). Another new solution ($X_{\text{new}3}$) is developed in order to have another shot at developing a decent solution. It is defined as follows:

$$\text{if } \text{rand} < \varpi$$

$$\chi_{\text{new}3} = (\chi_{\text{new}2} - \text{rand} \times \chi_{\text{new}2}) +$$

$$+ SF \times (\text{rand} \times \chi_{RK} + (\nu \times \chi_b - \chi_{\text{new}2})) \quad (13)$$

$$\text{end}$$

where ν is a random number with a value of two multiplied by rand [31].

Algorithm 1. The OBE pseudo-code

Phase One. Initialization

Set up variables a and b to their default values

OBE population χ_n ($j = 1, 2, \dots, D$) should be generated.

Determine each population member's objective function.

Find the χ_w , x_b , and χ_{best} solutions.

Phase Two. Operators of OBE

for $s = 1:\text{max}S$

for $j = 1:D$

for $t = 1:L$

Eq. (1) is used to determine the position $\chi_{j+1,k}$

End for

Enhance the solution quality

if $\text{rand} < 0.5$

Determine position $\chi_{\text{new}2}$ using Eq.(9)

if $g(\chi_j) > g(\chi_{\text{new}2})$

if $\text{rand} < \varpi$

Determine position $\chi_{\text{new}3}$ using Eq. (13)

```

        end
    end
    end
    Positions  $\chi_w$  and  $\chi_b$  should be updated.
end for
    Position  $\chi_{best}$  should be updated.
     $j=j+1$ 
end
Phase Three. Return  $\chi_{best}$ .
    
```

Here we reach the end of a detailed explanation of the method, and in the next part we deal with the numerical results of the OBE technique [31].

All computations are performed in MATLAB 2021b on a Windows 10 HP system with an Intel Core i5 CPU, 4 GB of RAM, and 500 GB of hard disk space, with a maximum iteration count of 500 for solution stopping.

The limits of work in solving fuzzy nonlinear equations have been narrowed, since only nonlinear equations have been solved without solving exponential or trigonometric equations.

5. Numerical results of the examples

Throughout this section, we will use numerical examples to demonstrate the efficiency and applicability of the OBE technique, which is applied to solve fuzzy nonlinear equations. The following are some instances from [8]. Table 1 shows some examples of fuzzy nonlinear equations solved by the Optimization Algorithm Based on the Euler Method (OBE) technique.

Table 1
Examples of fuzzy nonlinear equations solved by the new algorithm

Examples	Range	fmin
$f_1 = (3, 4, 5)\underline{\chi}^2 + (1, 2, 3)\underline{\chi} = (1, 2, 3)$ [8], $(3 + \varsigma)\underline{\chi}^2(\varsigma) + (1 + \varsigma)\underline{\chi}(\varsigma) - (1 + \varsigma) = 0,$ $(5 - \varsigma)\underline{\chi}^2(\varsigma) + (3 - \varsigma)\underline{\chi}(\varsigma) - (3 - \varsigma) = 0.$ For $\varsigma = 1$ $4\underline{\chi}^2(1) + 2\underline{\chi}(1) - 2 = 0.4\underline{\chi}^2(1) + 2\underline{\chi}(1) - 2 = 0.$ For $\varsigma = 0$ $3\underline{\chi}^2(0) + \underline{\chi}(0) - 1 = 0.5\underline{\chi}^2(0) + \underline{\chi}(0) - 3 = 0$	[0, 2]	0
$f_2 = (3, 4, 5)\underline{\chi}^2 + (1, 2, 3)\underline{\chi} = (1, 2, 3)$ [8], $(4 + 2\varsigma)\underline{\chi}^2(\varsigma) + (2 + \varsigma)\underline{\chi}(\varsigma) - (3 + 3\varsigma) = 0,$ $(8 - 2\varsigma)\underline{\chi}^2(\varsigma) + (4 - \varsigma)\underline{\chi}(\varsigma) - (9 - 3\varsigma) = 0.$ For $\varsigma = 1$ $6\underline{\chi}^2(1) + 3\underline{\chi}(1) - 6 = 0,$ $6\underline{\chi}^2(1) + 3\underline{\chi}(1) - 6 = 0.$ For $\varsigma = 0$ $4\underline{\chi}^2(0) + 2\underline{\chi}(0) - 3 = 0.8\underline{\chi}^2(0) + 4\underline{\chi}(0) - 9 = 0$	[0, 2]	0
$f_3 = (1, 2, 3)\underline{\chi}^3 + (2, 3, 4)\underline{\chi}^2 + (3, 4, 5) = (5, 8, 13)$ [8], $(1 + \varsigma)\underline{\chi}^3(\varsigma) + (2 + \varsigma)\underline{\chi}^2(\varsigma) - (2 + 2\varsigma) = 0,$ $(3 - \varsigma)\underline{\chi}^3(\varsigma) + (4 - \varsigma)\underline{\chi}^2(\varsigma) - (8 - 4\varsigma) = 0.$ For $\varsigma = 1$ $2\underline{\chi}^3(1) + 3\underline{\chi}^2(1) - 4 = 0,$ $2\underline{\chi}^3(1) + 3\underline{\chi}^2(1) - 4 = 0.$ For $\varsigma = 0$ $\underline{\chi}^3(0) + 2\underline{\chi}^2(0) - 2 = 0.3\underline{\chi}^3(0) + 4\underline{\chi}^2(0) - 8 = 0$	[0, 2]	0

Table 1 gives examples of fuzzy nonlinear equations that have been solved by the Optimization Algorithm Based on the Euler Method (OBE) technique.

5. 1. Efficiency of the Optimization Algorithm Based on the Euler Method (OBE) technique

The minimum fuzzy nonlinear equations were found as shown in the fourth column of Table 2, where the Optimization Algorithm Based on the Euler Method (OBE) technique was very effective in reaching the global solution to the above problems.

Table 2

Numerical results

Example	Iterations	X-Best	F-Best(OBE)	F-Best Grey Wolf Optimizer (GWO)
1	500	0.43426	6.153e-012	9.041e-06
		0.5		
		0.5		
		0.53066		
2	500	0.65139	2.0313e-010	4.1164e-05
		0.78078		
		0.78078		
		0.83978		
3	500	0.83929	2.9951e-011	4.7088e-05
		0.91082		
		0.91082		
		1.0564		

5. 2. F-Best

The aim of the paper was to find the best value for the given functions, and by using the Optimization Algorithm Based on the Euler Method (OBE) technique, the best values for the fuzzy nonlinear equations were obtained, as found in the fourth column of Table 2, thus achieving the desired purpose of the Optimization Algorithm Based on the Euler Method (OBE) technique.

At the end of this section, we note that the Optimization Algorithm Based on the Euler Method (OBE) technique is an effective way to find roots for equations, reach the global, and avoid the local solution of the given functions.

5. 3. Best Root Value

We got the roots of the given equations where the best value was found for each variable as shown in the third column of Table 2 and also shown in Fig. 1–3, where these graphics are figures of the values of the variables in the three problems. Where Fig. 1 refers to plotting the solution to the first equation, Fig. 2 refers to plotting the solution to the second equation. The OBE algorithm has been compared with the Grey Wolf Optimizer (GWO) algorithm [32]. There are previous papers that dealt with the topic of solving fuzzy nonlinear equations using swarms, for more see [33, 34].

Fig. 3 shows the solution of three equations when using the Optimization Algorithm Based on the Euler Method (OBE) technique. The plot reaches the peak when the value is 0.92 and it starts to decrease reaching zero when the value is 1.05.

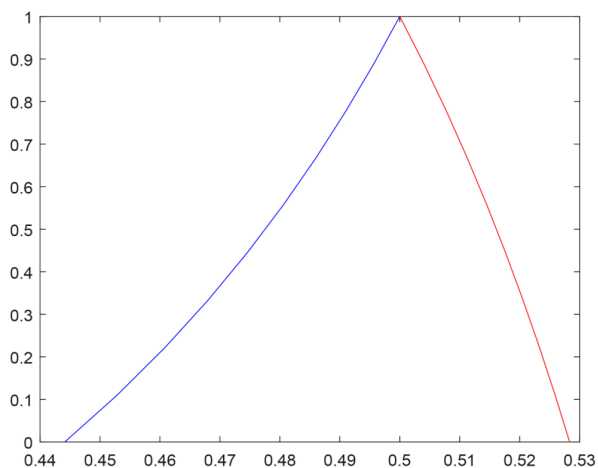


Fig. 1. Solution to problem 1 using Optimization Algorithm Based on the Euler Method (OBE) technique

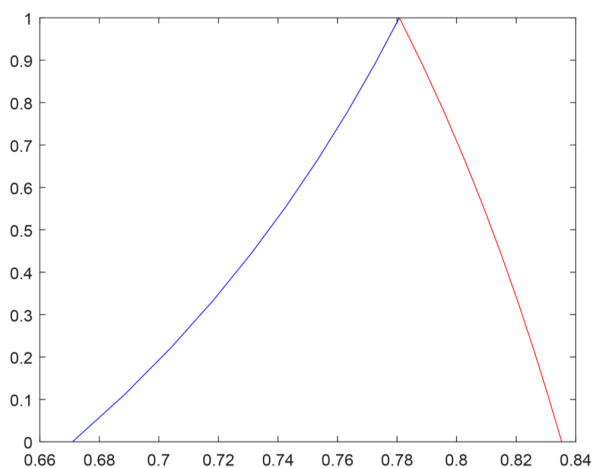


Fig. 2. Solution to problem 2 using Optimization Algorithm Based on the Euler Method (OBE) technique

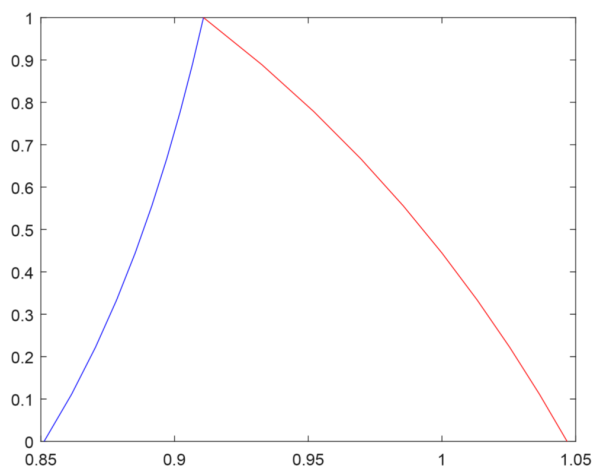


Fig. 3. Solution to problem 3 using Optimization Algorithm Based on the Euler Method (OBE) technique

6. Discussion of the results of developing metaheuristic techniques for solving fuzzy nonlinear equations using numerical methods

In this paper, one of the metaheuristic methods for solving fuzzy nonlinear equations is proposed, which is an effective optimization algorithm method that depends on the Euler method. This method was proposed because it is very effective in finding the roots of fuzzy nonlinear equations and has global convergence, unlike some numerical algorithms whose convergence is local and needs an initial point close to the exact solution and may diverge from the approximate solution if the initial point is outside the range of the approximate solution.

At the beginning of the paper, it deals with the issue of local and global convergence, and after obtaining the numerical results in Table 2 and Fig. 1–3, we prove that the Optimization Algorithm Based on the Euler Method (OBE) technique has global convergence compared to the GWO algorithm. The researchers did not address the solution of fuzzy nonlinear equations with metaheuristic algorithms before.

Table 2 indicates the numerical results of the new technique, where the best value of the variables and the best value of the functions were recorded, as the method converged towards the global in all the given functions compared to the GWO algorithm.

The limits of the study are the minimization of functions to obtain the required roots. One of the difficulties that we encountered in the paper is how to program the OBE algorithm and the problems using Matlab.

One of the disadvantages of the algorithm is that it needs many iterations to reach the solution, and we will overcome this problem in the future.

7. Conclusions

1. The new method is more efficient than the intelligent swarming methods because the intelligent swarming methods work in a random way.
2. We obtained encouraging results and convergence towards global in solving all the fuzzy nonlinear equations, as the results show that the solution of the equations reached the global minimum of the equations.
3. We can develop a method to use in artificial neural networks for data classification and approximate functions, and we can get encouraging results based on solving nonlinear equations, and they can be applied also in several other areas, including video cameras, smart cars, air conditioners, boilers, ships, and automatic washing machines.

Acknowledgments

The authors are very grateful to the University of Mosul/Collage of Education for Pure Science for their provided facilities, which helped to improve the quality of this work.

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