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OPTIMIZING THE PARTIAL GEAR RATIOS OF THE TWO-STAGE WORM GEARBOX FOR MINIMIZING TOTAL GEARBOX COST

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Optimizing the design of a worm gearbox is complex to get due to considering multiple objectives and numerous main design parameters. Hence, a more consistent and robust optimization technique will be considered in obtaining the optimized results. This paper presents the optimization process of the Two-Stage Worm Gearbox with the objective function of minimizing total gearbox cost. Ten main design parameters are chosen as input parameters for evaluating their impacts on the response of the partial gear ratio u_2. In this study, the simulation experiments were used, which do not need cost to perform all potential tests. In order to do this, a 2^(10-3) model and using 1/16 fractional model were selected due to the limitation of the built-in function in Minitab@18. Moreover, the screening experiments are purposely used to determine the number of parameters, which has a minor influence on the response. Compared to using the Taguchi technique, the model of 2^11 corresponding to L32 or 32 tests is a simple method to achieve the objectives.

The results show that Total gearbox ratio exhibits the biggest effect on the response compared to others. Furthermore, the interactions between these factors to the remaining are significant. The high reliability of the proposed model is verified by simulation experiments. The random tendency of data shows that u_2 is not crucially influenced by other than the input parameters. The data in versus order prove that the response is not varied to the time factor. Moreover, the coefficients of adjusted R^2 and R^2 are both greater than 99 %, it can be concluded that the proposed regression model is appropriate. The proposed optimization process in this study is reliable and the optimal design method can provide a useful reference on performance improvement of other worm gears

Keywords: bearing cost, gearbox cost, screen experiment, two-stage worm gearbox

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1. Introduction

The size, mass, hence total cost of a gearbox are significantly affected by gear ratios. Accordingly, mechanical designers typically consider the contribution of the partial gear ratios [1–3]. In particular, in the optimal design problems, finding out the optimal ratios of the gearbox is an important task. An example documented by [4] showing the dependence of the gearbox mass on the optimal partial ratios is presented in Fig. 1. It is clearly visualized that the total gear mass reaches 233 kg when u_2=6. The corresponding value of gear mass is 233 kg if u_2 of 1 is utilized. Inversely, the mass of gears minimizes at 175 kg if u_2 of 2 is adopted. Hence, it can be said that u_2 of 2 is the optimal value of the partial gearbox ratios in this study.

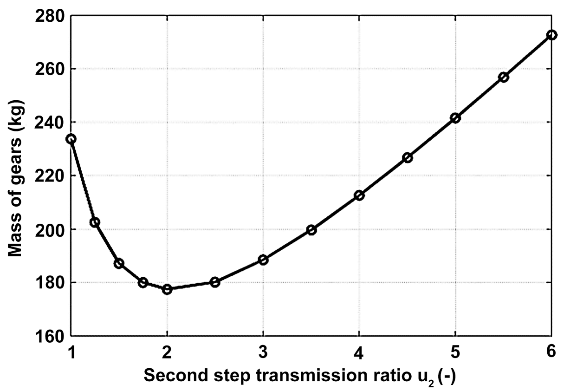

Fig. 1. The dependence of the mass of gears on the partial gear ratios [4]

Optimal determination of the partial gearbox ratios has been conducted by several researchers so far [5, 6]. The methodology used in gearbox optimization is methods based on graph, practice, and model. For the first one, the optimal gear ratios can be found from the graphical relations between the partial and total ratios. For instance, consider the intersection between the u_1 curve and the u_2 curve as exhibited in Fig. 2 in which the first and second stages u_1 and u_2 of a helical gearbox with three-stage are identified in the graphical form. The method based on practice is the fact that the ratios of gears can be optimally determined by the real data given by gearbox companies. The results presented in [7] reveal that the gearbox mass can be minimized if the center distance of the second stage is 1.4–1.6 times that of the first stage.

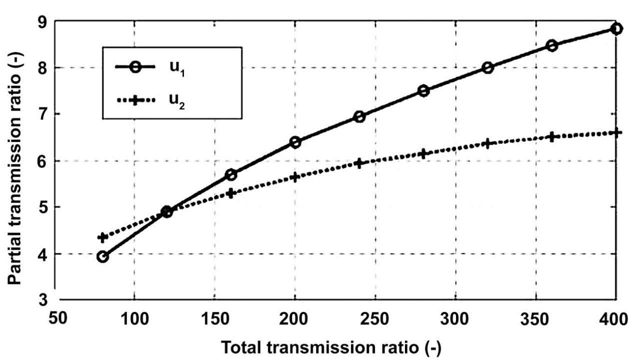

Fig. 2. The relation between partial and total transmission ratios [7]

The model method used for optimizing the gearbox ratios is widely applied by research communities [8–10]. Various types of output responses being minimizing gearbox width [11–13], shaft mass [14] or cross-section area [15–17] make this method attractive for many researchers. This may be due to the significant development of computers, experimental design methods, which can serve well for the treatment of huge analyzed data. With the strong development of information technology, optimization tools for gearboxes are increasingly useful when applied to modeling methods. That is why modeling methods are interested by many researchers as mentioned above.

2. Literature review and problem statement

There have been few studies dealing with optimizing worm gearbox, which are utilized in transmission systems due to the high reduction ratio with rigid size. The model method has been applied to optimize the worm gearbox. Optimization of the volume of two-stage gear drive and simple planetary gear by using a genetic algorithm was conducted by [18]. The weight of worm gears was minimized by applying the artificial immune algorithm presented in [19]. The impacts of lubricant viscosity appearing on the worm gear were conducted in [20]. Regarding the energy resulting from worm gear, the power loss in worm gears was considered in [21]. In this case, the genetic algorithm was used. The authors [22] presented a new and useful method for the calculation of the total transmission ratio of two-step worm reducers for the best reasonable gearbox housing structure. In that study, based on the moment equilibrium condition of the mechanic system including the two units and their regular resistance condition, an effective model for splitting the total ratio of two-step worm reducers was found. By giving explicit models, the transmission ratio of two steps can be calculated fast and effectively. However, it was shown that the proposed method cannot be widely utilized in other cases of gearbox optimization. With a similar objective function, it was revealed that the optimized gear ratios have a strong effect on the size of the gearbox [23]. Determination of the optimal values of gear ratios of a two-stage worm gearbox with the objective function of the reasonable gearbox structure was carried out [24]. It can be seen that the above-mentioned studies do not take into account the cost parameters, which is very important in the problem of optimal design of the gearbox. Hence, this issue should be more crucially concerned by research communities.

3. The aim and objectives of the study

The aim of the study is to find the optimal set of the main design parameters, which can minimize the total cost of gearbox.

To achieve the aim, the following objectives were set:

– to investigate the effects of the main design parameters on the u_2 response;

– to determine the regression equation to calculate u_2;

– to evaluate the fitness of the proposed model.

4. Materials and methods

4. 1. Cost analysis

In fact, the total gearbox cost is significantly affected by the cost of components like bearings, gears, shafts and casing. The calculation of bearing cost will be ignored because of its complication herein. For a two-speed helical gearbox, the cost, C_{gb}, can be calculated as follows:

$$C_{gb} = C_g + C_w + C_{wh} + C_{gh} + C_s + C_b, \quad (1)$$

where, C_{gh}, C_g, C_w, C_{wh}, C_s, and C_b are the cost of the gearbox housing, gears, worm, worm wheel, shafts, and bearing pairs of the gearbox. These cost components can be calculated by:

$$C_g = C_{g,m} \cdot m_g, \quad (2)$$

$$C_w = C_{w,m} \cdot m_w, \quad (3)$$

$$C_{wh} = C_{wh,m} \cdot m_{wh}, \quad (4)$$

$$C_{gh} = C_{gh,m} \cdot m_{gh}, \quad (5)$$

$$C_s = C_{s,m} \cdot m_s, \quad (6)$$

$$C_b = 2\sum_{i=0}^{n} C_{b,i}, \quad (7)$$

where, $C_{g,m}$, $C_{w,m}$, $C_{wh,m}$, $C_{gh,m}$, $C_{s,m}$ are the cost per kilogram (USD/kg) of gear, worm, worm wheel, housing, and shaft, respectively. $C_{b,i}$ is the cost of a bearing on the i^{th} shaft (i=1÷3). This cost component will be defined later. m_g, m_w, m_{wh}, m_{gh}, and m_s (kg) are noted for gear, worm, worm wheel, housing, and shaft mass. The mentioned components of mass can be calculated in the following sub-sections.

4. 2. Determining the mass of the gearbox housing

In this study, the housing mass, m_{gh} is identified as follows:

$$m_{gh} = \rho_{gh} \cdot V_{gh}, \quad (8)$$

where, ρ_{gh} is the density of the material, for cast iron, ρ_{gh}=7.2 (kg/dm^3); V_{gh} is assigned to the housing volume (m^3). The housing shape can be schematically determined by different subareas as shown in Fig. 1, 3:

$$V_{gh} = 2V_{A1} + V_{B1} + V_{T1} + V_{T2} + V_{S1} + V_{S2} + V_{S3}, \quad (9)$$

where, the subareas noted by V_b, V_{A1} and V_{A2} are calculated by:

$$V_{A1} = (L + 2S_G) \cdot H + L_1 \cdot H_1, \quad (10)$$

$$V_{B1} = (B - 2S_G) \cdot (L + 2S_G + L_1), \quad (11)$$

$$V_{T1} = (B - 2S_G) \cdot (L + 2S_G), \quad (12)$$

$$V_{T2} = (B - 2S_G) \cdot L_1, \quad (13)$$

$$V_{S1} = (B - 2S_G) \cdot (H - 2S_G), \quad (14)$$

$$V_{S2} = (B - 2S_G) \cdot (H - S_G - H_1), \quad (15)$$

$$V_{S3} = (B - 2S_G) \cdot (H_1 - 2S_G), \quad (16)$$

where, L, H, B_1, B_2 and S_G can be estimated by:

$$L = d_{w22} + 20, \quad (17)$$

$$H = \frac{d_{w22}}{2} + a_{w2} + \max\left(\frac{d_{w21}}{2}; \frac{d_{w12}}{2}\right) + 8.5S_G, \quad (18)$$

$$L_1 = b_{w1} + 4S_G, \quad (19)$$

$$H_1 = \frac{d_{w11}}{2} + a_{w1} + \frac{d_{w21}}{2} + 8.5S_G, \quad (20)$$

$$B = \max(d_{w21}; d_{w12}) + 20 + 2S_G, \quad (21)$$

$$S_G = 0.005L + 4.5. \quad (22)$$

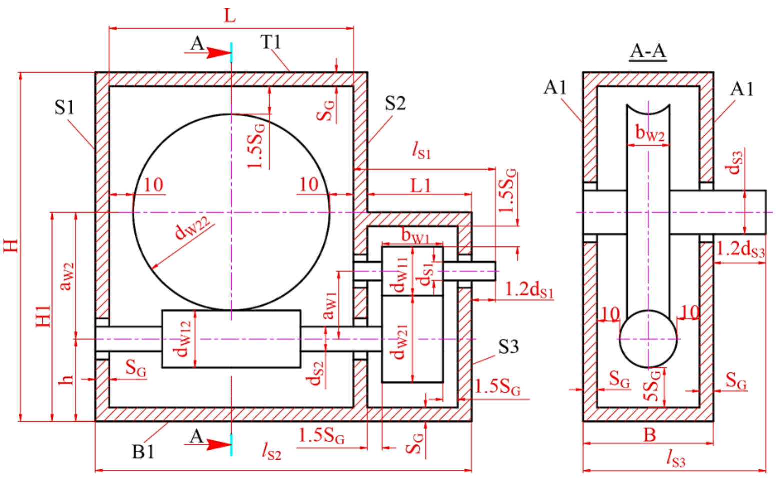

Fig. 3. Schema for determination of gearbox mass

In the above equations, d_{w11}, d_{w12}, d_{w21}, and d_{w22} are the pitch diameters of the pinion and the gear of the worm and the helical gear sets. These diameters will be specified in the coming parts of this study.

4. 3. Determining the mass of gears

It is discerned that the gear mass in a gearbox can be identified by totalizing the mass of element gears:

$$m_g = \rho_g \cdot \left(\pi \cdot e_1 \cdot d_{w11}^2 \cdot \frac{b_{w1}}{4} + \pi \cdot e_2 \cdot d_{w21}^2 \cdot \frac{d_{w1}}{4}\right), \quad (23)$$

where, ρ_g is the material density, ρ_g=7.8 (kg/dm^3) for steel [25]; for the first state e_1 and e_2 are the volume coefficients of the drive and driven gear.

In fact, e_1 and e_2 are chosen as 1 and 0.6 respectively; b_{w1} (mm), the gear width, is determined by:

$$b_{w1} = X_{ba1} \cdot a_{w1}, \quad (24)$$

where, dw11 and dw21 are the pitch diameters determined by [26]:

$$d_{w11} = 2 \cdot \frac{a_{w1}}{u_1 + 1}, \quad (25)$$

$$d_{w21} = 2a_{w1} \cdot \frac{u_1}{u_1 + 1}, \quad (26)$$

In (25) and (26), a_{w1} is the center distance of the helical gear provided by [26]:

$$a_{w1} = k_a \cdot (u_1 + 1) \cdot \sqrt[3]{T_{11} \cdot \frac{k_{H\beta}}{([\sigma_{H2}]^2 \cdot u_1 \cdot X_{ba})}}, \quad (27)$$

where, $k_{H\beta}$ is the contact load factor for pitting resistance; $k_{H\beta}$=1.16 for the helical gear set; $[\sigma_{H2}]$ is the allowable contacting stress of the first stage (MPa); k_a=43 is the material coefficient for steel of gear materials; X_{ba} is the coefficient of wheel face width of the helical gear set; T_{11} is the torque on the pinion (N·mm):

$$T_{11} = \frac{T_{out}}{u_g \cdot \eta_w \cdot \eta_{hg} \cdot \eta_b^3}, \quad (28)$$

in which, η_{hg}, η_b, η_w are the transmission efficiency of the helical gear unit, rolling bearing pair, and worm set, respectively. Choosing η_{hg}=0.97; η_b=0.992; η_w=0.76 and substituting them into (28) gives:

$$T_{11} = 1.3896 \cdot \frac{T_{out}}{u_g}, \quad (29)$$

where, T_{out} is the torque on the outcome shaft of the system.

4. 4. Calculating the mass of the worm

The mass of the worm can be determined by:

$$m_w = \rho_w \cdot \pi \cdot d_{w12}^2 \cdot \frac{l_w}{4}, \quad (30)$$

in which, ρ_w=7.8 (kg/dm^3) is the steel density used for the worm, [27]; l_w is the length of the worm. In practice, l_w can be approximated by $l_w=2d_{w11}$.

The pitch diameter of the worm d_{w11} can be found by [27]:

$$d_{w12} = \frac{d_{w22}}{u_2}, \quad (31)$$

with

$$d_{w22} = 2a_{w2} \cdot \frac{z_2}{z_2 + q}, \quad (32)$$

where, a_{w2} is the center distance of the worm gear set determined as follows:

$$a_{w2} = K_a \cdot \left(\frac{K_{HV} \cdot K_{H\beta} \cdot T_{22}}{[\sigma_{H2}]^2} \right)^{\frac{1}{3}}, \quad (33)$$

where, K_a=610 [34]; K_{HV}=1.2 is the internal dynamic coefficient [34]; $K_{H\beta}$=1.2 is the load concentration coefficient [26]; T_{22} is the wheel torque (Nmm); $T_{22}=T_{\text{out}}$.

In (33), [σ_{H2}] is the allowable contacting stress of the worm unit (N/mm^2); [σ_{H2}] is influenced by the materials of the wheel. If tinless bronze or soft grey iron is adopted, [σ_{H2}] is determined by the regression model below that is obtained by data in [26] (with the determination coefficient R^2=0.9906):

$$[\sigma_{H2}] = 5.0515 v_{sl}^2 - 49.742 v_{sl} + 189.9, \quad (34)$$

wherein, v_{sl} is the slip velocity, which is found by [26]:

$$v_{sl} = 0.0088 \left(P_1 \cdot u \cdot n_w^2 \right)^{\frac{1}{3}}, \quad (35)$$

where, P_1 is the power on the worm shaft; P_1 can be found by:

$$P_1 = T_{12} \cdot \frac{n_w}{9.66 \cdot 10^6}, \quad (36)$$

in which, n_w is the rotational speed of the worm; T_{12} is the worm torque (N·mm):

$$T_{12} = \frac{T_{out}}{u_w \cdot \eta_w \cdot \eta_b^2}. \quad (37)$$

Choosing η_w=0.76; η_b=0.992 and substituting them into (37) gives:

$$T_{12} = 1.3371 \cdot \frac{T_{out}}{u_w}. \quad (38)$$

In case of tin bronze materials, [σ_{H2}] is determined by:

$$[\sigma_{H2}] = K_{HL} \cdot v_{sl} \cdot [\sigma_{H0}], \quad (39)$$

where [σ_{H2}] is the allowable contacting stress as its varying cycle is 10^7:

$$[\sigma_{H0}] = (0.7 \div 0.9)\sigma_t, \quad (40)$$

σ_t is the tension stress (N/mm^2); the value of σ_t depends on the slip velocity v_{sl}:σ_t=260 if v_{sl}=5...8; σ_t=230 if v_{sl}=8...12; and σ_t=285 if v_{sl}=8...25.

K_{HL} is the servicing life ratio calculated by [26]:

$$K_{HL} = \left(\frac{10^7}{N_{HE}} \right)^{\frac{1}{8}}, \quad (41)$$

where, N_{HE} is the number of equivalent load cycles for the teeth of the wheel:

$$N_{HE} = 60 \cdot n_{wl} \cdot t_{\Sigma}, \quad (42)$$

in which, t_{Σ} is the service lifetime (h); n_{wl} is the wheel speed (rpm).

4. 5. Calculating the mass of the worm wheel

The mass of the worm wheel is calculated by:

$$m_{wh} = \rho_{wh} \cdot \pi \cdot d_{w12}^2 \cdot \frac{b_{w1}}{4}, \quad (43)$$

in which, ρ_{wh} is the worm wheel material density (kg/m^3), ρ_{wh}=8.88 (kg/m^3) for bronze worm wheel [25]; b_{w1} is the width of the worm wheel; b_{w1} is determined by $b_{w1}=0.75d_{11}$ [26].

4. 6. Calculating the mass of shafts

The mass of the shafts (m_s) can be determined as follows:

$$m_s = m_{s1} + m_{s2} + m_{s3}, \quad (44)$$

where,

$$m_{s1} = \rho_s \cdot \pi \cdot d_{s1}^2 \cdot \frac{l_{s1}}{4}, \quad (45)$$

$$m_{s2} = \rho_s \cdot \pi \cdot d_{s2}^2 \cdot \frac{l_{s2}}{4}, \quad (46)$$

$$m_{s3} = \rho_s \cdot \pi \cdot d_{s3}^2 \cdot \frac{l_{s3}}{4}, \quad (47)$$

where l_{s1}, l_{s2}, l_{s3} and m_{s1}, m_{s2}, m_{s3} are the length and mass of the shaft 1, 2, 3, respectively; ρ_s is the shaft material density (kg/m^3).

$$l_{s1} = L_1 + 1.2 \cdot d_{s1} + S_G, \quad (48)$$

$$l_{s2} = L + L_1 - 0.5 S_G, \quad (49)$$

$$l_{s3} = d_{w12} + 1.2 \cdot d_{s3} + 20, \quad (50)$$

where:

$$d_{s1}=\left[\frac{T_{11}}{0.2[\tau]}\right]^{\frac{1}{3}}, \tag{51}$$

$$d_{s2}=\left[\frac{T_{12}}{0.2[\tau]}\right]^{\frac{1}{3}}, \tag{52}$$

$$d_{s3}=\left[\frac{T_{13}}{0.2[\tau]}\right]^{\frac{1}{3}}, \tag{53}$$

where $[\tau]=17$ (MPa) is the allowable shearing stress (MPa) [26].

4. 7. Bearing cost calculation

From the data in [6], a regression formula (with $R^2=0.9877$) to determine the cost of a medium-sized deep groove ball bearing on the i^{th} shaft with the inner diameter of 5 $N_{e,i}$ has been proposed:

$$C_{b,mi}=0.2707N_{e,i}^2-1.2566N_{e,i}+3.3056, \tag{54}$$

in which, $C_{b,i}$ is the maximum retail price of the bearing (USD); $N_{e,i}$ is the value of the last two digits of the bearing's designation. $N_{e,i}$ can be easily determined from the value of the shaft diameter $d_{s,i}$.

The cost of a bearing on the i^{th} shaft $C_{b,i}$ is determined by:

$$C_{b,i}=k_{cb}\cdot C_{b,mi}, \tag{55}$$

wherein, k_{cb} is the bearing selling coefficient ($k_{cb}<1$).

4. 8. Optimization problem

From the above analysis, the definition of the optimization issue follows:

$$\text{Minimize } C_{gb}. \tag{56}$$

And including the constraints below:

$$\begin{gathered} 1\le u_1\le 9; \\ 8\le u_2\le 80; \\ a_{w1}+\frac{d_{w11}}{2}+2.5S_G<a_{w2}. \end{gathered} \tag{57}$$

To evaluate the effects of the main design parameters or investigated parameters on the u_2 partial gear ratio to get minimal gearbox cost, a screening experiment is conducted by using ten parameters. The investigated degrees of these parameters are listed in Table 1. To reduce the number of the required tests and still well evaluate the influence of the input parameters, we use partial experimental planning of $(2^{k-p})\cdot 2^{11-4}=128$ tests (a 2^(11-4) model and using 1/16 fractional model). The chosen experimental design is at 5 resolution degrees, and at this resolution no major factor or two-way interaction is coincident with any other major factor or 2-way interaction. The utilization of screen experiment can generate the largest number of experiments. Additionally, using this method can provide mathematical models that cannot be obtained by the Taguchi method.

To determine the experimental plan, Minitab®18 software is applied by 2^{10-3}, and the details of this design are described as presented in Table 2.

Table 1

The levels and denotation of input parameters

Parameter name	Minitab®18	Code	Unit	Low	High
Total gearbox ratio	A	u_g	–	60	120
Output torque	B	T_{out}	Nm	1,000	8,000
Coefficient of the face width of the first gear stage	C	X_{ba}	–	0.2	0.25
Allowable contact stress of stage 1	D	AS_1	MPa	350	420
Cost of gearbox housing	E	C_{gh}	USD/kg	1.5	5
Cost of a kilogram of gears	F	C_g	USD/kg	2	9
Cost of the worm	G	C_w	–	3	10
Cost of the worm wheel	H	C_{wh}		8	15
Cost of a kilogram of shafts	J	C_s	USD/kg	1.5	5
Bearing selling price coefficient	K	k_{cb}	–	0.7	1

Table 2

Testing plans and calculated partial gear ratio of u_2

Std Order	Run Order	CenterPt	Blocks	u_g	T_{out}	X_{ba}	AS_1	C_{gh}	C_g	C_w	C_{wh}	C_s	k_{cb}	u_2
81	1	1	1	60	1000	0.2	350	5	2	10	8	1.5	1	8.08
117	2	1	1	60	1000	0.25	350	5	9	10	15	5	1	8.00
86	3	1	1	120	1000	0.25	350	5	2	10	8	5	1	13.33
88	4	1	1	120	8000	0.25	350	5	2	10	15	1.5	1	13.33
27	5	1	1	60	8000	0.2	420	5	2	3	8	1.5	0.7	8.00
107	6	1	1	60	8000	0.2	420	1.5	9	10	15	5	1	8.00
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
18	127	1	1	120	1000	0.2	350	5	2	3	8	1.5	0.7	13.33
93	128	1	1	60	1000	0.25	420	5	2	10	15	1.5	1	8.00

It is noticed that the values of u_2 response corresponding to each test are displayed in the last column.

5. Results of research of finding the optimal set of the main design parameters

5. 1. The effects of the main design parameters on the u_2 response

The effects of the main design parameters on the u_2 response can be evaluated by the results given in Fig. 4. It is visualized that u_2 is increased with increasing Total gearbox ratio (u_g) and Cost of a kilogram of gears (C_g), while u_2 is reduced when Output torque (T_{out}), Allowable contact stress of stage 1 (AS_1), and Cost of the worm wheel (C_{wh}) increase. Moreover, Coefficient of the face width of the first gear stage (X_{ba}), Cost of gearbox housing (C_{gh}), Cost of the worm (C_w), Cost of a kilogram of shafts (C_s) and Bearing selling price coefficient (k_{cb}) have a small effect on u_2.

The effects of input parameters on the u_2 response can be also presented by Pareto Chart as in Fig. 5. It shows both the impact of the main design parameters and interactions on the response u_2. The influence level of each parameter is exhibited by the blue column length. In this case, the parameters with exceeding magnitude, the red reference line, are those that have a significant effect on u_2 corresponding to the significance level α of 0.05. Specifically, it is seen that the influence of A factor (u_g) on u_2 is the largest. The parameters having a dominant influence on u_2 including interactions are: B (T_{out}), D (AS_1), F (C_g), H (C_{wh}) and interactions BD (T_{out}*AS_1), BF (T_{out}*C_g), BH (T_{out}*C_{wh}), DF (AS_1*C_g), DH (AS_1*C_{wh}), FH (C_g*C_{wh}), GK (C_w*k_{cb}).

Taking into account the impacts of interactions and parameters on u_2 is crucial. These results can be seen in Fig. 6.

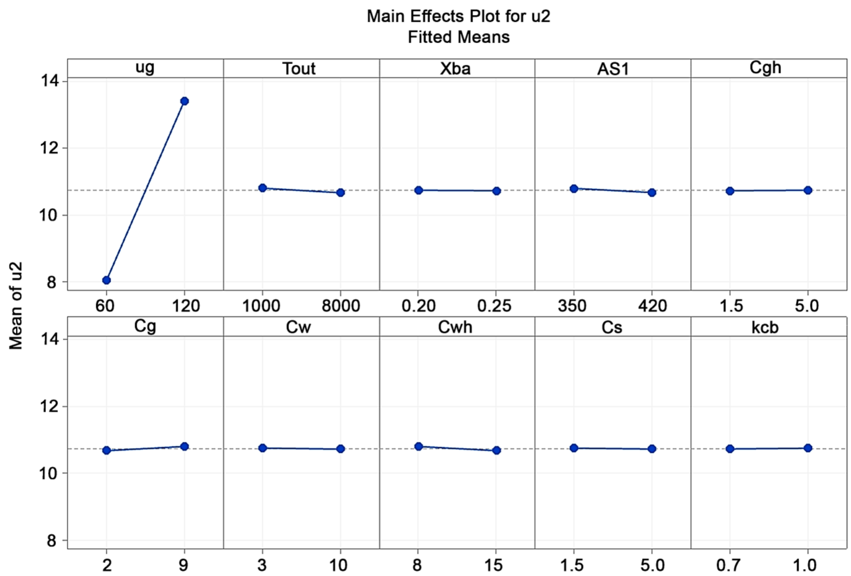

Fig. 4. Influence of the main design parameters on the u_2 partial gear ratios

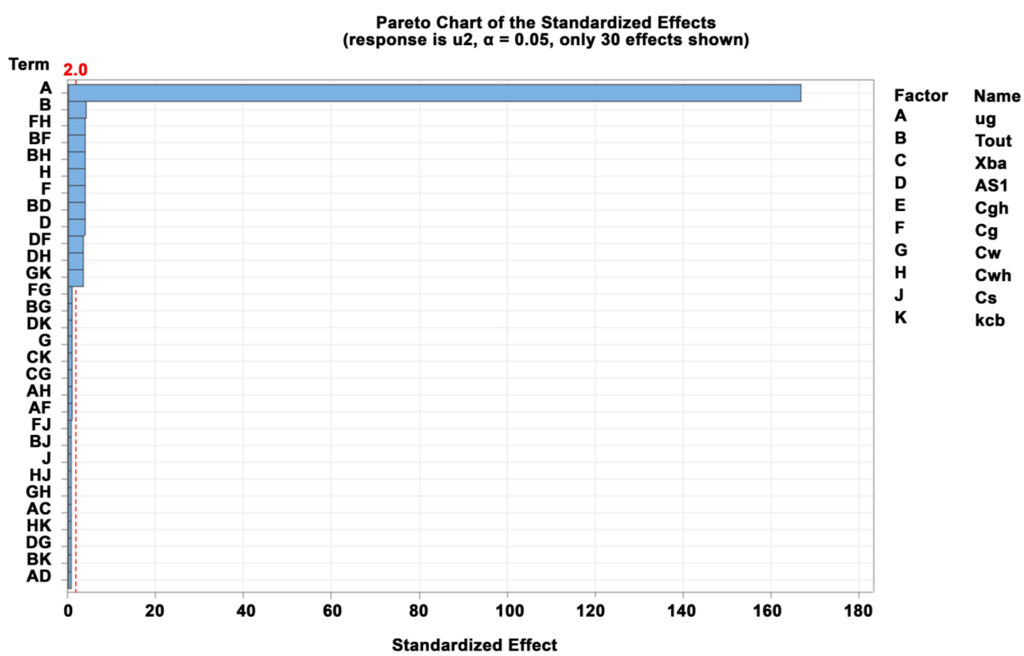

Fig. 5. Normalized Pareto diagram for the impact of both main design parameters and interactions on u_2

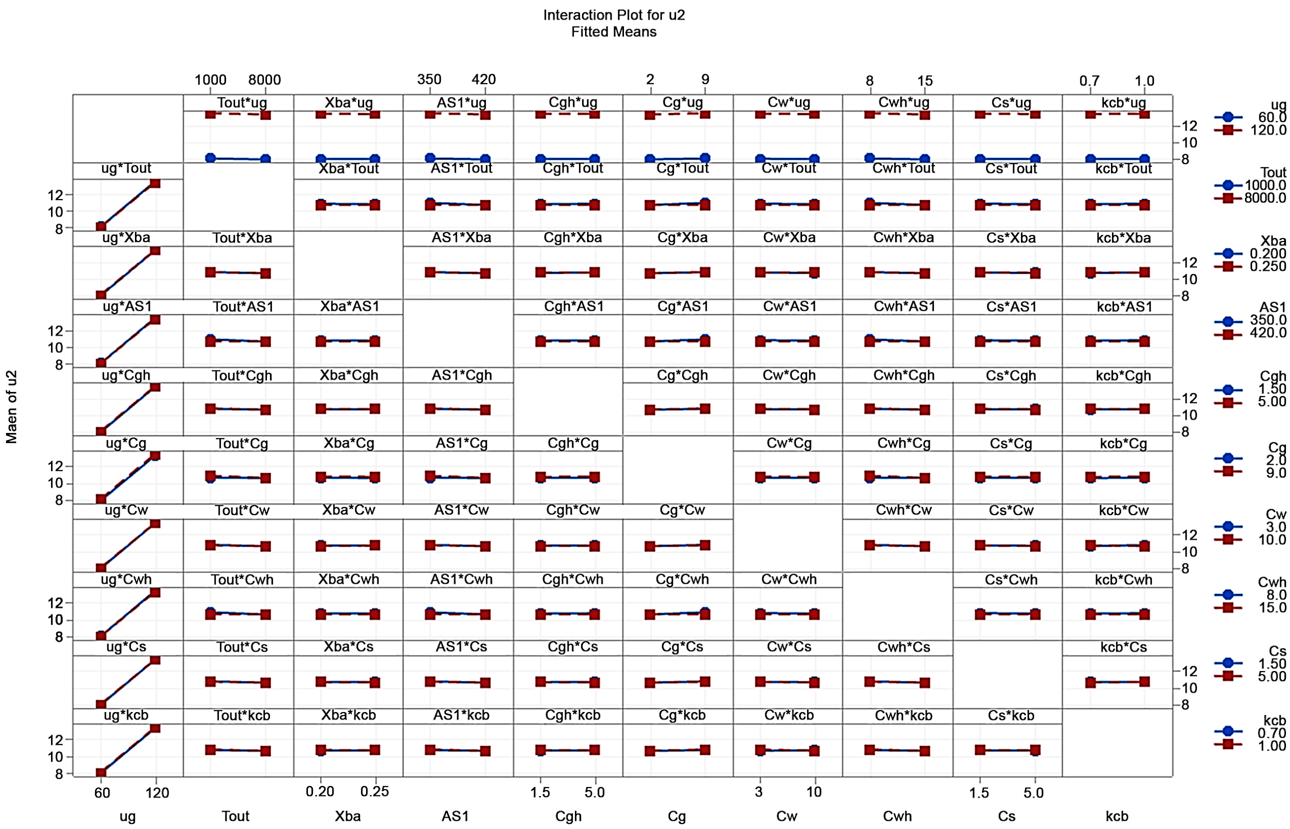

Fig. 6. Effects of interactions on u_2

There is the fact that the Total gearbox ratio factor shows the dominant influence on u_2 even though this factor interacts with nine remaining parameters where all of these interactions, in the leftmost column of Fig. 6, have positive impacts on u_2.

To determine the tendency and impacts of the parameters and their interactions, one can refer to the normal distribution chart in Fig. 7.

It is seen that the parameters highlighted in red have a significant impact on u_2. The parameters further from the reference line are the ones having a greater impact on u_2. The parameters to the right of the reference line have a positive influence on u_2 including: A (u_g), F (C_g), and interactions such as BD (T_{out}*AS_1), BH (T_{out}*C_{wh}), DH (AS_1*C_{wh}). The parameters to the left of the reference line have a negative influence on u_2, including: B (T_{out}), D (AS_1), H (C_{wh}) and interactions BF (T_{out}*C_g), DF (AS_1*C_g), FH (C_g*C_{wh}), GK (C_w*k_{cb}).

5. 2. Determining the regression equation to calculate u_2

Proceed to removing the parameters that have no or very little influence on u_2. A regression model will be proposed for predicting u_2 by using Minitab®18 software. This is because of the large number of variables, so manual calculation is not feasible. The estimating coefficients of the main design parameters and their interactions are presented in Table 3. The summary of the model showed that R-square and R-square (adjusted) are higher than 99 %.

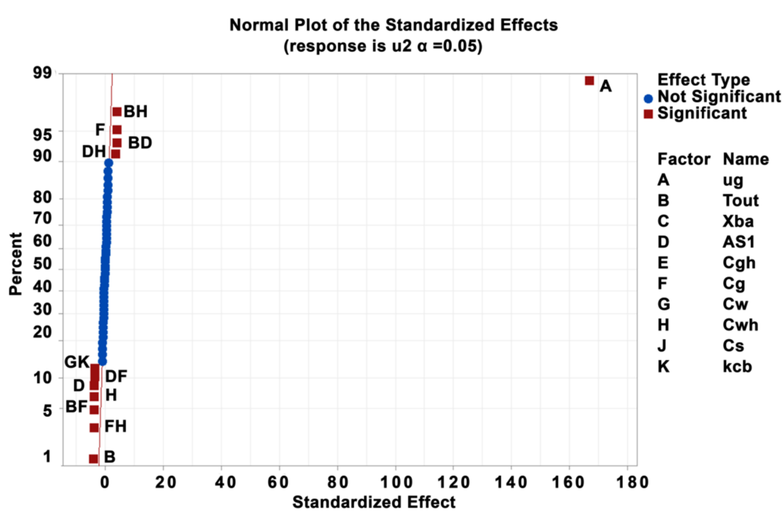

Fig. 7. Histogram of the normal distribution showing the influence of parameters and interactions on u_2

Table 3

Estimating coefficients of the main design parameters and interactions to determine u_2. Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	14	920.516	65.751	2592.65	0.000
Linear	7	917.226	131.032	5166.77	0.000
u_g	1	915.064	915.064	36082.17	0.000
T_{out}	1	0.594	0.594	23.42	0.000
AS_1	1	0.498	0.498	19.62	0.000
C_g	1	0.510	0.510	20.11	0.000
C_w	1	0.035	0.035	1.38	0.242
C_{wh}	1	0.510	0.510	20.11	0.000
k_{cb}	1	0.015	0.015	0.59	0.445
2-Way Interactions	7	3.290	0.470	18.53	0.000
$T_{out}*AS_1$	1	0.498	0.498	19.62	0.000
$T_{out}*C_g$	1	0.510	0.510	20.11	0.000
$T_{out}*C_{wh}$	1	0.510	0.510	20.11	0.000
AS_1*C_g	1	0.421	0.421	16.60	0.000
AS_1*C_{wh}	1	0.421	0.421	16.60	0.000
C_g*C_{wh}	1	0.510	0.510	20.11	0.000
C_w*k_{cb}	1	0.421	0.421	16.60	0.000
Error	113	2.866	0.025	–	–
Total	127	923.382	–	–	–

The coefficients of the regression model are obtained as follows:

$$u_2=4.736+0.089125u_g-0.000246T_{out}-0.00688AS_1+$$
$$+0.2807C_g+0.0881C_w-0.1931C_{wh}+0.782k_{cb}+$$
$$+0.000001T_{out}*AS_1-0.000005T_{out}*C_g+0.000005T_{out}*C_{wh}-$$
$$-0.000468AS_1*C_g+0.000468AS_1*C_{wh}-$$
$$-0.00515C_g*C_{wh}-0.1092C_w*k_{cb}.$$

5. 3. Evaluating the fitness of the proposed model

The model fitness is evaluated through the residual distribution chart to determine deviations between the experiments and prediction of u_2 in Fig. 8. It is revealed that on the Normal Probability Plot, comparing graph the probability distribution of errors (shown by points in blue) for the normal distribution (solid line), these deviations are distributed very near the normal distribution. The Versus fits graph discloses that the relation between the residual and fitted value of the model is random. This shows that the response u_2 is not impacted by any rule control factors other than the input variables. The histogram shows the frequency of residual values around the center of the distribution, which can be considered as a normal distribution. Finally, the Versus Order also exhibits the random relationship between residual and order of data point. This reveals that the response u_2 is not influenced by the time factor.

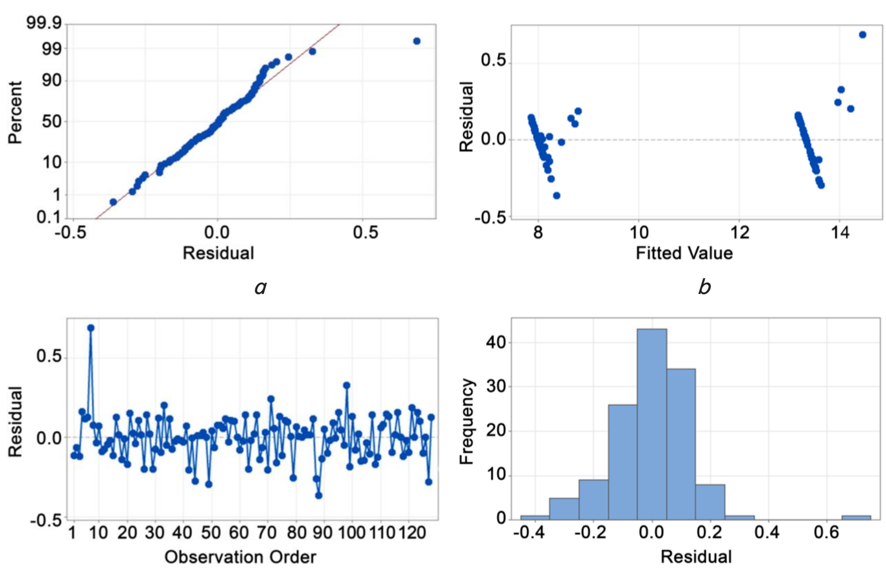

Fig. 8. Residual evaluation distribution charts: *a* – Normal Probability Plot; *b* – Versus Fits; *c* – Histogram; *d* – Versus Order

This result is also clearly shown in the error frequency graph. The near zero error dominates for the majority (ranging from −0.4 to 0.4). It is observed that the graph showing the relationship between the errors and the calculated values of the proposed model shows that the data are arbitrarily distributed.

6. Discussion of the results of finding the optimal set of the main design parameters

Due to the simulation experiments instead of physical one where there is no limiting of the cost, it is desired to perform all tests as possible in this study. However, the available function given by Minitab@18 could not execute as expected. For this reason, the authors have selected a 2^(10-3) model and using 1/16 fractional model, maximum ability to conduct the number of tests (128 tests). Furthermore, the screening experiments are adopted to reduce the number of parameters, which has a minor influence on the response. This is a simple method to get the mentioned aims. Compared to using Taguchi technique, for the model of 2^11 corresponding to L32 or 32 tests. This is not satisfied the maximum tests as previously mentioned (128 tests). Additionally, Taguchi design could not provide the regression equation. Hence, the method used in this study is better than simply using Taguchi method. For example, it is seen that there are seven single factors and seven interaction factors having a significant influence on the u_2 response. P-value of each parameter mentioned in Table 3 if higher than significant of a (α=0.05) will be eliminated because of its minor effect on the response.

The random tendency of data presented in Fig. 8 reveals that u_2 is not significantly impacted by other than the main design parameters. Identically, the data in versus order is also arbitrarily dispensed, showing u_2 is not varied to the time factor. Based on the previous analysis of error assessment, combined with the calculation results, the coefficients of adjusted R^2 and R^2 are both greater than 99 % (Table 3), it can be concluded that the calculated regression model is appropriate.

The aim of this study is to investigate the influence of the main design parameters on the optimal gear ratio, which is the constraint of the objective function. Hence, the influence of the assumptions is beyond the target of this study. Moreover, the cost of a given gearbox is decided by some cost factors such as the cost of manufacturing, the cost of the assembly process, the cost of operation, the cost of bearing, the cost of wear and reliability, and the cost of materials. Besides, there are other factors related to the cost such as the kind of surface treatment, the shape of the gear body, etc. In this study, the cost of materials is only considered, but others are beyond the scope here. The cost of material of the studied gearbox is indicated herein by C_{gb}. For future research, the previously mentioned factors should be considered in the gearbox optimization problems.

7. Conclusions

1. Total gearbox ratio has the biggest effect on the u_2 response, while the remaining parameters have minor. These results are both shown in Main Effects Plot and Pareto Chart of the Standardized Effects. The interactions between input parameters also have a significant impact on the response. In particular, the interaction between Total gearbox ratio and the remaining factors strongly influences the partial gear ratios.

2. The proposed regression model for u_2 is crucially consistent with validated data with the coefficients of adjusted R^2 and R^2 both greater than 99 %.

3. The fitness of the proposed model was reliably confirmed in Normal Probability Plot, Versus Fits, Histogram, and Versus Order, and ANOVA. For this reason, it can be concluded that the proposed optimization process can be applied to minimally calculate gearbox cost.

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