This paper examines the process of operative planning of the production of an industrial company under conditions of random fluctuations in current demand. It is shown that under these conditions there are losses, the size of which depends on the adopted policy of operation activity. The policy of operation activity is understood as the rule of making decisions on current production volumes based on information about incoming orders, probable volumes of future demand, and possible losses due to the deviation of capacity load from the normative one.

In the paper, it is proposed to assess the effectiveness of each policy of operation activity using the indicator of the limit average economic effect per unit of time for an infinite number of periods. An original approach to assessing the effectiveness of the policy of operation activity with product reservation was developed. It was shown that when using this policy, there is an effect of product "overstock" on the chains of successive periods. It was proposed to select the initial reserve so that the probability of completion of the reservation chain for a given number of periods should be close to unity. Such an approach creates an opportunity to determine the expected economic effect on the chains of reservation of various product types and, as a result, to assess the policy effectiveness in general.

An assessment of the effectiveness of the policy with reservation in the form of the dependence of the policy effectiveness indicator on the values of cost indicators was obtained. Comparison of this assessment with a similar assessment of the effectiveness of the policy of fulfilling incoming orders allowed finding a condition under which the policy with reservation is more profitable. It involves ensuring that the magnitude of losses per unit of production associated with the product stock storage does not exceed half the sum of the magnitude of losses per unit of production due to downtime and excess capacity load

Keywords: operative planning, policy of operation activity, random demand, risks, product reservation

UDC 303.725.35:658.5

DOI: 10.15587/1729-4061.2022.252667

ANALYSIS OF THE POLICY OF OPERATION ACTIVITY OF AN ENTERPRISE WITH PRODUCT RESERVATION

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Received date 11.01.2021
Accepted date 16.02.2022
Published date 28.02.2022

How to Cite: *Zaruba, V., Potrashkova, L., Guryanova, L., Sokol, K., Kuksa, I. (2022). Analysis of the policy of operation activity of an enterprise with product reservation. Eastern-European Journal of Enterprise Technologies, 1 (3 (115)), 31–42. doi: https://doi.org/10.15587/1729-4061.2022.252667*

1. Introduction

One of the urgent problems of management is to balance the production resources of an enterprise and the demand for its products, which is characterized by instability and random changes. The tasks of managing production resources are distributed between three organizational levels: strategic, tactic, and operational. These levels ensure the achievement of the company's objectives in the long, medium- and short-term perspectives, respectively [1].

The following things are developed at the strategic level: a long-term forecast of demand for the company's products; commodity policy, involving the introduction of new types of products into production; a program for the development of production capacities, taking into consideration the development of new technologies. In the course of tactic planning, a company develops a program for manufacturing products for a medium-term period, coordinates with suppliers the terms of supply of circulating material resources, establishes outsourcing volumes, and dynamics of the staff number of production personnel. Operative production planning is usually divided into volume-nomenclature and calendar. Operative volume-nomenclature plans determine current production volumes based on the incoming orders for the company's products, its production facilities, and forecasts of future orders. In the course of calendar planning, the volume-nomenclature plan is detailed to the terms of manufacture of certain products, assembly units, and parts [1, 2].

During the implementation of the production program under conditions of unstable demand, losses arise due to the discrepancy between the production volumes established by the program and random magnitudes of the volumes of incoming orders. These losses arise either due to the lack of sale of part of the finished product (storage costs, funds "freezing") or due to lost profit related to insufficient manufacturing of products if there is a demand for it.

Losses that arise during the implementation of the production program can be reduced during operative planning by changing the production volumes planned by the program. However, this adjustment of production volumes is associated with other losses. Losses from the downward change in production volume cause the payment of "unproductive" salary to personnel under conditions of downtime, the cost of storing unused working material resources, and "freezing" funds for their purchase. Losses from changes in the production volume towards increasing cause the need for additional payment to personnel for overtime work and the purchase of an additional amount of circulating material resources at increased prices [2].

Thus, under conditions of random demand, enterprises inevitably incur various kinds of losses. At the same time, a decrease in one type of loss leads to an increase in the losses of other types. That is why the problem of minimizing the total losses of an enterprise caused by the emerging deviations of the current volumes of demand from the normative load of production capacities is relevant.

In practice, different management approaches are used to reduce these losses, corresponding to different policies of operation activity. Each policy of operation activity is characterized by some ratio of different types of losses. That is why the choice of a set of policies of operation activity should be based on assessments of their effectiveness, taking into consideration losses caused by unstable demand. In this regard, studies on assessing the effectiveness of different policies of operation activity (including the policy with product reservation) under conditions of unstable demand are relevant.

2. Literature review and problem statement

The problem of reducing losses that arise in the course of operation activity under conditions of unstable demand is closely related to risk management. The basic concepts and conceptual apparatus of risk management in organizations are defined by the ISO 31000:2009 standard "Risk management. Principles and guidelines" [3]. The standard can be applied to the entire organization and at all levels, as well as to specific functions, projects, and activities.

Modern methods of balanced management of production resources correspond to the ERP (Enterprise Resource Planning) standards of information systems that provide comprehensive support of management for large and medium-sized enterprises. Dozens of publications are devoted to the problems of implementation and development of information systems that perform the ERP tasks. In particular, in paper [4], ERP is considered as a management concept aimed at improving business efficiency in general. However, one of the critical problems is the coordination of functional technologies that ensure the production process. These technologies include presentation of production plans in the context of calendar periods (Master Planning Scheduling); Material Requirement Planning; planning the requirements to ensure timely fulfillment of orders (Capacity Resource Planning). Note that the technologies of ERP systems really provide ample opportunities for solving the calculation problems of production planning under conditions of given levels of demand. However, they do not directly manage the risks that arise in the face of random fluctuations in current demand.

The concept of Sales & Operations Planning (S&OP) is based on a systemic approach to operation activity management. In [5], the S&OP process is considered at three levels depending on the planning horizon. The long-term planning horizon for a typical S&OP process spans more than 18–36 months. The Annual Action Plan (AOP) is the company's medium-term goal of sales and supply. Short-term (monthly) plans of sales and operations are the means of gradually achieving the AOP goals. The S&OP goal for short-term periods of time is to determine the general level of production (production plan) and other activities to achieve the overall goals of profitability, productivity, and competitive time of order fulfillment.

The S&OP process is focused on establishing production rates that will allow achieving the goal of maintaining, increasing, or decreasing inventories or accumulated reserves while maintaining the relative stability of personnel [5]. In paper [6], it is noted that many professionals in the supply chain in recent years have been concerned with improving the link between supply and demand. An important task of the S&OP is to reduce losses from the imbalance in time of the capabilities of the sales and production subsystems of an enterprise. However, the S&OP does not take into consideration the fact that the way of balancing the capabilities of these subsystems is determined by the policy of operation activity applied at an enterprise. S&OP does not offer methods and models to evaluate and choose the most effective operation policy.

Economic and mathematical methods for optimizing decisions under conditions of incomplete information are often considered as mathematical models of management of risks of operation activity. They include deterministic approximation, stochastic programming, Markovian decision-making models, simulation modeling, etc. [7].

Most of the studies are devoted to the following aspects of operation activity management:

– determining the number of purchases of circulating material resources and their stocks;

– determining the optimal size of a batch size of finished products or components;

– planning the operation activity with reservation of finished products.

Stocks of materials and components make it possible to maintain the smooth operation of an enterprise in situations of supply failure, equipment breakdown, and demand fluctuations. At the same time, the availability of stocks is accompanied by the cost of their storage. The problems of optimizing the size of purchases of circulating material resources and their stocks were first studied in the deterministic formulation, later – taking into consideration uncertainty factors, in addition, taking into consideration non-deterministic demand [7].

The task of optimizing the size of a batch of finished products or components arises in cases where the transition to the production of the next batch of products requires the reconfiguration of machines. In this regard, there are contradictions between the goals of reducing the cost of storing products and reducing the readjustment costs. The first goal is achieved by reducing the size of the produced batches of products, and the second, on the contrary, requires an increase in the batch size.

The studies on the planning of operation activity with the reservation of finished products take into consideration various factors that affect reservation effectiveness. Such factors include the cost of storing the manufactured products, the cost of stock creation, restrictions on production capacities, losses due to incomplete meeting the demand. To solve the problem of multi-product production planning, research [8, 9] uses the method of two-stage stochastic programming. Research [8] determines the optimal decisions on supplies, production, and stocks over several planning periods, and the criterion is the minimum total costs of the system, taking into consideration the cost of storing stocks of materials and finished products. In article [9], the optimal distribution of the number of workers and production volumes are determined.

Studies [2, 10] explore the problem of choosing the current production volumes under conditions of incompletely determined demand. At the same time, possible losses associated with the lack of sale of part of the finished product and with the lost profits from underproduction in the presence of demand are taken into consideration. It was supposed that the goal of an organization was to obtain the maximum effect over a given period. The effect in the general form can be presented by the value of the known function $f(\eta, y)$, the arguments of which are random magnitude η with probability density $p(x)$ and parameter y of the made decision. In this case $x\in X$, $y\in Y$, where X is the set of possible implementations of random magnitude η, Y is the set of possible values y. In this situation, the effect (the result of achieving the goal) when making any choice y turns out to be a random magnitude. The magnitude $H(y)=\int_{x\in X} f(x,y)p(x)\mathrm{d}x$ will match the mathematic expectation of the effect for the chosen value of y. The solution $\hat{y}$, which is optimal by the criterion of a maximum of the expected effect, was determined from the following formula: $H(\hat{y})=\max\{H(y)\,|\,y\in Y\}$.

A more complex problem of choosing the current volumes of production is presented in the form of controlling the random process taking place in the production system of an enterprise. The states of the production system are partly random and partly under the control of the decision-maker. In each period t, the system is in a certain state x_t and a decision-maker can choose any action y_t, which is accessible for him in state x_t of the system. In the following period t+1, the system randomly transfers to the new state x_{t+1}. In this case, a decision-maker gets the award $E_{t+1}=f_y(x_{t+1}, y_t)$ in the period t+1. In the problem of choosing actions (decisions) y_t in recurrent periods t=1, 2, ..., T, the aim of a decision-maker is to get the maximum award (effect) E in the time interval T, $E=\sum_{t=1}^{T} E_t$.

If for any period t, $1<t<T$, random magnitude x_{t+1} is independent on states x_s and decisions y_s, $s<t$, the controlled random process corresponds to the Markovian decision-making process. If the decision-making model for controlling some random process corresponds to the Markovian decision-making process, this model is called.

The Markovian decision-making model was used in papers [11, 12] to find the optimal control of a dynamic system under the influence of random factors. Thus, in paper [11], demand was described by a Markovian chain with two states, and the problem of finding the optimal batch size was formulated as a dynamic programming problem. Research [12] describes the probabilistic problem of optimal control with continuous time, in which parameters of output intensity and pricing policy act as management. In this case, demand is modeled by the Poisson probability distribution. The maximum long-term profit of an enterprise is the optimality criterion.

The policy of operation planning (decision-making) in paper [13] implies rule φ of choosing decision y_t in each period t, depending on the known state x_t:$y_t=\varphi(x_t)$ (t=1, 2,..., T). Designate as Φ the set of such policies φ in which the values of parameter x_t (t=1, 2,..., T) of the state of the system are implementations of some random magnitude and do not depend on decision y_t in all periods. In this case, policies $\varphi\in\Phi$ turn out to be Markovian decision-making models. For them, it is fundamentally possible to find the magnitudes of the expected total effect E, and then choose the most effective policy from set Φ [13].

If policy φ is that decision y_t in period t has an impact not only on state x_{t+1}, but can also have an impact on subsequent situations, the corresponding decision-making process will not be Markovian. The study of such processes causes fundamental difficulties and is effective only in some cases.

3. The aim and objectives of the study

The purpose of this study was to analyze and assess the effectiveness of the policy of operation activity of an enterprise with the reservation of finished products. The obtained results of the study will allow increasing the validity of management decisions on the choice of the policy of operation activity at an enterprise, taking into consideration the problem of minimizing losses under conditions of random demand.

To achieve the aim, the following tasks were set and solved:

– to analyze the process of operation activity with product reservation, taking into consideration the emergence of chains of activity periods with a delay in selling created reserves under conditions of random fluctuations in demand;

– to determine the method for finding the maximum safe magnitude of the product stock on the condition of its full sale within a given number of planning periods;

– to carry out numerical calculations of finding the maximum safe magnitude of the product stock for the case of normal distribution of a random magnitude of demand;

– to determine the method for finding the limit values of the intensities of occurrence of various chains of activity periods for the selected maximum number of periods covered by reservation;

– to evaluate the efficiency of operation activity with the reservation of finished products based on the indicator of maximum effectiveness of operation activity.

4. The study materials and methods

4. 1. Research hypothesis

The object of the study was the processes of operation activity of an enterprise with the reservation of finished products. The main hypothesis of the study was the assumption that the policy of operative planning of production with the reservation of finished products is effective under many conditions of operation activity of an enterprise.

The following assumptions were accepted:

– the normative production capacity of an enterprise coincides with the mathematical expectation of the magnitude of demand;

– the parameters of the law of distribution of the probability of demand magnitude quantity do not change when using the selected policy.

The source materials of the research were:

– the model of operative planning used directly or with minor modifications in papers [2, 12, 8, 10, 13];

– the results of the assessment of the effectiveness of policies of operative planning, given in [13] and used for their comparative analysis with the results of this research.

4. 2. The model of operative planning

In accordance with the model of operative planning, the process of receipt and execution of orders for the company's products is considered for T periods having the same duration. In each period t–1, orders for the company's products are collected and the production volume is planned for the next period t. Periods for which the production volume is planned are called planning periods.

For each planning period, the model determines the set of permissible (implementable) decisions and the dependence of the operation effect (profit) on the decisions made.

The following designations were used in the model: u_0 is the number of products produced by an enterprise for the planned period at the normal (normative) load of production capacities; η is the total volume of orders received by the beginning of each period. When the volume of production u_0 and intensity of demand η are deterministic constants, and u_0=η, the production capacity of an enterprise is used evenly with the normative load. If the magnitude of demand η is a variable random magnitude, the resources of an enterprise and the flow of orders will be balanced if $u_0=\lambda_\eta$, where λ_η is mathematical expectation η. However, in this situation, there are losses associated with overload and an insufficient load of production capacity.

The following designations were introduced:

– x_t is the volume of orders received by the beginning of planning period t;

– u_t is the volume of production within planning period t;

– z_t is the magnitude of residues of the finished product by the beginning of period t;

– y_t is the total amount of finished products which will be available within the considered period, $y_t=u_t+z_t$.

The dependence of the operation effect E_t, received at the end of period t, on magnitude x_t of incoming orders and the planned amount y_t of the finished product is determined by function $f(x_t, y_t)$:

$$E_t = f(x_t, y_t) = f_1(x_t, y_t) = dy_t - d(x_t - y_t) - q(u_t),$$

if

$$x_t \geq y_t; \qquad (1)$$

$$E_t = f(x_t, y_t) = f_2(x_t, y_t) = dx_t - a(y_t - x_t) - q(u_t),$$

if

$$x_t \leq y_t, \qquad (2)$$

where $f_1(x_t, y_t)$, $f_2(x_t, y_t)$ are functions that determine effect E_t respectively in cases of lost profits and of the presence of unsold products; d is the magnitude of profit from the sale of a unit of product at its production under conditions of a normative load of production capacity; a is the amount of losses associated with the storage of the stock of finished products during one planning period, calculated per unit of production; $d(x_t-y_t)$ is the sum of losses (lost profits) from underproduction when there is a demand; $q(u_t)$ is the magnitude of losses caused by downtime and over normative load of production capacities, in this case, $q(u_t)=b(u_0-u_t)$, if $u_0 \geq u_t$, $q(u_t)=c(u_t-u_0)$, if $u_0 \leq u_t$; b is the magnitude of losses per unit of production caused by downtime; c is the magnitude of losses per unit of product caused by over normative load of production capacities.

Magnitudes x_t (t=1, 2, ..., T) of the volume of orders were considered as sales in periods t=1, 2, ..., T of random magnitude η with the known function of probability distribution $F_\eta(x)=P\{\eta \leq x\}$ taking positive values in the interval $[0, x^{\max}]$ of the values of its argument. Paper [2] described the algorithm of construction of discrete function of probability density η based on retrospective information about the volumes of orders for its products.

For certainty, the functions of distribution of the probability of random magnitudes η were assumed to be symmetric with respect to their mathematical expectation:

$$\lambda_\eta = \frac{x^{\max}}{2},$$

$$F_\eta(\lambda_\eta + \varepsilon) - F_\eta(\lambda_\eta) = F_\eta(\lambda_\eta) - F_\eta(\lambda_\eta - \varepsilon),$$

if

$$0 \leq \varepsilon \leq \lambda_\eta. \qquad (3)$$

The symmetry condition is met by a wide class of probability distribution laws, including normal, uniform, "triangular". The use of the properties of symmetric laws makes it possible to simplify the form of mathematical formulas, their research, and numerical calculations in accordance with them.

We introduced designations T_m^+ and T^+, which describe, respectively, the set and the number of these planning periods t, in which $\eta_t \geq \lambda_\eta$. Similarly, T_m^- and T^- are the set and the number of operation periods t, in which $\eta_t \leq \lambda_\eta$ For the number of periods T^-, T^+ in the total number of planning periods are, respectively, made up of magnitudes $P^- = T^-/T$, $P^+ = T^+/T$.

If the function of distribution of probabilities of magnitude η is a symmetric function, $P^-=P\{\eta \leq \lambda_\eta\}=P^+=P\{\lambda_\eta \leq \eta\}=0{,}5$. In this case, magnitudes P^-, P^+ have the sense of probabilities that for an arbitrary planning period t, it will turn out that either $t \in T_m^-$, or $t \in T_m^+$.

The task of studying the properties of the processes of receipt and execution of orders with random volumes caused the need to use the following two magnitudes:

$$\rho^- = \frac{1}{T^-}\sum_{t \in T_m^-} x_t, \quad \rho^+ = \frac{1}{T^+}\sum_{t \in T_m^+} x_t, \qquad (4)$$

where ρ^- is the mean value of T^- implementations $x_t, t \in T_m^-$ of random magnitude η, ρ^+ is the mean value of T^+ implementations $x_t, t \in T_m^+$ of random magnitude η At the infinitely high value of the number T of planned periods, magnitudes ρ^-, ρ^+ have the sense of mathematical expectation of random magnitude η on condition that it gets, respectively, in intervals $[0, \lambda_\eta]$, $[\lambda_\eta, x^{\max}]$. It is not difficult to see that $\rho^-+\rho^+=2\lambda_\eta$. Besides, in [2] it was shown that at a decrease in the variance of magnitude η to zero, magnitudes $\lambda_\eta-\rho^-$, $\rho^+\lambda_\eta$ also decrease to zero.

4. 3. Evaluation of the effectiveness of the policies of orders fulfillment and production with constant intensity

Paper [13] analyzes the policies for the execution of orders and production with constant intensity in accordance with the considered model of operative planning. Here are the results of the evaluation of the effectiveness of these policies for the possibility of their subsequent comparison with the evaluation of the effectiveness of the policy with reservation of finished products.

Magnitude ζ of the sum of effects E_t within periods t=1, 2, ..., T, attributed to the maximum expected effect $d\lambda_\eta T$ for these periods was used as the indicator of the effectiveness of operation activity of an enterprise in T planning periods:

$$\zeta = \zeta(T) = \frac{1}{T}\sum_{t=1}^{T}\frac{E_t}{d\lambda_\eta}. \tag{5}$$

At an infinitely large value of T, indicator ζ is transformed into indicator ζ^* of maximum effectiveness of operation activity:

$$\zeta^* = \lim_{T\to\infty}\zeta(T). \tag{6}$$

In accordance with the policy of fulfilling incoming orders, production volume u_t for the current operation period t is selected as equal to volume x_t of incoming orders. Assuming that there are no residues z_0 of the finished product at the beginning of planning, we have $y_t=x_t$, $z_t=0$, $E_t=dx_t-q_t$ in each period t=1, 2, ..., T.

The ζ_{PFO} indicator of operation activity in T planning periods for the policy of fulfillment of incoming orders can be represented in the following form: $\zeta_{\mathrm{PFO}}=1-S_1-S_2$, where

$$S_1 = \frac{c}{d\lambda_\eta T}\sum_{t\in T_m^+}\left(x_t-\lambda_\eta\right) = \frac{cP^+}{d\lambda_\eta T^+}\left(\sum_{t\in T_m^+}x_t - T^+\lambda_\eta\right) = \frac{cP^+}{d\lambda_\eta}\left(\rho^+-\lambda_\eta\right); \tag{7}$$

$$S_2 = \frac{b}{d\lambda_\eta T}\sum_{t\in T_m^-}\left(\lambda_\eta - x_t\right) = \frac{bP^-}{d\lambda_\eta T^-}\left(T^-\lambda_\eta - \sum_{t\in T_m^-}x_t\right) = \frac{bP^-}{d\lambda_\eta}\left(\lambda_\eta-\rho^-\right). \tag{8}$$

Thus, we obtained:

$$\zeta_{\mathrm{PFO}} = 1-\frac{1}{d\lambda_\eta}\left(cP^+\left(\rho^+-\lambda_\eta\right)+bP^-\left(\lambda_\eta-\rho^-\right)\right). \tag{9}$$

Since the function of distribution of the probability of magnitude η is supposed to be symmetric, $P^-=P^+=0.5$, $\rho^-+\rho^+=2\lambda_\eta$, $\rho^+=2\lambda_\eta\rho^-$. Then

$$\zeta_{\mathrm{PFO}} = 1-\frac{1}{2d\lambda_\eta}\left(c\left(\lambda_\eta-\rho^-\right)+b\left(\lambda_\eta-\rho^-\right)\right) = 1-\frac{\lambda_\eta-\rho^-}{2d\lambda_\eta}(b+c). \tag{10}$$

From (10) it can be seen that at an increase in indicators b, c of specific costs from 0 to d, the ζ^*_{PFO} indicator of maximum effectiveness decreases from 1 to $\dfrac{\rho^-}{\lambda_\eta}$.

In the case of applying the constant-intensity production policy, the volume of production u_t for the operation period t is determined from the formula: $u_t=\min\{x_t, u_0\}$. That is why $y_t=u_t$, $z_t=0$,

$$E_t = \begin{cases} d\lambda_\eta - d\left(x_t-\lambda_\eta\right), & \text{if } \lambda_\eta \le x_t,\ t\in T_m^+, \\ dx_t - b\left(\lambda_\eta - x_t\right), & \text{if } \lambda_\eta \ge x_t,\ t\in T_m^-. \end{cases} \tag{11}$$

Based on this, the ζ^*_{PCI} indicator of maximum effectiveness of operation activity for the policy with constant intensity production was expressed through the formula:

$$\zeta^*_{\mathrm{PCI}} = 1-P^+\frac{\rho^+-\lambda_\eta}{\lambda_\eta}-P^-\frac{b\left(\lambda_\eta-\rho^-\right)}{d\lambda_\eta}. \tag{12}$$

Considering that $P^-=P^+=0.5$, $\rho^+=2\lambda_\eta\rho^-$, we obtained:

$$\zeta^*_{\mathrm{PCI}} = 1-\frac{\left(\lambda_\eta-\rho^-\right)}{2d\lambda_\eta}(b+d). \tag{13}$$

Since $d>c$, in the case of the policy of constant intensity production, the value of the ζ_{PCI} indicator of effectiveness ζ is lower than its value ζ_{PFO} at the policy of execution of incoming orders. This is caused by the losses due to lost profit that accompany the constant intensity of production.

4. 4. Methods of research

The features of this study are determined by the methods used to conduct it. The general approach to the study of controlled random processes is based on the method for assessing the effectiveness of decision-making policies using the indicator of maximum efficiency ζ^*, determined from the formula (6). This indicator expresses the ratio of mathematical expectation of the operation effect to the maximum possible effect in the planning period. Effectiveness indicators for a limited number of periods, used in many cited works, reflect the peculiarities of the realization of demand in these periods. In comparison with them, the indicator of maximum effectiveness has an undoubted advantage, since it determines the estimate of effectiveness objectively, regardless of the selected periods. In addition, the effectiveness evaluation based on the ζ^* indicator does not depend on the value of mathematical expectation of demand. That is why the ζ^* indicator makes it possible to assess the quality of planned solutions regardless of the intensity of demand. Due to losses arising during the planning periods due to the deviation of the volume of orders from the value of their mathematical expectation λ_η the value of the ζ^* indicator does not exceed 1.

Since the random process determined by the policy with the reservation of finished products does not have the Markov property, its research required the development of special research methods. During the operational activity of an enterprise with the reservation of products, there emerge reservation chains, consisting of successive periods of operative planning, at which a reserve of products is created and maintained due to an insufficient volume of incoming orders. The study of the corresponding random activity process is based on the method of limiting the duration of the "deposition" of the product reserve and on the balancing method to determine the expected intensities of the occurrence of chains.

In accordance with the method for limiting the product "deposition" duration, its initial reserve should be selected so that the reservation chain should be completed in a given number of periods with a probability close to unity. At the same time, the larger the number of periods allowed for reservation, the greater the maximum magnitude of the safe reserve. The method of limiting the "deposition" duration makes it possible to detail the reservation policy depending on the estimated maximum number of reservation periods. The application of the balancing method is caused by the need to determine the expected economic effect on the aggregates of the chains of reservation of separate product kinds and, as a result, to assess the effectiveness of the reservation policy in general.

5. Results of studying the policy of operation activity with product reservation

5. 1. Analysis of the process of operation activity with product reservation

In accordance with the policy of reservation of finished products for some planning periods $t=k+1$ with expected downtime $\{k+1\}\in T_m^-$, the production volume is set at a rate of u_{k+1}, which may exceed volume x_{k+1} of incoming orders:

$$u_{k+1}=\min\left\{x_{k+1}+\delta,\lambda_\eta\right\}, \tag{14}$$

where δ is the maximum magnitude of the reserve of finished products specified by the production policy with reservation. The choice of production volumes in accordance with (14) makes it possible to exclude the possibility of losses associated with the overload of production capacities during the $k+1$ period.

Thus, by the beginning of the transition to the beginning of planning period $k+2$, the reserve of finished products is created in the volume $z_{k+1}=u_{k+1}x_{k+1}\geq 0$. If in the period $k+2$, it turns out that $x_{k+2}z_{k+1}\geq 0$, the production volume in this period is established in volume $u_{k+2}=x_{k+2}z_{k+1}$. If $x_{k+2}z_{k+1}<0$, it is accepted that $u_{k+2}=0$ and the process of "zeroing" of production volumes will continue in the following intervals $t\in T_m^-$ the period $k+r$ that

$$\sum_{t=k+1}^{k+r-1} x_t < z_{k+1} \leq \sum_{t=k+1}^{k+r} x_t. \tag{15}$$

In the research it is proposed to interpret the sequence $k+1, k+2, ..., k+r$ of the planning periods as a complete reservation chain and sequence $k+1, k+2, ..., k+p$, $p<r$ as an incomplete chain. The number of r periods covered by the complete reservation chain is proposed to be called the chain length. The sequence $k+1, k+2, ..., k+r$ will be a complete reservation chain of the first type if $\{k+r\}\in T_m^-$, and a complete reservation chain of the second type if $\{k+r\}\in T_m^+$. For the convenience of presentation, planning periods $t\in T_m^+$, not included in reservation chains are called 0 type chains.

Indicators of operation activity in the periods of the reservation chain with length r take the following values:

$$u_{k+p}=0\left(p=2,3,...,r-1\right),$$
$$u_{k+r}=x_{k+r}-\left(z_{k+1}-\sum_{t=k+2}^{k+r-1}x_t\right); \tag{16}$$

$$z_{k+p}=z_{k+1}-\sum_{t=k+2}^{k+p}x_t\left(p=2,3,...,r-1\right),\quad z_{k+r}=0; \tag{17}$$

$$a_{k+p}=az_{k+p}\left(p=1,2,...,r\right); \tag{18}$$

$$b_{k+1}=b\left(\lambda_\eta-u_{k+1}\right),\quad b_{k+p}=b\lambda_\eta\left(p=2,...,r-1\right),$$
$$b_{k+r}=b\max\left\{\lambda_\eta-u_{k+r},0\right\}; \tag{19}$$

$$c_{k+p}=0\left(p=1,2,...,r-1\right),\quad c_{k+r}=c\max\left\{u_{k+r}-\lambda_\eta,0\right\}, \tag{20}$$

where a_t, b_t, c_t are the losses associated, respectively, with the storage of the product stock, downtime, and over normative load of capacities in the t-th planning period.

As a result of comparing losses on two-period chains under the reservation policy, on the one hand, and in the same two periods for the policy of order fulfillment, on the other hand, the following conclusions were made. In the type-1 chain periods, the magnitudes of downtime losses for both policies are the same. When using the policy with reservation on type-2 chains, both losses from downtime on $k+1$ periods and losses from an excess load of capacities on $k+2$ periods are reduced. However, the application of the policy with reservation results in losses associated with the lack of sales of finished products in the amount of z_{k+1}. At the same time, losses due to the reservation of finished products increase with an increase in the lengths of reservation chains.

5. 2. Finding the maximum safe value of the product stock

To assess the effectiveness of the policy of production with reservation, the choice of magnitude δ of reserve of finished products should provide an opportunity to foresee the maximum length of the reservation chain. Since the laws of probability distribution η are assumed to be symmetric, at any magnitude $z_{k+1}\leq\delta<\lambda_\eta$, the probability that a chain of length r will be completed is not less than magnitude $1-(0.5)^r$. Theoretically, chains of "almost infinite" length can emerge. However, the probability of their occurrence will be extremely small to take it into consideration in practice. If you specify the minimum probability Q^*, which should be taken into consideration, the maximum length $r=r(Q^*)$ of the chain, the occurrence of which must be considered, will be determined.

Probability Q_p that under conditions of an incomplete reservation chain in the periods $k+1, k+2, ..., k+p-1$, $p\geq 2$, it will remain incomplete in period $k+p$ is determined from the following formula:

$$Q_p=P\left\{\sum_{t=k+2}^{k+p}\eta_t<\delta\right\}=P\left\{\chi\left(p-1\right)<\delta\right\}=F_{p-1}\left(\delta\right), \tag{21}$$

where $\chi(p1)$ is the random magnitude of the sum of demand intensity η_t for $p1$ periods $k+2, k+3, ..., k+p$, $\chi\left(p-1\right)=\sum_{t=k+2}^{k+p}\eta_t$, $F_{p-1}\left(x\right)$ is the function of distribution of random magnitude $\chi(p-1)$, $F_1(x)=F_\eta(x)$.

However, so that a reservation chain starting in the $k+1$ period could finish in the $k+p$, $p\geq 3$ periods, it must not end in earlier periods. The probability of having an incomplete reservation chain in the periods $k+1, k+2, ..., k+p-1$, and its continuation after the period $k+p$, is determined by magnitude $U_{p-1}Q_p$, where U_{p-1} is the probability of occurrence of an incomplete reservation chain in periods $k+1, k+2, ..., k+p-1$,

$$U_{p-1}=P\left\{\eta_{k+1}\leq\lambda_\eta\right\}*Q_2*Q_3*...*Q_{p-1}=$$
$$=0.5*F_1\left(\delta\right)*F_2\left(\delta\right)*...*F_{p-2}\left(\delta\right). \tag{22}$$

The reservation chain that ends in the $k+p$ period is a 1-type chain with a probability

$$R_{p1}=U_{p-1}*P\left\{\delta-\sum_{t=k+1}^{k+p-1}x_t\le\eta_{k+p}\le\lambda_\eta\right\}=$$
$$=U_{p-1}\left(F_{p-1}(\lambda_\eta)-F_{p-1}(\delta)\right)=$$
$$=U_{p-1}\left(0.5-Q_p\right). \quad (23)$$

The reservation chain that ends in the $k+p$ period is a 2-type chain with a probability

$$R_{p2}=U_{p-1}*P\left\{\eta_{k+p}>\lambda_\eta\right\}=0.5U_{p-1}. \quad (24)$$

If an enterprise expects that the chain length will not exceed r periods, magnitude δ should meet the requirement: $\delta\le\delta^*_{r-1}$, where δ^*_{r-1} is the maximum safe magnitude of the stock of the product created in period $k+1$ from the calculation of its complete sale in $k+r$. According to formulas (22), (24), magnitude δ^*_{r-1} is found as a solution to the equation:

$$0.5*F_1(\delta)*F_2(\delta)*...*F_{r-2}(\delta)*F_{r-1}(\delta)=Q^*, \quad (25)$$

where Q^* is the boundary probability that in the r-th period of the reservation chain, the stock of product will not be sold completely. Obviously, boundary probability Q^* must be a negligibly small magnitude. If the magnitude δ^*_{r-1} is established, magnitudes $Q^*_p=F_{p-1}\left(\delta^*_{r-1}\right)$ determine the probability that the reservation chain that consists of p periods, p=2, 3,..., r, will continue in (p+1)-th period.

Since the policy of operation activity with reservation involves the choice of the maximum length of the reservation chain, for clarification in the paper, it is proposed to use the term "policy with r-period reservation", during which there can emerge reservation chains covering p planning periods, p=1, 2,..., r.

Following (25) the maximum safe magnitude δ^*_1 of the product stock for two-period reservation is determined from conditions: $0{,}5F_1(\delta)=0{,}5F_\eta(\delta)=Q^*$. Hence, it follows that

$$\delta^*_1=F^-_\eta\left(2Q^*\right), \quad (26)$$

where $F^-_\eta(...)$ is the function that is inverse to function $F_\eta(...)$. A reservation chain will be the 1-type chain with probability $0.5\left(0.5-F_\eta\left(\delta^*_1\right)\right)=0.5\left(0.5-2Q^*\right)\approx0.25$ and the 2-type chain with probability 0.5*0.5=0.25.

The maximum safe magnitude δ^*_2 of product stock for three-period reservation is determined from the condition: $0.5F_1(\delta)*F_2(\delta)=Q^*$. Assume that P_{ip} is the probabilities of emerging in arbitrary period t of operational activities of the i-th type reservation chains, i=1, 2, with lengths p=2, 3, and P_p is the probabilities of emerging of reservation chains of both types with lengths p=2, 3 Then, in accordance with (22) to (24):

$$P_{12}=0.5\left(0.5-Q^*_2\right);\ \ P_{22}=0.25;$$

$$P_2=P_{12}+P_{22}=0.5\left(1-Q^*_2\right); \quad (27)$$

$$P_{13}=0.5Q^*_2\left(0.5-Q^*_3\right)\approx0.25Q^*_2;\ \ P_{23}=0.25Q^*_2;$$

$$P_3=P_{13}+P_{23}=0.5Q^*_2, \quad (28)$$

where $Q^*_2=F_\eta\left(\delta^*_2\right)$, $Q^*_3=F_2\left(\delta^*_2\right)$, $0.5Q^*_2Q^*_3=Q^*\approx0$.

As you can see, $P_2+P_3=0.5$. Therefore, on condition that $t\in T^-_m$, the sum of probabilities of emerging of all 1-type and 2-type chains in this period makes up unity: $2(P_2+P_3)=1$.

The policy with product reservation, unlike other policies, involves the use of not only information about the volume of incoming orders, but also the forecasts of demand, which are reflected in the functions $F_p(x)$, p=1, 2,.... The information basis for finding these functions is the probability density of magnitude η of demand intensity, which can be determined from retrospective information on the volume of incoming orders.

Obviously, the problem of finding functions $F_p(x)$, p=1, 2,... boils down to finding the density of probability of random magnitude $\chi(p)=\sum_{t=1}^{p}\eta_t$ by the assigned density of magnitude probability η of demand intensity. The idea of the algorithm of calculation of the probability of the sum of some discrete magnitudes, assigned by their probability densities, is the following. Each summand η_t of magnitude $\chi(p)$ is considered as source t of receiving orders with volumes $\bar{h}_{rt}$ and probabilities p_{rt}, r=1, 2,..., R, where R is the number of values that can be taken by discrete random magnitude η of demand intensity. Pair $s_t=\left(\bar{h}_{rt},p_{rt}\right)$ determines the state of the t-th source, and vector $s=(s_t,\ t=1, 2,..., p)$ is the state of all p sources. The totality of all vectors s is matched by the set of mutually inclusive events within the time interval, covering p periods. In this case, the vector of state s uniquely determines the total volume of orders for time interval p and its probability, which are correlated with the value of $\chi(p)$ and its probability.

The adequacy of forecasting can be increased by clarifying the probability density η in each current planning period by using not only statistical but also expert estimates that take into consideration the specific features of formation of demand in the near future.

5. 3. Examples of numerical calculations of maximum safe values of product stock

To test the proposed approach, the paper considered numerical examples of finding values of δ^*_1. The assumption that random variable η has a normal distribution with assigned mathematical expectation λ_η and variance σ^2_η. was used. For calculations, we used table values of function $\Phi_0(t)=\Phi(t)-0.5$, where $\Phi(t)$ is the function of normalized and centered normal variance $\Phi(t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{t}e^{\frac{-u^2}{2}}\mathrm{d}u$.

The relevant tabular data are given in reference books. With these data, the value of function $F(x)$ of any normal variance with mathematical expectation λ and variance σ^2 is found for $x\le\lambda$ in the following way: $F(x)=0.5-\Phi_0\left(t=\frac{\lambda-x}{\sigma}\right)$.

To find the values of the argument of function $F(x)$, at which it takes the assigned value $Q\le0.5$, the following formula was used:

$$x=\lambda-\sigma\Phi^{-1}_0\left(0.5-Q\right), \quad (29)$$

where $\Phi^{-1}_0(...)$ is the function that is inverse to function $\Phi_0(t)$. Indeed, magnitude $t_y=\Phi^{-1}_0\left(0.5-Q\right)$ matches such magnitude $y=\lambda+\sigma t_y$, that $F(y)=0.5+\Phi_0(t_y)=1-Q$. From the symmetry property of function $F(x)$ it follows that:

$$F(y=\lambda+z)-F(\lambda)=0.5-Q=F(\lambda)-F(\lambda-z), \quad (30)$$

where $z=y-\lambda=\sigma t_y$. That is $F(\lambda-z)=Q$, $x=\lambda z=\lambda-\sigma t_y$.

We can suppose with high accuracy that function $\Phi(t)$ takes positive values only in the interval $[t^{\min}, t^{\max}]$, where

t^{min}=5, t^{max}=5. That is why it is possible to consider that arbitrary function $F(x)$ of normal variance takes positive values in the interval [x^{min}, x^{max}], где $x^{min}=\lambda+\sigma t^{min}$, $x^{max}=\lambda+\sigma t^{max}$.

For considered functions $F_\eta(x)$, for which $x^{min}=0$, it is true that $\lambda_\eta=\sigma_\eta t^{\max}=5\sigma_\eta$, $x^{\max}=2\sigma_\eta t^{\max}=10\sigma_\eta$. From (29) it follows that

$$\delta_1^*=\lambda_\eta-\sigma_\eta t^*=\sigma_\eta\left(t^{\max}-t^*\right)=\sigma_\eta\left(5-t^*\right),$$

where $t^*=\Phi_0^{-1}(0.5-Q)$, and according to formula (26) $Q=2Q^*$. Thus, the value δ_1^* is completely determined by magnitudes σ_η, Q^*. As it can be seen, at a decrease in root mean square deviation σ_η to 0, magnitude δ_1^* decreases to 0. At a decrease of boundary probability Q^* to 0, magnitude t^* increases up to t^{max}=5, and value δ_1^* decreases to 0. Magnitude υ of relative maximum product reserve depends only on boundary probability Q^*:

$$\upsilon=\frac{\delta_1^*}{\lambda_\eta}=1-\frac{t^*}{t^{\max}}=1-0.2t^*. \tag{31}$$

Table 1 gives absolute δ_1^* and relative υ values of maximum product stock for two-period reservation depending on the values of boundary probability Q^* and root mean square deviation σ_η.

Table 1

Values of product stock for a two-period reservation chain and the normal law of distribution η

$Q=2Q^*$	t^*	σ_η	λ_η	δ_1^*	υ
0.005	2.58	1	5	2.42	0.484
0.01	2.33	1	5	2.67	0.534
0.05	1.65	1	5	3.35	0.670
0.005	2.58	2	10	4.84	0.484
0.01	2.33	2	10	5.34	0.534
0.05	1.65	2	10	6.70	0.670
0.005	2.58	5	25	12.10	0.484
0.01	2.33	5	25	13.35	0.534
0.05	1.65	5	25	16.75	0.670

Above Table 1 shows that the creation of product stock δ_1^* demands an additional load of production capacity in the volume that is close to half its normative magnitude $u_0=\lambda_\eta$. With high probability, production volumes $u_{k+1}=x_{k+1}+\delta_1^*$ will be close to the normative capacity magnitude or even exceed it. That is why under the normal law of distribution η there is no need to increase δ_1^*.

5. 4. Estimation of intensities of occurrence of chains of periods

Assume that N is the number of planning periods in which the process of operation activity is considered. Since any chain of periods is uniquely determined by its initial period k+1 at the end of period T it is possible to determine sets U_{ip}^m of the chains of the similar type i, i=0, 1, 2, and of the same length p (p=1 for all 0-type chains, p=2, 3,..., r for 1-type and 2-type chains). These sets uniquely determine the number of K_{ip} chains of different types and different durations, as well as of intensity $k_{ip}=\frac{1}{T}K_{ip}$ of the occurrence of chains, which are the numbers of chains of various types that fall on one planning period.

Thus, in the process of operation activity with product reservation, chains of periods of a certain type and duration are formed. In this regard, the process of operational activity can be considered as the process of forming a complex of various chains. Information about the course of this process during T periods contain sets U_{ip}^m. The current state of the process at the end of current period T will display a vector composed of the magnitudes of intensities k_{ip} of occurrence of chains.

Imagine an experiment in which, in time interval with the duration of T periods, values x_t, t=1, 2,..., T, corresponding to distribution law η were generated, and decisions on production volumes were made in accordance with the reservation policy. Suppose that experiment duration T is long enough for statistical characteristics of random variable η to be manifested, in particular, condition $T^-\approx T^+\approx 0.5T$ to be met. Then, in the case of a repeated experiment with the same duration T, it will turn out that the composition of sets U_{ip}^m will change noticeably, however, there will be insignificant changes in quantities of K_{ip} of various chains and intensities k_{ip} of their occurrence.

It is proposed to call the values of intensities k_{ip} at $T\to\infty$ the maximum values of intensities of chains' occurrence (i, p). They represent the mathematical expectation of mean values of the number of chains in the time interval covering T planning periods. It is clear that during initial periods of operation activity, intensities k_{ip} vary significantly. At an increase in T, the deviations of current intensities from the limit values decrease, and at large values of T, the state of the chain formation process gets stabilized in accordance with the limit values of intensities.

To assess the effectiveness of reservation policies, it becomes necessary to assess the limit intensities of the emergence of chains of various types and to statistically assess the economic effect obtained on the totality of chains of each type. To find the maximum intensities, the research used the balancing method, which assumes matching the total number of periods $t\in T_m^-$ with the number of periods $t\in T_m^+$, required for the formation of various chains.

The application of this method for the two-period reservation was considered. Since for the final period t of the two-period chain, the probabilities that $t\in T_m^-$ and $t\in T_m^+$ are the same is 0.5, the number K_{12} of 1-type reservation chains is the same as the number K_{22} of 2-type reservation chains. In this case, $K_{12}+K_{22}=G$, where G is the number of all initial periods $t\in T_m^-$ in reservation chains. That is why the total number of periods $t\in T_m^-$ in reservation chains is magnitude $K_{12}+G=1.5G$ and this magnitude must coincide with half the number of periods T. Therefore, $1.5G=0.5T$, $G=T/3$, $K_{12}=K_{22}=T/6$. The number of 0-type chains (periods $t\in T_m^+$, not included in reservation chains), is determined by magnitude $K_{01}=\left(\frac{1}{2}-\frac{1}{6}\right)T=\frac{T}{3}$. Then limit intensities of occurrence of chains are magnitudes: $k_{01}=1/3$, $k_{12}=k_{22}=1/6$. If, for example, we assume that planning interval T includes 12 periods, this interval will contain 2 two-period 1-type chains, 2 two-period 2-type chains, and 4 single-period chains with periods $t\in T_m^+$.

5. 5. Assessment of operation activity with product reservation

To solve the problem of assessing the effectiveness of operation activity with product reservation, the following designations were introduced: H_i is the average effect on the i-th type chain i=0, 1, 2 in T planning periods; E_i is the mathematic expectation of effect on the i-th type chain, i=0, 1, 2.

It is proposed to calculate indicator ζ_{PPR} of the effectiveness of operation activity in T planning periods in the form of three constituent parts:

$$\zeta_{PPR} = S_0 + S_1 + S_2, \tag{32}$$

where S_0 is the constituent of effectiveness indicator, formed by planning periods $t \in T_m^+$, not included in reservation chains; S_1 is the constituent of effectiveness indicator formed by 1-type reservation chains; S_2 is the constituent of effectiveness indicator formed by 2-type reservation chains. In this case

$$S_0 = \frac{k_{01}}{d\lambda_\eta} E_0, \quad S_1 = \frac{k_{12}}{d\lambda_\eta} E_1, \quad S_2 = \frac{k_{22}}{d\lambda_\eta} E_2. \tag{33}$$

Then magnitudes H_i, E_i (i=0, 1, 2) were found. To simplify the calculations, it was accepted that reserve magnitude δ, chosen by an enterprise, does not exceed.

$\lambda_\eta - \rho^-$: $\delta = \min\{\delta_1^*, \lambda_\eta - \rho^-\}$, where ρ^- is the mathematic expectation of random magnitude η provided it falls in the interval $[0, \lambda_\eta]$.

The effect on the 0-type chain that is average for N planning periods is determined by the magnitude

$$H_0 = \frac{1}{K_{01}} \sum_{t \in U_{01}^m} \left(dx_t - c\left(x_t - \lambda_\eta\right)\right) = \frac{1}{K_{01}} \left((d-c) \sum_{t \in U_{01}^m} x_t + c\lambda_\eta U_0 \right) = (d-c) I + c\lambda_\eta, \tag{34}$$

where $I = \frac{1}{K_{01}} \sum_{t \in U_{01}^m} x_t$ is the mean value U_0 of implementations $x_t \in T_m^+$ of random magnitude η.

At an infinitely large value T, magnitude I meets mathematical expectation ρ^+ of random magnitude η provided it falls in the interval $[\lambda_\eta, x^{max}]$. Then

$$E_0 = (d-c)\rho^+ + c\lambda_\eta,$$
$$S_0 = \frac{k_{01}}{d\lambda_\eta} E_0 = \frac{1}{3d\lambda_\eta} \left((d-c)\rho^+ + c\lambda_\eta \right). \tag{35}$$

It was accepted to designate the volumes of orders in each reservation chain t as x_{k+1}^t, x_{k+2}^t. Since the effect on the 1-type reservation chain is determined by the magnitude

$$H_{1t} = d\left(x_{k+1}^t + x_{k+2}^t\right) - b\left(2\lambda_\eta - \left(x_{k+1}^t + x_{k+2}^t\right)\right) - a\delta, \tag{36}$$

the average effect on the 1-type chain is expressed as follows:

$$H_1 = \frac{1}{K_{12}} \sum_{t \in U_{12}^m} H_{1t} = \frac{1}{K_{12}} \sum_{t \in U_{12}^m} \left((d+b)\left(x_{k+1}^t + x_{k+2}^t\right) - 2b\lambda_\eta - a\delta_1^* \right) = (d+b)(I_1 + I_2) - 2b\lambda_\eta - a\delta_1^*, \tag{37}$$

where I_1 is the mean value of K_{12} implementations x_{k+1}^t, $\{k+1\} \in T_m^-$, of magnitude η,

$$I_1 = \frac{1}{K_{12}} \sum_{k \in U_{12}^m} x_{k+1}^t; \tag{38}$$

I_2 is the mean value of K_{12} implementations x_{k+2}^t, $\{k+2\} \in T_m^-$, of magnitude η,

$$I_2 = \frac{1}{K_{12}} \sum_{k \in U_{12}^m} x_{k+2}^t. \tag{39}$$

At an infinitely large value of T, it turns out that $I_1 = I_2 = \rho^-$. Therefore

$$E_1 = 2(d+b)\rho^- - 2b\lambda_\eta - a\delta_1^*, \tag{40}$$

$$S_1 = \frac{k_{12}}{d\lambda_\eta} E_1 = \frac{1}{3d\lambda_\eta} \left(d\rho^- - 0.5a\delta_1^* - b\left(\lambda_\eta - \rho^-\right) \right). \tag{41}$$

Given that $\rho^- + \rho^+ = 2\lambda_\eta$, $\rho^+ = 2\lambda_\eta - \rho^-$, it was obtained:

$$S_0 + S_1 = 2(d-c)\rho^+ + 2c\lambda_\eta + 2d\rho^- - 2b\left(\lambda_\eta - \rho^-\right) - a\delta_1^* = 4d\lambda_\eta + \left(-4\lambda_\eta - 2\rho^- + 2\lambda_\eta\right)c - 2b\left(\lambda_\eta - \rho^-\right) - a\delta_1^* = 4d\lambda_\eta - 2\left(\lambda_\eta - \rho^-\right)c - 2b\left(\lambda_\eta - \rho^-\right) - a\delta_1^*. \tag{42}$$

The effect on the type-2 reservation chain t is expressed by the following magnitude:

$$H_{2t} = d\left(x_{k+1}^t + x_{k+2}^t\right) - b\left(\lambda_\eta - x_{k+1}^t\right) - c\left(x_{k+2}^t - \lambda_\eta\right) + \delta_1^*(b+c-a). \tag{43}$$

Then

$$H_2 = \frac{1}{K_{22}} \sum_{t \in U_{22}^m} H_{2t} = (d+b)I_3 + (d+c)I_4 + \delta_1^*(b+c-a) - (b-c)\lambda_\eta, \tag{44}$$

where I_3 is the mean value of K_{22} implementations x_{k+1}^t, $\{k+1\} \in T_m^+$, of magnitude η,

$$I_3 = \frac{1}{K_{22}} \sum_{k \in U_{22}^m} x_{k+1}^t; \tag{45}$$

I_4 is the mean value of K_{22} implementations x_{k+2}^t, $\{k+2\} \in T_m^+$, of magnitude η,

$$I_4 = \frac{1}{K_{22}} \sum_{k \in U_{22}^m} x_{k+2}^t. \tag{46}$$

At an infinitely large value T, magnitudes I_3, I_4 meet mathematic expectations ρ^-, ρ^+ of the values of random magnitude η respectively in intervals $[0, \lambda_\eta]$, $[\lambda_\eta, x^{max}]$. Therefore

$$E_2 = d\rho^- - b\left(\lambda_\eta - \rho^- - \delta_1^*\right) + d\rho^+ - c\left(\rho^+ - \lambda_\eta - \delta_1^*\right) - a\delta_1^*. \tag{47}$$

Since $\rho^- + \rho^+ = 2\lambda_\eta$, $\rho^+ = 2\lambda_\eta - \rho^-$,

$$S_2 = \frac{k_{22}}{d\lambda_\eta} E_2 = \frac{1}{6d\lambda_\eta} \begin{pmatrix} 2d\lambda_\eta - b\left(\lambda_\eta - \rho^-\right) - \\ -c\left(\rho^+ - \lambda_\eta\right) + \delta_1^*(b+c-a) \end{pmatrix} = \frac{1}{6d\lambda_\eta} \begin{pmatrix} 2d\lambda_\eta - b\left(\lambda_\eta - \rho^-\right) - \\ -c\left(\lambda_\eta - \rho^-\right) + \delta_1^*(b+c-a) \end{pmatrix}. \tag{48}$$

As a result, an indicator ζ^*_{PPR} of limit effectiveness of operation activity for the policy with reserves of finished products was expressed by the following formulas:

$$\zeta^*_{\text{PPR}} = \frac{1}{6d\lambda_\eta}\begin{pmatrix} 6d\lambda_\eta - 3(\lambda_\eta - \rho^-)c - \\ -3b(\lambda_\eta - \rho^-) + \delta_1^*(b+c-2a) \end{pmatrix} =$$

$$= 1 - \frac{\lambda_\eta - \rho^-}{2d\lambda_\eta}(b+c) + \frac{1}{6d\lambda_\eta}\delta_1^*(b+c-2a). \tag{49}$$

From the comparison of the value of the indicator

$$\zeta^*_{\text{PFO}} = 1 - \frac{\lambda_\eta - \rho^-}{2d\lambda_\eta}(b+c), \tag{50}$$

of limit effectiveness for the policy of fulfillment of incoming orders with the obtained value ζ^*_{PPR} of this indicator for the policy with product reservation, the following conclusions were made. The values of the constituent parts corresponding to the profit from the sale of products are the same, equal to 1. The magnitudes of losses caused by downtime and over normative capacity load under the policy with reservation of finished products are less than under the policy of fulfilling incoming orders, by a magnitude

$$e^+ = \frac{\delta_1^*}{6d\lambda_\eta}(b+c). \tag{51}$$

However, when using the reservation policy, there occur additional losses due to the lack of sale of part of the finished product. A reservation policy will be more beneficial to an enterprise if $a<0.5(b+c)$. When this condition is met, magnitude e^+ of a loss reduction increases in proportion to the magnitude δ_1^* of the maximum allowable reserve of finished products $\delta_1^* = F_\eta^-(Q^*)$. In turn, magnitude δ_1^* increases at an increase in dispersion of the law of distribution F_η of magnitude η of the volume of orders and an increase in probability Q^* of occurrence of the chains with the length of more than two periods.

The effectiveness of the finished product reservation policy can be improved by forecasting the volume of orders in periods $k+2$ of reservation chains. This requires finding distribution function $F^*(\eta_{k+2})$, reflecting the peculiarities of the formation of demand for this period, before the beginning of each period $k+2$.

6. Discussion of results of studying the policy of operation activity with product reservation

The policy of operation activity of an enterprise with the creation of reserves of finished products is considered as a model of decision-making in a controlled random process. Policy effectiveness is estimated by the limit average economic effect per unit of time over an infinite number of periods.

Representation of operation activity in the form of a controlled random process is different from the approach to operation activity based on the method of stochastic programming. This method is aimed at finding optimal values of production intensity in one or more adjacent planning periods. At the same time, the used concept makes it possible to assess the effectiveness of decision-making policies in general, regardless of the specific situation in the periods under consideration. At the same time, the choice of decisions based on a certain policy is in practice more convenient.

The policy of product reservation is different from the policy of fulfilling incoming orders and the production policy with constant intensity. When it is applied, the value of E_t in planning period t depends not only on the random volume of orders in this period but also on the product reserves created in previous periods. That is why the random process E_t, $t=1, 2,..., T$, corresponding to the reservation policy, does not meet the Markov property. For this reason, there is a need to consider reservation chains, to search for intensities of their appearance and represent the non-Markov process E_t, $t=1, 2,..., T$, in the form of the sum of Markov processes of effects on reservation chains of separate types.

It was shown that during the operational activity of an enterprise with product reservation, there can occur reservation chains consisting of successive periods of operative planning, in which a product reserve is created and maintained due to an insufficient volume of incoming orders. As a result of analysis of the process of operation activity with product reservation, we obtained formulas (21) to (24), expressing the probabilities of occurrence of various reservation chains in accordance with the law of distribution of demand probability.

The proposed method for selecting the magnitude of the initial stock of products on the reservation chain is that the boundary probability of continuation of this chain was an assigned magnitude close to zero. This condition is reflected in formula (25). It implicitly determines the maximum value of the product stock, at which the duration of the reservation chain is guaranteed not to exceed the specified number of periods. In the case of a two-period reservation, such a maximum safe magnitude of the reserve is found from formula (26) as the value of the function inverse of the original function of the demand probability distribution. Finding the maximum magnitude of a safe reserve for more than two reservation periods requires information on the laws of distribution of the probability of the sum of several random magnitudes of demand. That is why a general scheme of the algorithm for calculating the density of probability of the sum of several random magnitudes assigned by their probability densities was presented.

For two-period reservation, in the case of normal distribution of demand probability, numerical calculations of the maximum safe magnitude of product stock were performed. We considered the functions of the normal distribution with probability densities taking "practically" non-zero values only in the interval $[0, 2\lambda_\eta]$, where λ_η is the mathematical expectation of the demand magnitude. It was shown that in this case, root mean square deviation σ_η is related to λ_η by a directly proportional relationship.

For the convenience of analyzing the results, we introduced the magnitude of the relative maximum stock of products, which is the ratio of the maximum (absolute) stock of products to mathematical expectation of the demand magnitude, which by definition coincides with the normative capacity of an enterprise. Table 1 shows absolute δ_1^* and relative υ values of the maximum product stock, depending on the values of boundary probability Q^* and root mean square deviation σ_η. Table 1 shows that magnitude υ

increases at an increase in σ_η and at values from 1 to 5, magnitude υ accepts values close to 0.5. Thus, it was shown that up to half its standard production capacity can be used to create a safe stock of products δ_1^*, and this will allow providing an enterprise with an acceptable uniformity of production.

To assess the effectiveness of reservation policy, it becomes necessary to find the expected intensities of the emergence of chains of various kinds. To solve this problem, we proposed the balancing method, which assumes the coincidence of the total number of periods with the number of periods required to form the chains with different probabilities of their occurrence. Subsection 5.4 shows the formulas that determine the intensities of occurrence of chains of various kinds in the case of two-period reservation.

An estimate of the effectiveness of the two-period reservation policy, which is pursued in the case of the normal law of distribution of the demand magnitude, was obtained. The expression of this estimate depending on the values of cost indicators in the model of operative planning is described by formula (49). Its comparison with formula (50), which determines the estimate of the effectiveness of the policy of execution of incoming orders, makes it possible to assert that the policy with reservation is more profitable if $a<0.5(b+c)$, where a, b, c are the magnitudes of losses per unit of production, due, respectively, to the storage of the stock of finished products, downtime, and excess capacity load. The above condition quite clearly determines the scope of the possible application of the reservation policy.

Subsequent research should be focused on assessing the effectiveness of operation activity with reservation for the laws of demand magnitude distribution that differ from normal.

7. Conclusions

1. The policy with the reservation of finished products makes it possible to meet increased demand by creating stocks of finished products produced during periods of reduced demand. During the operational activity of an enterprise, there can emerge reservation chains, consisting of successive periods of operative planning, in which a reserve of products is created and maintained due to an insufficient volume of incoming orders. Formulas that make it possible to find the probabilities of occurrence of reserve chains that correspond to the laws of demand probability distribution were obtained.

2. In order to limit the negative effect of “depositing” of finished products, it was proposed to limit the number of periods covered by reservation chains. That is why the proposed method for selecting the initial stock on a reservation chain implies that the boundary probability of the continuation of this chain is an assigned magnitude close to zero. Formulas that make it possible to find the maximum safe size of the product stock from the condition of its full sale during a given number of planning periods were obtained.

3. It was shown that in the case when the law of demand magnitude distribution is normal, sufficiently large values of the safe reserve correspond to the two-period reservation, ensuring acceptable uniformity of production.

4. To find the expected intensities of the emergence of chains of various types, the balancing method was proposed. It is assumed that the total number of periods coincides with the number of periods required to form chains with different probabilities of their occurrence. Formulas determining the intensities of occurrence of chains of various types in the case of two-period reservation were obtained.

5. The effectiveness of operation activity with the reservation of finished products was evaluated. The formula expressing the value of the indicator of limit effectiveness depending on the values of cost indicators in the model of operative planning was obtained. A comparative analysis of the effectiveness of the reservation policy with the policies for the execution of incoming orders and production with constant intensity was performed. The correlation of values of cost indicators, at which the policy with a reservation has higher effectiveness than other considered variants of policies of operation activity, was determined.

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