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# BUILDING A MODEL OF SUPPLY CHAINS DUOPOLY TAKING INTO ACCOUNT THE MARKETING AND INNOVATIVE ACTIVITIES OF MANUFACTURING ENTERPRISES

**Yuliia Kurudzhy**
*Corresponding author*
PhD*
E-mail: yulia.kurudzhi@ukr.net
**Iryna Mayorova**
Senior Lecturer*
**Iryna Moskvichenko**
PhD, Associate Professor*
*Department of Management and Marketing
Odessa National Maritime University
Mechnikova str., 34, Odessa, Ukraine, 65029

***This paper reports the construction and analysis of the economic and mathematical model of the duopoly of supply chains, based on the model of optimization of plans for the release and delivery of multi-range articles, taking into consideration the marketing and innovative activities of industrial enterprises. Demand for goods is supposed to be an increasing function of advertising costs. In this case, marketing investments affect only the base selling prices of articles and do not affect competitive discounts. The explicit form of this dependence can be established as a result of marketing research. It is also assumed that investments in innovative technological projects could reduce industrial costs; production costs are decreasing functions of the size of the investment. It is believed that the demand function is linearly dependent on the total volume of output produced. The criterion of optimality for supply chains is the maximum of the total profit received from the sale and delivery of finished products to points of consumption, taking into consideration the costs of production and advertising. As a result of this study, equilibrium solutions of the duopoly according to Cournot and Stackelberg were found. That has made it possible to determine the optimal values of product volumes for output, the size of investment investments, as well as product advertising costs. The model helped study the impact of investment deductions and advertising costs on the acquisition of competitive advantages by manufacturing enterprises. A numerical illustration of the results obtained is given. The proposed approach could be used to build and analyze dynamic optimization models taking into consideration the innovation and marketing activities of enterprises, as well as to study other market structures***

***Keywords: supply chain, duopoly, equilibrium solution, marketing activity, innovation activity, industrial enterprise, competitive environment***

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## 1. Introduction

Modern trends in the development of logistics theory and practice, which are expressed in the desire of market participants to extract the maximum benefit from the interaction of logistics, marketing, and innovation strategies of enterprises, pose new challenges for researchers. One of these tasks is the task of determining the agreed plans of production and transport enterprises – participants in supply chains. Finding optimal investment and marketing plans, plans for the supply of raw materials for production, production of finished products, and its delivery to end-users helps ensure the effective operation of both individual enterprises and supply chains as a whole. Equally important is the issue of competition. Supply chain participants are forced to look for new sources of competitive advantage and use the integration of marketing and supply chain management concepts to improve the efficiency of their activities. Therefore, research on the development and analysis of mathematical models of duopoly, taking into consideration the joint impact on the equilibrium solution of supply chains of the innovative policy of enterprises and their marketing activity, is relevant.

## 2. Literature review and problem statement

In the field of competition studies between enterprises, economic and mathematical methods and models have become quite widespread. For example, in work [1], linear models of competition in the supply chain are considered, and finding Pareto equilibrium is reduced to solving a linear system. Paper [2] simulates a two-stage supply chain and analyzes the impact of competitors' strategies on the performance of the entire chain. The duopoly of supply chains consisting of one producer and one seller is considered in work [3]. To study competition within a chain and between chains, a game-theoretic approach is used, the problem of two-level programming is solved, the Stackelberg-Nash equilibrium is analyzed. The disadvantage is that they are purely theoretical in nature and illustrate the general provisions of competition theory.

Study [4] examines the influence of interaction between buyers and suppliers of products in supply chains on the efficiency of new products on the example of the functioning of many international companies. The focus is on improving the interaction between links in supply chains. The authors of [5] propose a model of interaction between the processes of managing the stability of supply chains and the innovative activity of enterprises. However, it should be noted that in the cited works either an empirical or purely theoretical analysis of the problem of the influence of the innovative activity of the enterprise on the acquisition of competitive advantages is applied.

Paper [6] highlights the problems of implementation and synchronization of marketing and logistics solutions in supply chains. Work [7] simulates the activity of a supply company, which is a link in the supply chain, taking into consideration its marketing policy. Analytically, the optimal values of the size of purchase lots and advertising costs were found. However, the problem of practical implementation of theoretical research in the considered field of economic science has not yet been sufficiently developed.

Part of the question of the practical application of theoretical studies of oligopoly is revealed in work [8]. The results of the study of the duopoly of industrial enterprises with nonlinear cost functions, the participants of which are the manufacturer of original products and his follower, are given. In [9], the author considers a special case of competition with several leaders and followers with a nonlinear demand function, points to some applications of the model, for example, in the aircraft industry, which is an oligopoly. The leader-follower duopoly is explored in [10]. A solution algorithm is proposed for the optimal design of a supply chain in which the follower can make discrete decisions, for example, to choose technologies or open production lines. Work [11] contains many practical examples of German companies applying scientific knowledge in the logistics business.

However, in order to achieve practical goals, it is necessary to apply other approaches, based, for example, on methods of operations research. This makes it possible to build optimization models (maximizing profits or minimizing costs throughout the supply chain), taking into consideration a number of limitations. Constraints may be related, for example, to the availability of resources for production, the capacity of warehouses, the possibility of transport for the delivery of products. Thus, in work [12], optimization models were built and investigated, based on the combination of the classical model of the optimal production plan of the plant and the transport problem of linear programming. That approach made it possible to develop an economic and mathematical model close to the practice of planning work of enterprises for the analysis of the duopoly of the «industrial enterprise – distribution network» type. The influence of the competitive environment on the optimal distribution of material flows that move from product manufacturers to the place of their final consumption has been studied. In article [13], that approach was used to study the impact of the innovative activity of two enterprises – manufacturers of the same product range on the equilibrium solution of the duopoly. Work [14] reports the study of the influence of the marketing activity of an enterprise that produces multi-nomenclature products on the equilibrium solution of the duopoly of supply chains. The disadvantage of the cited works [13, 14] is that the demand in them was assumed to be either unlimited or random with given probability distributions. At the same time, in practice, the amount of demand for finished products is influenced not only by the marketing activity of enterprises but also by their innovation strategy.

But the issue of the simultaneous influence of the innovation and marketing activity on the competitive strategies of enterprises remains unresolved. This gives grounds to assert that it is advisable to conduct a study into the economic and mathematical models of competition based on the synthesis of the innovation policy of enterprises, as well as marketing and logistics management concepts.

## 3. The aim and objectives of the study

The aim of this work is to build an economic and mathematical model of the duopoly of supply chains, taking into consideration the marketing and innovative activities of manufacturing enterprises. This will make it possible to determine the equilibrium solutions of industrial enterprises participating in competing supply chains, taking into consideration additional advertising costs and investments in production.

To accomplish the aim, the following tasks have been set:

– to choose the structure of the economic and mathematical model of the duopoly of supply chains and build a system of restrictions, taking into consideration the marketing and innovative activities of production enterprises – links of supply chains;

– to find equilibrium solutions to the duopoly according to Cournot and Stackelberg;

– to study the impact of the size of investment deductions and advertising costs on the acquisition of competitive advantages by manufacturers.

## 4. The study materials and methods

Underlying the construction of a model of the duopoly of supply chains, the model of optimization of the plan for the release of multi-nomenclature products by an enterprise and its delivery to consumers, proposed in [15], is considered. It is believed that two enterprises produce goods of $K$ names and use for this purpose resources of $R$ types. An enterprise with number $i$, $i=1,2$ for the production of one unit of goods of the $k$-th type ($k=1,2,...,K$) uses $a_{rk}^{(i)}$ units of a resource of type $r$, the stock of which in the warehouse is $b_r^{(i)}$, $r=1,2,...,R$. The manufactured products are delivered to $M$ points of consumption $D_1, D_2,...,D_M$. The demand $d_{km}$ for a product of type $k$ is known at the point of consumption $m$. The control parameters for the described problem are: $x_k^{(i)}$ is the number of products of the $k$-th type, planned by the $i$-th enterprise for release, $y_{km}^{(i)}$ is the number of finished products of the $k$-th type, planned for delivery to the destination $D_m$. We also assume that the delivery of finished products is executed at the expense of their buyer.

To account for the marketing activity of manufacturers, it is assumed that the prices of products $p_{km}$ are some increasing functions from the size of advertising costs $u_k^{(i)}$. The explicit form of the dependence can be determined as a result of, for example, marketing research. In particular, paper [14] states that in the simplest case it can be a linear dependence: $p_{km}\left(u_k^{(i)}\right)=p_{0km}\left(1+\alpha_k^{(i)}u_k^{(i)}\right)$. Here, the multiplier $p_{0km}$ is the size of the base price for products of the $k$-th type at destination $m$ without taking into consideration additional advertising costs, and the parameter $\alpha_k^{(i)}$ determines the degree of influence of these costs on the increase in demand.

As innovative projects for competing enterprises, we can consider, for example, the introduction of advanced transportation and transshipment technologies, the replacement of vehicles with newer ones. Innovation activity in modeling is accounted for by the assumption that the costs of producing a unit of output $s_k^{(i)}$ are some decreasing functions of the size of the investment $v_k^{(i)}$. In [13], it is noted that it is possible to take $s_k^{(i)}\left(v_k^{(i)}\right)=s_{0k}^{(i)}/\left(1+\gamma_k^{(i)}v_k^{(i)}\right)$, where $s_{0k}^{(i)}$ is the value of costs for obsolete technology, $\gamma_k^{(i)}$ is the coefficient that characterizes the degree of efficiency of innovation.

Equilibrium solutions of the duopoly are found for three options for the actions of competitors: according to Cournot and according to Stackelberg. The Cournot equilibrium assumes that manufacturing plants make decisions independently of each other. In the case of equilibrium according to Stackelberg, one of the enterprises acts as a leader, the second chooses the strategy of the follower and takes into consideration the actions of the leader.

To analyze the impact of the size of investment deductions and advertising costs, a numerical experiment was conducted on the acquisition of competitive advantages by producers. The values of the equilibrium solutions of the duopoly are found by solving a system of nonlinear equations. Microsoft Excel (USA) software package was used for calculations.

## 5. Results of studying the duopoly of supply chains, taking into consideration the innovative and marketing activities of manufacturers

### 5. 1. Selection of the structure for the economic-mathematical model of the duopoly, taking into consideration the innovative and marketing activities of producers

As limitations for the optimization model of production and delivery of products in relation to duopoly, consider the following:

$$\sum_{k=1}^{K} a_{kr}^{(i)} x_k^{(i)} \le b_r^{(i)},\ r=1,\dots,R,\ i=1,2,$$

$$y_{km}^{(1)}+y_{km}^{(2)} \le d_{km},\ k=1,\dots,K,\ m=1,\dots,M,$$

$$x_k^{(i)}=\sum_{m=1}^{M} y_{km}^{(i)},\ k=1,\dots,K,\ i=1,2.$$

$$x_k^{(1)}, x_k^{(2)}, y_{km}^{(1)}, y_{km}^{(2)} \ge 0\ \forall k, m.$$

They reflect the fact that in production the enterprise cannot use more resources than is available in stock. Also, all manufactured products must be exported to their destinations while the demand at each point should not be exceeded. Let us assume that the demand function for products of the $k$-th type at the $m$-th destination takes the following form:

$$p_{km}\left(y_{km}^{(1)}, y_{km}^{(2)}\right)=p_{km}-g_{km}\left(y_{km}^{(1)}+y_{km}^{(2)}\right),$$

where $p_{km}$ is the maximum possible price of products of the $k$-th type at the $m$-th point of consumption, $g_{km}$ is the parameter that determines the elasticity of demand. This type of demand function is chosen as a result of the assumption that the prices of the products of any of the competitors depend on the volume of products sold by all enterprises and decrease with an increase in the number of products entering the market. Obviously, the demand function cannot be negative, so the following conditions must be met: $y_{km}^{(1)}+y_{km}^{(2)} \le p_{km}/g_{km}$, $k=1,2,\dots,K$, $m=1,2,\dots,M$. Therefore, let us assume that the demand is $d_{km}=p_{km}/g_{km}$.

Under such conditions, the profit of production enterprises will equal:

$$\Pi^{(i)}=\sum_{k=1}^{K}\sum_{m=1}^{M}\left[\begin{pmatrix} p_{km}-g_{km}\times \\ \times\left(y_{km}^{(1)}+y_{km}^{(2)}\right)\end{pmatrix} y_{km}^{(i)}\right]-\sum_{k=1}^{K} s_{0k}^{(i)}\cdot x_k^{(i)},$$
$$i=1,2,$$

where $s_{0k}^{(i)}$ is the cost of producing a unit of output.

Consider the possibility of increasing the profits of enterprises due to additional marketing costs. Suppose that each enterprise, in order to increase sales of the $k$-th type goods, allocates cash in the amount of $u_k^{(i)}$. We also assume that marketing costs affect only the base selling prices of products and do not affect competitive discounts. This means that prices are some increasing function of $u_k^{(i)}$. Then the profit of the $i$-th enterprise takes the form:

$$\Pi^{(i)}=\sum_{k=1}^{K}\sum_{m=1}^{M}\left[\left(p_{km}\left(u_k^{(i)}\right)-g_{km}\left(y_{km}^{(1)}+y_{km}^{(2)}\right)\right)y_{km}^{(i)}\right]-$$
$$-\sum_{k=1}^{K} s_{0k}^{(i)}\cdot x_k^{(i)}-\sum_{k=1}^{K} u_k^{(i)},$$
$$i=1,2.$$

In work [13], the authors considered the possibility of implementing innovative projects at production enterprises in order to reduce production costs and increase profits. Applying the approach proposed in [13], we obtain the following expressions for the profit $\Pi^{(i)}$:

$$\Pi^{(i)}=\sum_{k=1}^{K}\sum_{m=1}^{M}\left[\left(p_{km}\left(u_k^{(i)}\right)-g_{km}\left(y_{km}^{(1)}+y_{km}^{(2)}\right)\right)y_{km}^{(i)}\right]-$$
$$-\sum_{k=1}^{K} s_k^{(i)}\left(v_k^{(i)}\right)\cdot x_k^{(i)}-\sum_{k=1}^{K} v_k^{(i)}-\sum_{k=1}^{K} u_k^{(i)},\ i=1,2, \qquad (1)$$

where $s_k^{(i)}\left(v_k^{(i)}\right)$ is the cost of producing a unit of products of the $k$-th type at the enterprise with number $i$ taking into consideration investments in the implementation of technological innovations $v_k^{(i)}$. Since

$$x_k^{(i)}=\sum_{m=1}^{M} y_{km}^{(i)},$$

we exclude the variables $x_k^{(1)}$, $x_k^{(2)}$ in expression (1) and derive the following values for the profits of manufacturing enterprises:

$$\Pi^{(1)}=\sum_{k=1}^{K}\sum_{m=1}^{M}\left[\left(p_{km}\left(u_k^{(1)}\right)-g_{km}\left(y_{km}^{(1)}+y_{km}^{(2)}\right)\right)y_{km}^{(1)}\right]-$$
$$-\sum_{k=1}^{K}\left(s_k^{(1)}\left(v_k^{(1)}\right)\cdot\sum_{m=1}^{M} y_{km}^{(1)}\right)-\sum_{k=1}^{K}\left(v_k^{(1)}+u_k^{(1)}\right), \qquad (2)$$

$$\Pi^{(2)}=\sum_{k=1}^{K}\sum_{m=1}^{M}\left[\left(p_{km}\left(u_k^{(2)}\right)-g_{km}\left(y_{km}^{(1)}+y_{km}^{(2)}\right)\right)y_{km}^{(2)}\right]-$$
$$-\sum_{k=1}^{K}\left(s_k^{(2)}\left(v_k^{(2)}\right)\cdot\sum_{m=1}^{M} y_{km}^{(2)}\right)-\sum_{k=1}^{K}\left(v_k^{(2)}+u_k^{(2)}\right). \qquad (3)$$

In this case, the restrictions for the two optimization problems are as follows:

$$\sum_{k=1}^{K} a_{kr}^{(i)}\sum_{m=1}^{M} y_{km}^{(i)} \le b_r^{(i)},\ r=1,\dots,R,$$

$$y_{km}^{(1)}+y_{km}^{(2)} \le p_{km}\left(u_k^{(i)}\right)/g_{km},\ k=1,\dots,K,\ m=1,\dots,M. \qquad (4)$$

Each manufacturing enterprise strives to maximize its profits on variables $y_{km}^{(i)}, v_k^{(i)}, u_k^{(i)} \geq 0,\; i=1,2,$ taking into consideration constraints (4). Note that there may be some dependence between variables $x_k^{(1)}, y_{km}^{(1)}, u_k^{(1)}$ and $x_k^{(2)}, y_{km}^{(2)}, u_k^{(2)}$ due to the presence of competition between plants but in the simplest case, such dependence can be neglected.

**5. 2. Determining equilibrium solutions of the duopoly according to Cournot and Stackelberg**

Our study of competition between supply chains involving manufacturing plants starts by determining the equilibrium solution of the Cournot duopoly.

It is assumed that enterprises make decisions on the volume of production independently of each other, considering the output of a competitor to be constant, and the function of market demand is known. To find the equilibrium solution of the duopoly in the Cournot sense, in which the profits of enterprises are maximum possible, we determine the necessary conditions for the extremum of profit functions (2), (3):

$$\frac{\partial \Pi^{(1)}}{\partial y_{km}^{(1)}} = p_{km}\left(u_k^{(1)}\right) - 2g_{km}y_{km}^{(1)} - g_{km}y_{km}^{(2)} - s_k^{(1)}\left(v_k^{(1)}\right) = 0,$$

$$\frac{\partial \Pi^{(2)}}{\partial y_{km}^{(2)}} = p_{km}\left(u_k^{(2)}\right) - g_{km}y_{km}^{(1)} - 2g_{km}y_{km}^{(2)} - s_k^{(2)}\left(v_k^{(2)}\right) = 0,$$

$$\frac{\partial \Pi^{(i)}}{\partial v_k^{(i)}} = -\sum_{m=1}^{M} y_{km}^{(i)} \cdot \frac{\partial s_k^{(i)}\left(v_k^{(i)}\right)}{\partial v_k^{(i)}} - 1 = 0,\; i=1,2,$$

$$\frac{\partial \Pi^{(i)}}{\partial u_k^{(1)}} = y_{km}^{(i)} \cdot \frac{\partial p_{km}\left(u_k^{(i)}\right)}{\partial u_k^{(i)}} - 1 = 0,\; i=1,2.$$

Let us express the optimal level of output of each enterprise through the optimum output of its competitor. Then the equilibrium solution of the duopoly in the Cournot sense can be defined as follows:

$$y_{km}^{(1)} = \frac{2p_{km}\left(u_k^{(1)}\right) - p_{km}\left(u_k^{(2)}\right) - 2s_k^{(1)}\left(v_k^{(1)}\right) + s_k^{(2)}\left(v_k^{(2)}\right)}{3g_{km}}, \tag{5}$$

$$y_{km}^{(2)} = \frac{-p_{km}\left(u_k^{(1)}\right) + 2p_{km}\left(u_k^{(2)}\right) + s_k^{(1)}\left(v_k^{(1)}\right) - 2s_k^{(2)}\left(v_k^{(2)}\right)}{3g_{km}}, \tag{6}$$

$$\sum_{m=1}^{M} y_{km}^{(i)} = -1 \Big/ \left[s_k^{(i)}\left(v_k^{(i)}\right)\right]', \tag{7}$$

$$y_{km}^{(i)} = 1 \Big/ \left[p_{km}\left(u_k^{(i)}\right)\right]' = 0,$$

$$i=1,2,\; k=1,\ldots,K,\; m=1,\ldots,M. \tag{8}$$

In this case, for $y_{km}^{(1)}$, $y_{km}^{(2)}$, $k=1,\ldots,K$, $m=1,\ldots,M$, constraints (4) must be met. In the case of violation of these conditions, the equilibrium solution of a pair of problems (2), (4), and (3), (4) should employ known optimization algorithms [16]. In this case, $\partial y_{km}^{(1)} / \partial y_{km}^{(2)} = 0,\; \partial y_{km}^{(2)} / \partial y_{km}^{(1)} = 0,$ that is, the fulfillment of Cournot's conditions for the alleged variations is required.

Let us now turn to determining the equilibrium solution of the duopoly in the sense of Stackelberg. In this case, duopolies can choose two lines of behavior: leader or follower. The leader is the first to set its production volume and believes that its decision would lead to the follower's response. The follower, when determining the number of goods for production, is guided by the leader's decision and assumes that the leader does not react to its actions.

Suppose that the first manufacturing plant ($i=1$) is the leader and assumes that the second enterprise ($i=2$) will react to its actions in accordance with the direct reaction of Cournot (6). Then the presumptive variation is $\partial y_{km}^{(2)} / \partial y_{km}^{(1)} = 1/2$. In this case,

$$\frac{\partial \Pi^{(1)}}{\partial y_{km}^{(1)}} = p_{km}\left(u_k^{(1)}\right) - \frac{3}{2}g_{km}y_{km}^{(1)} - g_{km}y_{km}^{(2)} - s_k^{(1)}\left(v_k^{(1)}\right). \tag{9}$$

Having equated the derivative (9) to zero, we obtain the equation of the direct reaction of the first enterprise:

$$y_{km}^{(1)} = \frac{2\left(p_{km}\left(u_k^{(1)}\right) - g_{km}y_{km}^{(2)} - s_k^{(1)}\left(v_k^{(1)}\right)\right)}{3g_{km}}. \tag{10}$$

The first enterprise, the leader, assumes that the second (follower) uses the Cournot reaction (6). Then, taking into consideration (6), (10), as well as conditions (7), (8), the solution of the duopoly will be the Stackelberg equilibrium for the first producer:

$$y_{km}^{(1)} = \frac{\left[2p_{km}\left(u_k^{(1)}\right) - p_{km}\left(u_k^{(2)}\right)\right] - 2s_k^{(1)}\left(v_k^{(1)}\right) + s_k^{(2)}\left(v_k^{(2)}\right)}{2g_{km}},$$

$$y_{km}^{(2)} = \frac{\left[-2p_{km}\left(u_k^{(1)}\right) + 3p_{km}\left(u_k^{(2)}\right)\right] + 2s_k^{(1)}\left(v_k^{(1)}\right) - 3s_k^{(2)}\left(v_k^{(2)}\right)}{4g_{km}},$$

$$\sum_{m=1}^{M} y_{km}^{(i)} = -1 \Big/ \left[s_k^{(i)}\left(v_k^{(i)}\right)\right]',$$

$$y_{km}^{(i)} = 1 \Big/ \left[p_{km}\left(u_k^{(i)}\right)\right]' = 0,$$

$$i=1,2,\; k=1,\ldots,K,\; m=1,\ldots,M.$$

In the case where the leader is the second production plant, the solution of the duopoly will be the Stackelberg equilibrium:

$$y_{km}^{(1)} = \frac{\left[3p_{km}\left(u_k^{(1)}\right) - 2p_{km}\left(u_k^{(2)}\right)\right] - 3s_k^{(1)}\left(v_k^{(1)}\right) + 2s_k^{(2)}\left(v_k^{(2)}\right)}{4g_{km}},$$

$$y_{km}^{(2)} = \frac{\left[-p_{km}\left(u_k^{(1)}\right) + 2p_{km}\left(u_k^{(2)}\right)\right] + s_k^{(1)}\left(v_k^{(1)}\right) - 2s_k^{(2)}\left(v_k^{(2)}\right)}{2g_{km}},$$

$$\sum_{m=1}^{M} y_{km}^{(i)} = -1 \Big/ \left[s_k^{(i)}\left(v_k^{(i)}\right)\right]',$$

$$y_{km}^{(i)} = 1 \Big/ \left[p_{km}\left(u_k^{(i)}\right)\right]' = 0,$$

$$i=1,2,\; k=1,\ldots,K,\; m=1,\ldots,M.$$

Note that for the values $y_{km}^{(1)}$, $y_{km}^{(2)}$, defined in this way, constraints (4) must be met.

**5. 3. Studying the impact of the size of investment and marketing deductions on the acquisition of competitive advantages by manufacturers**

Let us conduct a study of the behavior of manufacturing enterprises – links of competing supply chains for the simplest case. Suppose that manufacturers produce one type of product using two types of raw materials and deliver these products to two destinations ($K=1$, $R=2$, $M=2$). Let us find the optimal production plans of each enterprise for three options for their action: according to Cournot and according to Stackelberg: $y_{11}^{(1)}$, $y_{12}^{(1)}$, $y_{11}^{(2)}$, $y_{12}^{(2)}$.

We shall assume that the cost of advertising products $(u_1^{(1)}, u_1^{(2)})$ affects the basic sales prices and does not affect competitive discounts. The dependence of the price on the size of marketing investments is as follows: $p_{km}(u_k^{(i)}) = p_{0km}(1+\alpha_k^{(i)}\sqrt{u_k^{(i)}})$. Also, inves tments in innovative projects $(v_1^{(1)}, v_1^{(2)})$ make it possible to reduce production costs. Unit production costs will decrease as the size of the investment increases: $s_k^{(i)}(v_k^{(i)}) = s_{0k}^{(i)} e^{-\gamma_k^{(i)} v_k^{(i)}}$. The initial data for calculations are given in Table 1.

Table 1

Initial data for determining equilibrium solutions

| Designation | Parameter value | Designation | Parameter value |
|---|---|---|---|
| $p_{011}^{(1)}$ | 10 | $a_{11}^{(1)}$ | 0.2 |
| $p_{012}^{(1)}$ | 12 | $a_{12}^{(1)}$ | 0.15 |
| $p_{011}^{(2)}$ | 11 | $a_{11}^{(2)}$ | 0.5 |
| $p_{012}^{(2)}$ | 11 | $a_{12}^{(2)}$ | 0.1 |
| $g_{11}$ | 0.6 | $b_1^{(1)}$ | 80 |
| $g_{12}$ | 0.5 | $b_2^{(1)}$ | 80 |
| $\gamma_1^{(1)}$ | 0.3 | $b_1^{(2)}$ | 60 |
| $\gamma_1^{(2)}$ | 0.2 | $b_2^{(2)}$ | 50 |
| $s_{01}^{(1)}$ | 5.0 | $\alpha_1^{(1)}$ | 0.07 |
| $s_{01}^{(2)}$ | 5.5 | $\alpha_1^{(2)}$ | 0.07 |

Our calculations to determine the equilibrium solutions of the duopoly were performed using the Microsoft Excel software package. The results of calculating the volume of output, investment and marketing investments, and profits for each of the competing enterprises for the three options for their actions are given in Tables 2, 3.

Data in Tables 2, 3 allow enterprises to make decisions about the volume of output, the size of investment and marketing investments in the cases of various options for the behavior of competitors.

A comparison of the volume of investment and marketing investments of each enterprise is shown in Fig. 1, 2.

Table 2

Results of control parameter calculation

| Designation | | $y_{11}$ | $y_{12}$ | $v_1$ | $u_{11}$ | $u_{12}$ |
|---|---|---|---|---|---|---|
| Cournot equilibrium | enterprise No. 1 | 5.65 | 6.27 | 7.61 | 2.03 | 2.55 |
| | enterprise No. 2 | 6.30 | 7.02 | 8.93 | 2.69 | 3.39 |
| Stackelberg equilibrium (leader – No. 1) | enterprise No. 1 | 10.13 | 11.47 | 9.48 | 6.55 | 8.56 |
| | enterprise No. 2 | 3.54 | 3.79 | 6.54 | 0.85 | 0.99 |
| Stackelberg equilibrium (leader – No. 2) | enterprise No. 1 | 2.34 | 2.39 | 4.72 | 0.35 | 0.37 |
| | enterprise No. 2 | 11.55 | 13.13 | 11.40 | 9.02 | 11.89 |

Table 3

Results of profit calculation of competing enterprises

| Designation | | Value |
|---|---|---|
| Cournot equilibrium | $\Pi_1$ | 28.52 |
| | $\Pi_2$ | 35.93 |
| | $\Pi_1+\Pi_2$ | 64.44 |
| Stackelberg equilibrium (leader – No. 1) | $\Pi_1$ | 42.42 |
| | $\Pi_2$ | 7.05 |
| | $\Pi_1+\Pi_2$ | 49.47 |
| Stackelberg equilibrium (leader – No. 2) | $\Pi_1$ | 0.97 |
| | $\Pi_2$ | 55.17 |
| | $\Pi_1+\Pi_2$ | 56.13 |

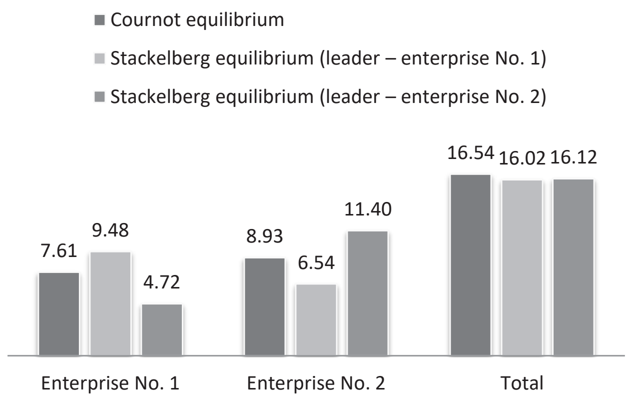


Fig. 1. Investments of competing enterprises, monetary units

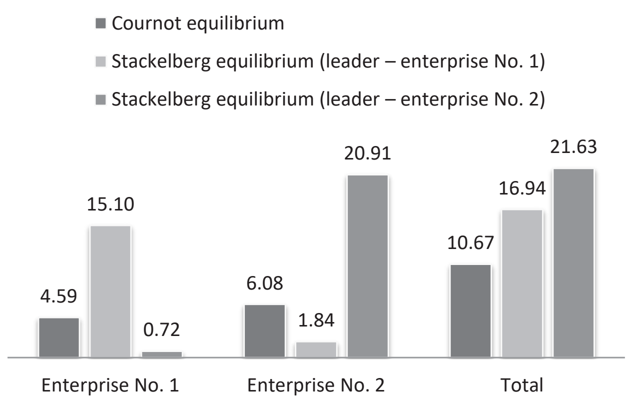


Fig. 2. Marketing investments of competing enterprises, monetary units

It is also indicative to compare the profits of the duopolies, taking into consideration their marketing and innovation activities and without it (Table 4).

Table 4

Results of calculation of profits of enterprises – competitors, monetary units

| Total profit value | Profit including investment and advertising | Profit without investment and advertising |
|---|---|---|
| Cournot equilibrium | 64.44 | 42.79 |
| Stackelberg equilibrium (leader – enterprise No. 1) | 49.47 | 35.32 |
| Stackelberg equilibrium (leader – enterprise No. 2) | 56.13 | 36.93 |

Data in Table 4 allow us to conclude that the total profit for different variants of the behavior of the duopolies (according to Cournot and Stackelberg) with the introduction of innovations and advertising investments increased by 1.4–1.5 times.

## 6. Discussion of results of duopoly modeling, taking into consideration the marketing and innovative activities of manufacturers

To find equilibrium solutions for industrial enterprises participating in competing supply chains, an economic and mathematical model of the duopoly has been built, taking into consideration the marketing and innovative activities of manufacturers. The features of the proposed model are that, unlike the existing ones, it takes into consideration both the innovative and marketing activity of supply chain participants. One of the links in competing supply chains are manufacturing enterprises, which are considering the possibility of increasing their profits through innovative projects in production and additional advertising costs. Our model represents two problems of nonlinear programming.

Our calculations have made it possible to infer that the role of the leader allows manufacturing enterprises to increase investments in technological innovations while the position of the follower forces to reduce the amount of innovative investments. As regards advertising costs, their volume also depends on the behavior of duopolies. Acting as a follower, the company is forced to significantly reduce marketing costs compared to the Cournot equilibrium. Acting as a leader, the manufacturing enterprise gets the opportunity to allocate more money for promotional activities. The calculations, given in Tables 2, 3, show that for enterprise No. 1, the position of leader is preferable (the second option for the behavior of the duopolies). In this case, it will receive a profit of the largest size of 42.42 monetary units; this is 13.9 monetary units more than in the case of Cournot equilibrium. The position of the follower entails the lowest value of profit. For company No. 2, the position of leader by Stackelberg is also the best. The largest total profit of the two competing enterprises corresponds to the equilibrium solution of the duopoly in the sense of Cournot: 64.44 monetary units. This value is higher than the total profit for the leader/follower options by 14.97 monetary units and 8.31 monetary units, respectively. At the same time, the total volume of output for the Cournot solution is the smallest: 25.23 units against 28.94 units, and 29.41 units for Stackelberg behavior options.

Note that in order to calculate the values of equilibrium solutions of the duopoly, it is necessary to solve a system of nonlinear equations. With a small number of variables, the Microsoft Excel software package can be used for the solution. With a large number of types of raw materials for production and a wide range of products, numerical difficulties are possible. These can be overcome by using specialized computing programs, such as MATLAB.

Our proposed model considers the simplest supply chain, taking into consideration the supply of raw materials to enterprises, as well as the production and delivery of finished products to consumers. The model assumes a series of simplifications. For example, possible restrictions on available financial resources are not taken into consideration, the case of lack of raw materials for production is not considered, the limited capacity of warehouses for storing raw materials and finished products is not taken into consideration. These shortcomings can be eliminated by adding appropriate constraints to the optimization problem.

In the future, it is possible to summarize the above results, for example, the construction and study of dynamic optimization models, taking into consideration the innovation and marketing activities of production enterprises, as well as research for other market structures.

## 7. Conclusions

1. To build an economic and mathematical model of the duopoly of supply chains, a model for optimizing plans for the release and delivery of multi-nomenclature products was used. It is assumed that in the production of products the enterprise cannot use resources more than is available. Also, all manufactured products from the manufacturer's warehouse must be exported to their destinations while the demand at the destinations should not be exceeded. The innovative activity of product manufacturers in the developed model is taken into consideration by introducing values that reflect the amount of investment in the implementation of technological innovations. To take into consideration the marketing activity of enterprises, it is assumed that they direct part of the profit to advertising their products.

2. Based on the constructed model, equilibrium solutions of the duopoly of supply chains according to Cournot and Stackelberg have been determined. The optimal values of product volumes for output, the size of investments, and the cost of advertising products are found by solving a nonlinear system of equations for each case.

3. We analyzed the impact of the size of investment and marketing deductions on the acquisition of competitive advantages by manufacturing enterprises by using a numerical experiment. The calculations have shown that the role of the leader allows manufacturing enterprises to increase investments in technological innovations and allocate more money for promotional activities. The position of the follower forces the company to reduce the volume of innovative investments and significantly reduce marketing costs compared to the Cournot equilibrium. At the same time, the total profit for different variants of the behavior of duopolies (according to Cournot and Stackelberg) with the introduction of innovations and advertising investments increases by 1.4–1.5 times.

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