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COMPUTER MODELING IN THE STUDY OF THE EFFECT OF NORMALIZED QUANTITIES ON THE MEASUREMENT ACCURACY OF THE QUADRATIC TRANSFORMATION FUNCTION

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The research of the systems of equations of quantities describing, respectively, 5 and 6 measurement cycles revealed the peculiarities of redundancy formation. It is proved that the normalized temperature T_1 has the greatest effect on the measurement result for both systems. In addition, it was found that in both systems, an increase in the reproduction accuracy of the normalized temperature T_1 (with a constant reproduction error of T_2) does not lead to a significant improvement in the results. Due to this, it can be argued on the use of non-precision normalized sources to reproduce the temperature T_1. However, an order of magnitude increase in the reproduction accuracy of both normalized quantities T_1 and T_2 also increases the measurement accuracy by an order of magnitude. Computer modeling confirmed that for the redundant measurement equation (11) at the ratio $T_1=T_i(0.0005 \cdot T_i+1)$ in the range (10÷200) °C, measurement with a relative error (0.01÷0.00003) % is provided. When applying the redundant measurement equation (13), the accuracy increases to 0.0059 % only at the end of the range. Based on the results obtained, it was found that the accuracy of redundant measurements is influenced by the type of equation itself, not their number. Processing of the results based on the redundant measurement equation, by the way, ensures the independence of the measurement result from the influence of absolute values of the transformation function parameters, as well as their deviations from nominal values under the influence of external destabilizing factors.

Thus, there is reason to believe that it is possible to increase the accuracy of measurement in a wide range by observing the ratio between normalized and controlled quantities

Keywords: redundant methods, measurement equations, accuracy improvement, normalized quantities, reproduction errors of quantities

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1. Introduction

The vast majority of production and technical processes involve measuring one or more desired physical quantities, the accuracy and reliability of which determine the quality of finished products or the final result of research. Therefore, for laboratory research or industrial application, sensors are used that have high requirements for accuracy, linearity and sensitivity. In this regard, relevant research and techniques are aimed at improving and further developing existing measurement methods and means to ensure the quality of measurement [1–3]. Thus, digital or analog methods are used to reduce the error from the nonlinearity of the transformation function. Linearization algorithms are applied when using digital methods, additional linearization schemes are applied when using analog methods. However, the use of these methods leads to a narrowing of the measurement range. The accuracy of the measurement result is also affected by the instability of the sensor transformation function caused by external destabilizing factors and aging of the materials of the sensitive elements. This, in turn, leads to the need for frequent sensor checks or calibration, which requires additional material and time costs.

Thus, research aimed at a comprehensive solution of the problem of improving measurement accuracy with nonlinear and unstable sensor transformation function is considered relevant.

2. Literature review and problem statement

The process of improving sensors and related research methods determines the overall quality of measurement.

Therefore, it is important to create high-precision sensors and improve methods for reliable measurements. Various measures and methods are applied to improve the accuracy of measurements in a wide range: stable materials and calibration are used, systematic errors are corrected, linearization measures are carried out, sensitivity is increased, and so on. Thus, in [4], an increase in sensitivity was achieved using low-current base-collector voltage, and in [5] elements of solid microelectronics and multifunctional signal converters based on calorimetric methods were used. Ways to increase the speed of transistors based on materials science structures and schematic aspects of selectively doped heterotransistors are given in [6]. However, the issues related to reducing the systematic measurement error remained unresolved. An option to overcome these difficulties may be to use recursive methods. This approach is used in [7], where obtaining individual values of the resistances of resistors in an array of sensors made it possible to reduce cross-noise. The issue of reducing the systematic error of the measuring channel was also considered in [8], where the accuracy was improved by introducing corrections. However, the issue of obtaining reliable and useful information from sensors was not considered. So, in [9], to effectively obtain reliable information from an array of gas sensors, hardware data processing (analog interface) was used. Improving the efficiency of signal processing was also considered in [10], where the received signals were processed according to the proposed HD (hyperdimensional) computing. In [11], the informativeness of sensor signals was increased by combining thermal and capacitive measurement methods. However, there are still unresolved issues related to the nonlinearity of sensor characteristics, which leads to a narrowing of the range or an increase in the measurement error. A way to overcome these difficulties may be to use the linearization method. This approach is used in [12], which proposed to form a compensatory measuring current and change the transformation factor of the output scaling amplifier in certain temperature measurement ranges. In [13], by introducing the correction of nonlinear curvature at several reference temperatures, an increase in the sensor accuracy was achieved. Ways to expand the linear range by improving the signal processing algorithm were presented in [14]. However, the issues of improving accuracy by minimizing the impact of errors caused by the influence of destabilizing factors on measurement elements remain unresolved. An option to overcome these difficulties may be to use calibration methods. In [15], the calibration process is based on continuous temperature deployment while recording calibration data. The issue of accuracy improvement by calibration was also considered in [16]. The work found that the quality of calibration is affected by the number of coefficients in the calibration equation. Therefore, based on nine additional calibration equations, a new calibration equation was proposed, which provides high accuracy and reliability of the information obtained. The issue of improving accuracy through linearization and calibration was presented in [17], where a CMOS temperature sensor with integrated calibration for curve linearization was proposed. But such accuracy improvement approaches require the use of exemplary sources (materials) or high-precision components, as in [18], which require certain material costs for their implementation. Despite the practical significance of the results obtained, the issue of the scattering of characteristics from measurement to measurement on the one hand, and the nonlinearity of characteristics on the other, was not sufficiently considered. As an option to overcome difficulties in improving accuracy for unstable and nonlinear transformation functions in a wide range of values, redundant measurement methods can be recommended. Thus, the main theoretical aspects of redundant methods were considered in [19]. In [20], a structural analysis of the equations of redundant and super-redundant measurements describing redundant methods was performed in order to study the statistical characteristics before and after modification of their structure. However, of practical interest is also the mathematical form of redundant measurement equations. In [21], a universal measurement equation is mathematically presented. Despite the fact that various forms of redundant measurement equations were given, their practical implementation for various sensor transformation functions in order to obtain a highly accurate result was not demonstrated. Further development of measurement redundancy was obtained in [22], where the possibility of high-precision measurement of the resistance of resistive sensors by obtaining not one but several basic equations of redundant measurements was considered. However, such results were obtained with a linear transformation function. Therefore, to solve the problem of applying the redundant measurement method for a nonlinear transformation function, a sensor with a logarithmic transformation function was considered in [23]. The efficiency of the presented methods for improving the measurement accuracy and the possibility of providing metrological self-control are shown. However, the effect of the random component of the measurement error on the measurement result was not considered. An option to overcome the relevant difficulties may be the approach proposed in [24]. Processing of measurement results according to the proposed approaches allows us to assert an increase in measurement accuracy by eliminating the systematic component of the error caused by changing the parameters of the transformation function, as well as reducing the random component of the error. However, the issue of the influence of the values of normalized quantities and their relationship with the desired quantity on the final measurement result for a nonlinear transformation function remains unresolved. Therefore, further research on redundant measurement methods presented in [25] established a relationship between the normalized and desired quantities, which significantly expands the range of high-precision measurements. The results obtained demonstrated the high efficiency of redundant measurement methods in improving measurement accuracy by adjusting the values of normalized quantities for the logarithmic transformation function. However, sensors such as a transistor and a thermistor have quadratic transformation functions. Thus, it is of scientific interest to further identify ways to improve measurement accuracy for quadratic and unstable transformation functions.

This allows us to argue about the feasibility of research to determine the effect of normalized quantities and their relationships with the controlled quantity on the measurement accuracy with a quadratic sensor transformation function.

3. The aim and objectives of the study

The aim of the study is to determine the relationship between controlled and normalized quantities, which provides an increase in accuracy for the quadratic transformation function of the sensor. This will make it possible to control the measurement process with high accuracy over a wide measurement range by adjusting the values of normalized quantities.

To achieve the aim, the following objectives were accomplished:

– to carry out computer modeling of a system of nonlinear equations of quantities consisting of 5 equations and the corresponding equation of redundant measurements to study the effect of normalized quantities on the measurement result;

– to conduct a similar computer modeling of a system of nonlinear equations of quantities consisting of 6 equations and the corresponding equation of redundant measurements to study the effect of normalized quantities on the measurement result;

– to make a comparative analysis of the results obtained applying each of the proposed systems with the corresponding equation of redundant measurements.

4. Research materials and methods

4. 1. Research materials and modeling tools

The primary temperature converter, KT3132 A-2 silicon bipolar transistor (Ukraine), was chosen as the test material.

The Mathcad15.0 (USA) and MS Excel (USA) applications were used for mathematical modeling.

4. 2. Method for studying redundant measurements

Redundant measurement methods (RMM) have proven themselves well for a comprehensive solution of the problem of improving accuracy over the entire measurement range. The essence of RMM is a time-separated measurement transformation, in addition to the desired physical quantity, of several normalized physical quantities, which are associated with the desired specific pattern. The number of such measurements depends on the number of parameters of the transformation function (TF). So, if the TF consists of (n) parameters, the number of measurement cycles must be at least (n+1). A feature of using RMM for nonlinear TF is the presence of measurement cycles in which the desired and normalized quantity (quantities) are simultaneously measured. In general, the mathematical model of RMM is a system of equations of quantities describing the measurement cycles, the solution of which gives the equations of redundant measurements of the desired physical quantity. Generally, a nonlinear TF:

$$y'_x = f\left(x_i, S'_H, S'_L, \Delta y\right), \tag{1}$$

has 4 parameters, so it is necessary to make a system of at least 5 equations of quantities:

$$\begin{cases} y'_{H1} = f\left(x_1, S'_H, S'_L, \Delta y'\right); \\ y'_{H2} = f\left(x_2, S'_H, S'_L, \Delta y'\right); \\ y'_{H3} = f\left(x_3, S'_H, S'_L, \Delta y'\right); \\ y'_{H4} = f\left(x_i, x_2, S'_H, S'_L, \Delta y'\right); \\ y'_{H5} = f\left(x_i, x_3, S'_H, S'_L, \Delta y'\right). \end{cases} \tag{2}$$

The solution of this system relative to the desired quantity x_i allows obtaining its value:

$$x_i = F\left(y'_{H1}, \ldots, y'_{H5}, x_1, x_2, x_3\right), \tag{3}$$

where y'_{Hj} – sensor output signals (j=5);

x_i – desired physical quantity;

x_1, ..., x_3 – normalized physical quantities, the values of which are related by a certain law, from a source with normalized characteristics;

S'_H, S'_L – sensitivity (steepness) of the transformation of the nonlinear and linear components of the transformation function;

$\Delta y'$ – parameter (offset) of the transformation function taking into account the additive component of the error.

As can be seen from the redundant measurement equation (3), the obtained value of the desired quantity x_i does not depend on the values of the parameters S'_H, S'_L, $\Delta y'$ of the nonlinear transformation function, and hence on their deviations from the nominal values.

The application of RMM for quadratic TF and the effect of normalized quantities on the measurement result in cases where the system consists of (n+1) and (n+2) redundant measurement equations describing measurement cycles are considered below.

A quadratic TF has the form:

$$y'_H = S'_H x_i^2 + S'_L x_i + \Delta y'. \tag{4}$$

Since the quadratic TF (4) has 4 parameters, two variants of systems (5), (6) were studied, consisting of 5 and 6 measurement cycles, described by the corresponding equations of quantities. Thus, if it is possible to form normalized physical quantities x_1, $2x_1$ and x_2, a system of nonlinear equations of quantities takes the form:

$$\begin{cases} y'_{H1} = S'_H x_1^2 + S'_L x_1 + \Delta y'; \\ y'_{H2} = S'_H x_2^2 + S'_L x_2 + \Delta y'; \\ y'_{H3} = S'_H x_i^2 + S'_L x_i + \Delta y'; \\ y'_{H4} = S'_H \left(x_i + x_1\right)^2 + S'_L \left(x_i + x_1\right) + \Delta y'; \\ y'_{H5} = S'_H \left(2x_1\right)^2 + S'_L \left(2x_1\right) + \Delta y'. \end{cases} \tag{5}$$

If it is possible to form normalized physical quantities x_1, $2x_1$ and $2x_2$, a system of nonlinear equations of quantities takes the form:

$$\begin{cases} y'_{H1} = S'_H x_1^2 + S'_L x_1 + \Delta y'; \\ y'_{H2} = S'_H x_2^2 + S'_L x_2 + \Delta y'; \\ y'_{H3} = S'_H x_i^2 + S'_L x_i + \Delta y'; \\ y'_{H4} = S'_H \left(x_i + x_1\right)^2 + S'_L \left(x_i + x_1\right) + \Delta y'; \\ y'_{H5} = S'_H \left(2x_1\right)^2 + S'_L \left(2x_1\right) + \Delta y'; \\ y'_{H6} = S'_H \left(2x_2\right)^2 + S'_L \left(2x_2\right) + \Delta y'. \end{cases} \tag{6}$$

As a result of solving systems (5) and (6), the corresponding equations of redundant measurements of the desired physical quantity x_i were obtained:

$$x_i = \frac{\left(y'_{H5} - y'_{H3}\right)\left(3x_1x_2 - 2x_1^2 - x_2^2\right) - \left(y'_{H4} - y'_{H1}\right)\left(x_1x_2 - x_2^2\right) - 2x_1^2\left(y'_{H2} - y'_{H1}\right)}{2\left[\left(x_2 - x_1\right)\left(y'_{H4} - y'_{H1}\right) - x_1\left(y'_{H2} - y'_{H1}\right)\right]} \tag{7}$$

and

$$x_i = \frac{2\left(y'_{H4}-y'_{H3}\right)\left(x_2^2-x_1^2\right)-x_1\left(x_1+x_2\right)\left[4\left(y'_{H2}-y'_{H1}\right)-\left(y'_{H6}-y'_{H5}\right)\right]}{2x_1\left[\left(y'_{H6}-y'_{H5}\right)-2\left(y'_{H2}-y'_{H1}\right)\right]}-\frac{x_1}{2}. \quad (8)$$

In equations of redundant measurements (7), (8) due to differences in the sensor output signals, the additive component of the measurement error is excluded, and due to their division, the multiplicative component of the systematic error is excluded. Thus, the influence of the parameters S'_H, S'_L, $\Delta y'$ on the measurement result is excluded, which confirms the effectiveness of using RMM in quadratic TF.

5. Results of computer modeling to improve measurement accuracy

5. 1. Computer modeling of a system of equations consisting of 5 equations of quantities

As is known [11], the dependence of the base-emitter voltage of a transistor on temperature is described by the equation:

$$U'_{bet} = U'_{bet0} - \Delta U'_A T_i - \Delta U'_B T_i^2, \quad (9)$$

where U_{bet0} – value of the base-emitter voltage at t=0 °C;

$\Delta U'_A$ – linear coefficient of change in the base-emitter voltage from temperature;

$\Delta U'_B$ – quadratic coefficient of change in the base-emitter voltage from temperature;

T_i – value of the desired temperature.

If the temperature values T_1, $2T_1$ and T_2 are formed using standard sources with normalized characteristics based on system (5), we obtain the following system of equations:

$$\begin{cases} U'_{bet1} = U'_{bet0} - \Delta U'_A T_1 - \Delta U'_B T_1^2; \\ U'_{bet2} = U'_{bet0} - \Delta U'_A T_2 - \Delta U'_B T_2^2; \\ U'_{bet3} = U'_{bet0} - \Delta U'_A T_i - \Delta U'_B T_i^2; \\ U'_{bet4} = U'_{bet0} - \Delta U'_A 2T_1 - \Delta U'_B \left(2T_1\right)^2; \\ U'_{bet5} = U'_{bet0} - \Delta U'_A \left(T_1+T_i\right) - \Delta U'_B \left(T_1+T_i\right)^2. \end{cases} \quad (10)$$

As a result of solving system (10), the equation of redundant measurements of the desired temperature T_i was obtained:

$$T_i = \frac{\left(U'_{bet5}-U'_{bet3}\right)\left(3T_1T_2-2T_1^2-T_2^2\right)-\left(U'_{bet4}-U'_{bet1}\right)\left(T_1T_2-T_2^2\right)-2T_1^2\left(U'_{bet2}-U'_{bet1}\right)}{2\left(T_2-T_1\right)\left(U'_{bet4}-U'_{bet1}\right)-T_1\left(U'_{bet2}-U'_{bet1}\right)}. \quad (11)$$

As can be seen from the redundant measurement equation (11), it does not include the parameters $\Delta U'_A$, $\Delta U'_B$ and U'_{bet0}, i. e. their influence on the measurement result is excluded.

For computer modeling, the limits of change for the parameters $\Delta U'_A$, $\Delta U'_B$ and U'_{bet0} lie within ±10.0 %. The values of the reproduction error of the normalized temperatures T_1 and T_2 were chosen as Δt=0.001 °C. Thus, the system (10) takes the form:

$$\begin{cases} U'_{bet1} = 1.1U'_{bet0} - 1.1\Delta U'_A\left(T_1+0.001\right) - \\ -1.1\Delta U'_B\left(T_1+0.001\right)^2; \\ U'_{bet2} = 1.1U'_{bet0} - 1.1\Delta U'_A\left(T_2+0.001\right) - \\ -1.1\Delta U'_B\left(T_2+0.001\right)^2; \\ U'_{bet3} = 1.1U'_{bet0} - 1.1\Delta U'_A\left(T_i+0.001\right) - \\ -1.1\Delta U'_B\left(T_i+0.001\right)^2; \\ U'_{bet4} = 1.1U'_{bet0} - 1.1\Delta U'_A 2\left(T_1+0.001\right) - \\ -1.1\Delta U'_B\left(2\left(T_1+0.001\right)\right)^2; \\ U'_{bet5} = 1.1U'_{bet0} - 1.1\Delta U'_A\left(T_1+T_i+0.001\right) - \\ -1.1\Delta U'_B\left(T_1+T_i+0.001\right)^2. \end{cases} \quad (12)$$

During the modeling, a KT3132 A-2 transistor with a measurement range from 10 °C to 200 °C, at U_{bet0}=0.6 V and at ΔU_A=1.882 mV/°C, ΔU_B=0.41 μV/°C^2 was used [11]. To study the effect of the value of each of the normalized temperatures T_1 and T_2 on the measurement result of T_i by equation (12), 3 cases were considered:

1) T_1=T_2=(10÷100) °C with a step of 30 °C;
2) T_1=(10÷100) °C, T_2=(1÷10) °C with a step of 30 °C;
3) T_1=(1÷10) °C, T_2=(10÷100) °C with a step of 30 °C.

As a result of computer modeling in the Mathcad15.0 environment, the following values of relative measurement errors were obtained in three sections of the operating range (at the beginning, middle and end), presented in Tables 1–3.

As can be seen from the tables above (Tables 1–3), there is a certain relationship between T_i and normalized temperatures T_1 and T_2. It was found that a more significant relationship was found between the values of T_1 and T_i than between T_2 and T_i. Thus, the lowest value of the relative error is obtained when T_1 is close to T_i. The best accuracy results were obtained for the second case (Table 2) at T_1=(10÷100) °C, T_2=(1÷10) °C. For this case, the influence of reproduction errors of normalized temperatures T_1 and T_2 at different values was considered. It was found that increasing the reproduction accuracy of the normalized temperature T_1 (with a constant reproduction error of T_2) does not significantly improve the results, which gives grounds to use a non-precision normalized source for T_1 reproduction. On the other hand, an order of magnitude increase in the reproduction accuracy of both normalized temperatures T_1 and T_2 also increases the measurement accuracy by an order of magnitude.

Since the best accuracy results were obtained for the second case (Table 2) at T_1=(10÷100) °C, T_2=(1÷10) °C, it was considered in more detail. Studies were performed at T_1=(10÷100) °C with a step of 10 °C and at T_2=(1÷10) °C with a step of 1 °C, as well as at T_i=50 °C. The obtained calculations of the relative measurement error for the second case are given in Table 4.

As can be seen from Table 4, the results obtained confirm the fact that the value of T_1 should be equal to T_i or higher, while the value of T_2 has little effect under these conditions. Thus, in subsequent calculations, the value T_2=10 °C was chosen. To find the desired relationship between the values of T_1 and T_i, which provide high-precision measurement results, the MS Excel data analysis package for the nonlinear function was used. Using the solution search option and constructing a trend line, the optimal ratio between T_1 and T_i was found, at which the measurement error decreases.

Thus, a relationship was determined between the values of T_1 and T_i·T_1=T_i(0.0005·T_i+1), which provides high-precision temperature measurement with a relative error (0.01÷0.00003) % in the range of measured temperatures (10÷200) °C.

Table 1

Relative measurement errors (%) at T_1=T_2=(10÷100) °C

T_i=10 °C				
T_{ij}	T_{21}=10 °C	T_{22}=40 °C	T_{23}=70 °C	T_{24}=100 °C
T_{11}=10 °C	0.0095	0.010	0.010	0.010
T_{12}=40 °C	0.491	0.871	3.504	1.810
T_{13}=70 °C	0.302	0.399	0.577	1.023
T_{14}=100 °C	0.217	0.262	0.326	0.430
T_i=100 °C				
T_{11}=10 °C	3.974	2.141	0.849	0.532
T_{12}=40 °C	0.101	0.177	0.703	0.359
T_{13}=70 °C	0.017	0.021	0.030	0.053
T_{14}=100 °C	0.0009	0.0009	0.0009	0.0009
T_i=200 °C				
T_{11}=10 °C	4.194	2.261	0.897	0.562
T_{12}=40 °C	0.134	0.235	0.937	0.479
T_{13}=70 °C	0.034	0.045	0.064	0.112
T_{14}=100 °C	0.013	0.016	0.019	0.025

Table 2

Relative measurement errors (%) at T_1=(10÷100) °C, T_2=(1÷10) °C

T_i=10 °C				
T_{ij}	T_{21}=1 °C	T_{22}=4 °C	T_{23}=7 °C	T_{24}=10 °C
T_{11}=10 °C	0.0094	0.0093	0.0091	0.0089
T_{12}=40 °C	0.433	0.451	0.470	0.491
T_{13}=70 °C	0.282	0.288	0.295	0.302
T_{14}=100 °C	0.207	0.210	0.214	0.217
T_i=100 °C				
T_{11}=10 °C	2.133	2.523	3.087	3.974
T_{12}=40 °C	0.090	0.093	0.097	0.101
T_{13}=70 °C	0.016	0.016	0.016	0.017
T_{14}=100 °C	0.0009	0.0009	0.0009	0.0009
T_i=200 °C				
T_{11}=10 °C	2.251	2.662	3.258	4.194
T_{12}=40 °C	0.119	0.123	0.128	0.134
T_{13}=70 °C	0.032	0.033	0.034	0.034
T_{14}=100 °C	0.013	0.013	0.013	0.013

Table 3

Relative measurement errors (%) at T_1=(1÷10) °C, T_2=(10÷100) °C

T_i=10 °C				
T_j	T_{21}=10 °C	T_{22}=40 °C	T_{23}=70 °C	T_{24}=100 °C
T_{11}=1 °C	121.219	12.446	6.595	4.503
T_{12}=4 °C	81.548	2.244	1.140	0.765
T_{13}=7 °C	4.248	0.775	0.352	0.226
T_{14}=10 °C	0.0089	0.011	0.010	0.010
T_i=100 °C				
T_{11}=1 °C	133.385	13.704	7.267	4.964
T_{12}=4 °C	130.543	3.606	1.840	1.240
T_{13}=7 °C	13.149	2.434	1.121	0.731
T_{14}=10 °C	3.974	2.141	0.849	0.532
T_i=200 °C				
T_{11}=1 °C	134.061	13.774	7.304	4.990
T_{12}=4 °C	133.265	3.682	1.878	1.266
T_{13}=7 °C	13.643	2.526	1.164	0.759
T_{14}=10 °C	4.194	2.261	0.897	0.562

Table 4

Relative measurement errors (%) at T_1=(10÷100) °C, T_2=(1÷10) °C

T_i=50 °C										
Temperature	T_{21}	T_{22}	T_{23}	T_{24}	T_{25}	T_{26}	T_{27}	T_{28}	T_{29}	T_{210}
T_{11}	1.897	2.000	2.115	2.244	2.390	2.555	2.746	2.966	3.226	3.535
T_{12}	0.356	0.365	0.375	0.386	0.397	0.408	0.421	0.434	0.448	0.462
T_{13}	0.107	0.109	0.111	0.112	0.115	0.117	0.119	0.121	0.123	0.126
T_{14}	0.032	0.032	0.032	0.033	0.033	0.034	0.034	0.034	0.035	0.035
T_{15}	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
T_{16}	0.011	0.011	0.011	0.012	0.012	0.012	0.012	0.012	0.012	0.012
T_{17}	0.017	0.018	0.018	0.018	0.018	0.018	0.018	0.019	0.019	0.019
T_{18}	0.020	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.022	0.022
T_{19}	0.022	0.022	0.022	0.022	0.022	0.022	0.023	0.023	0.023	0.023
T_{10}	0.022	0.022	0.022	0.022	0.023	0.023	0.023	0.023	0.023	0.023

Since the results given in Table 1 are close to those in Table 2, this case was also considered in more detail. Thus, in a detailed study of the first case at T_1=T_2=(10÷100) °C and T_i=100 °C, which corresponds to Table 1, a functional dependence of the relative measurement error on the normalized values of T_1 and T_2 was constructed. Fig. 1 shows a surface graph, where the relative error values are on the z-axis, the step number (from 1 to 10) for T_1=(10÷100) °C on the i-axis, j – the step number (from 1 to 10) for T_2=(10÷100) °C.

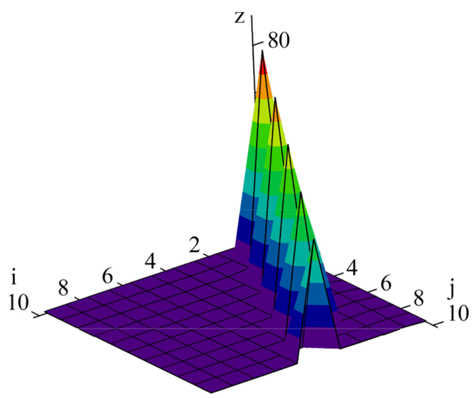

Fig. 1. Graph of the relative error against the values of normalized temperatures

As can be seen from Fig. 1, there is a certain undesirable relationship between T_1 and T_2, at which measurements become unacceptable. Based on Fig. 1 and calculations performed in the Mathcad15.0 environment, it was found that $T_2 \neq 2 \cdot T_1$. Moreover, as the difference between T_1 and T_i increases, this ratio will not be point, but have a certain range with a maximum error at the point $T_2 \neq 2 \cdot T_1$. This indicates the expediency of observing the previously found relationship between the values of the normalized temperature T_1 and controlled temperature T_i, which is T_1=T_i (0.0005·T_i+1). In addition, when measuring at the beginning of the range at T_1=T_2=(10÷100) °C and T_i=100 °C, it is necessary to adhere more precisely to the desired ratio between the normalized temperatures T_1 and T_2.

The worst accuracy results were obtained for the third case (Table 3) at T_1=(1÷10) °C, T_2=(10÷100) °C, where even the proximity of T_2 to T_i does not lead to the desired result. Thus, it was found that when applying the redundant measurement equation (11), the value of T_2 should not exceed the value of T_1.

5. 2. Computer modeling of a system consisting of 6 equations of quantities

Based on the dependence of the base-emitter voltage of a transistor on temperature described by equation (9), the system of nonlinear equations of quantities (6) has the following form:

$$\begin{cases} U'_{bet1} = U'_{bet0} - \Delta U'_A T_1 - \Delta U'_B T_1^2; \\ U'_{bet2} = U'_{bet0} - \Delta U'_A T_2 - \Delta U'_B T_2^2; \\ U'_{bet3} = U'_{bet0} - \Delta U'_A T_i - \Delta U'_B T_i^2; \\ U'_{bet4} = U'_{bet0} - \Delta U'_A (T_1 + T_i) - \Delta U'_B (T_1 + T_i)^2; \\ U'_{bet5} = U'_{bet0} - \Delta U'_A 2T_1 - \Delta U'_B (2T_1)^2; \\ U'_{bet6} = U'_{bet0} - \Delta U'_A 2T_2 - \Delta U'_B (2T_2)^2. \end{cases} \quad (13)$$

As a result of solving system (13), the equation of redundant measurements of the desired temperature T_i was obtained:

$$T_i = \frac{2(U'_{bet4} - U'_{bet3})(T_2^2 - T_1^2) - T_1(T_1 + T_2)\left[4(U'_{bet2} - U'_{bet1}) - (U'_{bet6} - U'_{bet5})\right]}{2T_1\left[(U'_{bet6} - U'_{bet5}) - 2(U'_{bet2} - U'_{bet1})\right]} - \frac{T_1}{2}. \quad (14)$$

For computer modeling, the limits of changes similar to those for system (10) were set for the parameters $\Delta U'_A$, $\Delta U'_B$ and U'_{bet0}, lying within ±10.0 %. The value of the reproduction error of the normalized temperatures T_1 and T_2 was selected as Δ_t=0.001 C. Thus, the system (13) takes the form:

$$\begin{cases} U'_{bet1} = 1.1U'_{bet0} - 1.1\Delta U'_A (T_1 + 0.001) - \\ -1.1\Delta U'_B (T_1 + 0.001)^2; \\ U'_{bet2} = 1.1U'_{bet0} - 1.1\Delta U'_A (T_2 + 0.001) - \\ -1.1\Delta U'_B (T_2 + 0.001)^2; \\ U'_{bet3} = 1.1U'_{bet0} - 1.1\Delta U'_A (T_i + 0.001) - \\ -1.1\Delta U'_B (T_i + 0.001)^2; \\ U'_{bet4} = 1.1U'_{bet0} - 1.1\Delta U'_A (T_1 + T_i + 0.001) - \\ -1.1\Delta U'_B (T_1 + T_i + 0.001)^2; \\ U'_{bet5} = 1.1U'_{bet0} - 1.1\Delta U'_A 2(T_1 + 0.001) - \\ -1.1\Delta U'_B (2(T_1 + 0.001))^2; \\ U'_{bet6} = 1.1U'_{bet0} - 1.1\Delta U'_A 2(T_2 + 0.001) - \\ -1.1\Delta U'_B (2(T_2 + 0.001))^2. \end{cases} \quad (15)$$

To logically compare the results obtained by equations (11) and (14), studies were conducted with the same initial data for three cases:

1) T_1=T_2=(10÷100) °C with a step of 30 °C;
2) T_1=(10÷100) °C, T_2=(1÷10) °C with a step of 30 °C;
3) T_1=(1÷10) °C, T_2=(10÷100) °C with a step of 30 °C.

As a result of computer modeling in the Mathcad15.0 environment, the following values of relative measurement errors were obtained, presented in Table 5–7.

Table 5

Relative measurement errors (%) at T_1=T_2=(10÷100) °C

	T_i=10 °C			
T_{ij}	T_{21}=10 °C	T_{22}=40 °C	T_{23}=70 °C	T_{24}=100 °C
T_{11}=10 °C	2.300	2.308	2.311	2.312
T_{12}=40 °C	0.574	0.579	0.581	0.582
T_{13}=70 °C	0.328	0.331	0.333	0.334
T_{14}=100 °C	0.230	0.232	0.233	0.235
	T_i=100 °C			
T_{11}=10 °C	0.230	0.236	0.238	0.239
T_{12}=40 °C	0.056	0.058	0.059	0.059
T_{13}=70 °C	0.032	0.033	0.033	0.034
T_{14}=100 °C	0.022	0.023	0.023	0.023
	T_i=200 °C			
T_{11}=10 °C	0.115	0.121	0.123	0.123
T_{12}=40 °C	0.027	0.029	0.030	0.030
T_{13}=70 °C	0.015	0.016	0.017	0.017
T_{14}=100 °C	0.011	0.011	0.012	0.012

Table 6

Relative measurement errors (%) at T_1=(10÷100) °C, T_2=(1÷10) °C

	T_i=10 °C			
T_{ij}	T_{21}=1 °C	T_{22}=4 °C	T_{23}=7 °C	T_{24}=10 °C
T_{11}=10 °C	2.287	2.293	2.297	2.300
T_{12}=40 °C	0.572	0.573	0.573	0.574
T_{13}=70 °C	0.327	0.327	0.328	0.328
T_{14}=100 °C	0.229	0.229	0.229	0.230
	T_i=100 °C			
T_{11}=10 °C	0.221	0.225	0.228	0.230
T_{12}=40 °C	0.055	0.055	0.056	0.056
T_{13}=70 °C	0.031	0.032	0.032	0.032
T_{14}=100 °C	0.022	0.022	0.022	0.022
	T_i=200 °C			
T_{11}=10 °C	0.107	0.111	0.113	0.115
T_{12}=40 °C	0.026	0.027	0.027	0.027
T_{13}=70 °C	0.015	0.015	0.015	0.015
T_{14}=100 °C	0.011	0.011	0.011	0.011

Table 7

Relative measurement errors (%) at T_1=(1÷10) °C, T_2=(10÷100) °C

	T_i=10 °C			
T_{ij}	T_{21}=10 °C	T_{22}=40 °C	T_{23}=70 °C	T_{24}=100 °C
T_{11}=1 °C	23.038	23.055	23.057	23.058
T_{12}=4 °C	5.755	5.767	5.769	5.770
T_{13}=7 °C	3.287	3.297	3.299	3.300
T_{14}=10 °C	2.300	2.309	2.311	2.312
	T_i=100 °C			
T_{11}=1 °C	2.377	2.391	2.393	2.394
T_{12}=4 °C	0.585	0.595	0.597	0.598
T_{13}=7 °C	0.331	0.339	0.340	0.341
T_{14}=10 °C	0.230	0.236	0.238	0.239
	T_i=200 °C			
T_{11}=1 °C	1.230	1.243	1.245	1.246
T_{12}=4 °C	0.298	0.308	0.310	0.310
T_{13}=7 °C	0.167	0.174	0.176	0.177
T_{14}=10 °C	0.115	0.121	0.123	0.123

As can be seen from Tables 5–7, the best accuracy results were obtained for the second case (Table 6) at T_1=(10÷100) °C, T_2=(1÷10) °C in the middle and end of the range. The worst accuracy results, as for equation (11), were obtained for the third case (Table 7) at T_1=(1÷10) °C, T_2=(10÷100) °C even at the end of the range. Thus, we can summarize that the relationship between T_i and T_1 is more significant than between T_i and T_2, and the best accuracy results are obtained at a high (more than 100 °C) value of T_1. The lowest value of the relative error is obtained when the total value of T_1 and T_i increases. It was found that for the second case (at T_1=(10÷100) °C, T_2=(1÷10) °C) when the value of T_i increases, the relative measurement error decreases. In addition, when the reproduction accuracy of the normalized temperature T_1 increases (with a constant reproduction error of T_2), there was no significant improvement in the results, which gives grounds to use a non-precision normalized source for T_1 reproduction. On the other hand, an order of magnitude increase in the reproduction accuracy of both normalized temperatures T_1 and T_2 also increases the measurement accuracy by an order of magnitude.

As a result of computer modeling in the Mathcad15.0 environment, calculations of the relative measurement error at T_1=(100÷190) °C and T=(100÷190) °C and with step of 10 °C were carried out, presented in Table 8. Since the parameter T_2 does not have a significant effect on the measurement result, T_2=10 °C was taken for calculations (as for the redundant measurement equation (11)).

Table 8

Relative measurement errors, %

	T_2=10 °C									
Temperature	Ti_1	Ti_2	Ti_3	Ti_4	Ti_5	Ti_6	Ti_7	Ti_8	Ti_9	Ti_{10}
T_{11}	0.0222	0.0202	0.0184	0.0169	0.0157	0.0146	0.0136	0.0128	0.0120	0.0113
T_{12}	0.0202	0.0183	0.0167	0.0154	0.0142	0.0132	0.0124	0.0116	0.0109	0.0103
T_{13}	0.0185	0.0168	0.0153	0.0141	0.0130	0.0121	0.0113	0.0106	0.0100	0.0094
T_{14}	0.0171	0.0155	0.0141	0.0130	0.0120	0.0112	0.0104	0.0098	0.0092	0.0087
T_{15}	0.0159	0.0144	0.0131	0.0121	0.0112	0.0104	0.0097	0.0091	0.0085	0.0081
T_{16}	0.0148	0.0134	0.0122	0.0112	0.0104	0.0097	0.0090	0.0085	0.0080	0.0075
T_{17}	0.0139	0.0126	0.0115	0.0105	0.0097	0.0091	0.0085	0.0079	0.0075	0.0070
T_{18}	0.0130	0.0118	0.0108	0.0099	0.0092	0.0085	0.0080	0.0075	0.0070	0.0066
T_{19}	0.0123	0.0112	0.0102	0.0094	0.0087	0.0080	0.0075	0.0070	0.0066	0.0062
T_{10}	0.0117	0.0106	0.0096	0.0089	0.0082	0.0076	0.0071	0.0067	0.0063	0.0059

Thus, processing of measurement results according to the redundant measurement equation (14) is recommended for measuring high temperatures $T_i \geq 100$ °C and $T_1=190$ °C. During the analysis of Table 8, it was found that the best accuracy results were obtained at $T_1=T_i=190$ °C.

It should be noted that for the case shown in Table 8, an order of magnitude increase in the reproduction accuracy of the normalized temperature T_1 (with a constant reproduction error of T_2) does not increase the measurement accuracy. So, this gives grounds to use a non-precision normalized source for T_1 reproduction.

The calculations confirmed that an order of magnitude increase in the reproduction accuracy of normalized temperatures T_1 and T_2 leads to an order of magnitude increase in the measurement accuracy.

5. 3. Comparative analysis of the two systems

Comparisons of the results obtained in the study of redundant measurement equations (11) and (14) were made according to the following criteria:

1) measurement capability over the entire operating range;
2) measurement accuracy;
3) number of measurement cycles;
4) ease of use.

The redundant measurement system (10) is considered below, which describes and consists of 5 measurement cycles in relation to the specified criteria:

1) applies to the entire range provided that $T_1=T_i(0.0005 \cdot T_i+1)$ and $T_1>T_2$ (recommended range of normalized temperatures $T_1=(10 \div 100)$ °C and $T_2=(1 \div 10)$ °C);
2) in the measurement range from 10 °C to 200 °C, provides high-precision measurement with a relative error of 0.0100 % to 0.00003 % at $T_1=T_i(0.0005 \cdot T_i+1)$;
3) to implement it, you need to perform 5 measurement cycles;
4) high-precision measurement in a wide range requires the relationship between the desired temperature T_i and the normalized temperature $T_1=T_i(0.0005 \cdot T_i+1)$.

The redundant measurement system (13) is considered below, which describes and consists of 6 measurement cycles in relation to the specified criteria:

1) it is recommended to use for measuring high temperatures $T_i \geq 100$ °C;
2) in the measurement range from 100 °C to 190 °C at $T_1=190$ °C, provides high-precision measurement with a relative error from 0.0117 % to 0.0059 %;
3) to implement it, you need to perform 6 measurement cycles;
4) high-precision measurement of high temperatures requires compliance with the condition for setting the normalized temperature value T_1, which corresponds to the end of the operating range ($T_1=190$ °C).

Thus, when using a sensor with an unstable quadratic TF to increase the accuracy in the whole range, a system of nonlinear equations of quantities (10) and the equation of redundant measurements (11) should be used to process the obtained measurement results.

6. Discussion of the results of computer modeling of the quadratic transformation function

When applying RMM, it is necessary to generate redundancy or additional data, as in [16], which, when processed according to a certain algorithm, as in [9, 14], will improve accuracy. Similarly, for quadratic TF, two variants of redundancy formation systems were studied, consisting of 5 and 6 equations of quantities, each of which describes the measurement cycle. It should be noted that in each cycle, there is a measurement of the desired temperature, normalized temperatures T_1 and T_2, and their combination. To create normalized quantities (temperatures), it is necessary to have standardized sources with normalized characteristics, because accurate knowledge of normalized quantities during their further processing helps to improve accuracy, as noted in [12, 13]. In turn, the use of high-precision elements in measurements, as in [18], reduces the methodological error of RMM.

As a result of solving the proposed systems (10) and (13), the corresponding equations of redundant measurements (11) and (14) were obtained. Processing of measurement results in accordance with the proposed equations provides automatic exclusion of the additive and multiplicative components of the error, i.e. error correction, which does not contradict the data presented in [19–25]. In addition, due to the application of the proposed redundant measurement equations (11) and (14), it is possible to directly use a nonlinear TF without special measures for its linearization, which are typical for studies published in [12–14].

When studying the effect of normalized quantities on the measurement result for systems (10) and (13), as can be seen from Tables 1–3 and Tables 5–7, it was found that T_1 has a greater impact than T_2. This is especially true for the system (13) with the corresponding redundant measurement equation (14). This effect of normalized quantities on the measurement result, according to the authors, is due to the type of equations of quantities: so in equation (11) there are two quantities T_1 and T_2 in the denominator, and for equation (14) – only T_1. This means that such a mechanism of influence of the normalized quantity T_1 is the process regulation factor that can affect the relative measurement error. The influence of the values of certain parameters on the measurement result does not differ from the practical data presented in [7, 12]. It was also found that for both systems (10) and (13), the best accuracy results (Tables 2, 6) were obtained for the case of $T_1=(10 \div 100)$ °C, $T_2=(1 \div 10)$ °C. For the best accuracy, studies were conducted on the effect of reproduction errors of normalized temperatures, which are part of the methodological error of RMM on the accuracy of the result. The studies were carried out first with an order of magnitude increase in the reproduction accuracy of the normalized temperature T_1, and then with an order of magnitude increase in the reproduction accuracy of both normalized temperatures T_1 and T_2. As a result, it was found that increasing the reproduction accuracy of the normalized temperature T_1 (with a constant reproduction error of T_2) does not lead to a significant improvement in the results. This gives grounds to use a non-precision normalized source for T_1 reproduction. On the other hand, an order of magnitude increase in the reproduction accuracy of both normalized temperatures T_1 and T_2 also increases the measurement accuracy by an order of magnitude.

Since the best accuracy results were obtained at $T_1=(10 \div 100)$ °C, $T_2=(1 \div 10)$ °C, more detailed studies of the system (10) and the corresponding equation (11) are of particular interest. In further research (Table 4), a certain relationship was found for the controlled T_i and normalized T_1 quantities, at which the smallest error was observed.

Thus, it becomes possible to measure with a relative error of 0.002 %, with the proximity of T_1 to T_i, namely T_1=T_i=50 °C, and T_2=(1÷10) °C. Therefore, to identify the relationship between the controlled T_i and normalized T_1 quantities, at which an increase in accuracy is observed, computer analysis was performed. Thus, it was found that the ratio between the values of T_1 and T_i is $T_1=T_i(0.0005 \cdot T_i+1)$, which provides high-precision measurement with a relative error of 0.01 % to 0.00003 % in the range of measured temperatures from 10 to 200 °C. Thus, compliance with the ratio between the controlled T_i and normalized T_1 quantities provides high-accuracy results in a wide range of values. This also agrees with the conclusions of [8], where an increase in measurement accuracy is also associated with compliance with the established parameter corrections.

For system (10) with equation (11), detailed studies were also conducted for another relationship between T_1 and T_2: T_1=T_2=(10÷100) °C and at the value of the controlled temperature T_i=100 °C, i.e. without observing the found ratio $T_1=T_i(0.0005 \cdot T_i+1)$. Thus, the functional dependence of the relative measurement error on the normalized values of T_1 and T_2 and at the value of the controlled temperature T_i=100 °C was constructed (Fig. 1). The analysis of the resulting surface revealed a certain undesirable ratio between T_1 and T_2, at which measurements become unacceptable. Therefore, to identify the undesirable ratio, computer analysis was performed, which found that $T_2 \neq 2 \cdot T_1$. It should be noted that when the difference between T_1 and T_i increases, this ratio has a certain range with a maximum error at the point $T_2 \neq 2 \cdot T_1$. This indicates the expediency of observing the found relationship between the values of the normalized T_1 and controlled T_i temperature, which is $T_1=T_i(0.0005 \cdot T_i+1)$.

When determining the nature of the effect of the normalized quantities T_1 and T_2 for the system (13) with the corresponding redundant measurement equation (14), the study was conducted with similar initial data as for equation (11). The best accuracy results (Tables 5–7), in contrast to the results obtained for equation (11), are obtained only at large (more than 100 °C) values of T_1. But, at the same time, the range of normalized temperatures T_1=(10÷100) °C and T_2=(1÷10) °C, which provides the smallest measurement error, was the same as for equation (11). For this range, studies were also conducted on the effect of reproduction errors of normalized temperatures T_1 and T_2 on the measurement accuracy. The results of this study were similar to those for equation (11). This indicates the possibility of using a non-precision normalized source for T_1 reproduction. In the further analysis of the obtained data (Tables 5–7), it was found that when the total value of T_1 and T_i increases, the relative error value decreases. Therefore, to identify the relationship between the normalized T_1 and controlled T_i temperature, computer modeling was performed at T_1=(100÷190) °C and T_i=(100÷190) °C. Since, as can be seen from Tables 1–3, 5–7, the parameter T_2 does not have a significant effect on the measurement result, T_2=10 °C was taken for calculations. As a result of computer modeling, it was found that the smallest error (0.0059 %) is obtained at T_1=T_i=190 °C (Table 8). This means that when applying the redundant measurement equation (14), it is necessary to set the value of the normalized temperature T_1, which corresponds to the end of the operating range (T_1=190 °C).

The analysis of the results obtained, presented in p. 5. 3, showed that the system of equations of quantities (10) with 5 equations proved to be the best compared to the system (13) with 6 equations. It should be noted that in terms of statistical data processing, the opposite results were expected. This indicates that the increase in the accuracy of RMM measurements is influenced not by the number of measurements, but by the type of redundant measurement equation. In addition, the ability to use systems with a different number of equations of quantities indicates the flexibility of using RMM depending on the possibility of their implementation, as well as the possibility of reproducing normalized quantities.

Thus, the obtained data on the application of the system (10) with the redundant measurement equation (11) for an unstable quadratic TF allow us to state the following:

1) by observing the relationship between the values of the normalized T_1 and controlled T_i temperature, which is $T_1=T_i(0.0005 \cdot T_i+1)$, high-precision measurement in a wide range becomes possible,

2) there is no reason to use a precision normalized source for T_1 reproduction.

Such conclusions can be considered expedient from the practical point of view as they allow high-precision measurement in a wide range while observing the found ratio between the values of normalized T_1 and controlled T_i temperature. In addition, the use of a non-precision normalized source to reproduce the normalized temperature T_1 allows reducing the requirements for its power and accuracy. It should also be noted that the application of the redundant measurement equation (11) ensures the independence of measurement results from the absolute values of the parameters of the transformation function and their deviations from the nominal values. Due to this, inexpensive sensors can be used for RMM (with low stability requirements). From a theoretical point of view, the relationship between T_1 and T_i of the form $T_1=T_i(0.0005 \cdot T_i+1)$ allows us to argue about the possibility of improving the accuracy of measurement in a wide range of values, which is a certain advantage of this study. Another advantage is determining the impact of T_1 and T_2 on the measurement result. It was found that T_1 has a greater influence on the measurement result than T_2. In addition, the best accuracy results for equations (11) and (14) were obtained for the range of normalized temperatures T_1=(10÷100) °C and T_2=(1÷10) °C. However, it should be noted that such accuracy results are obtained when changes in the parameters of the transformation function remain constant during the measurement cycles. Given the found ratio between T_1 and T_i, knowledge of the current value of the controlled temperature is required, which is not always known. In addition, when applying RMM, there is a methodological error due to the error of reproduction of the normalized temperature T_2. The inability to remove these limitations in this study creates a potentially interesting direction for further research.

7. Conclusions

1. Computer modeling of the presented system of nonlinear equations of quantities consisting of 5 equations was carried out. The modeling was performed at different values of normalized temperatures T_1 and T_2 and with changes in the parameters of the transformation function within ±10.0 %. The studies have shown that T_1 has the greatest influence on the measurement error among the normalized quantities. Based on the research, it was found

that $T_1>T_2$, and the recommended range of normalized temperatures is $T_1=(10\div100)$ °C and $T_2=(1\div10)$ °C. In addition, it was found that for the redundant measurement equation (11), the relationship between T_1 and T_2 should be observed, where $T_2\neq2\cdot T_1$. The studies of the effect of reproduction errors of normalized temperatures found that increasing the reproduction accuracy of the normalized temperature T_1 (with a constant reproduction error of T_2) does not significantly improve the results. So, it can be argued on the use of non-precision normalized sources for T_1 reproduction. An order of magnitude increase in the reproduction accuracy of both normalized temperatures T_1 and T_2 also increases the measurement accuracy by an order of magnitude, which indicates the relationship of these normalized quantities with controlled T_i. The relationships between the controlled and normalized quantities were determined, which reduce the measurement error in a wide range. It is found that when applying the redundant measurement equation (11), it is recommended to adhere to such a relationship between T_1 and T_i at which $T_1=T_i(0.0005\cdot T_i+1)$. This provides high-precision measurement with a relative error of 0.01 % to 0.00003 % in the range of measured temperatures from 10 °C to 200 °C. This indicates the possibility of increasing the measurement accuracy with an unstable quadratic transformation function by correctly setting the normalized quantity T_1.

2. Computer modeling of the presented system of nonlinear equations of quantities consisting of 6 equations, with the same initial data as for the system of 5 equations was carried out. The studies have shown that T_1 has the greatest influence on the measurement error among the normalized quantities, as for the system of 5 equations. The studies found that $T_1>T_2$, and the recommended range of normalized temperatures is $T_1=(10\div100)$ °C and $T_2=(1\div10)$ °C. It was found that an order of magnitude increase in the reproduction accuracy of the normalized temperature T_1 (with a constant reproduction error of T_2) does not increase the measurement accuracy. Therefore, we can conclude on the use of a non-precision normalized source for T_1 reproduction, as for the system of 5 equations. An order of magnitude increase in the reproduction accuracy of both normalized temperatures T_1 and T_2 also increases the measurement accuracy by an order of magnitude, which indicates the relationship of these normalized quantities with controlled T_i. The studies have shown that when applying the redundant measurement equation (14) and at $T_1=T_i=190$ °C, the error is reduced to 0.0059 %. This indicates the need to set the value of the normalized temperature T_1, which corresponds to the end of the operating range. Thus, it is recommended to use a system of 6 equations of quantities and the corresponding redundant measurement equation (14) when measuring large values of the controlled temperature T_i.

3. A comparative analysis of the proposed systems with the corresponding redundant measurement equations was carried out. When comparing the results obtained according to equations (11) and (14), the prospects of conducting 5 measurement cycles in accordance with the system (10) were evaluated. Compared to the system of 6 equations, processing of the results using equation (11) makes it possible to obtain high-precision results over the entire measurement range of temperatures, and also requires fewer measurement cycles.

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