This paper analyzes the influence of kinetic and physical-mechanical parameters of systems on the characteristics of dynamic processes in moving one-dimensional nonlinear-elastic systems. Improved convenient calculation formulas have been derived that describe the laws of changing the amplitude-frequency characteristics of systems for both a non-resonant case and a resonant one. An important issue of studying the influence of the speed of movement of elements of mechanisms on the oscillations of one-dimensional nonlinear-elastic systems has not been considered in detail until now in the scientific literature. This issue relates to the vibrations of shafts in gears, pipe strings when drilling oil and gas wells, the oscillations of turbine blades and rotating turbine discs, the longitudinal vibrations of the beam as an element of structures. The main reason for this in the analytical study of dynamic processes were the shortcomings of the mathematical apparatus for solving the corresponding nonlinear differential equations that describe the laws of motion of those systems.

It was found that in the case of longitudinal oscillations in the moving beam with an increase in the longitudinal speed of the medium to 10 m/s, the amplitude of the oscillation also increases by 13.5 %. However, when the longitudinal velocity of the beam is 5 m/s, the amplitude will increase by only 3 %. It is established that with the growth of the amplitude, the frequency of longitudinal oscillations decreases sharply, and if the system moves at a higher speed, for example, 20 m/s, it reduces the frequency of oscillation by about 13 %.

The results reported here make it possible to assess the effect of kinetic and physical-mechanical parameters on the frequency and amplitude of oscillations. The research that involved the asymptotic method makes it possible to predict resonant phenomena and obtain engineering solutions to improve the efficiency of technological equipment

Keywords: nonlinear oscillations, asymptotic method, elastic beam, longitudinal oscillations, torsional oscillations

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ADVANCING ASYMPTOTIC APPROACHES TO STUDYING THE LONGITUDINAL AND TORSIONAL OSCILLATIONS OF A MOVING BEAM

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1. Introduction

With the development of new technology, with increasing speeds, that is, during the transition to high-speed engineering, the role of fluctuations in the elements of mechanisms has become especially relevant. In particular, this concerns the processes of oscillation of the drilling part of the rig (column) when drilling oil and gas fields [1], fluctuations in the ring combustion chamber in cars [2], vibrations in the elements of power plants [3]. The stricter operational requirements for the safe and efficient operation of modern machines lead to the fact that increased attention in engineering calculations is paid to solving problems associated with longitudinal and torsional oscillations. Based on the theory of nonlinear oscillations, important problems of machine building balancing machines, torsional oscillations of shafts and gears, oscillations of turbine blades and rotating turbine disks, longitudinal oscillations of the beam as an element of structures, etc. were investigated. Important models relating to nonlinear oscillations are considered in the problems of helioseismology [4]. Of practical importance are these studies in models of nonlinear fluctuations of the railroad track [5]. Experimental research shows that even minor speeds of movement (including angular ones) lead to changes in both quantitative and qualitative characteristics of dynamic processes [6]. Such differences are considered between the nonlinear-elastic one-dimensional system and their analogs, which are not characterized by longitudinal (for longitudinal oscillations) or rotational (for torsional oscillations) movements [7].

Only after numerous experiments related to the problems of dynamic processes in mechanical systems, the difference between the mechanics of nonlinear oscillations and the mechanics of linear oscillations, which is fully preserved even when considering weakly nonlinear oscillations, became apparent [8]. Mathematical models of such oscillations are described by differential equations, which differ from linear equations with constant coefficients only by the presence of sufficiently small terms [9]. Therefore, effective in the study of models of such systems are asymptotic methods of nonlinear mechanics [10].

All real physical systems are nonlinear. The peculiarity of nonlinear systems is the failure to fulfill the principle of superposition in them. This means that individual harmonic vibrations interact with each other. Significant difficulties are that it is not always possible to use the Poincare method to obtain results that would be suitable for studying movement over a long enough period [11].

Therefore, deriving and using convenient calculation formulas that describe the laws of changing amplitude-frequency characteristics (AFCs) and take into consideration kinematic and physical characteristics is a relevant task. Such studies are a determining factor in investigating the dynamics of moving environments. They are relevant both at the design stage and in the operation of mechanisms.

2. Literature review and problem statement

Actual oscillatory systems are characterized by various physical parameters, in particular, rigidity, mass, and characteristics of damping. For such one-dimensional systems, dynamic processes are studied in detail (for practical purposes) if the body material matches the linear or similar law of elasticity [12]. If, in addition, such systems move at constant or variable speeds along their geometric axis, then the study of the corresponding oscillations, even for the case of linearly elastic properties of the material, is associated with significant mathematical difficulties.

There are several analytical methods for constructing and studying solutions to nonlinear differential equations describing the movements of mechanical systems [13]. Note that they reject the assumption of the error in these solutions.

Methods that have already become classical are most effectively applied to nonlinear systems with one degree of freedom and are generalized to systems with a finite number of degrees of freedom [14]. However, with an increase in the number of generalized coordinates of the system, the possibility of obtaining analytical solutions is significantly complicated. However, the use of computing equipment in some cases makes it possible to overcome such difficulties.

Many approximate methods have been devised to calculate the periodic movements of nonlinear systems. In particular, quite often there is a method of harmonious balance [15]. However, the possibility of applying this method to stationary systems is determined by the proximity of the periodic movement of the system to the harmonic one. This condition is usually satisfied only when the linear parts of the system are low-frequency filters, that is, they filter out high harmonics well.

Variational methods are also used. In particular, the Ostrohrad-Hamilton principle is used to solve the equations of longitudinal, torsional, and transverse vibrations of the rod [16]. By establishing the equivalence of solving boundary problems to solving problems about the extremum of the functionality, this principle opens up the possibility for application to vibration calculations of some special methods of variational calculus. These include, first of all, the so-called direct methods of variational calculus, the use of which is effective only in approximate calculations of their natural frequencies and shapes of oscillations of rods of the variable cross-section with an uneven distribution of rigidity and mass.

The most widespread in engineering computing practice are the methods by Relay, Ritz, Galerkin [17]. The essence of these methods is the use in the calculation of transverse oscillations of the heterogeneous rod of the Ostrohrad-Hamilton functionality. However, in this case, the influence of linear velocity (with longitudinal fluctuations of the beam) and angular (with torsional oscillations of shafts, pipe strings when drilling oil and gas wells, etc.) was neglected.

These properties of nonlinear systems with concentrated masses and distributed parameters greatly simplify their research procedures. Underlying the study of such systems is the principle of single-frequency oscillations in nonlinear systems with many degrees of freedom and distributed parameters [18], the asymptotic methods for constructing solutions to some classes of differential equations with partial derivatives. However, the presence of dissipative and other nature of nonlinear forces, as well as external perturbing forces in real-world systems, leads to the rapid disappearance of higher harmonic oscillations. In addition, the dynamics of movement with a frequency close to the frequency of external perturbing force or basic harmonics are established.

Our review of literary sources [12–18] reveals that such an important issue as the influence of the speed of movement of elements of mechanisms on the oscillations of one-dimensional nonlinear elastic systems was not considered in detail. The main reason for this in the analytical study of dynamic processes was the lack of a mathematical apparatus for solving appropriate nonlinear differential equations that describe the laws of motion of those systems. It is necessary to study both a non-resonant case and a resonant one. This makes it possible to determine the influence of perturbing force with a frequency close to the frequency of oscillations of the system. In this regard, it becomes necessary to conduct research and do something in this area.

3. The aim and objectives of the study

The purpose of our study is to determine the influence of kinetic and physical-mechanical parameters of systems on the characteristics of dynamic processes of moving one-dimensional nonlinear-elastic systems. This makes it possible to predict resonant zones and establish the most effective modes of operation of the equipment, to establish less stringent requirements for the system and its elements.

To accomplish the aim, the following tasks have been set:

– to suggest a procedure for constructing mathematical models that describe the dynamic processes of mechanical systems characterized by longitudinal (for longitudinal oscillations) movement;

– to offer a procedure for constructing mathematical models that describe the dynamic processes of mechanical systems characterized by rotational (for torsional oscillations) motion;

– to derive mathematical relations that determine the laws of changing the amplitude, frequency (period) of oscillation, as functions from parameters that characterize the physical-mechanical and kinematic properties of the medium;

– to conduct numerical modeling of the influence of kinematic and physical-mechanical quantities on the nature of changes in amplitude and frequency.

4. The study materials and methods

Using an example of longitudinal oscillations of the moving beam, the influence exerted on the dynamics of the oscillatory process by the physical-mechanical, kinematic, and force factors is investigated. In particular, the speed of its longitudinal movement, nonlinearly elastic characteristics of the beam material, external periodic perturbations are analyzed.

In the study of single- and multifrequency modes of longitudinal oscillations of the beam, practical tasks are often encountered for the case of hinged ends [19]. In a linearly elastic statement, the Fourier method can be applied to them and reduce the initial problem to the study of ordinary differential equations or a system of ordinary differential equations. The current paper deals with more complex issues:

a) the beam moves along its undeformed axis at a constant speed;

b) the material of the beam satisfies the elasticity close to the linear law;

c) external periodic disturbances act on the beam.

The one-dimensional system (beam) is investigated, which is described by the function of two variables $u=u(x, t)$ coordinates x and time t. Such oscillations can be described by the differential equation [20]:

$$\rho\frac{\partial^2 u}{\partial t^2}-E\frac{\partial^2 u}{\partial x^2}=\varepsilon X_1, \qquad (1)$$

where E is a module of elasticity of the first kind (Young modulus); X_1 is the first natural shape of oscillations (amplitude function, which is unchanged over time); ε is some small positive parameter; ρ is the specific mass of the rod material, kg/m^3.

The oscillation equation (1) for individual practical cases can be built by using kinematic hypotheses (flat cross-sections, straight normals, etc.). However, the disadvantage of such methods is the inability to derive a solution for resonant cases. These solutions can be obtained only taking into consideration internal or external friction. Internal friction in the case of linear nonstationary or stationary oscillations can be taken into consideration using the Boca-Schlippe-Colar or Kelvin-Foigt hypotheses. In addition, for a stationary case, one can also use the method of complex elasticity modules (Sorokin hypothesis). The Kelvin-Foigt hypothesis is employed for stationary or nonstationary oscillations, even though it is not experimentally confirmed for metallic materials. Sorokin's linear hysteresis hypothesis corresponds to experimental results but is used only in the case of fluctuations that have already been established. Its application on nonstationary oscillations is not mathematically correct because there are both stable and unstable partial solutions to the equation. When one discards unstable partial solutions based on physical reasons, the principle of superposition is violated.

The wave equation of the free longitudinal oscillations of the beam, which has a constant cross-section, can be written as [21]:

$$\frac{\partial^2 u}{\partial x^2}-\frac{1}{s^2}\frac{\partial^2 u}{\partial t^2}=\varepsilon f\left(\varepsilon,x,\eta,\frac{\partial u}{\partial x},\frac{\partial^2 u}{\partial x^2}\right), \qquad (2)$$

where $s=\sqrt{E/\rho}$ is the coefficient that determines the frequency of the system, phase velocity (the rate of propagation of longitudinal waves in the rod); ρ is the specific mass of the rod material, kg/m^3; E is the elasticity module; ε is the small positive parameter; t – time; $f\left(\varepsilon,x,\eta,\frac{\partial u}{\partial x},\frac{\partial^2 u}{\partial x^2}\right)$ is the function, infinitely differentiable in all its arguments, periodic relative to η with a period of 2π; $d\eta/dt=\nu(t)$ is the positive function. When there is no perturbation ($\varepsilon=0$), we obtain a purely harmonious oscillation $u(x,t)=aX(x)\cos(\omega t+\varphi)$, where $d\varphi/dt=\omega$. The shape of oscillations is determined by the function $X(x)$, then $du(x,t)/dt=-a\omega X(x)\sin(\omega t+\varphi)$. In this case, a and φ are some constants, the amplitude will be constant $da/dt=0$ with a uniform phase angle φ. Thus, when $f\left(\varepsilon,x,\eta,\frac{\partial u}{\partial x},\frac{\partial^2 u}{\partial x^2}\right)=0$, equation (2) can be solved by standing waves (Fourier method) and running waves (d'Alembert's method).

To build an asymptotic solution to the perturbed system in which single-frequency oscillations occur, it is necessary to meet the following conditions:

1) in an undisturbed system, non-damping harmonic oscillations with a frequency $\omega_1(\tau)$ are possible, which depends only on two free constants;

2) the only solution to the equation of an undisturbed system is trivial;

3) there are no internal resonances in the undisturbed system, that is, $\omega_1(\tau)\neq\omega_k(\tau)$ ($k=2, 3,...N$);

4) initial conditions ensure the existence of a single-frequency oscillation mode, that is, $u|_{t=0}=pX_i(x)$, $\partial u/\partial t|_{x=0}=qX_i(x)$, where p and q are real numbers, $X_i(x)$ is a fundamental function of the undisturbed boundary problem, which is described by a wave equation and linear homogeneous boundary conditions ($\varepsilon=0$). Under such assumptions, the solution to the equation of the disturbed system is to be derived in the form [22]:

$$u(x,t)=a(t)X(x)\cos(\psi)+$$
$$+\varepsilon u_1(a,\eta,\psi,x)+\varepsilon^2 u_2(a,\eta,\psi,x), \qquad (3)$$

where $a(t)$ is the amplitude of single-frequency oscillations; $u_1(a,\eta,\psi,x)$, $u_2(a,\eta,\psi,x)$ are the 2π-periodic functions with variables ψ and η.

Only those cases for which the length of the longitudinal waves of oscillations is large compared to the size of the cross-sections of the beam are considered. In such cases, it is possible to neglect the influence of transverse movements on the nature of longitudinal movements.

5. Results of studying the longitudinal and torsional oscillations of the moving beam

5. 1. Differential equation of the longitudinal oscillations of the moving beam

Among the different types of natural fluctuations arising in the nonlinearly elastic rod, longitudinal fluctuations occupy a significant place. They are the easiest to investigate. We assume that the cross-section of the beam is flat and each point of such a cross-section executes only axial movements (moving only along the axis). Longitudinal stretching and compression, which occur with such fluctuations in the beam, are accompanied by the occurrence of transverse deformations. Fig. 1 shows a load-free prismatic beam with a length of l, an infinitesimally small element of which is equal to dx and placed at a distance of x from the left end. Through u, the longitudinal movement of the cross-sectional point with the coordinate x is indicated.

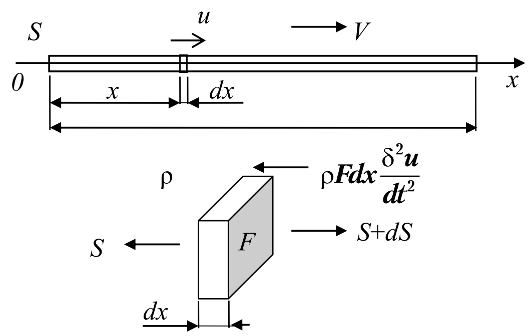

Fig. 1. Diagram of forces acting on a beam element moving along its axis

The beam moves along its axis at a constant speed, so:

$$\frac{d}{dt}=\frac{\partial}{\partial x}\frac{dx}{dt}+\frac{\partial}{\partial t}=V\frac{\partial}{\partial x}+\frac{\partial}{\partial t}, \tag{4}$$

$$\frac{d^2}{dt^2}=\frac{\partial^2}{\partial x^2}\left(\frac{dx}{dt}\right)^2+2\frac{dx}{dt}\frac{\partial^2}{\partial x\partial t}+\frac{\partial^2}{\partial t^2}=$$
$$=V^2\frac{\partial^2}{\partial x^2}+2V\frac{\partial^2}{\partial x\partial t}+\frac{\partial^2}{\partial t^2}. \tag{5}$$

When the beam fluctuates longitudinally, the sum of the longitudinal forces acting on an infinitesimal element of the beam (Fig. 1), in accordance with the d'Alembert's principle, takes the following form:

$$\frac{\partial S}{\partial x}-\rho F\left[\frac{\partial^2 u}{\partial t^2}+2V\frac{\partial^2 u}{\partial x\partial t}+V^2\frac{\partial^2 u}{\partial x^2}\right]=$$
$$=\varepsilon f\left(u,\frac{\partial u}{\partial t},\frac{\partial u}{\partial x},\frac{\partial^2 u}{\partial x^2}\right), \tag{6}$$

where S is the equivalent to internal stresses arising in cross-section with the coordinate x, which is directed along the axis; ρ is the density of material; F is the cross-sectional area of the beam; V is the speed of moving the beam along its undeformed axis; $\varepsilon f\left(u,\frac{\partial u}{\partial t},\frac{\partial u}{\partial x},\frac{\partial^2 u}{\partial x^2}\right)$ is the function that takes into consideration the nonlinear elastic properties of the beam material, as well as dissipative forces and resistance forces. Below it is accepted that they are small compared to nonlinearly elastic forces.

The internal force appearing in equation (6) is equal to the product of the material density by the volume of the small segment Fdx. Using Hooke's law, the longitudinal force S can be expressed through longitudinal stress and through axial deformation in the form of:

$$S=g(\sigma)=EF\varepsilon_x+\delta_1(\sigma,\varepsilon,\varepsilon_x), \tag{7}$$

where E is the Young module; δ_1 is a function that characterizes the nonlinearity of the system. Substituting expressions (4), (5) in equation (6), taking into consideration (7), we obtain after transformations:

$$\frac{\partial^2 u}{\partial t^2}-\alpha^2\frac{\partial^2 u}{\partial x^2}+2V\frac{\partial^2 u}{\partial x\partial t}+V^2\frac{\partial^2 u}{\partial x^2}=$$
$$=\varepsilon f\left(u,\frac{\partial u}{\partial t},\frac{\partial u}{\partial x},\frac{\partial^2 u}{\partial x^2}\right), \tag{8}$$

where $\alpha=\sqrt{E/\rho}$. Equation (8) describes the longitudinal oscillations of the moving beam. It can be called a one-dimensional wave to indicate that during longitudinal oscillations, the contour of movements spreads in the axial direction, that is, at the speed of sound propagation in the material.

It can be noted that equation (8) is similar to the equation of transverse oscillations of the string. The difference is only in the physical content of some coefficients while the principle remains the same. After all mathematical transformations, the solution to equation (8) is defined as follows:

– for a non-resonant case:

$$\frac{da}{dt}=\varepsilon\frac{1}{p}\frac{1}{4\omega\pi^2}\int_0^l\int_0^{2\pi}F(a,x,\psi)\sin\frac{\pi}{l}x\sin\psi\,\mathrm{d}x\mathrm{d}\psi, \tag{9}$$

$$\frac{d\psi}{dt}=\omega-\left(\frac{k\pi}{l}\right)^2\frac{V^2}{\omega}+$$
$$+\varepsilon\frac{1}{p}\frac{1}{4\omega\pi^2 a}\int_0^l\int_0^{2\pi}F(a,x,\psi)\sin\frac{\pi}{l}x\cos\psi\,\mathrm{d}x\mathrm{d}\psi;$$

– for a resonant case

$$(\omega-\nu)\frac{\partial^2 a}{\partial t\,\partial\varphi}-2a\omega\frac{\partial\varphi}{\partial t}=\frac{1}{p}\frac{1}{4\pi^2}\times$$
$$\times\sum_s e^{is\varphi}\int_0^l\int_0^{2\pi}F(a,x,\psi,\theta)\sin\frac{k\pi}{l}xe^{-is\varphi}\cos\psi\mathrm{d}x\mathrm{d}\psi, \tag{10}$$

$$a\frac{\partial^2\psi}{\partial t\,\partial\varphi}(\omega-\nu)-2a\frac{\partial a}{\partial t}+\frac{V^2}{\omega}\frac{\pi^2}{l^2}=\frac{1}{p}\frac{1}{4\pi^2}\times$$
$$\times\sum_s e^{is\varphi}\int_0^l\int_0^{2\pi}F(a,x,\psi,\theta)\sin\frac{k\pi}{l}xe^{-is\varphi}\cos\psi\mathrm{d}x\mathrm{d}\psi,$$

where $d\psi/dt=\omega$ and $d\eta/dt=\nu(t)$ are the positive frequencies of natural and perturbing oscillations, respectively.

5. 2. Differential equation of the torsional oscillations of the moving beam

Fig. 2 illustrates the torsional oscillations of a rectilinear shaft, which rotates around its axis. Through θ, the angle of torsion (around the axis of the shaft) of an arbitrary cross-section is indicated.

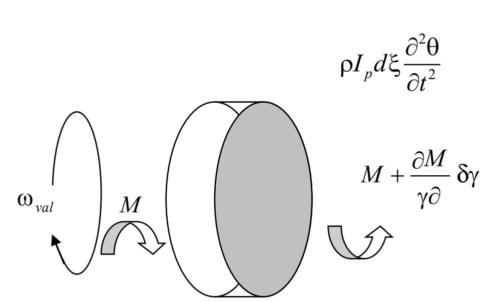

Fig. 2. Diagram of forces acting on the element of the shaft

With torsional oscillations, the equilibrium condition of elastic and inertial moments acting on a small element of the shaft, in accordance with the d'Alembert's principle, is written in the form of:

$$\frac{\partial}{\partial x}\left(GJ_p\frac{\partial\theta}{\partial x}\right)-\rho J_p\left[\frac{\partial^2\theta}{\partial t^2}+2\omega_{val}\frac{\partial^2\theta}{\partial t\partial x}+\omega_{val}{}^2\frac{\partial^2\theta}{\partial x^2}\right]=$$
$$=\varepsilon f\left(\theta,\frac{\partial\theta}{\partial t},\frac{\partial\theta}{\partial x},\frac{\partial^2\theta}{\partial x^2}\right), \quad (11)$$

where $\omega_{val}=dx/dt$ is the angular speed of rotation of the shaft around its axis; G is the elasticity module of the second kind; J_p is the polar moment of inertia of the cross-section; M is a moment that is equivalent to the internal forces acting in the cross-section; $\partial^2\theta/\partial t^2$ is the angular acceleration. According to the introduced designations, the moment of inertia of mass is equal to $\rho I_p d\xi\frac{\partial^2\theta}{\partial t^2}$. From the theory of simple torsion, we obtained the ratio:

$$M=\frac{\partial}{\partial x}\left(GJ_p\frac{\partial\theta}{\partial x}\right). \quad (12)$$

Substituting expression (12) in equation (11), after the transformations, we obtain:

$$\frac{\partial^2\theta}{\partial t^2}+\lambda^2\frac{\partial^2\theta}{\partial x^2}+2\omega_{val}\frac{\partial^2\theta}{\partial x\partial t}+\omega_{val}^2\frac{\partial^2\theta}{\partial x^2}=\varepsilon f\left(\theta,\frac{\partial\theta}{\partial t},\frac{\partial^2\theta}{\partial x^2}\right),$$
$$\lambda^2=\frac{G}{\rho}. \quad (13)$$

Equation (13) is a one-dimensional wave equation of torsional oscillations of the rotating shaft. Again, there is a certain similarity between equations (13) and (8).

Our equations coincide in form with similar equations and formula for longitudinal oscillations of the prismatic beam (8), if in the latter the values u, α, and E are replaced by θ, λ, and G, respectively. Therefore, all the results for the problem of longitudinal oscillations of prismatic beams can be extended to the problems of torsional oscillations of shafts of the circular cross-section by simply replacing the designations. Therefore, the solution to equation (13) is defined similar to (9), (10) as follows:

– for a non-resonant case:

$$\frac{d\theta}{dt}=\varepsilon\frac{1}{p}\frac{1}{4\omega\pi^2}\int_0^l\int_0^{2\pi}F(a,x,\psi)\sin\frac{\pi}{l}x\sin\psi\,\mathrm{d}x\,\mathrm{d}\psi, \quad (14)$$
$$\frac{d\psi}{dt}=\omega-\left(\frac{k\pi}{\theta}\right)^2\frac{\omega_{2val}^2}{\omega}+$$
$$+\varepsilon\frac{1}{p}\frac{1}{4\omega\pi^2 a}\int_0^l\int_0^{2\pi}F(a,x,\psi)\sin\frac{\pi}{l}x\cos\psi\,\mathrm{d}x\,\mathrm{d}\psi;$$

– for a resonant case:

$$(\omega-\nu)\frac{\partial^2\theta}{\partial t\,\partial\varphi}-2\theta\omega\frac{\partial\varphi}{\partial t}=\frac{1}{p}\frac{1}{4\pi^2}\times$$
$$\times\sum_s e^{is\varphi}\int_0^l\int_0^{2\pi}F(\theta,x,\psi,\theta_1)\sin\frac{k\pi}{l}xe^{-is\varphi}\cos\psi\,\mathrm{d}x\,\mathrm{d}\psi, \quad (15)$$
$$\theta\frac{\partial^2\psi}{\partial t\,\partial\varphi}(\omega-\nu)-2\theta\frac{\partial\theta}{\partial t}+\omega_{val}{}^2\frac{\pi^2}{\theta^2}=\frac{1}{p}\frac{1}{4\pi^2}\times$$
$$\times\sum_s e^{is\varphi}\int_0^l\int_0^{2\pi}F(\theta,x,\psi,\theta_1)\sin\frac{k\pi}{l}xe^{-is\varphi}\cos\psi\,\mathrm{d}x\,\mathrm{d}\psi,$$

where θ_1 is the frequency of the perturbing force that acts on the shaft.

Equations (14), (15) are similar in structure to equations (9), (10). The only difference is to change some coefficients but the very nature of the dynamic process is similar to the longitudinal oscillations of the beam. This is due to similar laws that describe the vibrations of the shaft or beam, and the same nonlinear differential equations that characterize this system.

5. 3. Laws of changing the amplitude and frequency of oscillation as functions of parameters that characterize the properties of the environment

With the help of the theory of nonlinear oscillations and the asymptotic method used in the current work, it is possible to establish the optimal characteristics of nonlinearly-elastic systems. Employing mathematical models (9), (10), and (14), (15), it is possible to expand the operating conditions of machines. Our results make it possible to utilize the equipment more efficiently in the event of fluctuations that almost always occur during operation.

Increased attention in engineering calculations is paid to solving problems associated with longitudinal and torsional oscillations. This is due to an increase in size and an increase in the speed of operation of modern machines. It is known that such important problems were investigated on the basis of the theory of nonlinear fluctuations. These include balancing machines, torsional vibrations of shafts and gears, the vibrations of turbine blades and rotating turbine discs, the longitudinal oscillations of the beam as an element of structures, etc. As a rule, with the help of this theory, it is possible to establish the optimal characteristics of nonlinear elastic systems. Such characteristics expand the operating conditions of machines, make it possible to use the equipment more efficiently in the event of fluctuations that almost always occur during the operation of the equipment.

An example of using the above defining ratios for determining the AFC of a dynamic process is considered. The longitudinal vibrations of the movable prismatic beam on which the harmonic force acts are investigated, provided that the beam material matches the nonlinear technical law of elasticity. The differential equation of motion of such a system is written in the form of:

$$\frac{\partial^2 u}{\partial t^2}-\alpha^2\frac{\partial^2 u}{\partial x^2}=$$
$$=-\frac{\partial^2 u}{\partial x^2}V^2-2\frac{\partial^2 u}{\partial x\partial t}V-\varepsilon\frac{\partial}{\partial x}\left(b\frac{\partial u}{\partial x}\right)^3+\varepsilon H\sin\nu t,$$

where $[b]=\mathrm{m}^2/\mathrm{s}^2$ is the coefficient having the dimensionality of the velocity square; the value of H is defined as the maximum value of the perturbing force per unit of mass of the beam.

If we consider that the boundary conditions for equations (6), (13) correspond to the hinged ends, then a single-frequency oscillatory process under a mode close to the frequency of external disturbances can be described as the following dependence:

$$u(x,t)=a\sin\frac{\pi}{l}x\cos(\nu t+\varphi),$$

moreover, the parameters a and φ for a resonant case are determined by a system of differential equations (10).

The law of changing the frequency of longitudinal oscillations of the beam (in accordance with equation (9)) is found from the following system:

$$\frac{da}{dt}=0, \qquad (16)$$

$$\frac{d\varphi}{dt}=\omega-\varepsilon\left(\frac{3}{8}\beta\frac{\pi^4}{l^4}\frac{a^2}{m\omega}+\left(\frac{\pi}{l}\right)^2\frac{V^2}{2\omega}\right).$$

A resonant case similar to the previously obtained system of equations (16) was obtained in a similar way:

$$\frac{da}{dt}=-\frac{\varepsilon H}{m\pi(\omega+\nu)}\cos\varphi, \qquad (17)$$

$$\frac{d\varphi}{dt}=\omega-\nu-\varepsilon\left(\frac{3}{8}\frac{\beta\pi^4 a^2}{\omega m l^4}+\frac{V^2\pi^2}{2\omega l^2}\right)+\varepsilon\frac{H}{m\pi(\omega+\nu)a}\sin\varphi.$$

System (17) demonstrates that as the speed increases, the frequency of longitudinal oscillations of the beam falls on the parabola.

5. 4. Numerical modeling of the influence of kinematic and physical-mechanical quantities on the nature of changes in amplitude and frequency

For the study, the following parameters are adopted: l=2 m, F=0.12·0.085 m^2, H=500 H, ρ=7.900 kg/m^3, E=2.06·10^{11} N/m^2, a=0.02 m, m=80.54 kg/m, I_0=6.1·10^{-6} m^4, and the frequency of the perturbing force $\omega_1=\pi/l\sqrt{E/\rho}=8017$ rad/s is the angular frequency of the first mode of longitudinal vibrations of the rod with fixed ends.

Fig. 3 shows how longitudinal velocity reduces the frequency of longitudinal oscillations. This decrease follows the parabolic law because in formula (16) there is a square of magnitude V, and, therefore, the effect of speed will be significant at high speeds (already at a speed of 20 m/s – the frequency of oscillations decreases by 7 %). Similarly, the initial amplitude affects such fluctuations. If the longitudinal velocity reaches 30 m/s, and the initial amplitude is 1 cm, then the oscillation rate of the dynamic system will be 4.5 kHz. This is 44 % less than with the natural oscillations of the beam, which does not move along its axis.

Fig. 4 shows a 3D plot of the dependence of the oscillation frequency on the length and initial amplitude at a speed of 5 m/s.

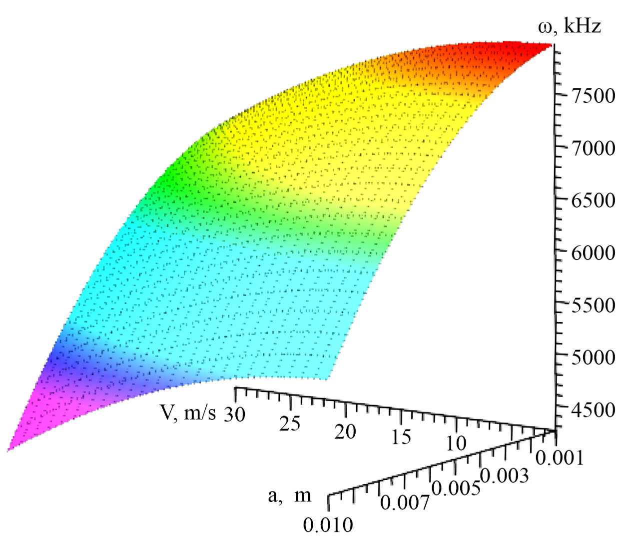

Fig. 3. Dependence of the frequency of oscillation of the system on speed and amplitude

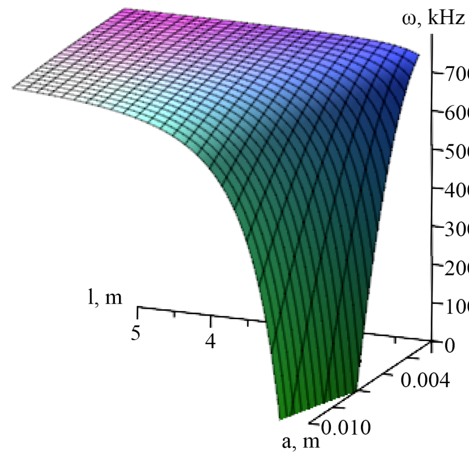

Fig. 4. Dependence of the frequency of longitudinal oscillations on the length of the beam and the initial amplitude at a speed of 5 m/s

Fig. 4 shows how, as the amplitude increases, the frequency of longitudinal oscillations decreases sharply. The length of the beam does not affect the oscillations so much if it is greater than 2 m; with a decrease in the length of the beam, the frequency also drops sharply and there may even be a breakdown of oscillations at a length of 0.5 m and an initial amplitude of 7 mm. If the system moves at a higher speed, for example, V=20 m/s (Fig. 5), then such a failure will occur even earlier. After all, speed also reduces the frequency by about 13 %.

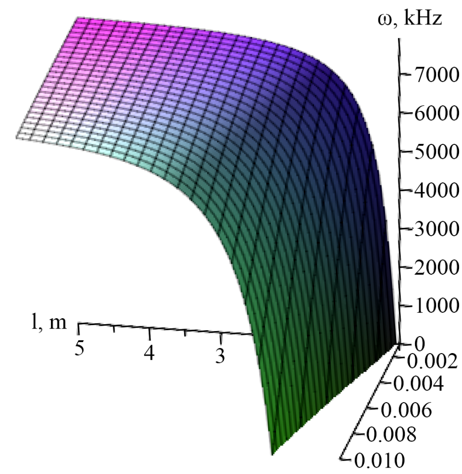

Fig. 5. Dependence of the frequency of longitudinal oscillations on the length of the beam and the initial amplitude at a speed of 20 m/s

The system of equations (16) demonstrates that the constant speed of the environment affects only the frequency of its transverse oscillations since the system is conservative.

As one can see from Fig. 6, the change in amplitude depends on the longitudinal speed of the beam. Although its impact at low speed is not very significant. When the longitudinal speed increases to 10 m/s, the amplitude increases by 13.5 %. However, when the speed is equal to 5 m/s, the amplitude value will increase by only 3 %, that is, there will be no such tangible impact. Consequently, with a further increase in the speed of the beam, the amplitude increases sharply.

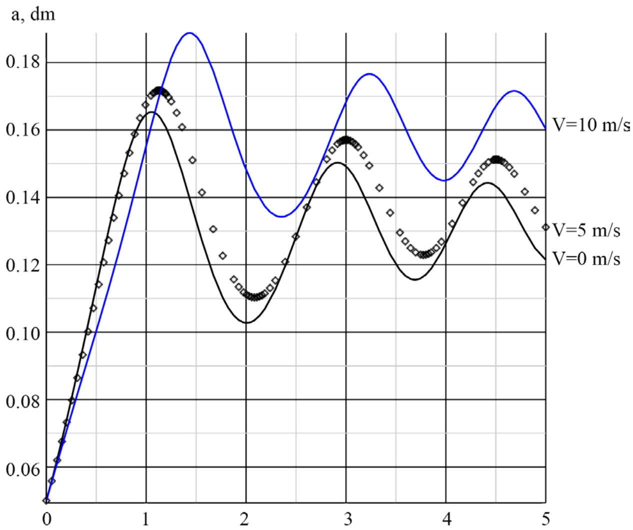

Fig. 6. Transient processes that occur during longitudinal oscillations of the beam for different speeds

As one can see from Fig. 7, the increase in longitudinal velocity has almost no effect on changing the phase of the beam oscillations. The nature of change in φ remains the same, the speed increase only slightly shifts the curve to the right, that is, the value of the φ changes later by 0.2 s.

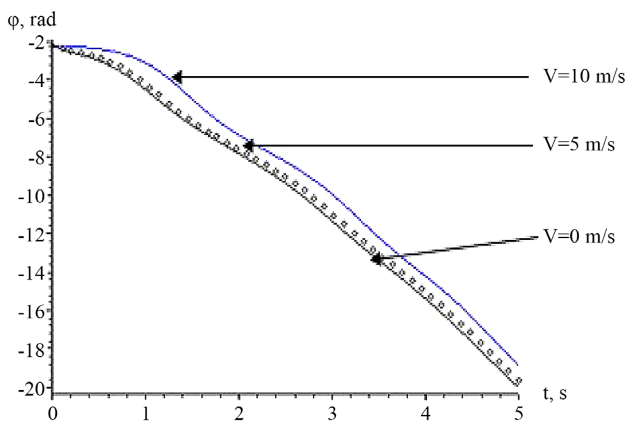

Fig. 7. Dependence of the phase of oscillations of the system for different speeds in a resonant case (V=10 m/s, V=5 m/s, V=0 m/s)

For such a system (17), one can build different graphic dependences.

6. Discussion of the influence of kinematic and physical-mechanical quantities on the amplitude-frequency characteristics of oscillations

In contrast to [17] where models of oscillatory processes are analyzed without taking into consideration the influence of linear velocity (with longitudinal oscillations of the beam) and angular velocity (with torque oscillations of shafts), our mathematical dependences (16), (17) make it possible to determine the change in frequency and amplitude depending on the kinematic parameters. This is made possible using the asymptotic method. Built on the basis of this method, computational algorithms make it possible to more accurately describe the dynamic process, in contrast to numerical approaches [14] where the derivation and analysis of solutions is much more complicated. Asymptotic approaches make it possible to establish an approximate solution with sufficient accuracy for engineering calculations, to determine various dynamic oscillation modes, in particular, resonant (17). This, in turn, makes it possible to avoid resonantly dangerous zones and set the optimal values of the parameters of the moving element, as well as reduce strict requirements for the system and its elements.

Our results illustrate the following properties of the considered oscillatory systems:

1. As follows from (16), the constant longitudinal velocity of the environment affects only the frequency of its transverse oscillations, since the system is conservative.

2. With increasing speed, the frequency of longitudinal oscillations of the beam falls according to the parabolic law because in formula (16) there is a square of the velocity value. Therefore, the impact of speed will be significant at high speeds (already at a speed of 20 m/s – the frequency of oscillations decreases by 7 %). This result follows from Fig. 3, which is a graphical representation of the solution to the equation system (16).

3. For the resonant case described by system (17), the change in amplitude depends on the longitudinal velocity. With an increase in the longitudinal speed of the environment to 10 m/s, the amplitude also increases by 13.5 %. However, when the longitudinal velocity of the beam is 5 m/s, the amplitude will increase by only 3 %, that is, there will be no such tangible effect. Consequently, with a further increase in the speed, the amplitude increases sharply. This is due to the form of the approximate solution obtained, as well as its graphic representation in Fig. 6.

4. The effect of longitudinal speed of movement on the change in amplitude and frequency of longitudinal oscillations of the beam is not so noticeable. However, in engineering and design calculations, even for such fluctuations, it is impossible to neglect such a kinematic quantity as the longitudinal speed of movement. The result is explained by the form of the right-hand part (16).

5. The approaches and results in our work can be extended for the case of mathematical models of torsional oscillations represented by dependences (14), (15). Similarly, it is easy to construct approximate solutions with the desired accuracy and corresponding graphical dependences.

The proposed approach has limitations related to the possibility of its use in the study of tasks with a sufficiently low speed of movement. In addition, the problems under consideration imply the presence of «small» nonlinear terms in mathematical models (the right-hand part (6)). Subsequently, our results and proposed approaches can be used to analyze fluctuations in nonlinearly dissipative systems.

7. Conclusions

1. With the help of the asymptotic method, functional dependences have been derived that determine the influence of physical and kinematic parameters on longitudinal oscillations for a beam that moves along its axis. Unlike the earlier reported results in [17], the mathematical models discussed in the current paper make it possible to take into consideration the influence of these parameters on a change in the amplitude and frequency of oscillation. The effectiveness

of the suggested procedure is, in particular, in more precise comparison with numerical methods [14] for predicting resonant modes of oscillatory process.

2. Mathematical models have been constructed that describe the dynamic processes of mechanical systems characterized by rotational motion oscillations for torsional oscillations. The peculiarity of our result is the ability to take into consideration the influence of angular velocity, shear module, material density, and diameter on the amplitude-frequency characteristics of torsional oscillations. This makes it possible to more accurately establish the oscillation amplitude for nonlinear-elastic moving systems for resonant and non-resonant cases.

3. We have derived dependences, convenient for engineering practice, which are more informative, compared to those reported earlier [17]. Such ratios make it possible to investigate the influence of the parameters of the moving environment on the nature of changes in the frequency and amplitude of oscillations and with the necessary accuracy to predict the dynamic phenomena in them. With appropriate use in engineering calculations of industrial equipment, our dependences can become the basis for the synthesis and optimization of the parameters of the screw and other similar structural elements.

4. Numerical simulation was carried out in the MAPLE 15 programming environment, as a result of which it was found that at a speed of 20 m/s, the frequency of oscillations decreases by 7 %. The initial amplitude similarly affects such fluctuations. If the longitudinal velocity reaches 30 m/s, and the initial amplitude is 1 cm, then the frequency of oscillations of the system is 4.5 kHz. This is 44 % less than with its the natural oscillations of the beam, which does not move along its axis. When the longitudinal speed increases to 10 m/s, the amplitude increases by 13.5 %. However, when the speed is equal to 5 m/s, the amplitude value will increase by only 3 %, that is, there will be no such tangible impact. Consequently, with a further increase in the speed of the beam, the amplitude increases sharply.

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