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SELECTION AND RESEARCH OF STABILITY OF THE STEADY STATE MOTIONS OF A SINGLE-MASS RESONANCE VIBROMATING MACHINE WORKING ON THE SOMERFELD EFFECT

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This paper has defined and investigated for stability the steady state modes of motion of a single-mass resonant vibratory machine. The vibratory machine has a platform that is supported by viscoelastic supports. The platform moves rectilinearly translationally. A vibration exciter is installed on the platform. The vibration exciter consists of N identical loads – balls, rollers, or pendulums. The center of mass of each load can move in a circle of a certain radius with a center on the longitudinal axis of the rotor. Each load, when moving relative to the body of the vibration exciter, is exposed to a viscous resistance force.

It was established theoretically that with small forces of viscous resistance and any number of loads, the vibratory machine has jamming modes under which the loads that are collected form a conditional combined load and lag behind the rotor. In this case, there are two bifurcation speeds of the rotor. At speeds less than the first bifurcation speed, the vibratory machine has one single (first) jamming mode. When the first bifurcation speed is exceeded, the second and third jamming modes appear. When the second bifurcation speed is exceeded, the first and second jamming modes disappear. The first jamming mode is resonant.

In the cases of two or more loads, the vibratory machine also has an auto balancing mode (no vibrations), under which the loads rotate synchronously with the body of the vibration exciter and mutually balance each other.

With small forces of viscous resistance, the computational experiment found that odd jamming modes are stable if they are numbered in ascending order of the frequency of load jamming. An auto-balancing mode is stable at the rotor speeds above the resonance. For the onset of a resonant mode of motion of the vibratory machine, it is enough to slowly accelerate the rotor to a speed lower than the second bifurcation speed.

The results reported here are applicable in the design of resonant single-mass vibratory machines with inertial vibration exciters of the ball, roller, or pendulum type

Keywords: inertial vibration exciter, resonant vibratory machine, steady movement, Sommerfeld effect, auto balancing, stability of motion

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1. Introduction

In powerful resonant vibratory machines, the intense vibrations of the platforms are provided by relatively small-mass inertial vibration exciters [1]. This increases the productivity of vibratory machines, reduces material intensity, improves energy efficiency.

The simplest resonant inertial vibration exciters are operated on the Sommerfeld effect [2]. In the corresponding resonant vibratory machines, there are several possible steady state motions at the same time. Investigating their stability is a difficult mathematical problem. In addition, the stability of a certain steady state motion can be local. Therefore, the question remains how to implement such a motion in practice.

Once all possible steady state motions of the vibratory machine are known, then the study of their stability and the possibility of implementation is effectively carried out by a computational experiment.

Information on the stability of a certain steady state mode of motion and the ability to ensure the desired mode is necessary for the design of vibratory machines. It is significant that the rotor speed can be an object of control. Its change can ensure the onset of the desired mode of motion of the vibratory machine, control the amplitude of the oscillations of the platform, etc.

2. Literature review and problem statement

The Sommerfeld effect [2] has been investigated in various rotary machines with passive auto balancers in the following works:

– [3] – for a rotor that performs spatial motion and a ball auto balancer;

– [4] – within the framework of a flat rotor model and a ball auto balancer;

– [5] – for a rotor that performs spatial motion and two pendulum auto balancers.

It was found that loads in the form of balls [3, 4] or pendulums [5] can get stuck at one of the resonant frequencies of oscillations of the rotor, performing both flat [4] and spatial [3, 5] motions. At the same time, the loads are collected, which creates a conditional combined load. Jamming modes occur with small resistance forces in the system. Jamming modes do not allow an auto balancing mode to be established (under which loads balance the rotor and rotor oscillations are absent).

Ways to use the Sommerfeld effect to design resonant vibratory machines were investigated in the following works:

– [6] – for a two-mass system, one of the platforms of which hosts a low-power DC electric motor with a pendulum rigidly mounted on the shaft;

– [7] – for a three-mass system whose one platform hosts a wind wheel with unbalanced mass.

It was found that the pendulum [6], a wind wheel with unbalanced mass [7] get stuck at one of the resonant frequencies of oscillations of the platform, which excites intense vibrations. It is significant that intense vibrations occur only with small resistance forces in the supports of vibratory platforms.

In [3–5], the effect of stuck loads in the auto balancer was considered undesirable. Taking into consideration the results reported in [6, 7], paper [8] proposed using the ball, roller, or pendulum auto balancers as exciters of two-frequency vibrations. At the same time, intense resonance oscillations excite loads in the auto balancer when stuck at a resonant speed. Rapid oscillations are induced by the unbalanced mass on the body of the auto balancer. It is clear that when a vibratory machine is operated under a resonant mode, the auto balancing mode is undesirable.

In [9], for a single-mass vibratory machine, possible steady state two-frequency modes of motion were analytically found. In [10], a computational experiment determined the motion of the vibratory machine, which over time will be established when the body of the vibration exciter is overclocked to any speed. At the same time, the stability of all possible steady state motions of the vibratory machine was not investigated. As a result, no recommendations were given to ensure the necessary steady state mode of motion. In addition, in [10], only two-frequency vibrations were investigated. However, to design purely resonant vibratory machines, it is necessary to investigate vibration exciters of the ball, roller, pendulum type without unbalanced mass on the body of the vibration exciter.

It should be noted that the effect of the stuck unbalanced masses in a single-mass vibratory machine, as undesirable, was investigated in the following works:

– [11] – using the method of energy balance;

– [12] – using the method of direct separation of motions;

– [13] – using the averaging method for partially severely fading systems.

The methods used in [11–13] are somewhat cumbersome and time-consuming to use. They did not make it possible to investigate the form of all possible steady state motions of the vibratory machine, their stability, the possibility of ensuring over time the implementation of a certain (necessary) mode of motion by a vibratory machine. Therefore, the methods used below are those whose effectiveness was proven in [9, 10]. They are based on the method of small parameter [14], elements of the theory of bifurcation of motions [15], information on the theory of auto balancing systems [16].

3. The aim and objectives of the study

The aim of this work is to define possible steady state modes of motion of a single-mass resonant vibratory machine that works on the Sommerfeld effect and to study their stability. The obtained results will make it possible to design such vibratory machines with stable desirable steady state modes of motion.

To accomplish the aim, the following tasks have been set:

– to theoretically find the steady state modes of motion of the vibratory machine, highlight those observed in practice and pre-evaluate their stability based on the general theory;

– through a computational experiment, investigate the stability of possible steady state motions of the vibratory machine with two loads;

– via a computational experiment, investigate the stability of possible steady state motions of a vibratory machine with one load.

4. The study materials and methods

In theoretical studies, a previously developed model of a single-mass resonant vibratory machine with a rectilinear translational motion of the platform (without unbalanced mass on the body of the vibration exciter) is used [9].

The vibratory machine (Fig. 1) includes a platform, mass M. The platform hosts a vibration exciter – ball, roller (Fig. 1, *b*), or pendulum (Fig. 1, *c*). The platform moves progressively in a vertical direction. The platform is supported by a viscoelastic support with a coefficient of rigidity k and viscosity b. The position of the platform is determined by the coordinate y, and in the position of static equilibrium of the platform $y=0$.

The body of the vibration exciter (housing) has mass M_c and rotates around the shaft, point K, at a constant angular velocity ω. The center of mass of the body is at point K. The position of the body is determined relative to the X_K, Y_K axes by the angle ωt, where t is time.

The vibration exciter has N identical loads. The weight of one load is m. The center of mass of the load moves in a circle of radius R with the center at point K (Fig. 1, *b, c*). The position of the load number j relative to the body is determined by the angle φ_j, $/ j=\overline{1,N} /$. The load is exposed when moving relative to the body to the force of viscous resistance, which has a module $b_W R|\varphi_j' - \omega|$, $/ j=\overline{1,N} /$, where b_W is the coefficient of viscous resistance force and the bar behind the value denotes the time derivative t. The action of the forces of weight is neglected.

The differential equations of motion of a single-mass vibratory machine in dimensionless form are as follows [9]:

$$\ddot{y}+2h\dot{y}+y+\ddot{s}_y=0,$$

$$\ddot{\varphi}_j+\varepsilon\beta\left(\dot{\varphi}_j-n\right)+\varepsilon\ddot{y}\cos\varphi_j=0,\ \ /j=\overline{1,N}\,/. \tag{1}$$

In (1):

– dimensionless variables and time:

$$y=\frac{YM_\Sigma}{NmR},\ \ s_x=\frac{1}{N}\sum\nolimits_{j=1}^{N}\cos\varphi_j,$$

$$s_y=\frac{1}{N}\sum\nolimits_{j=1}^{N}\sin\varphi_j,\ \ \tau=\tilde{\omega}t; \tag{2}$$

– dimensionless parameters:

$$h=\frac{b}{2M_\Sigma\tilde{\omega}},\ \ n=\frac{\omega}{\tilde{\omega}},\ \ \varepsilon=\frac{Nm}{\kappa M_\Sigma},$$

$$\beta=\frac{b_W M_\Sigma}{Nm^2\tilde{\omega}},\ \ \tilde{\omega}=\sqrt{\frac{k}{M_\Sigma}}. \tag{3}$$

In turn, in (2), (3):

– $M_\Sigma=M+M_c+Nm$ – the mass of the entire system;

– κ – dimensionless coefficient equal to 7/2 for a ball, 3/2 for a roller, and $1+J_C/mr^2$ for a pendulum.

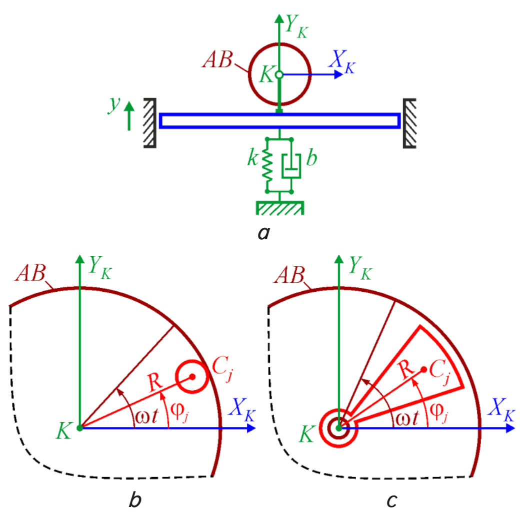

Fig. 1. Single-mass vibratory machine, model, and the kinematics of motion [9]: *a* – platform; *b* – ball or roller; *c* – pendulum

Using the system of equations (1), possible steady state modes of motion of the vibratory machine are sought and classified. The small parameter method [14] is used. A small parameter is ε. The stability of steady state modes of motion is preliminarily evaluated using elements of the theory of bifurcation of motions [15], information from the theory of auto balancing systems [16].

The system of equations (1) in normal form is as follows [10]:

$$\dot{z}_0=z_1,\ \ \dot{z}_{2j}=z_{2j+1},\ \ /j=\overline{1,N}/,\ \ \left(\dot{z}_1,\dot{z}_3,...,\dot{z}_{2N+1}\right)^T=A^{-1}B. \tag{4}$$

In (4):

– new variables:

$$z_0=y,\ \ z_1=\dot{y}=\dot{z}_0,\ \ z_2=\varphi_1,\ \ z_3=\dot{\varphi}_1=\dot{z}_2,...,z_{2j}=\varphi_j,$$

$$z_{2j+1}=\dot{\varphi}_j=\dot{z}_{2j},...,z_{2N}=\varphi_N,\ \ z_{2N+1}=\dot{\varphi}_N=\dot{z}_{2N}; \tag{5}$$

– matrix and vector:

$$A=\begin{pmatrix} 1 & \eta\cos z_2 & \eta\cos z_4 & \dots & \eta\cos z_{2N-2} & \eta\cos z_{2N} \\ \varepsilon\cos z_2 & 1 & 0 & \dots & 0 & 0 \\ \varepsilon\cos z_4 & 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \varepsilon\cos z_{2N-2} & 0 & 0 & \dots & 1 & 0 \\ \varepsilon\cos z_{2N} & 0 & 0 & 0 & 0 & 1 \end{pmatrix},$$

$$B=\begin{pmatrix} -2hz_1-z_0+\eta\sum_{j=1}^{N}z_{2j+1}^2\cos z_{2j}+\delta n^2\cos n\tau \\ -\varepsilon\beta(z_3-n) \\ \vdots \\ -\varepsilon\beta(z_{2N+1}-n) \end{pmatrix}, \tag{6}$$

where $\eta=1/N$.

The equation system (4) is used for integration. The entire integration interval:

$$\tau\in[0,T],\ \ T>0. \tag{7}$$

The interval at which the steady state motion is examined:

$$\tau\in[T-\Delta\tau,T],\ \ 0<\Delta\tau\ll 1. \tag{8}$$

The values for the parameters T and $\Delta\tau$ are selected by attempts. At the time interval $[0, T-\Delta\tau]$, the motion must be established. At the time interval $[T-\Delta\tau, T]$, the rotor must make several revolutions.

The average frequency of rotation of the load under the mode of jamming is calculated by averaging:

$$\bar{\omega}_j=\bar{z}_{2j+1}=\frac{1}{\Delta\tau}\int_{T-\Delta\tau}^{T}z_{2j+1}(\tau)\,\mathrm{d}\tau,\ \ /j=\overline{1,N}/, \tag{9}$$

or calculating the roots of equation (22).

To devise recommendations for ensuring a certain steady state mode of motion, we shall slowly accelerate the rotor under any initial conditions under the following law:

$$n(\tau)=\begin{cases}2n_0\tau/T\ \text{ if }\ \tau<T/2;\\ n_0\ \text{ otherwise}.\end{cases} \tag{10}$$

In (10), n_0 is the working rotor speed.

Computational experiments were conducted for the case of 2 ($\eta=0.5$) and 1 ($\eta=1=1$) loads. At the same time, it was assumed that the qualitative pattern of well-established motions and their stability for the cases of 2 or more loads is the same.

In the nonlinear system, there can be several simultaneously stable established motions [15]. In this case, the steady state motions are locally asymptotically stable. With a change in the rotor speed, the areas of attraction of various stable steady state motions may change. In the vicinity of bifurcation speeds of the rotor, the attraction area can be «infinitely» small.

To check the global asymptotic stability of a certain steady state mode of motion, we shall accelerate the rotor according to law (10) under any initial conditions. In the case of global asymptotic stability of a certain steady state motion, it will be established over time under any initial conditions.

To check the local asymptotic stability (or instability) of a certain steady state mode of motion, we shall set the initial conditions corresponding to this mode. In the case of local asymptotic stability, the vibratory machine will continue to carry out the specified steady state mode of motion. In the case of instability, the vibratory machine will eventually leave the unstable established motion and move to another (stable) steady state mode of motion.

5. Results of studying the stability of the steady state motions of the vibratory machine

5. 1. Classification of steady state modes of motion of vibratory machines, possible conditions for their stability

At $\varepsilon=0$, the system of differential equations (1) takes the form:

$$\ddot{y}+2h\dot{y}+y+\ddot{s}_y=0,\ \ddot{\varphi}_j=0,\ /j=\overline{1,N}/. \qquad (11)$$

The last N equations in (11) have the following solution:

$$\tilde{\varphi}_j=\Omega\tau+\psi_j,\ /j=\overline{1,N}/, \qquad (12)$$

where the ψ_j parameters are stable and it is taken into consideration that the loads in the vibration exciter on the installed motion can rotate only at the same angular speeds Ω.

From (2), after the transformations, we find:

$$s_y=s_0\sin(\Omega\tau+\psi_0), \qquad (13)$$

where

$$s_0=\frac{1}{N}\sqrt{\left(\sum_{j=1}^{N}\cos\psi_j\right)^2+\left(\sum_{j=1}^{N}\sin\psi_j\right)^2},$$

$$\tan\psi_0=\sum_{j=1}^{N}\sin\psi_j\Big/\sum_{j=1}^{N}\cos\psi_j. \qquad (14)$$

Taking into consideration (13), the first equation in (1) takes the form:

$$\ddot{y}+2h\dot{y}+y=\Omega^2 s_0\sin(\Omega\tau+\psi_0). \qquad (15)$$

The partial solution to the differential equation (14) is:

$$y_0=\frac{\Omega^2 s_0}{\left(1-\Omega^2\right)^2+4h^2\Omega^2}\begin{bmatrix}\left(1-\Omega^2\right)\sin(\Omega\tau+\psi_0)-\\ -2h\Omega\cos(\Omega\tau+\psi_0)\end{bmatrix}. \qquad (16)$$

Introduce:

$$\cos\vartheta(\Omega)=\frac{\left(1-\Omega^2\right)}{\sqrt{\left(1-\Omega^2\right)^2+4h^2\Omega^2}},$$

$$\sin\vartheta(\Omega)=\frac{-2h\Omega}{\sqrt{\left(1-\Omega^2\right)^2+4h^2\Omega^2}},$$

$$\tan\vartheta(\Omega)=-2h\Omega\big/\left(1-\Omega^2\right). \qquad (17)$$

Then:

$$y_0=\frac{\Omega^2 s_0}{\sqrt{\left(1-\Omega^2\right)^2+4h^2\Omega^2}}\sin\left[\Omega\tau+\psi_0+\vartheta(\Omega)\right]. \qquad (18)$$

By law (18), the platform moves on any possible steady state motion. Note that in zero approximation it is impossible to find the parameters Ω and ψ_j.

Substitute (18) in N last equations in (1). We obtain such equations:

$$\varepsilon\beta(\Omega-n)-\varepsilon\frac{\Omega^4 s_0\cos\left(\Omega\tau+\psi_j\right)}{\sqrt{\left(1-\Omega^2\right)^2+4h^2\Omega^2}}\times$$
$$\times\sin\left[\Omega\tau+\psi_0+\vartheta(\Omega)\right]=0,\ /j=1,N/. \qquad (19)$$

In (19), aperiodic components create in the following approximations age components that interfere with the onset of steady state motion. We distinguish the aperiodic components by averaging equation (19) for time at the interval $[0, 2\pi/\Omega]$. We obtain that on possible steady state motions:

$$\varepsilon\left\{\begin{matrix}\beta(\Omega-n)+\dfrac{\Omega^4 s_0}{2\sqrt{\left(1-\Omega^2\right)^2+4h^2\Omega^2}}\times\\ \times\sin\left[\psi_0-\psi_j+\vartheta(\Omega)\right]\end{matrix}\right\}=0,\ /j=\overline{1,N}/. \qquad (20)$$

We are looking for possible solutions to the system of equations (20). In practice, with small internal resistance forces, only the auto balancing mode and jamming modes are implemented, under which the loads are collected (forming a conditional combined load) [3–5, 9, 10]. Let's limit ourselves to these cases:

1. Auto balancing mode:

$$n=\Omega,$$

$$s_0=\frac{1}{N}\sqrt{\left(\sum_{j=1}^{N}\cos\psi_j\right)^2+\left(\sum_{j=1}^{N}\sin\psi_j\right)^2}=0\ (y=0). \qquad (21)$$

The mode exists in the cases where $N\geq 2$. This is a single ($N=2$) or multiparametric ($N>2$) family of steady state motions. On these motions, the total imbalance of loads is zero, the loads rotate synchronously with the rotor, there are no fluctuations in the platform.

From the theory of auto balancing systems, it is known that the auto balancing mode can be stable only at the rotor speeds above the resonance ($n>1$) [16]. Therefore, we enter this speed ($n_r=1$) and check the stability of the auto balancing mode at the rotor speeds above the resonance.

2. Jamming modes under which loads are collected:

$$\psi_0=\psi_j=\tilde{\psi},\ /j=\overline{1,N}/,\ s_0=1, \qquad (22)$$

and the angular load jamming velocities are the roots of the equation:

$$\beta(\Omega-n)+\frac{\Omega^5 h}{\left(1-\Omega^2\right)^2+4h^2\Omega^2}=0. \qquad (23)$$

Equation (23) is investigated in [9]. The main results are as follows.

The solution to equation (23) in parametric form is as follows:

$$n(\Omega)=\Omega+\frac{\chi\Omega^5}{\left(1-\Omega^2\right)^2+4h^2\Omega^2},\ \chi=\frac{h}{\beta},\ \Omega\in(0,+\infty). \qquad (24)$$

In critical cases, $dn(\Omega)/d\Omega=0$, whence the following equation is obtained to search for bifurcation (multiple) frequencies of load jamming:

$$F(\Omega)=(1+\chi)\Omega^8-2(1-2h^2)(2+3\chi)\Omega^6+$$
$$+\left[5\chi+4(1-2h^2)^2+2\right]\Omega^4-4(1-2h^2)\Omega^2+1=0. \quad (25)$$

With small forces of viscous resistance in the supports, there are two bifurcation speeds of rotor n_1, n_2 [9]. At the same time, $1<n_1<n_2$ and:

– $\forall n\in(0, n_1)$ there is a single frequency of load jamming Ω_1, and $0<\Omega_1<1$;

– $\forall n\in(n_1, n_2)$ there are three frequencies of load jamming $\Omega_{1,2,3}$, such that $1<\Omega_1<\Omega_2\ll\Omega_3<n$;

– $\forall n\in(n_2, +\infty)$ there is a single frequency of load jamming Ω_3, such that $1\ll\Omega_3<n$.

According to the theory of bifurcations of motions, different modes of stuck composite load can acquire or lose stability only when crossing the points of bifurcations of motions [15]. Bifurcation parameter is the rotor speed n.

5. 2. Results of studying the stability of the steady state modes of motion of the vibratory machine for the case of 2 loads

Regardless of the number of loads, a computational experiment will be conducted for the following calculation data:

$$\chi=0.1;\ h=0.1;\ \varepsilon=0.05;\ \beta=1.0;$$

$$\sigma=1;\ T=2{,}000,\ \Delta\tau=0.005. \quad (26)$$

In (26), the dimensionless parameters χ, h, ε are an order of magnitude less than 1 and, therefore, are small. They correspond to the case of small resistance forces in the system.

From (25), we find two bifurcation speeds of a stuck load:

$$\Omega_{c1}=1.0176618;\ \Omega_{c2}=1.4050126. \quad (27)$$

From (24), we find two bifurcation speeds of rotor:

$$n_1=n(\Omega_{c2})=1.9377474;\ n_2=n(\Omega_{c1})=3.5741110. \quad (28)$$

The bifurcation speeds of jammed composite load (27) and rotor rotation (28) do not depend on the number of loads in the vibration exciter.

In the case of two (or more) identical loads in the vibratory machine, in addition to the jamming modes, there is an auto balancing mode. Therefore, computational experiments test the stability of both jamming modes and auto balancing mode.

Our experiments show that at rotor speeds below the resonance ($n<1$), a globally asymptotically stable jamming mode is Ω_1. It occurs at any constant pre-resonant rotor speed.

Fig. 2 shows the process of setting the jamming mode Ω_1 in the limit case. The rotor accelerates to a resonant frequency of $n_0=1$ according to law (10) (Fig. 2, *a*). Even during the acceleration of the rotor, the loads come together, rotate as one combined load (Fig. 2, *b*) and, at the same time, lag behind the rotor (Fig. 2, *c*).

Immediately after the end of the acceleration of the rotor, the jamming mode Ω_1 is executed. The actual speeds of rotation of loads fluctuate around their average value of Ω_1 (Fig. 2, *d*). At $n=1$, from (23) we find $\Omega_1=0.797023$.

During the acceleration of the rotor, the amplitude of the oscillations of the platform increases (Fig. 2, *d*). After setting the jamming mode Ω_1, the platform fluctuates almost according to the ideal harmonic law (Fig. 2, *f*).

The first jamming mode Ω_1 is stable with a gradual further acceleration of the rotor to any speed less than the second bifurcation speed n_2.

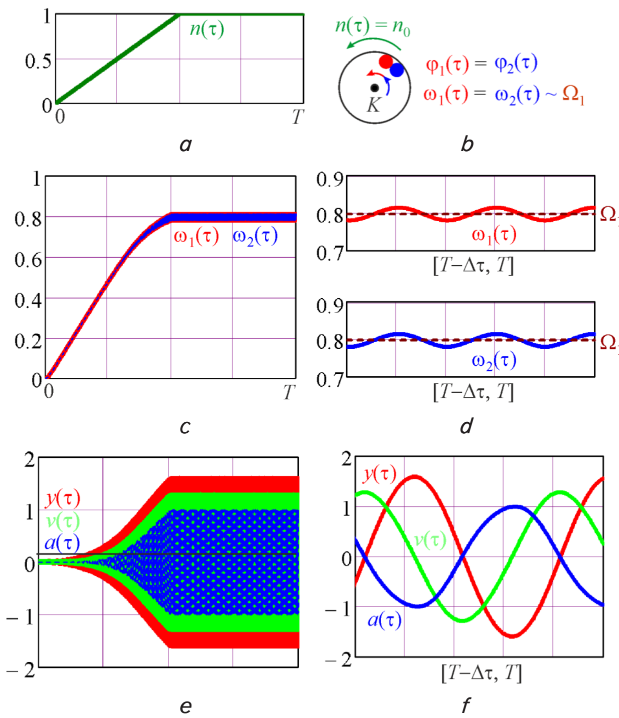

Fig. 2. The onset of the stuck mode Ω_1 when the rotor accelerates to a resonant frequency ($n_0=1$): *a* – a plot of changing the rotor speed; *b* – motion of loads relative to the body of the vibration exciter under the mode of jamming Ω_1; *c* – plots of changes in the speed of rotation of loads at the interval [0, *T*]; *d* – at the interval [*T*–Δτ, *T*]; *e* – plots of motion change (τ), velocity *v*(τ) and acceleration *a*(τ) of the platform at the interval [0, *T*], *f* – at the interval [*T*–Δτ, *T*]

Fig. 3 shows the occurrence of the jamming mode Ω_1 in the limit case. The rotor accelerates according to law (10) to the speed $n_0=3.57$, slightly less than the second bifurcation speed n_2 but greater than the additional characteristic speed n^*. Even during the acceleration of the rotor, the loads come together, rotate as one combined load (Fig. 2, *b*) and, at the same time, lag behind the rotor (Fig. 3, *a*).

Immediately after the end of the acceleration of the rotor, the jamming mode Ω_1 is executed. The actual speed of rotation of loads varies around its average value Ω_1 (Fig. 3, *b*). At $n_0=3.57$, from (23) we find

$$\Omega_1=1.0134222,\ \Omega_2=1.0219888,$$

$$\Omega_3=3.1802558. \quad (29)$$

Thus, $\Omega_1>1$. During the acceleration of the rotor, the amplitude of the oscillations of the platform increases (Fig. 3, *c*). After setting the jamming mode Ω_1, the platform fluctuates almost according to the ideal harmonic law (Fig. 3, *d*).

Note that at $n_0=3.57$ the amplitude of oscillations of the platform is almost 3 times greater than at $n_0=1$. At $n_0=3.57$, the velocity of load jamming is closer to the resonant speed than at $n_0=1$.

At the rotor speeds above the resonance ($n>1$), the auto balancing mode becomes locally asymptotically stable. If at the initial moment of time the auto balancing mode of motion is carried out, then in the future the vibratory machine carries out this mode of motion. The auto balancing mode with increasing rotor speed increases the attraction area. Therefore, the auto balancing mode may occur with a gradual acceleration of the rotor, for example, provided that the loads balance each other during launch. This case of the onset of auto balancing mode is shown in Fig. 4.

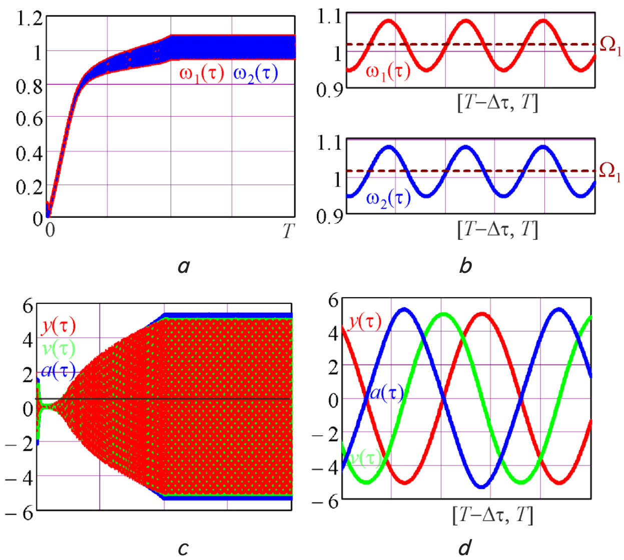

Fig. 3. The onset of the jamming mode Ω_1 when accelerating the rotor to the speed n_0=3.57, a slightly lower than the second bifurcation speed n_2: *a* — plots of changes in the speed of rotation of loads at the interval [0, *T*]; *b* — at the interval [*T*–Δτ, *T*]; *c* — plots of changes in the motion *y*(τ), speed *v*(τ), and acceleration *a*(τ) of the platform at the interval [0, *T*]; *d* — at the interval [*T*–Δτ, *T*]

The rotor accelerates to a frequency of n_0=3.57 by law (10). During rotor acceleration, the loads do not have time to come together. Therefore, the stuck mode Ω_1 does not have time to establish. After rotor acceleration, the loads line up opposite each other, balance each other, and rotate synchronously with the rotor (Fig. 4, *a*). During rotor acceleration, the loads first begin to lag behind (Fig. 4, *b*) and try to get together. However, with the increase in the rotor speed, they are suddenly captured in motion and catch up the rotor. Next, the loads rotate synchronously with the rotor (Fig. 4, *c*). In addition, loads during the acceleration of the rotor first excite intense vibrations (Fig. 4, *d*). However, after rotor acceleration and the onset of auto balancing, the vibrations disappear (Fig. 4, *d*).

At rotor speeds greater than the first bifurcation speed (n>n_1), the third jamming mode Ω_3 was locally asymptotically stable. This mode of jamming does not occur with a gradual acceleration of the rotor and, therefore, it is difficult to implement in practice.

Fig. 5 shows the onset of the stuck mode Ω_3 at the initial conditions close to this mode. The rotor rotates at a constant speed slightly less than the second bifurcation speed n_2 (Fig. 5, *a*). At the initial moment, the loads rotate synchronously with the rotor and are assembled, the platform is deviated from the position of static equilibrium. On the steady state motion Ω_3, the loads are collected and rotate as one combined load (Fig. 5, *b*) and, at the same time, lag behind the rotor (Fig. 5, *c*). The actual speed of load rotation varies around its average value of Ω_3 (Fig. 5, *d*). The value of Ω_3 is given in (29).

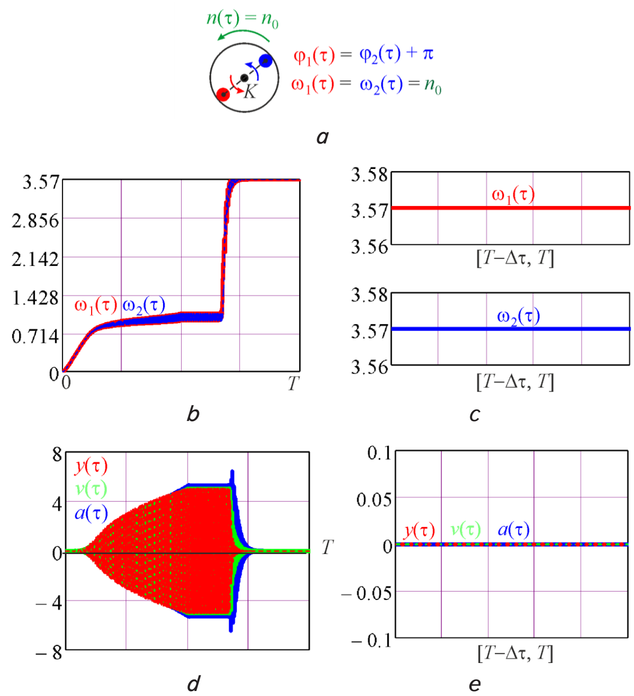

Fig. 4. The onset of auto balancing mode when accelerating the rotor to the speed n_0=3.57, slightly smaller than the second bifurcation speed n_2, provided that during the start the loads balance each other: *a* — the motion of loads relative to the body of the vibration exciter under the auto balancing mode; *b* — plots of changes in the speed of rotation of loads at the interval [0, *T*]; *c* — at the interval [*T*–Δτ, *T*]; *d* — plots of changes in the motion *y*(τ), speed *v*(τ), and acceleration *a*(τ) of the platform at the interval [0, *T*]; *e* — at the interval [*T*–Δτ, *T*]

All the time of integration, the platform fluctuates intensively (Fig. 5, *d*). After setting the jamming mode W_3 the platform fluctuates almost according to the ideal harmonic law (Fig. 5, *e*).

In the rotor rotation range with speeds between the first and second bifurcation speeds ($n_1<n<n_2$), theoretically there is a second jamming mode Ω_2. However, it has proven unstable under any initial conditions. Even if at the initial moment there is a second mode of jamming, then the vibratory machine leaves this motion. In this case, over time, the first jamming mode is established.

Thus, in the range $n\in(n_1, n_2)$, the locally asymptotically stable are the first and third modes of jamming, as well as an auto balance mode. The onset of a certain regime depends on the initial conditions. The second jamming mode is not stable.

At rotor speeds greater than the second bifurcation speed, both the third jamming mode and the auto balancing mode are locally asymptotically stable. It depends on the initial conditions. However, with the gradual acceleration of the rotor, an auto balancing mode occurs. At the same time, at speeds at which there is no resonant mode, the vibration exciter is turned off. This can be used to «turn off» the vibration exciter when the rotor exceeds the second bifurcation speed.

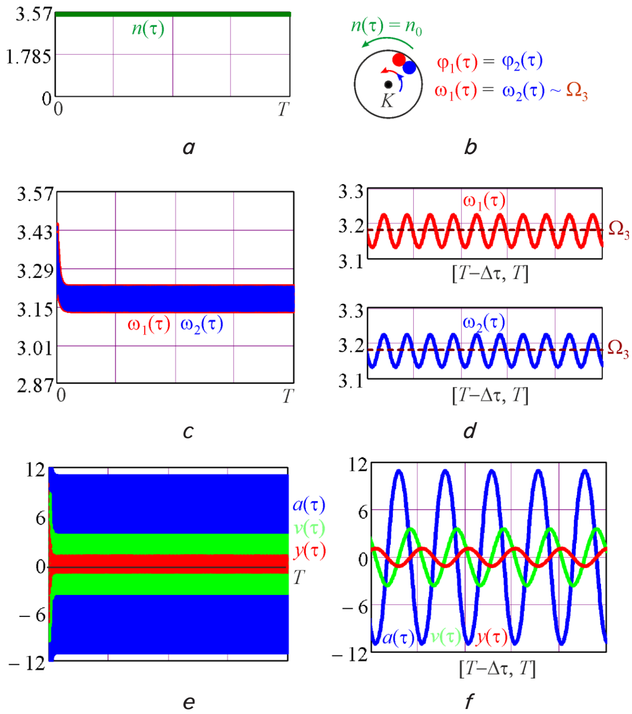

Fig. 5. The onset of the jamming mode Ω_3 when rotating the rotor at the speed $n_0=3.57$, slightly less than the second bifurcation speed n_2: *a* – a plot of changing the rotor speed; *b* – motion of loads relative to the body of the vibration exciter under the mode of jamming Ω_3; *c* – plots of changes in the speed of rotation of loads at the interval [0, *T*]; *d* – at the interval [$T-\Delta\tau$, *T*]; *e* – plots of changes in the motion $y(\tau)$, speed $v(\tau)$, and acceleration $a(\tau)$ of the platform at interval [0, *T*]; *f* – at the interval [$T-\Delta\tau$, *T*]

5. 3. Results of studying the stability of the steady state modes of motion of the vibratory machine for the case of 1 load

In the case of one load, the vibratory machine has only jamming modes.

Therefore, computational experiments test the stability of jamming modes, especially in the vicinity of bifurcation speeds.

When rotating the rotor with speeds less than the first bifurcation speed n_1, globally asymptotically stable is jamming mode Ω_1. It occurs under any initial conditions.

When rotating the rotor with speeds greater than the first bifurcation speed n_1 but less than the second bifurcation speed n_2, locally asymptotically stable are the odd jamming modes (Ω_1, Ω_3), and the even mode (Ω_2) is unstable. To establish the first (resonant) mode of jamming, it is enough to slowly accelerate the rotor. To set the third jamming mode, one needs to start from the initial conditions close to the third mode.

When rotating the rotor with speeds greater than the second bifurcation speed n_2, the globally asymptotically stable is jamming mode Ω_3. It occurs under any initial conditions. The jamming mode W_3 retains stability with a gradual decrease in the rotor speed to the first bifurcation speed.

The transitional processes in the event of a certain mode of jamming in the case of one load are similar to transient processes for the case of two loads.

6. Discussion of results of studying the stability of the steady state motions of the vibratory machine

With any number of loads, depending on the speed of the rotor n, the vibratory machine has 1 or 3 jamming modes. Only odd modes can be stable if they are numbered in ascending order of the frequency of load jamming.

In the cases of two or more loads, the vibratory machine has an auto balancing mode (no vibrations) under which the loads rotate synchronously with the body of the vibration exciter and mutually balance each other. The auto balancing mode can be stable only at the above-the-resonant speeds of the rotor ($n>1$).

With small viscous resistance forces in the case of two identical loads:

– when the rotor rotates with speeds less than the first bifurcation speed n_1, globally asymptotically stable is the mode of jamming Ω_1 (Fig. 2);

– when rotating the rotor with speeds greater than the resonant frequency n_r and less than the first bifurcation speed n_1, locally asymptotically stable are jamming mode Ω_1 and auto balance mode;

– when rotating the rotor with speeds greater than the first bifurcation speed n_1 but less than the second bifurcation speed n_2, locally asymptotically stable are the odd jamming modes Ω_1 (Fig. 3), Ω_3 (Fig. 5), and an auto balancing mode (Fig. 4), while the even mode Ω_2 is unstable;

– when rotating the rotor with speeds greater than the second bifurcation speed n_2, locally asymptotically stable are the jamming mode Ω_3 and auto balancing mode.

Note that the auto balancing mode attraction area is much larger than the attractive jamming area of the third mode. Therefore, in practice, it will be difficult to ensure a third jamming mode.

Thus, with a gradual acceleration of the rotor to a speed exceeding n_2, an auto balancing mode occurs. At the same time, at speeds at which there is no resonant mode, the vibration exciter «turns off». This can be used as a limiter when the rotor exceeds the second bifurcation speed. At the same time, the vibration exciter will be purely resonant.

With small viscous resistance forces for the case of one load:

– when rotating the rotor with speeds less than the first bifurcation speed n_1, the globally asymptotically stable is jamming mode Ω_1;

– when rotating the rotor with speeds greater than the first bifurcation speed n_1 but less than the second bifurcation speed n_2, locally asymptotically stable are the odd jamming modes (Ω_1, Ω_3), while the even mode (Ω_2) is unstable;

– when rotating the rotor with speeds greater than the second bifurcation speed n_2, the globally asymptotically stable is jamming mode Ω_3.

Consequently, when the rotor exceeds the second bifurcation speed n_2, the resonant jamming mode is replaced by a third jamming mode with a much higher frequency and a lower amplitude of the platform oscillations. This can be used to design a combined vibratory machine that works under both resonant and non-resonant modes. After setting the third mode of jamming, the rotor speed can be reduced to the first bifurcation speed. At the same time, both the frequency and amplitude (non-resonant) oscillations of the platform will change.

It should be noted that the stability of various steady state motions can be influenced by certain unaccounted factors, in particular a slight imbalance of the rotor, eccentricity, and roughness of the running track, etc. Therefore, in the future, it is planned to investigate by a full-scale experiment the stability of possible steady state modes of motion of a single-mass resonant vibratory machine with a straight-line translational motion of the platform.

7. Conclusions

1. With small forces of viscous resistance and any number of loads, depending on the speed of the rotor, the vibratory machine has 1 or 3 modes of load jamming and two bifurcation speeds of the rotor. When the first bifurcation speed is reached, the second and third jamming modes appear. When reaching the second bifurcation speed, the first and second modes of jamming disappear. Resonant is the first jamming mode. Only odd jamming modes can be stable if they are numbered in ascending order of the frequency of load jamming.

In the cases of two or more loads, the vibratory machine has an auto balancing mode (no vibrations) under which the loads rotate synchronously with the body of the vibration exciter and mutually balance each other. An auto balancing mode can be stable only at the above-the-resonant speeds of the rotor.

2. With small viscous resistance forces for the case of two identical loads:

– when rotating the rotor with speeds less than the first bifurcation speed, the first jamming mode is globally asymptotically stable;

– when rotating the rotor with speeds greater than the resonant speed and less than the first bifurcation speed, the first jamming mode and auto balancing mode are locally asymptotically stable;

– when rotating the rotor with speeds greater than the first bifurcation speed, but less than the second one, locally asymptotically stable are the odd jamming modes and an auto balancing mode while the even jamming mode is unstable;

– when rotating the rotor with speeds greater than the second bifurcation speed, the locally asymptotically stable is the mode of jamming and auto balancing mode.

With a gradual acceleration of the rotor to speeds:

– smaller than the second bifurcation speed of the rotor, the first – resonant jamming mode – is set;

– greater than the second bifurcation speed of the rotor, the auto balancing mode is set.

3. With small viscous resistance forces for the case of one load:

– when rotating the rotor with speeds less than the first bifurcation speed, the first jamming mode is globally asymptotically stable;

– when rotating the rotor with speeds greater than the first bifurcation speed, but less than the second one, locally asymptotically stable are the odd jamming modes while the even mode is unstable;

– when rotating the rotor with speeds greater than the second bifurcation speed, the third jamming mode is globally asymptotically stable.

With a gradual acceleration of the rotor to speeds:

– smaller than the second bifurcation speed of the rotor, is the first – resonant – jamming mode is set;

– greater than the second bifurcation speed of the rotor, a third jamming mode is set.

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