
#### Abstract

The object of this study is a structural member made from a C-shaped cold-formed profile, investigated to search for optimal cross-sectional dimensions. The parametric optimization task is stated as the problem to find the optimal cross-sectional dimensions of the structural member under axial compression conditions, taking into account its post-buckling behavior (local buckling of the web and flanges, as well as a distortional buckling of the cross-section) and structural requirements. In this case, the material consumption and mechanical characteristics of steel, as well as the design lengths of the structural member, were considered constant and predefined. The considered criterion of optimality zas the maximization of the load-carrying capacity of the structural member for the overall buckling under the axial compression. The stated optimization problem is solved using the method of exhaustive search while applying the developed software. Additionally, for fixed steel consumption, compromise solutions zeere searched that do not depend on the thickness of the profile and the design lengths of the structural member. The resulting cold-formed C-shaped profiles with optimal cross-sectional dimensions are characterized by a higher load-carrying capacity for the overall buckling under axial compression (to $24.45 \%$ and 22.19 \%) at the same steel consumption compared to the profiles offered by the manufacturer. Analysis of the reported results made it possible to devise recommendations for optimal ratios of dimensions and geometric characteristics of the structural members made from $C$-shaped profiles operating under axial compression. The ratios could be used both at the stage of selection of cross-sections of structural members from cold-formed profiles, and in the development of effective assortments of cold-formed profiles


Keywords: cold-formed profile, load-carrying capacity, flexural-torsional buckling, post-buckling behavior, parametric optimization

## UDC 624.04.012.4.044 <br> DOI: 10.15587/1729-4061.2022.261037 <br> OPTIMIZATION OF CROSS-SECTION DIMENSIONS OF STRUCTURAL MEMBERS MADE OF COLD-FORMED PROFILES USING COMPROMISE SEARCH

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## 1. Introduction

The use of cold-formed thin-walled profiles was previously limited to cases where reducing the weight of the structure was a top priority, as, for example, in the aviation or automotive industry. Nevertheless, due to the development of manufacturing technology, corrosion protection, product availability, as well as an understanding of the design behavior and improvement of technical standards for the analysis of thin-walled rod elements, the use of coldformed structural members is gradually expanding [1].

A variety of structural systems made of thin-walled coldformed profiles are widely used in the construction industry. The introduction into the building practice of steel structures made from thin-walled cold-formed profiles is relevant and economically justified. The area of use, where their efficiency is the highest, covers buildings for administrative and commercial purposes, covered sports facilities, trade and exhibition pavilions, crane production workshops, warehouses, hangars, farms, greenhouses, one-story residential buildings [2].

The high degree of technology flexibility for manufacturing structural members from cold-formed profiles relative
to the shape of cross-sections, together with the optimization of their sizes, provides a unique opportunity to obtain the most effective structures [3]. In view of this, scientific research on the optimal design of structures from cold-formed profiles is considered relevant. The results of such studies will be in practical demand among manufacturers of coldformed profiles.

## 2. Literature review and problem statement

A large number of studies are tackling the problem of finding the optimal cross-sectional sizes of structural members from cold-formed profiles. Paper [4] provides an overview of various statements of optimization problems for the considered class of structures, computational techniques, and optimization algorithms, which cover gradient methods, stochastic search methods, evolutionary algorithms, etc.

Methodologies for finding the optimal cross-sectional dimensions of structural members from cold-formed profiles are proposed in [5,6]. As part of the mathematical model, geometric constraints on the dimensions of the
profile cross-section were considered, due to the practical application of cold-formed profiles and their manufacture technology [7], as well as constraints reflected structural requirements [8]. Optimization problems were stated as problems of maximizing the load-carrying capacity of the structural member from cold-formed profiles [9]. At the same time, their load-carrying capacity was calculated in accordance with the requirements of the building codes, taking into account the local buckling and/or distortional buckling of the cross-section [10].

Particular attention of scientists was paid to optimizing the size of edge folds that restrain the flanges of structural members made of cold-formed profiles. Thus, in paper [1], the problem of finding the optimal length of a single edge fold for C-shaped cold-formed profiles operating under axial compression conditions is considered. The objective function and constraints of the mathematical model are presented by the author as continuously differentiated functions, which made it possible to solve the problem of parametric optimization by a nonlinear method of gradient projection [11, 12]. A similar optimization problem was also considered in [13], where the authors applied a genetic algorithm to solve it.

In works $[3,14]$ that tackle the search for optimal dimensions of sections of structural members from coldformed profiles, genetic algorithms were used based on the implementation of purposeful enumeration of a finite set of design solutions. The search strategy in such methods is based on the calculation and comparison of the values of some function for evaluating design solutions at the points of the search space under consideration. At the same time, the requirements for unimodality, continuity, differentiation of such a function are not put forward. However, when applying stochastic search methods and evolutionary algorithms, despite their high performance, it is possible to obtain, as you know, design solutions that are only close to optimum. This suggests that it is expedient to conduct research aimed at finding a global optimum for the class of problems under consideration.

The results of optimization calculations obtained by the authors of [3,10], in particular, the optimal dimensions of cold-formed profiles, depend on the thickness of the profile and the design lengths of the structural member. For example, in [3], optimal design solutions for C-shaped cold-formed profiles were obtained but the optimization results depend on the profile thickness ( 1.5 mm and 3 mm ) and on the design length of the member ( 3 m and 5 m ). The same applies to work [10], in which the authors obtained the optimal cross-sectional dimensions of the structural member from the C-shaped cold-formed profile, depending on the profile thickness of 16 mm and on the design length of the member $(0.0 \mathrm{~m}, 1.0 \mathrm{~m}$, and 3.0 m ). Given this, it is considered expedient to conduct a study aimed at finding optimal solutions that will not depend on the thickness of the profile and the design lengths of the structural member. Obtaining optimal dimensions of sections of cold-formed profiles that do not depend on the thickness of the profile and the design lengths of the structural member are especially important when developing an optimal range and assortment of coldformed profiles.

On the other hand, a critical review of the above studies showed that the issue of determining the optimal ratio of cross-section dimensions and geometric characteristics of structural members made of cold-formed profiles remained
unresolved. In view of this, it is advisable to analyze the obtained optimal solutions and devise recommendations for the optimal distribution of the material in the cross-sections of structural members made from cold-formed profiles.

## 3. The aim and objectives of the study

The aim of this study is to solve the problem of optimal design of structural members from cold-formed profiles and to study the properties of such structures with optimal parameters. This will make it possible to devise recommendations for designers on the optimal distribution of material in cross-sections of the structural members made of coldformed profiles.

To achieve the set aim, the following tasks have been solved:

- to propose a methodology for finding the optimal cross-sectional dimensions of thin-walled structural members from cold-formed profiles;
- to solve the problem of optimizing the cross-sectional sizes of structural members from C-shaped cold-formed profiles that are under axial compression conditions;
- to compile recommendations for the optimal distribution of the material in the cross-sections of the structural members from cold-formed profiles.


## 4. The study materials and methods

The object of this study is a structural member made of a C-shaped cold-formed profile, which is under conditions of axial compression. In the structural member made of cold-formed profiles, such features of the behavior under load are manifested that are not described by the classical theory of mechanics of thin-walled structures. The latter, as is known, is based on the hypothesis of non-deformability of the contour of the member cross-section and considers only three overall buckling modes by such structural members under the axial compression (flexural, torsional, and flexur-al-torsional).

In the presence of compression stresses in the cross-sections of the structural members made from cold-formed profiles, the phenomenon of local buckling occurs when individual thin plates that make up the cross-sectional contour of the member are buckled. In this case, the contact lines of adjacent plates remain rectilinear (Fig. 1). In addition, the structural member made of cold-formed profiles are characterized by a special local buckling mode (distortional buckling of the cross-section), which is manifested by the curvature of the cross-sectional contour. It occurs in cases where the edge stiffness elements (single or double folds) or intermediate stiffness elements are not able to resist the local displacement of the conjugation nodes of the cross-section elements (Fig. 2).

Numerical experiments of thin-walled structural members under load showed that their load-carrying capacity at the time of local buckling may not be exhausted. For the correct assessment of the load-carrying capacity of such members, it is necessary to take into account the post-buckling behavior of the compressed rod (the ability to resist applied loads after achieving the phenomenon of local buckling of the cross-sectional elements and the distortional buckling of the
cross-section). For the purpose of application in ordinary engineering practice, modern codes for the design of steel structures made from thin-walled cold-formed profiles use simplified models of the post-buckling behavior of such members. The influence of the local buckling on the load-carrying capacity of the thin-walled member is accounted for by constructing an "effective" cross-section, when the decrease in the load-carrying capacity of a separate cross-sectional element (thin plate) due to its buckling is replaced by the conditional "exclusion" from the work of the cross-section part of such an element.

In the reported studies, the task of optimal design of structural elements from coldformed profiles is interpreted as a problem of mathematical programming. To build a mathematical model and develop an algorithm for implementing this approach, numerical optimization methods were used, in particular, combinatorial calculus methods. The latter (unlike other methods) provide a global optimum for mathematical programming problems if the model contains non-smooth constraint functions, the criterion of optimality, and/or functional dependences between design variables and state variables. This is what justifies the chosen method of solving the problem of finding the optimal cross-sectional dimensions of the structural meber.


Fig. 2. Distortional buckling of the C-shaped cold-formed profile cross-section under axial compression


Fig. 1. Local buckling of the web and flanges of the C-shaped cold-formed profile under axial compression
> 5. Results of optimization of crosssectional dimensions of structural members from cold-formed profiles and study of their properties

## 5. 1. Development of a methodology

 for finding the optimal cross-sectional dimensions of structural members from cold-formed profilesApplied tasks of optimal design of building structures in some cases are stated as problems of searching for unknown structural parameters that provide an extreme value of a certain optimality criterion in the search space, outlined by a set of specified constraints. The mathematical model of parametric optimization problems covers a set of design variables, an objective function, as well as constraints that generally reflect nonlinear relationships between them.

The parametric optimization problem was stated as follows: it is required to find the optimal cross-sectional dimensions of the structural member from a C-shaped cold-formed profile operating under longitudinal compression conditions. In this case, steel consumption, steel strength characteristics, as well as the design lengths of such a structural member will be considered constant and predetermined.

The stated task was represented as a multiparametric one in the form of a problem of mathematical programming [11], namely the problem of finding such values of unknown structural parameters:

$$
\begin{equation*}
\vec{X}=\left\{X_{\imath}\right\}, \quad \mathrm{r}=\overline{1, N_{X}}, \tag{1}
\end{equation*}
$$

that provide the smallest (or largest) value of the selected (deterministic) optimality criterion:

$$
\begin{equation*}
f^{*}=f\left(\vec{X}^{*}\right)=\max _{\vec{X} \in \mathfrak{S}_{1}} f(\vec{X}), \tag{2}
\end{equation*}
$$

in the domain of permissible design solutions $\mathfrak{I}$, outlined by the system of constraints-inequalities:

$$
\begin{equation*}
\varphi(\vec{X})=\left\{\phi_{\eta}(\vec{X}) \leq 0 \mid \eta=\overline{1, N_{I C}}\right\} \tag{3}
\end{equation*}
$$

where $\vec{X}$ is the vector of design variables (desired design parameters); $N_{X}$ - the number of unknown structural parameters (design variables); $f, \varphi_{\eta}$ - functions of the vector argument; $\vec{X}^{*}$ - optimal solution (vector of optimal values of design variables); $f^{*}$ - the highest value of the optimality criterion; $N_{I C}$ - the number of constraints-inequalities $\varphi_{\eta}(\vec{X})$, which determine the regions of permissible design solutions in the search space $\mathfrak{I}$.

As design variables (1), we consider the overall dimensions of the C-shaped cold-formed profile: the web height $h$, the flange width $b$, and the single edge fold length $c$ (Fig. 3). The initial data for optimization calculation are the profile thickness $t$, the inner bending radius of the profile $r=1.5 t$, steel characteristics (yield strength $f_{y b}$ and modulus of elasticity $E$ ), the design lengths of the structural member, corresponding to the flexural buckling modes of the member in the main planes of inertia $l_{e f}=l_{e f, y}=l_{e f, z}$. In this case, the design length of the structural member corresponding to the torsional buckling modes $l_{e f, T}$ is taken as $l_{e f, T}=l_{e f, z}$.

We introduce to the cross-sectional plane of the thinwalled rod the $y O z$ coordinate system with the origin in the center of mass $C$ of the cross-section, the direction of the coordinate axes $y \mathrm{Oz}$ of which coincides with the direction of the main axes of inertia. Describe the cross-section of the thinwalled rod under consideration as a set of intersection points $\mathbf{P}=\left\{\mathbf{p}_{j}=\left\{y_{j}, z_{j}\right\} \mid j=\overline{0, n}\right\}\left(y_{j}\right.$ and $z_{j}$ are the coordinates of the $j$-th point of intersection in the $y O z$ coordinate system ) and sets of cross-sectional segments $\mathbf{S}=\left\{\mathbf{s}_{i}=\left\{\mathbf{p}_{i-1}, \mathbf{p}_{i}\right\} \mid i=\overline{1, n}\right\}$, which connect adjacent cross-sectional points, here $n$ is the number of cross-sectional segments; $n+1$ is the number of cross-sectional points. It should be noted that the coordinates of the cross-sectional points depend on the design variables of the stated optimization problem $\mathbf{P}=\mathbf{P}(\vec{X})$.

Integral geometric characteristics of the cross-section under consideration (such as $A_{g}$ - gross cross-sectional area; $I_{y}, I_{z}$ - moments of inertia relative to the main axes of inertia, coinciding with the axes of the global coordinate system $y O z ; i_{y}, i_{z}$ - the radii of inertia relative to the main axes of inertia; $I_{\omega}-$ sectoral moment of inertia; $I_{t}$ - moment of inertia of free torsion) can be calculated depending on the defined set $\mathbf{P}$ of the points of cross-section and the set $\mathbf{S}$ of the segments of the cross-section (Appendix C [15]). Since the coordinates of the cross-sectional points depend on the design variables, therefore, the integral geometric characteristics of the cross-section gross also depend on the design variables.

The design dimensions of flat cross-sectional elements (Fig. 3) of the C-shaped cold-formed profile are calculated in accordance with [15] depending on the design variables $h, b$, and $c$, as well as depending on the inner radius of the bend $r$ and the thickness of the profile $t$ as:

$$
\begin{align*}
& h_{p}=h-2 R+r_{m} \sqrt{2}  \tag{4}\\
& b_{p}=b-2 R+r_{m} \sqrt{2}  \tag{5}\\
& c_{p}=c-R+0.5 r_{m} \sqrt{2}, \tag{6}
\end{align*}
$$

where $h_{p}$ is the design web height; $b_{p}$ is the design flange width; $c_{p}$ is the design length of the single edge fold; $r_{m}$ - the median radius of the profile bend, $r_{m}=r+0.5 t ; R$ is the outer radius of the profile bend, $R=r+t$.


Fig. 3. Cross-section of the structural member of the C-shaped cold-formed profile: $P$ - the point that is located in the middle of the connection arc of the middle lines of section flat elements; $C$ - the center of gravity of gross cross-section; $t$ - profile thickness; $h_{p}, b_{p}, c_{p}$ - design dimensions of the cross-section; $C_{\text {eff }}$ - the center of gravity of the "effective"
cross-section; $h_{e 1}, h_{e 2}, b_{e 1}, b_{e 2}, c_{e f}$ - dimensions of the "effective" cross-section; $t_{r e d}$ - the reduced thickness of the stiffness element; $e_{z}$ - distance between the center of mass of the "effective" cross-section and the center of mass of the cross-section gross

The slenderness of the flange $\bar{\lambda}_{p b}$, web $\bar{\lambda}_{p h}$, and the single edge fold $\bar{\lambda}_{p c}$ of the C-shaped cold-formed profile are calculated according to $[15,16]$ as:

$$
\begin{align*}
& \bar{\lambda}_{p h}=\frac{h_{p}}{56.8 t \varepsilon} ;  \tag{7}\\
& \bar{\lambda}_{p b}=\frac{b_{p}}{56.8 t \varepsilon} ;  \tag{8}\\
& \bar{\lambda}_{p c}=\frac{c_{p}}{28.4 t \varepsilon \sqrt{\tilde{\mathbf{k}}_{\sigma \mathbf{c c}}\left(c_{p} / b_{p}\right)}}, \tag{9}
\end{align*}
$$

where $\varepsilon$ is the coefficient that takes into account the properties of steel; $\varepsilon=\sqrt{\frac{235}{f_{y b}[\mathrm{MPa}]}} ; \tilde{\mathbf{k}}_{\mathrm{\sigma c}}\left(c_{p} / b_{p}\right)$ is the buckling factor for a single edge fold, determined in accordance with the dependence proposed in [15].

The flanges and web of the C-shaped cold-formed profile cross-section are subject to local buckling (post-buckling behavior) if their slenderness exceeds the limit value, name-
ly: $\bar{\lambda}_{p h}>0.673$ and/or $\bar{\lambda}_{p b}>0.673$. In this case, the "effective" width of the web $h_{\text {eff }}$ and the "effective" width of the flanges $b_{e f f}$, as well as the dimensions $h_{e 1}, h_{e 2}, b_{e 1}, b_{e 2}$ of the "effective" cross-section are calculated according to [15, 16] as presented below:

$$
\begin{align*}
& h_{e 1}=h_{e 2}=\frac{h_{e f f}}{2}= \\
& =\frac{h_{p}}{2 \bar{\lambda}_{p h}}\left(1-\frac{0.22}{\bar{\lambda}_{p h}}\right)= \\
& =28.4 t \varepsilon\left(1-\frac{12.496 t \varepsilon}{h_{p}}\right)-\text { if } \bar{\lambda}_{p h}>0.673 ;  \tag{10}\\
& h_{e 1}=h_{e 2}=\frac{h_{p}}{2}-\text { if } \bar{\lambda}_{p h} \leq 0.673 ;  \tag{11}\\
& b_{e 1}=\frac{b_{e f f}}{2}=\frac{b_{p}}{2 \bar{\lambda}_{p b}}\left(1-\frac{0.22}{\bar{\lambda}_{p b}}\right) \\
& =28.4 t \varepsilon\left(1-\frac{12.496 \varepsilon t}{b_{p}}\right)-\text { if } \bar{\lambda}_{p b}>0.673 ;  \tag{12}\\
& b_{e 2}=\frac{b_{e f f}}{2}= \\
& =\frac{b_{p}}{2 \bar{\lambda}_{p b} \sqrt{\chi_{d}}}\left(1-\frac{0.22}{\bar{\lambda}_{p b} \sqrt{\chi_{d}}}\right)= \\
& =\frac{28.4 t \varepsilon}{\sqrt{\chi_{d}}}\left(1-\frac{12.496 t \varepsilon}{b_{p} \sqrt{\chi_{d}}}\right) \quad-\text { if } \bar{\lambda}_{p b}>0.673 ;  \tag{13}\\
& b_{e 1}=b_{e 2}=\frac{b_{p}}{2}-\text { if } \bar{\lambda}_{p b} \leq 0.673 ;
\end{align*}
$$

where $\chi_{d}$ is a coefficient that takes into account the distortional buckling of the cross-section, which is derived below.

The single edge fold of the cross-section of the C-shaped cold-formed profile is subject to local buckling (post-buckling behavior) if its slenderness exceeds the limit value ( $\bar{\lambda}_{p c}>0.748$ ). In this case, the "effective" width of the single edge fold $c_{e f f}$ is determined in accordance with [15] as:

$$
\begin{align*}
& c_{e f f}=\frac{28.4 t \varepsilon}{\sqrt{\chi_{d}}} \sqrt{\tilde{\mathbf{k}}_{\mathrm{c}}\left(\frac{c_{p}}{b_{p}}\right)} \times \\
& \times\left(1-\frac{5.3392 t \varepsilon}{c_{p} \sqrt{\chi_{d}}} \sqrt{\tilde{\mathbf{k}}_{\mathrm{c}}\left(\frac{c_{p}}{b_{p}}\right)}\right)-\text { if } \bar{\lambda}_{p c}>0.748  \tag{15}\\
& c_{e f f}=c_{p}-\text { if } \bar{\lambda}_{p c} \leq 0.748 . \tag{16}
\end{align*}
$$

The "effective" cross-section of the structural member under consideration will be described using the set of intersection points $\mathbf{P}_{e f f}=\left\{\mathbf{p}_{e f f, j}=\left\{y_{e f f, j}, z_{e f f, j}\right\} \mid j=\overline{0, n_{e f f}}\right\} \quad\left(y_{e f f, j}\right.$ and $z_{\text {eff } ; j}$ are the coordinates of the $j$-th point of cross-section in the yOz coordinate system above) and the set of cross-sectional segments $\mathbf{S}_{e f f}=\left\{\mathbf{s}_{e f f, i}=\left\{\mathbf{p}_{e f f, i-1}, \mathbf{p}_{e f f, i}\right\} \mid i=\overline{1, n_{e f f}}\right\}$, which connect adjacent points of cross-section, where $n_{\text {eff }}$ is the number of cross-sectional segments; $n_{\text {eff }}+1$ is the number of cross-sectional points. It should be noted that the coordinates of the cross-sectional points depend on the design variables of the stated optimization problem, as well as on
the "effective" cross-sectional dimensions $h_{e 1}, h_{e 2}, b_{e 1}, b_{e 2}$, $c_{e f f:}: \mathbf{P}_{e f f}=\mathbf{P}_{e f f}\left(\vec{X}, h_{e 1}, h_{e 2}, b_{e 1}, b_{e 2}, c_{e f f}\right)$.

The area $A_{\text {eff }}$ of the "effective" cross-section of the structural member from C-shaped cold-formed profile under axial compression conditions can be calculated using a defined set $\mathbf{P}_{\text {eff }}$ of the cross-sectional points and a defined set $\mathbf{S}_{e f f}$ of the cross-sectional segments (Appendix C [15]). It should be noted that the area $A_{e f f}$ depends on the design variables (1) of the optimization problem since the coordinates of the points of the "effective" cross-section, determined by the set $\mathbf{P}_{e f f}$, depend on them.

Single edge folds in C-shaped cold-formed profiles flanges provide their partial restraints. When analyzing a structural member for the distortional buckling of the cross-section, the assumption is used, according to which the stiffness element (for example, a single edge fold in the flange) behaves like a compressed member with continuous elastic support, simulated by an elastic spring. Such a spring is applied to the center of gravity of the stiffness element. In this case, part of the flange, which is restrained, is also involved in joint work with the stiffness element. Thus, the verification of compressed parts of cross-sections with stiffness elements (single or double edge folds, intermediate stiffeners) is based on the assumption that the stiffness element behaves as a compressed member with continuous restraining with elastic supports, the rigidity of which depends on the boundary conditions and the bending stiffness of adjacent flat elements of the cross-section.

To estimate such restraining, we consider the design cross-section of the stiffness element. This cross-section contains a single edge fold of the flange with effective width $c_{e f f}$ and part of the flange with an effective width $b_{e 2}$ (Fig. 4), adjacent to a single edge fold.


Fig. 4. The flat element of the flange of the C-shaped profile, stiffened by a single edge fold (the color shows the design cross-section of the stiffness element): $P$ is the point that is located in the middle of the connection arc of the middle lines of flat section elements; $a-a-$ horizontal axis passing through the center of gravity of the design cross-section of the stiffness element; $b, c$ - overall dimensions of the flange stiffened by a single edge fold; $b_{p}, c_{p}$ - design crosssectional dimensions; $b_{e 1}, b_{e 2}, c_{\text {eff }}$ - dimensions of the "effective" cross-section; $b_{1}$ is the distance from the middle line of the cross-sectional web to the center of gravity of the design cross-section of the stiffness element

In the case that the distortional buckling of the cross-section has not occurred ( $\chi_{d}=1$ ), the thickness of the design
cross-section of the stiffness element is assumed to be equal to the thickness of the C-shaped cold-formed profile $t$. Otherwise, in accordance with [15], the reduced thickness $t_{\text {red }}$ of the design cross-section of the single edge fold is calculated, taking into account the reduced resistance of the stiffness element due to the loss of the overall stability according to the flexural buckling mode:

$$
\begin{equation*}
t_{r e d}=\chi_{d} t \tag{17}
\end{equation*}
$$

We describe the design cross-section of the stiffness element using the set of intersection points

$$
\mathbf{P}_{s}=\left\{\mathbf{p}_{s, j}=\left\{y_{s, j}, z_{s, j}\right\} \mid j=\overline{0, n_{s}}\right\}
$$

( $y_{s, j}$ and $z_{s, j}$ are the coordinates of the $j$-th point of intersection in the $y O z$ coordinate system above) and the set of cross-sectional segments

$$
\mathbf{S}_{s}=\left\{\mathbf{s}_{s, i}=\left\{\mathbf{p}_{s, i-1}, \mathbf{p}_{s, i}\right\} \mid i=\overline{1, n_{s}}\right\}
$$

each of which connects two adjacent points of the section, where $n_{s}$ is the number of segments of the cross-section, $n_{s}+1$ - the number of cross-sectional points. It should be noted that the coordinates of the intersections from the set $\mathbf{P}_{s}$ depend on the design variables (1) of the stated optimization problem, as well as on the dimensions of the "effective" cross-section $b_{e 2}, c_{e f f}$ and the reduced thickness $t_{\text {red }}$ : $\mathbf{P}_{s}=\mathbf{P}_{s}\left(\vec{X}, b_{e 2}, c_{e f f}, t_{\text {red }}\right)$. For the design cross-section of the stiffness element defined on the set $\mathbf{P}_{s}$ of the cross-sectional points and the set $\mathbf{S}_{s}$ of the cross-sectional segments, it is possible to calculate the required geometric characteristics of the cross-section.

Partial flange restraining of the structural member of a C-shaped cold-formed profile, which is ensured by the presence of a single edge fold, is simulated using a linear spring. In the case of longitudinal compression, the rigidity of such a spring can be estimated in accordance with [15] as:

$$
\begin{equation*}
K=\frac{E}{3.64} \cdot \frac{t^{3}}{b_{c, s}^{2}\left(b_{c, s}+1.5 h-3 t\right)} \tag{18}
\end{equation*}
$$

where $b_{c, s}$ is the distance from the web-to-flange adjoining of the C-shaped profile to the center of gravity of the design cross-section of the single edge fold. It should be noted that the analytical expression (18) for the rigidity of the linear spring is limited to the case when only the structural members with a cross-section symmetrical relative to the main axis of inertia, perpendicular to the plane of the web, are considered.

The slenderness of the stiffness element $\bar{\lambda}_{d}$, corresponding to the flexural buckling of a single edge fold, is calculated in accordance with [15] as:

$$
\begin{equation*}
\bar{\lambda}_{d}=\sqrt{\frac{f_{y b} A_{s}}{2 \sqrt{K E I_{s}}}} \tag{19}
\end{equation*}
$$

where $A_{s}$ and $I_{s}$ are the area and moment of inertia of the design cross-section of the stiffness element.

The coefficient $\chi_{d}$, which takes into account the flexural buckling of the stiffness element under the axial compression (or a coefficient taking into account the distortional buckling of the cross-section), is calculated iteratively depending
on the slenderness $\bar{\lambda}_{d}$ according to the rules and dependences proposed in [15]:

$$
\begin{equation*}
\chi_{d}=\Xi\left(\bar{\lambda}_{d}\right) \tag{20}
\end{equation*}
$$

As an objective function (2), the criterion of maximizing the load-carrying capacity of the structural member for the overall buckling under the axial compression is considered. In this case, the objective function will be written as:

$$
\begin{equation*}
N_{b R d, \min }=\min \left\{N_{b y R d}, N_{b z R d}, N_{b T, R d}, N_{b T F, R d}\right\} \rightarrow \max \tag{21}
\end{equation*}
$$

where $N_{b R d, \min }$ is the minimum load-carrying capacity of the structural member from cold-formed profiles for overall buckling under the axial compression; $N_{b y, R d}, N_{b z, R d}-$ design resistances that correspond to the flexural buckling modes relative to the main inertia axes $y-y$ and $z-z$, calculated in accordance with [15, 17]; $N_{b T, R d}, N_{b T F, R d}-$ design resistances corresponding to the torsional and flexural-torsional buckling modes, calculated in accordance with [15, 17].

The objective function (2) is then rewritten as:

$$
\begin{equation*}
N_{b R d, \min }=\frac{A_{e f f} f_{y b}}{\gamma_{M 1}} \times \min \left\{\chi_{y}, \chi_{2}, \chi_{T}, \chi_{T F}\right\} \rightarrow \max \tag{22}
\end{equation*}
$$

where $\chi_{y}, \chi_{z}, \chi_{T}, \chi_{T F}$ are the buckling factors that take into account both the flexural buckling of the structural member from cold-formed profiles relative to the main axes of inertia $y-y$ and $z-z$, and the torsional and flexural-torsional buckling.

The buckling factors $\chi_{y}, \chi_{z}, \chi_{T}, \chi_{T F}$ are calculated from the stability curve $b$ in accordance with $[15,17]$ as:

with substitution instead of $\bar{\lambda}$ of the slendernesses $\bar{\lambda}_{y}$, $\bar{\lambda}_{z}, \bar{\lambda}_{T}, \bar{\lambda}_{T F}$, which correspond to the considered buckling mode and are calculated depending on the geometric characteristics of the "effective" cross-section of the structural member under axial compression conditions, in accordance with $[15,17]$ as:

$$
\begin{equation*}
\bar{\lambda}=\sqrt{\frac{A_{e f f} f_{y b}}{N_{c r}}} \tag{24}
\end{equation*}
$$

where $N_{c r}$ is the elastic critical force for the corresponding buckling mode, calculated depending on the design length of the structural member and the geometric characteristics of its cross-sectional gross in accordance with [17]. Slendernesses $\bar{\lambda}_{y}, \bar{\lambda}_{z}, \bar{\lambda}_{T}, \bar{\lambda}_{T F}$ are calculated with a substitution in (24) instead of $N_{c r}$ of the elastic critical forces $N_{c r, y}, N_{c r, z}$, $N_{c r, T}$ and $N_{c r, T F}$, corresponding to the flexural, torsional, and flexural-torsional buckling modes, respectively.

The system of constraints (3) for the stated optimization problem contains the limitation on the profile perimeter or on the strip width, which can be written as:

$$
\begin{equation*}
\frac{h+2 b+2 c}{P_{\max }}-1 \leq 0 \tag{25}
\end{equation*}
$$

where $P_{\text {max }}$ is the maximum cross-sectional perimeter value for a C-shaped cold-formed profile.

Limitations reflecting the design codes requirements [15] regarding the ultimate slenderness of cross-sectional elements of a C-shaped cold-formed profile with flanges restrained by single edge folds must be included in the system of constraints (3) as represented below:

$$
\begin{align*}
& \frac{h}{500 t}-1 \leq 0  \tag{26}\\
& \frac{b}{60 t}-1 \leq 0  \tag{27}\\
& \frac{c}{50 t}-1 \leq 0  \tag{28}\\
& 0.2-\frac{c}{b} \leq 0  \tag{29}\\
& \frac{c}{b}-0.6 \leq 0 \tag{30}
\end{align*}
$$

The system of constraints (3) also involved limitation on the gap between the ends of the single edge folds, which takes into account the need to provide access to the inner surface of the profile (in order to organize a bolted connection on the flanges [18, 19]):

$$
\begin{equation*}
\frac{h-2 c}{d_{\min }}-1 \leq 0, \tag{31}
\end{equation*}
$$

where $d_{\text {min }}$ is the minimum gap between the single edge folds ends.

We shall finally state the optimization problem - it is required to find the optimal dimensions ( $h, b$, and $c$ ) of the cross-section of the C-shaped cold-formed profile, which provide the maximum value of the objective function (21) in the domain of permissible solutions determined by the system of constraints (25) to (31). In this case, the perimeter and profile thickness, as well as the material strength characteristics and the design lengths of the structural member are taken as constant and predetermined. It should be noted that the stated problem is limited to the consideration of the structural member, which is under conditions of axial compression.

## 5. 2. Results of cross-sectional size optimization of

 structural members from C-shaped cold-formed profilesFor further optimization, $\mathrm{C} 70 \times 45 \times 15$ profiles were selected from the entire range of C-shaped cold-formed profiles manufactured by Light House Ukraine [20]. Other C-shaped profiles with a higher web height are more rational to use as beams or column-beam structural members. The width of the strip (profile perimeter) for the selected C-shaped profiles is 19.0 cm and 22.0 cm , respectively.

As the initial data for the optimization calculation, the following design lengths of the structural member were considered, corresponding to the flexural buckling of 1.2 m , $1.5 \mathrm{~m}, 1.8 \mathrm{~m}, 2.0 \mathrm{~m}$, and 2.2 m . It should be noted that the use of a single C-shaped cold-formed profile for larger design lengths is not rational.

Taking into account the small dimensionality of the stated problem of parametric optimization, it was solved by the method of exhaustive search (full enumeration of possibilities) using software developed in Python. As a result of
the optimization calculation, C-shaped cold-formed profiles with optimal cross-sectional dimensions depending on the thickness of the profile and the design length of the structural member were obtained. The resulting structural members from C-shaped cold-formed profiles with optimal cross-sectional dimensions are characterized by higher load-carrying capacity compared to members from profiles with the same steel consumption offered by the profile manufacturer [20]. As a result of optimization for the strip width of 19.0 cm , an increase in load-carrying capacity for overall buckling under the axial compression in the range of up to $26.38 \%$ inclusive was achieved (Table 1). For the strip width of 22.0 cm , the increase in load-carrying capacity was up to $22.19 \%$ inclusive (Table 2).

Tables 1, 2 give the optimal dimensions of cross-sections of the structural members made of a C-shaped cold-formed profile, depending on the profile thickness and on the design lengths, which corresponds to the flexural and flexural-torsional buckling under the axial compression. As can be seen from Tables 1, 2, for most of the optimal solutions obtained, the load-carrying capacity of the member for flexural-torsional buckling is decisive. For all obtained cross-sections of C-shaped cold-formed profiles with optimal dimensions, the phenomenon of web local buckling is characteristic. At the same time, the phenomenon of flanges local buckling was not observed for them. The phenomenon of distortional buckling of the cross-sectional shape was characteristic only for optimal design solutions for C-shaped profiles up to 0.15 mm thickness.

In order to obtain optimal solutions for the cross-sectional dimensions of a C-shaped cold-formed profile, which will not depend on the thickness of the profile and the design lengths of the structural member, a compromise solution was sought according to the following criterion:

$$
\begin{equation*}
\sum_{l_{e f}}\left(\sum_{t} 1-\frac{\hat{N}_{b R d \text { min }}\left(t, l_{e f}\right)}{N_{b R d, \text { min }}\left(t, l_{e f}\right)}\right) \rightarrow \min \tag{32}
\end{equation*}
$$

where $N_{b R d, \text { min }}\left(t, l_{e f}\right)$ - the minimum load-carrying capacity of the structural member with optimal cross-sectional dimensions in accordance with Tables 1, 2; $\hat{N}_{b R d, \text { min }}\left(t, l_{e f}\right)$ - the minimum load-carrying capacity of the structural member with "compromise" cross-sectional dimensions, calculated depending on the profile thickness $t$ and the design length $l_{e f}$ of the structural member.

The results of the search for a compromise solution are given in Tables 3, 4 for the strip width of 19.0 cm and 22.0 cm , respectively. As a result of the optimization calculation, two «compromise» solutions were obtained: C $72 \times 37 \times 22$ (for the strip width of 19.0 cm , which corresponds to the original profile C $70 \times 45 \times 15$ ) and $\mathrm{C} 92 \times 40 \times 24$ (for the strip width of 22.0 cm , which corresponds to the original profile C $100 \times 45 \times 15$ ).

For fixed steel consumption, structural members were obtained from C-shaped cold-formed profiles with "compromise" cross-sectional dimensions. They are characterized by a greater load-carrying capacity for overall buckling under axial compression compared to structural members from C-shaped cold-formed profiles offered by the profile manufacturer [20]. In this case, an increase in load-carrying capacity to $24.45 \%$ was achieved (for the strip width of 19.0 cm ) and up to $22.19 \%$ inclusive (for the strip width of 22.0 cm ).

Table 1
Structural members made of C-shaped cold-formed profiles with optimal cross-sectional dimensions (the strip width is $P_{\max }=19.0 \mathrm{~cm}$, which corresponds to the original profile $\mathrm{C} 70 \times 45 \times 15$ )

| Design length, m | $t, \mathrm{~cm}$ | Optimal dimensions of C-shaped cross section, cm | $\underset{\mathrm{kN}}{N_{b R d, \text { min }},}$ | Buckling mode | Web local buckling | Flange local buckling | Distortional buckling | Increase in load-carrying capacity, \% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.2 | 0.070 | $7.2 \times 3.7 \times 2.2$ | 17.584 | flex.-torsional | Yes | No | Yes | 23.15 |
|  | 0.100 | $7.8 \times 3.5 \times 2.1$ | 27.906 | flex.-torsional | Yes | No | Yes | 21.47 |
|  | 0.120 | $7.8 \times 3.5 \times 2.1$ | 34.468 | flex.-torsional | Yes | No | Yes | 19.65 |
|  | 0.150 | $7.8 \times 3.5 \times 2.1$ | 46.000 | flex.-torsional | Yes | No | No | 23.29 |
|  | 0.200 | $7.8 \times 3.5 \times 2.1$ | 61.864 | flex.-torsional | Yes | No | No | 19.11 |
|  | 0.250 | $7.6 \times 3.6 \times 2.1$ | 77.866 | flex.-torsional | Yes | No | No | 16.39 |
| 1.5 | 0.070 | $7.8 \times 3.5 \times 2.1$ | 13.726 | flex.-torsional | Yes | No | Yes | 24.72 |
|  | 0.100 | $7.8 \times 3.5 \times 2.1$ | 21.100 | flex.-torsional | Yes | No | Yes | 23.84 |
|  | 0.120 | $7.8 \times 3.5 \times 2.1$ | 25.956 | flex.-torsional | Yes | No | Yes | 22.60 |
|  | 0.150 | $8.2 \times 3.4 \times 2.0$ | 24.332 | flex.-torsional | Yes | No | No | 24.88 |
|  | 0.200 | $7.6 \times 3.6 \times 2.1$ | 46.877 | flex.-torsional | Yes | No | No | 20.01 |
|  | 0.250 | $7.2 \times 3.7 \times 2.2$ | 60.204 | flex.-torsional | Yes | No | No | 15.85 |
| 1.8 | 0.070 | $7.8 \times 3.5 \times 2.1$ | 10.669 | flex.-torsional | Yes | No | Yes | 25.74 |
|  | 0.100 | $7.8 \times 3.5 \times 2.1$ | 16.203 | flex.-torsional | Yes | No | Yes | 25.00 |
|  | 0.120 | $7.8 \times 3.5 \times 2.1$ | 20.002 | flex.-torsional | Yes | No | Yes | 23.99 |
|  | 0.150 | $7.8 \times 3.5 \times 2.1$ | 26.557 | flex.-torsional | Yes | No | No | 25.36 |
|  | 0.200 | $7.8 \times 3.7 \times 1.9$ | 36.726 | flex.-torsional | Yes | No | No | 18.77 |
|  | 0.250 | $7.4 \times 3.9 \times 1.9$ | 47.679 | flexural relative to the weak axis | Yes | No | No | 12.40 |
| 2.0 | 0.070 | $7.8 \times 3.5 \times 2.1$ | 9.096 | flex.-torsional | Yes | No | Yes | 26.13 |
|  | 0.100 | $7.8 \times 3.5 \times 2.1$ | 13.805 | flex.-torsional | Yes | No | Yes | 25.41 |
|  | 0.120 | $7.8 \times 3.5 \times 2.1$ | 17.121 | flex.-torsional | Yes | No | Yes | 24.49 |
|  | 0.150 | $7.8 \times 3.5 \times 2.1$ | 22.686 | flexural relative to the weak axis | Yes | No | No | 24.39 |
|  | 0.200 | $7.2 \times 3.7 \times 2.2$ | 31.835 | flexural relative to the weak axis | Yes | No | No | 17.37 |
|  | 0.250 | $7.8 \times 4.2 \times 1.4$ | 41.836 | flex.-torsional | Yes | No | No | 10.42 |
| 2.2 | 0.070 | $7.8 \times 3.5 \times 2.1$ | 7.830 | flex.-torsional | Yes | No | Yes | 26.38 |
|  | 0.100 | $7.8 \times 3.5 \times 2.1$ | 11.918 | flex.-torsional | Yes | No | Yes | 25.66 |
|  | 0.120 | $7.8 \times 3.5 \times 2.1$ | 14.866 | flex.-torsional | Yes | No | Yes | 24.79 |
|  | 0.150 | $7.6 \times 3.6 \times 2.1$ | 19.712 | flex.-torsional | Yes | No | No | 23.47 |
|  | 0.200 | $7.2 \times 3.8 \times 2.1$ | 27.830 | flex.-torsional | Yes | No | No | 15.07 |
|  | 0.250 | $7.8 \times 4.4 \times 1.2$ | 37.107 | flexural relative to the weak axis | Yes | No | No | 8.12 |

Table 2
Structural members made of C-shaped cold-formed profiles with optimal cross-sectional dimensions (the strip width is
$P_{\max }=22.0 \mathrm{~cm}$, which corresponds to the original profile C $100 \times 45 \times 15$ )

| Design length, m | $t, \mathrm{~cm}$ | Optimal dimensions of C-shaped cross section, cm | $\begin{gathered} N_{b R d, \text { min }}, \\ \mathrm{kN} \end{gathered}$ | Buckling mode | Web local buckling | Flange local buckling | Distortional buckling | Increase in load-carrying capacity, \% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 1.2 | 0.070 | $9.2 \times 4.0 \times 2.4$ | 21.272 | flex.-torsional | Yes | No | Yes | 22.19 |
|  | 0.100 | $7.6 \times 4.5 \times 2.7$ | 35.742 | flex.-torsional | Yes | No | Yes | 17.82 |
|  | 0.120 | $8.6 \times 4.2 \times 2.5$ | 45.441 | flex.-torsional | Yes | No | Yes | 15.66 |
|  | 0.150 | $9.2 \times 4.0 \times 2.4$ | 63.224 | flex.-torsional | Yes | No | No | 21.25 |
|  | 0.200 | $9.2 \times 4.0 \times 2.4$ | 83.711 | flex.-torsional | Yes | No | No | 13.72 |
|  | 0.250 | $9.2 \times 4.0 \times 2.4$ | 104.494 | flex.-torsional | Yes | No | No | 6.67 |
| 1.5 | 0.070 | $9.2 \times 4.0 \times 2.4$ | 17.902 | flex.-torsional | Yes | No | Yes | 19.97 |
|  | 0.100 | $8.6 \times 4.2 \times 2.5$ | 28.550 | flex.-torsional | Yes | No | Yes | 15.72 |
|  | 0.120 | $8.6 \times 4.2 \times 2.5$ | 35.684 | flex.-torsional | Yes | No | Yes | 14.18 |
|  | 0.150 | $9.2 \times 4.0 \times 2.4$ | 48.262 | flex.-torsional | Yes | No | No | 17.98 |
|  | 0.200 | $9.2 \times 4.0 \times 2.4$ | 65.045 | flex.-torsional | Yes | No | No | 12.77 |
|  | 0.250 | $9.0 \times 4.1 \times 2.4$ | 82.258 | flex.-torsional | Yes | No | No | 6.54 |

Continuation of Table 2

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.8 | 0.070 | $9.2 \times 4.0 \times 2.4$ | 14.631 | flex.-torsional | Yes | No | Yes | 17.95 |
|  | 0.100 | $9.2 \times 4.0 \times 2.4$ | 22.624 | flex.-torsional | Yes | No | Yes | 14.71 |
|  | 0.120 | $9.2 \times 4.0 \times 2.4$ | 28.044 | flex.-torsional | Yes | No | Yes | 13.52 |
|  | 0.150 | $9.2 \times 4.0 \times 2.4$ | 37.399 | flex.-torsional | Yes | No | No | 15.86 |
|  | 0.200 | $9.0 \times 4.1 \times 2.4$ | 50.991 | flex.-torsional | Yes | No | No | 10.61 |
|  | 0.250 | $8.8 \times 4.2 \times 2.4$ | 65.664 | flex.-torsional | Yes | No | No | 4.57 |
| 2.0 | 0.070 | $9.2 \times 4.0 \times 2.4$ | 12.730 | flex.-torsional | Yes | No | Yes | 16.92 |
|  | 0.100 | $9.2 \times 4.0 \times 2.4$ | 19.462 | flex.-torsional | Yes | No | Yes | 14.21 |
|  | 0.120 | $9.2 \times 4.0 \times 2.4$ | 24.090 | flex.-torsional | Yes | No | Yes | 13.13 |
|  | 0.150 | $9.2 \times 4.0 \times 2.4$ | 32.054 | flex.-torsional | Yes | No | No | 14.80 |
|  | 0.200 | $9.0 \times 4.1 \times 2.4$ | 44.314 | flex.-torsional | Yes | No | No | 9.73 |
|  | 0.250 | $8.8 \times 4.3 \times 2.3$ | 57.047 | flex.-torsional | Yes | No | No | 3.70 |
| 2.2 | 0.070 | $9.2 \times 4.0 \times 2.4$ | 11.101 | flex.-torsional | Yes | No | Yes | 16.12 |
|  | 0.100 | $9.2 \times 4.0 \times 2.4$ | 16.869 | flex.-torsional | Yes | No | Yes | 13.78 |
|  | 0.120 | $9.2 \times 4.0 \times 2.4$ | 20.895 | flex.-torsional | Yes | No | Yes | 12.75 |
|  | 0.150 | $9.2 \times 4.0 \times 2.4$ | 27.838 | flex.-torsional | Yes | No | No | 13.92 |
|  | 0.200 | $8.6 \times 4.2 \times 2.5$ | 38.491 | flex.-torsional | Yes | No | No | 7.33 |
|  | 0.250 | $8.8 \times 4.4 \times 2.2$ | 50.070 | flexural relative to the weak axis | Yes | No | No | 6.90 |

Table 3
Structural members made of C -shaped cold-formed profiles with optimal cross-sectional dimensions (strip width $P_{\max }=19.0 \mathrm{~cm}$, which corresponds to the original profile $\mathrm{C} 70 \times 45 \times 15$ )

| Design length, m | $t, \mathrm{~cm}$ | Optimal dimensions of C-shaped cross section, cm | $\underset{\mathrm{kN}}{N_{b R d, \text { min }},}$ | Compromise solution, cm | $\hat{N}_{b R d, \text { min }}, \mathrm{kN}$ | Reduction of load-carrying capacity compared to the optimal profile, \% | Increase in load-carrying capacity compared to the original profile, \% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.2 | 0.070 | $7.2 \times 3.7 \times 2.2$ | 17.584 | $7.2 \times 3.7 \times 2.2$ | 17.584 | - | 23.15 |
|  | 0.100 | $7.8 \times 3.5 \times 2.1$ | 27.906 |  | 27.840 | 0.24 | 21.18 |
|  | 0.120 | $7.8 \times 3.5 \times 2.1$ | 34.468 |  | 34.467 | 0.01 | 19.64 |
|  | 0.200 | $7.8 \times 3.5 \times 2.1$ | 61.864 |  | 60.662 | 1.94 | 16.80 |
|  | 0.250 | $7.6 \times 3.6 \times 2.1$ | 77.866 |  | 76.848 | 1.31 | 14.87 |
| 1.5 | 0.070 | $7.8 \times 3.5 \times 2.1$ | 13.726 | $7.2 \times 3.7 \times 2.2$ | 13.648 | 0.57 | 24.01 |
|  | 0.100 | $7.8 \times 3.5 \times 2.1$ | 21.100 |  | 20.886 | 1.01 | 22.59 |
|  | 0.120 | $7.8 \times 3.5 \times 2.1$ | 25.956 |  | 25.697 | 1.00 | 21.38 |
|  | 0.150 | $8.2 \times 3.4 \times 2.0$ | 24.332 |  | 33.548 | 2.28 | 22.02 |
|  | 0.200 | $7.6 \times 3.6 \times 2.1$ | 46.877 |  | 46.145 | 1.56 | 18.14 |
|  | 0.250 | $7.2 \times 3.7 \times 2.2$ | 60.204 |  | 60.204 | - | 15.85 |
| 1.8 | 0.070 | $7.8 \times 3.5 \times 2.1$ | 10.669 | $7.2 \times 3.7 \times 2.2$ | 10.553 | 1.08 | 24.38 |
|  | 0.100 | $7.8 \times 3.5 \times 2.1$ | 16.203 |  | 15.950 | 1.56 | 23.04 |
|  | 0.120 | $7.8 \times 3.5 \times 2.1$ | 20.002 |  | 19.661 | 1.70 | 21.88 |
|  | 0.150 | $7.8 \times 3.5 \times 2.1$ | 26.557 |  | 25.808 | 2.82 | 21.82 |
|  | 0.200 | $7.8 \times 3.7 \times 1.9$ | 36.726 |  | 36.509 | 0.59 | 18.07 |
|  | 0.250 | $7.4 \times 3.9 \times 1.9$ | 47.679 |  | 45.185 | 5.23 | 6.52 |
| 2.0 | 0.070 | $7.8 \times 3.5 \times 2.1$ | 9.096 | $7.2 \times 3.7 \times 2.2$ | 8.975 | 1.33 | 24.45 |
|  | 0.100 | $7.8 \times 3.5 \times 2.1$ | 13.805 |  | 13.548 | 1.86 | 23.07 |
|  | 0.120 | $7.8 \times 3.5 \times 2.1$ | 17.121 |  | 16.761 | 2.10 | 20.99 |
|  | 0.150 | $7.8 \times 3.5 \times 2.1$ | 22.686 |  | 22.152 | 2.36 | 21.46 |
|  | 0.200 | $7.2 \times 3.7 \times 2.2$ | 31.835 |  | 31.835 | - | 17.37 |
|  | 0.250 | $7.8 \times 4.2 \times 1.4$ | 41.836 |  | 37.763 | 9.74 | - |
| 2.2 | 0.070 | $7.8 \times 3.5 \times 2.1$ | 7.830 | $7.2 \times 3.7 \times 2.2$ | 7.710 | 1.54 | 24.43 |
|  | 0.100 | $7.8 \times 3.5 \times 2.1$ | 11.918 |  | 11.663 | 2.14 | 22.96 |
|  | 0.120 | $7.8 \times 3.5 \times 2.1$ | 14.866 |  | 14.499 | 2.47 | 21.71 |
|  | 0.150 | $7.6 \times 3.6 \times 2.1$ | 19.712 |  | 19.317 | 2.01 | 21.00 |
|  | 0.200 | $7.2 \times 3.8 \times 2.1$ | 27.830 |  | 26.953 | 3.15 | 11.45 |
|  | 0.250 | $7.8 \times 4.4 \times 1.2$ | 37.107 |  | 31.946 | 13.91 | - |

Table 4
Structural members made of C-shaped cold-formed profiles with optimal cross-sectional dimensions (strip width
$P_{\max }=22.0 \mathrm{~cm}$, which corresponds to the original profile $\mathrm{C} 100 \times 45 \times 15$ )

| Design length, $m$ | $t, \mathrm{~cm}$ | Optimal dimensions of C-shaped cross section, cm | $N_{b R d, \text { min }}$, <br> kN | Compromise solution, cm | $\hat{N}_{b R d, \text { min }}, \mathrm{kN}$ | Reduction of load-carrying capacity compared to the optimal profile, \% | Increase in load-carrying capacity compared to the original profile, \% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.2 | 0.070 | $9.2 \times 4.0 \times 2.4$ | 21.272 | $9.2 \times 4.0 \times 2.4$ | 21.272 | - | 22.19 |
|  | 0.100 | $7.6 \times 4.5 \times 2.7$ | 35.742 |  | 35.452 | 0.81 | 16.86 |
|  | 0.120 | $8.6 \times 4.2 \times 2.5$ | 45.441 |  | 45.101 | 0.75 | 14.79 |
|  | 0.150 | $9.2 \times 4.0 \times 2.4$ | 63.224 |  | 63.224 | - | 21.25 |
|  | 0.200 | $9.2 \times 4.0 \times 2.4$ | 83.711 |  | 83.710 | - | 13.72 |
|  | 0.250 | $9.2 \times 4.0 \times 2.4$ | 104.494 |  | 104.494 | - | 6.67 |
| 1.5 | 0.070 | $9.2 \times 4.0 \times 2.4$ | 17.902 | $9.2 \times 4.0 \times 2.4$ | 17.902 | - | 19.97 |
|  | 0.100 | $8.6 \times 4.2 \times 2.5$ | 28.550 |  | 28.538 | 0.04 | 15.67 |
|  | 0.120 | $8.6 \times 4.2 \times 2.5$ | 35.684 |  | 35.670 | 0.04 | 14.14 |
|  | 0.150 | $9.2 \times 4.0 \times 2.4$ | 48.262 |  | 48.262 | - | 17.98 |
|  | 0.200 | $9.2 \times 4.0 \times 2.4$ | 65.045 |  | 65.045 | - | 12.77 |
|  | 0.250 | $9.0 \times 4.1 \times 2.4$ | 82.258 |  | 80.269 | 2.42 | 3.96 |
| 1.8 | 0.070 | $9.2 \times 4.0 \times 2.4$ | 14.631 | $9.2 \times 4.0 \times 2.4$ | 14.631 | - | 17.95 |
|  | 0.100 | $9.2 \times 4.0 \times 2.4$ | 22.624 |  | 22.624 | - | 14.71 |
|  | 0.120 | $9.2 \times 4.0 \times 2.4$ | 28.044 |  | 28.044 | - | 13.52 |
|  | 0.150 | $9.2 \times 4.0 \times 2.4$ | 37.399 |  | 37.399 | - | 15.86 |
|  | 0.200 | $9.0 \times 4.1 \times 2.4$ | 50.991 |  | 50.930 | 0.12 | 10.48 |
|  | 0.250 | $8.8 \times 4.2 \times 2.4$ | 65.664 |  | 60.885 | 7.28 | - |
| 2.0 | 0.070 | $9.2 \times 4.0 \times 2.4$ | 12.730 | $9.2 \times 4.0 \times 2.4$ | 12.730 | - | 16.92 |
|  | 0.100 | $9.2 \times 4.0 \times 2.4$ | 19.462 |  | 19.462 | - | 14.21 |
|  | 0.120 | $9.2 \times 4.0 \times 2.4$ | 24.090 |  | 24.090 | - | 13.13 |
|  | 0.150 | $9.2 \times 4.0 \times 2.4$ | 32.054 |  | 32.054 | - | 14.80 |
|  | 0.200 | $9.0 \times 4.1 \times 2.4$ | 44.314 |  | 42.868 | 3.26 | 6.15 |
|  | 0.250 | $8.8 \times 4.3 \times 2.3$ | 57.047 |  | 51.181 | 10.28 | - |
| 2.2 | 0.070 | $9.2 \times 4.0 \times 2.4$ | 11.101 | $9.2 \times 4.0 \times 2.4$ | 11.101 | - | 16.12 |
|  | 0.100 | $9.2 \times 4.0 \times 2.4$ | 16.869 |  | 16.869 | - | 13.78 |
|  | 0.120 | $9.2 \times 4.0 \times 2.4$ | 20.895 |  | 20.895 | - | 12.75 |
|  | 0.150 | $9.2 \times 4.0 \times 2.4$ | 27.838 |  | 27.838 | - | 13.92 |
|  | 0.200 | $8.6 \times 4.2 \times 2.5$ | 38.491 |  | 36.443 | 5.32 | 1.62 |
|  | 0.250 | $8.8 \times 4.4 \times 2.2$ | 50.070 |  | 43.471 | 13.18 | - |

5.3. Recommendations for the optimal distribution of material in cross-sections of structural members from cold-formed profiles

Analysis of the obtained C-shaped cold-formed profiles with optimal cross-sectional dimensions allowed us to devise recommendations for the optimal distribution of the material in the cross-sections of structural members from cold-formed profiles operating under axial compression conditions. The average ratio of the optimal single edge folds length $c_{o p t}$ to the optimal flange width $b_{\text {opt }}$ was:

$$
\begin{equation*}
\frac{c_{\text {opt }}}{b_{\text {opt }}} \cong 0.56 \text {. } \tag{33}
\end{equation*}
$$

The average ratio of the optimal flange width $b_{\text {opt }}$ to the optimal height of the profile $h_{\text {opt }}$ was:

$$
\begin{equation*}
\frac{b_{\text {opt }}}{h_{\text {opt }}} \cong 0.46 . \tag{34}
\end{equation*}
$$

The optimal ratio of the moments of inertia of the cross-section of the profile relative to the main inertial axes $I_{z, o p t}$ and $I_{y, \text { opt }}$ was (the orientation of the axes is adopted in accordance with Fig. 3):

$$
\begin{equation*}
\frac{I_{y, o p t}}{I_{z, o p t}}=0.2 \ldots . . .0 .29 . \tag{35}
\end{equation*}
$$

The radii of inertia of the cross-section of the profile relative to the main axes of inertia $i_{z}$ and $i_{y}$ are obtained in the range:

$$
\begin{align*}
& i_{z, \text { opt }}=(0.38 \ldots 0.39) h ;  \tag{36}\\
& i_{y, \text { opt }}=(0.37 \ldots 0.41) b . \tag{37}
\end{align*}
$$

## 6. Discussion of results of optimization of cross-sectional dimensions of structural members from cold-formed profiles

Our results are explained by the type of the considered cross-section of the structural member, the stressed-strained state of the member, as well as the phenomenon of cross-sectional web local buckling, which is characteristic of all the optimal solutions is obtained. The reliability of our results of optimizing the size of cross-sections of structural members from C-shaped cold-formed profiles (Tables 1, 2) is confirmed by:

- rigor and correctness of the mathematical model of the problem of optimal design of the studied class of structures;
- stability of the obtained numerical solutions in relation to the initial data and analysis of the convergence of the iterative search process.

To solve the problem of optimizing the size of cross-sections of structural elements from cold-formed profiles, a method of finding a compromise has been developed, which ensures optimal solutions that do not depend on the thickness of the profile and the design lengths of the member. This is undoubtedly an advantage of this work compared to similar known studies. For example, in [3], optimal design solutions for C-shaped cold-formed profiles were obtained but the optimization results depend on the profile thickness ( 1.5 mm and 3 mm ) and on the design length of the element ( 3 m and 5 m ). The same applies to work [10], in which the authors obtained the optimal cross-sectional dimensions of the rod element from the C-shaped cold-formed profile, depending on the profile thickness of 1.6 mm and on the design length of the element ( $0.0 \mathrm{~m}, 1.0 \mathrm{~m}$, and 3.0 m ).

It should be noted that the results obtained regarding the optimal distribution of the material in the cross-sections of the structural members made of cold-formed profiles (33) to (37) apply only to the case of a C-shaped profile under axial compression conditions. However, the proposed method of finding the optimal cross-sectional dimensions of structural members from cold-formed profiles is also applicable for other types of cross-sections of profiles. With this in mind, in the development of this study, one can consider optimizing the cross-sectional dimensions of the structural members from cold-formed profiles of other types of profiles that are under conditions other than axial compression.

## 7. Conclusions

1. A procedure for searching for optimal cross-sectional dimensions of structural members from C-shaped coldformed profiles has been proposed, taking into account their post-buckling behavior and structural requirements. The proposed methodology provides optimal solutions that do not depend on the thickness of the profile and the design lengths of the structural member, which is its distinguishing feature.
2. The C-shaped profiles with optimal cross-sectional dimensions have been obtained, which are characterized by higher load-carrying capacity compared to structural members from profiles with the same steel consumption offered by the manufacturer. An increase in load-carrying capacity for overall buckling under axial compression of structural members from such profiles up to $24.45 \%$ (for a strip width of 19.0 cm ) and up to $22.19 \%$ (for a strip width of 22.0 cm ) was achieved. For all the resulting cross-sections of C-shaped profiles with optimal dimensions, the phenomenon of web local buckling is characteristic, which is a qualitative indicator of the study results.
3. Recommendations have been devised for designers on optimal dimensional ratios and geometric characteristics of the cross-section of the C-shaped profile operating under axial compression conditions. The resulting optimal ratios (33) to (37) can be used by design engineers at the stage of selection of cross-sections of structural elements from cold-formed profiles. Our studies also serve as the basis for the development of an effective range and assortment of cold-formed profiles, which is a qualitative indicator of the results of the current research.

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## Conflict of interests

The authors declare that they have no conflict of interest in relation to this research, whether financial, personal, authorship or otherwise, that could affect the research and its results presented in this paper.

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