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A STUDY OF HYDRODYNAMIC VISCOUS FLUID FLOW PARAMETERS CHANGE REGULARITIES IN CASE OF A CONICAL DIFFUSER

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Studies of patterns of changes in hydrodynamic parameters of the viscous incompressible fluid in a conical diffuser were conducted. The specificity of the viscous liquid flow in a conical diffuser is that the kinetic energy of the flow, depending on the opening angle, is converted into pressure energy. Depending on Reynolds numbers and diffuser opening angles, the velocity vector field is stationary. With an increase in the Reynolds number, the symmetry of the flow relative to the axis of the diffuser is broken. A general solution to the approximate Navier-Stokes equations is given, based on the diffuser opening angle and the Reynolds number. A method for integrating the boundary value problem has been developed, and the patterns of velocity changes across the diffuser length at a parabolic distribution of velocities in the inlet section are obtained. By integrating partial differential equations that match all boundary conditions, the solution to the boundary value problem can be found. Graphs of changes in radial and axial velocities along the length and with a fixed value of the opening angle are shown; the flow pattern and the transition of a single-mode flow to multimode regimes are obtained. For a fixed opening angle and Reynolds number, the conditions for flow separation from a fixed wall are derived, where the flow velocity changes the sign. A mixing process is observed in the multi-mode region, which is accompanied by numerous pulsation phenomena and an unstable diffuser operation, where the resulting solutions are inappropriate. Based on the results of the studies obtained, it is possible to correctly design a conical diffuser, namely, under the condition of non-separated flow, to choose the opening angle and its length

Keywords: conical diffuser, velocity profile, pressure distribution, breaking point, viscous fluid, fluid flow

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1. Introduction

Diffusers, either as nozzles or constituent elements, are frequently used in many mechanisms and machines designed to regulate or control the production cycle. The regulator's work depends mainly on the stable operation of the main unit, i.e., the conical diffuser. In this regard, the study of the viscous fluid flow in conical diffusers is relevant as the analysis results will reveal the conditions for mechanism unit construction, ensuring its reliable and durable operation. Given its practical importance, this task has attracted the attention of many researchers. The stable operation area of all equipment largely depends on the viscous fluid flow mode in conical diffusers. Therefore, the study of viscous fluid flow in diffusers aims to discover patterns of changes in the flow's hydrodynamic parameters that are not only relevant but also represent a great practical value.

2. Literature review and problem statement

The viscous fluid flow in conical diffusers is one of the classic problems of hydromechanics and has attracted the attention of many researchers. The classical problem statement was first formulated by [1, 2], who proposed the solution of equations of viscous fluid motion in diffusers, taking into account squares of components of velocities and their product multiplication. In further studies, justifications have been made about the effectiveness of this approach, and solutions have been proposed based on the results of experimental data. Despite the fundamental formulation of the issue and the conventional solutions offered, their practical implementation necessitates time-consuming computing work, making them difficult to implement.

The authors of papers [3, 4] also addressed these issues in terms of identifying stable and unstable traffic areas in conical diffusers, which is important for assessing their performance. However, the problem's solution was reduced to a system of nonlinear transcendental equations, where integration is loaded with difficulties. This approach did not allow efficient calculations for specific parameters of the diffuser. Therefore, the authors proposed more suitable methods for integrating the differential equations of motion in the cone diffuser area. Moreover, the solution of the boundary value problem was carried out at constant values of velocities in the inlet sections of the diffuser, which does not correspond to reality.

As for [5] reference, it is considered while solving the problem.

In the work [6] the authors studied the generalization of the Jeffrey-Hamel problem solution, obtained conditions for asymmetric stationary flows, and gave one-, two-, and three-mode bifurcation solutions. Conditions for ensuring stationary asymmetric and multi-mode solutions are found for specific intervals of Reynolds numbers and opening angles. The authors of [6] generalized the Jeffrey-Hamel problem solution and deduced conditions for stationary asymmetrical and multi-mode solutions for specific Reynolds numbers and diffuser opening angles.

In [7], the author is studying the evolution of the main single-mode stationary flow of the viscous incompressible fluid in the flat diffuser. The Jeffrey-Hamel problem solution is obtained based on the opening angle of the diffuser and Reynolds number. It is established that starting from some critical value of the Reynolds number, the existence of a stationary single-mode flow is impossible. The results of examining several laminar flow regimes in a flat diffuser/confuser with a small opening angle were presented by the authors in [8]. As a result, the regularities of changes in the hydrodynamic parameters of the viscous incompressible liquid by numerical simulation are determined based on the solution of the Navier-Stokes equations. The existence of stationary and non-stationary flow regimes was determined, depending on the Reynolds number. The Reynolds number values determining the ranges of the existence of these regimes of fluid flow for Newtonian and non-Newtonian fluids are obtained.

In [9], the author, based on a numerical solution of the Navier-Stokes equations for a viscous incompressible fluid, has studied the flow regimes in a flat diffuser with a small opening angle. Depending on the Reynolds number, the existence of stationary and non-stationary flow regimes has been identified. The conditions of transition of current regimes in the diffuser from symmetrical stationary to asymmetrical stationary and then to non-stationary asymmetrical were obtained. The values of the ranges of Reynolds numbers for the existence of these regimes are presented. The fluid flow in diffusers most often occurs in non-stationary and turbulent regimes, therefore, a significant part of the theoretical and experimental studies are devoted to these very regimes in flat diffusers [10].

In [11], the authors obtained the criteria for classifying separations in flat diffusers, as well as diagrams for determining them. Flows in channels and in the diffuser with a small opening angle and at low Reynolds numbers have similar features. Free-jet flows and flows in rapidly expanding channels are margin circumstances of the flow in diffusers.

In [12], an idealized solution of the Jeffrey-Hamel problem for an expanding channel is proposed. Numerical results for a two-dimensional flow in a wedge bounded by two circles are given. The outflow and bifurcation conditions, depending on the Reynolds number are shown. A mathematical model has been created based on studies of changes in the hydrodynamic parameter pattern of a viscous incompressible fluid in the transitional sections of flat pipes, which has allowed obtaining results with acceptable accuracy indicating motion dynamics patterns [13].

Despite a large number of works on the hydrodynamics of a viscous incompressible fluid, new approaches are required to investigate the change in patterns of hydrodynamic flow parameters in conical diffusers. The qualitative characteristic parameters determining the flow properties of the viscous incompressible liquid in a conical diffuser under flow stability conditions are the opening angle of the diffuser and the Reynolds number. In the cited sources [6–13], thorough research on structural changes in the hydrodynamic parameters of a viscous fluid in diffusers was carried out and bifurcation conditions were determined related to certain values of the geometric parameters of the diffuser, which limits the application of the results obtained for arbitrarily given diffuser dimensions.

From the above analyses, it can be concluded that it is necessary to formulate a boundary value problem for studying the patterns of changes in the hydrodynamic parameters of a viscous liquid in conical diffusers, which allows developing a universal method for computing the above parameters suitable for engineering calculations.

3. The aim and objectives of the study

The aim of the study is to reveal the patterns of changes in the hydrodynamic parameters of viscous fluid in conical diffusers depending on the opening angle and Reynolds number, which will enable to determine the dimensions of the diffuser from the condition of it's stable operation, which is of great practical importance.

To achieve this aim, the following objectives are completed:

– to formulate the boundary problem, set initial and boundary conditions and develop a method for its solution, identify regularities of hydrodynamic parameters change of a viscous flow in a conical diffuser;

– to design the graphs of changes in radial velocities along the cross section and along the length of the diffuser, as well as shear stresses on the wall of the stationary channel along the length;

– to determine the dependence of the bifurcation point on the diffuser opening angle and the Reynolds number;

– to identify the conditions for reliable and stable operation of the diffuser.

4. Materials and methods

4. 1. Choosing a calculation scheme

The problem of viscous fluid flow development in the conical diffuser is considered. The conical diffuser is made up of a conic surface with an opening angle (Fig. 1) that is directed along the axis to infinity. The motion in a conical diffuser will be considered in spherical coordinates, starting with the zero point (Fig. 1).

It is assumed that in the inlet sections of the diffuser at x=1, the velocities change according to a parabolic law. It is necessary to find patterns of change in the hydrodynamic parameters of a viscous fluid in a conical diffuser, assuming that it is axisymmetric and steady. Mass forces are neglected.

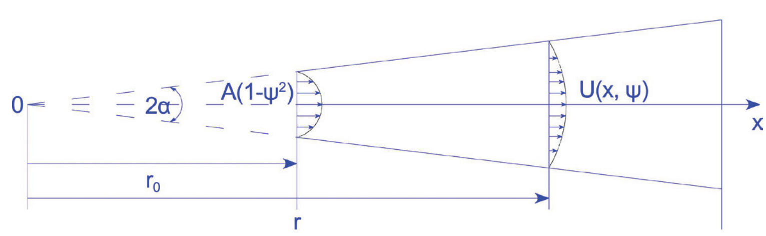

Fig. 1. On the study of a viscous fluid motion in a conical diffuser

4. 2. Statement of the problem and formulation of the system of differential equations for the study

The study of the patterns of change in the hydrodynamic parameters of a viscous fluid is carried out on the basis of the Navier-Stokes equations. However, exact solutions to these equations are possible only for limited problems. This explains the fact that such problems are mostly solved by the finite element method. In addition to computational methods, exact integration of the simplified Navier-Stokes equations is also effective. The task of studying the patterns of changes in the hydrodynamic parameters of viscous liquids in conical diffusers is carried out according to the approximating system of Navier-Stokes equations. Let us assume that the patterns of radial distribution of the liquid velocity at the inlet section of the diffuser is spherical, i.e.

$$\upsilon_r = A\left(1-\phi^2\right), \text{ at } r=r_0.$$

Let us consider the patterns of changes in viscous fluid motion, assuming that the motion is asymmetrical and steady. Neglecting the inertial members in equations of viscous flow expressed in spherical coordinates [3, 4], we get a system of approximate equations.

$$\upsilon_r \frac{\partial \upsilon_r}{\partial r} + \frac{\upsilon_\phi}{r} \cdot \frac{\partial \upsilon_r}{\partial \phi} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{\nu}{r^2}\frac{\partial^2 \upsilon_r}{\partial \phi^2}, \tag{1}$$

$$-\frac{\partial p}{\partial \phi} + \frac{2\mu}{r}\frac{\partial \upsilon_r}{\partial \phi} = 0, \tag{2}$$

$$\frac{\partial \upsilon_r}{\partial r} + \frac{\upsilon_r}{r} + \frac{1}{r}\frac{\partial \upsilon_\phi}{\partial \phi} = 0, \tag{3}$$

where $\nu=\mu/\rho$ is the kinematic viscosity coefficient, μ is the dynamic viscosity coefficient, υ_r is the fluid velocity in radial directions, r is the radius (Fig. 1), all the notations are well known [3, 4].

Having in mind, that transverse velocity υ_ϕ is a negligibly small value, we can take $\upsilon_\phi=0$, and the radial velocity υ_r can be replaced for a given section with an average flow rate U. In these conditions, the equation will take the form:

$$U\frac{\partial \upsilon_r}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{\nu}{r^2}\left(\frac{\partial^2 \upsilon_r}{\partial \phi^2} + \frac{1}{\phi}\frac{\partial v_r}{\partial \phi}\right). \tag{4}$$

(2)–(4) constitute a system of approximate equations of fluid motion to identify patterns of changes in the hydrodynamic parameters of a viscous fluid in a conical diffuser. The characteristic velocity is the velocity incorporated into the Reynolds number formula [3], $\mathrm{Re} = \frac{Ur\alpha}{\nu}$.

Based on the condition that is in this regime, maintaining a constant value of the number Re leads to the dependence of characteristic velocity on the radius

$$U = \frac{U_0 r_0}{r}. \tag{5}$$

The average flow rate in the inlet section will be:

$$U_0 = \frac{Q}{2\pi \cdot r_0^2 (1-\cos\alpha)}, \tag{6}$$

where Q is the fluid flow rate in the conical diffuser, $2\pi \cdot r_0^2(1-\cos\alpha)$ is the area of the inlet section of the spherical surface.

For a given initial velocity distribution, the flow rate of the liquid flowing into the diffuser will be:

$$Q = \int_0^\alpha \int_0^{2\pi} A\left(1-\phi^2\right) r_0^2 \sin d\phi d\theta = \\ = 2\pi \cdot r_0^2 A\left[2\alpha \sin\alpha + \cos\alpha\left(1-\alpha^2\right) - 1\right], \tag{7}$$

and the corresponding average velocity in the initial section of the diffuser according to (6) will be

$$U_0 = \frac{A\left[2\alpha\sin\alpha + \cos\alpha\left(1-\alpha^2\right)-1\right]}{1-\cos\alpha}. \tag{8}$$

In order to obtain universal solutions to the problem, we introduce dimensionless variables u, v, Ψ, x, p assuming that:

$$u = \frac{\upsilon_r}{U_0}, \quad v = \frac{\phi \upsilon_\phi}{\alpha U_0}, \quad \psi = \frac{\phi}{\alpha}, \quad x = \frac{r}{r_0}, \quad \overline{p} = \frac{p}{\rho U_0^2}. \tag{9}$$

Taking into account designation (9), equations (2)–(4) take the form:

$$\begin{cases} \frac{1}{x}\frac{\partial u}{\partial x} = -\frac{\partial \overline{p}}{\partial x} + \frac{\nu}{r_0 U_0 \alpha^2 x^2}\left(\frac{\partial^2 u}{\partial \psi^2} + \frac{1}{\psi}\frac{\partial u}{\partial \psi}\right), \\ -\frac{\partial \overline{p}}{\partial \psi} + \frac{2\nu}{U_0 r_0 x}\frac{\partial u}{\partial \psi} = 0, \\ \frac{\partial u}{\partial x} + \frac{2u}{x} + \frac{1}{x\alpha\psi}\frac{\partial v}{\partial \psi} = 0. \end{cases} \tag{10}$$

From the second equation of system (10), after integration over the angle Ψ, the equation is defined:

$$\overline{P}(x,\psi) = \frac{2\nu}{U_0 r_0}\frac{u}{x} + C(x), \tag{11}$$

where $C(x)$ is an unknown function of x, to be determined.

Differentiating (11) by x, we get:

$$\frac{\partial \overline{P}(x,\psi)}{\partial x} = 2\frac{\nu}{U_0 r_0}\frac{\partial}{\partial x}\left(\frac{u}{x}\right) + \frac{dC(x)}{dx}. \tag{12}$$

If we substitute $\frac{\partial \overline{p}(x,\psi)}{\partial x}$ into the right side of the first equation (10) and discard the member containing the value $\frac{\partial}{\partial x}\left(\frac{u}{x}\right)$ as a member of a lower order, the system of equations (10) is finally transformed to the form:

$$\begin{cases} \frac{1}{x}\frac{\partial u}{\partial x} = \frac{a^2}{x^2}\frac{\partial^2 u}{\partial \psi^2} - \frac{dC(x)}{dx}, \\ \frac{\partial u}{\partial x} + \frac{u}{x} + \frac{1}{x}\frac{dv}{d\psi}, \end{cases} \tag{13}$$

where designated $a^2 = \frac{\nu}{r_0 \alpha^2 U_0}$.

To integrate the system of equations (13), based on the essence of the problem, it is necessary to establish boundary conditions.

4. 3. Choice of boundary conditions

To integrate the system of equations (13), the boundary conditions are set:

1) conditions for adhesion between the liquid and the wall surface:

$$x>0,\ \psi=\pm 1,\ u=0,\ \upsilon=0; \tag{14}$$

2) conditions for the symmetry of the velocity profile along the flow section:

$$\psi=0,$$
$$\frac{\partial u}{\partial \psi}=0, \tag{15}$$

3) conditions for the radial velocity distribution over the input arc section are given by some function $f(\psi)$.

When $x=1$

$$-1\le\psi\le+1,\ u(1,\ \psi)=A(1-\psi^2). \tag{16}$$

Thus, to study the viscous fluid flow in a conical diffuser, the system of differential equations (13) with boundary conditions (14)–(16) is integrated.

5. Results of research to identify patterns of changes in hydrodynamic parameters

5. 1. Integration of the boundary value problem to identify patterns of change in radial and transverse velocities as well as pressure

From the solution of the equation of system (11), we get the form of the sum [5]

$$u(x,\psi)=\sum_{k=1}^{\infty}R_k(x)J_0(\lambda_k\psi)+\sum_{k=1}^{\infty}W_k(x)J_0(\lambda_k\psi), \tag{17}$$

where $R_k(x)$ and $W_k(x)$ are continuous functions to be determined.

From the boundary condition (16), it follows:

$$A(1-\psi^2)=\sum_{k=1}^{\infty}\left[R_k(1)+W_k(1)\right]\cdot J_0(\lambda_k\psi). \tag{18}$$

We choose an arbitrary function $W_k(x)$ so that the condition $W_k(1)=0$ is satisfied. To determine the value $R_k(1)$, both parts of equality (18) are multiplied by $\Psi J_0(\lambda_k\Psi)d\Psi$ and integrating in the interval (0; 1) we get:

$$A\int_0^1(1-\psi^2)\cdot J_0(\lambda_n\psi)\psi d\psi=$$
$$=\sum_{k=1}^{\infty}\int_0^1 R_k(1)\cdot J_0(\lambda_k\psi)J_0(\lambda_n\psi)\psi d\psi. \tag{19}$$

Using the orthogonality property of the function

$\{\Psi J_0(\lambda_k\Psi)\}$,

$$\int_0^1\psi J_0(\lambda_k\psi)J_0(\lambda_n\psi)d\psi=\begin{cases}0,\ \lambda_k\ne\lambda_n,\\ \frac{1}{2}J_1^2(\lambda_n),\ \lambda_k=\lambda_n.\end{cases}, \tag{20}$$

equation (19) is transformed into the form:

$$R_k(1)=\frac{2A}{J_1^2(\lambda_k)}\int_0^1(1-\psi^2)\cdot J_0(\lambda_n\psi)\psi\cdot d\psi=$$
$$=\frac{2A}{J_1^2(\lambda_k)}\left[\frac{J_1(\lambda_k)}{\lambda_k}-\frac{2J_2(\lambda_k)}{\lambda_k^2}\right]. \tag{21}$$

Using the properties of Bessel functions, equality (21) is transformed to the form:

$$R_k(1)=\frac{8A}{\lambda_k^3 J_1(\lambda_k)}=C_k, \tag{22}$$

where λ_k eigenvalues, positive roots of eigenfunctions $J_0(\lambda_k)=0$.

The first equation of system (13), taking into account (17), will be rewritten in the form:

$$\sum_{k=1}^{\infty}\left[R_k'(x)+W_k'(x)\right]J_0(\lambda_k\psi)=$$
$$=\frac{a^2}{x}\sum_{k=1}^{\infty}\left[R_k(x)+W_k(x)\right]\left[J_0''(\lambda_k\psi)+\frac{1}{\psi}J_0'(\lambda_k\psi)\right]-$$
$$-xC'(x). \tag{23}$$

We can put the function $C`(x)$ in a series as $J_0(\lambda_k\Psi)$ eigenfunctions, we will have:

$$C'(x)=\sum_{k=1}^{\infty}B_k(x)J_0(\lambda_k\psi), \tag{24}$$

where

$$B_k(x)=\frac{2C'(x)}{J_1^2(\lambda_k)}\int_0^1\sum_{k=1}^{\infty}\psi J_0(\lambda_k\psi)d\psi=\frac{4C'(x)}{\lambda_k J_1(\lambda_k)}. \tag{25}$$

From equations (23), taking into account (24), we can find:

$$\sum_{k=1}^{\infty}\left[R_k'(x)+W_k'(x)\right]J_0(\lambda_k\psi)=$$
$$=\frac{a^2}{x}\sum_{k=1}^{\infty}\left[R_k(x)+W_k(x)\right]\left[J_0''(\lambda_k\psi)+\frac{1}{\psi}J_0'(\lambda_k\psi)\right]-$$
$$-x\sum_{k=1}^{\infty}B_k(x)J_0(\lambda_k\psi). \tag{26}$$

Since $J_0(\Psi)$ is the root of the equation

$$J_0''(\psi)+\frac{1}{\psi}J_0'(\psi)+J_0(\psi)=0,$$

we will have:

$$J_0''(\lambda_k\psi)+\frac{1}{\psi}J_0'(\lambda_k\psi)=-\lambda_k^2 J_0(\lambda_k\psi). \tag{27}$$

Equation (26), taking into account (27), is transformed into the form:

$$R_k'(x)+W_k'(x)=-\frac{\beta_k^2}{x}\left[R_k(x)+W_k(x)\right]-xB_k(x),$$

where

$$\beta_k^2 = a^2\lambda_k^2,$$

from where

$$R_k'(x)+\frac{\beta_k^2}{x}R_k(x)=-\left[W_k'(x)+\frac{\beta_k^2}{x}W_k(x)+xB_k(x)\right]. \quad (28)$$

$R_k(x)$ and $W_k(x)$ are arbitrary differentiable functions from which we choose $W_k(x)$ so that the conditions will be satisfied

$$W_k'(x)+\frac{\beta_k^2}{x}W_k(x)=-xB_k(x). \quad (29)$$

Then

$$R_k'(x)+\frac{\beta_k^2}{x}R_k(x)=0. \quad (30)$$

Solving equations (30), it will be

$$R_k(x)=R_k(1)x^{-\beta_k^2}=C_k x^{-\beta_k^2}. \quad (31)$$

By solving equation (29), we will look for the method of variation of an arbitrary constant. A particular solution of the homogeneous equation corresponding to equation (29) will be:

$$\bar{W}_k(x)=x^{-\beta_k^2}. \quad (32)$$

We look for the corresponding general solution in the form:

$$W_k(x)=D_k(x)x^{-\beta_k^2}. \quad (33)$$

Solving equations (29) and (33) together, we get:

$$D_k'(x)=-x^{\beta_k^2+1}B_k(x),$$

from where

$$D_k(x)=-\int_1^x t^{\beta_k^2+1}B_k(t)\mathrm{d}t+D_k(1), \quad D_k(1)=W_k(1). \quad (34)$$

From the second equation of system (13), we have

$$-\frac{1}{\alpha\psi}\frac{d\text{v}}{d\psi}=x\frac{\partial u}{\partial x}+2u. \quad (35)$$

Bearing in mind equation (17), we get:

$$x\frac{\partial u}{\partial x}+2u=\sum_{k=1}^{\infty}\left\{\begin{array}{l}x\left[R_k'(x)+W_k'(x)\right]+\\+2\left[R_k(x)+W_k(x)\right]\end{array}\right\}J_0(\lambda_k\psi)=$$
$$=\sum_{k=1}^{\infty}\left\{\begin{array}{l}\left[xR_k'(x)+2R_k(x)\right]+\\+\left[xW_k'(x)+2W_k(x)\right]\end{array}\right\}J_0(\lambda_k\psi)=$$
$$=\sum_{k=1}^{\infty}F_k(x)J_0(\lambda_k\psi), \quad (36)$$

where it is indicated:

$$F_k(x)=\left[xR_k'(x)+2R_k(x)\right]+\left[xW_k'(x)+2W_k(x)\right]. \quad (37)$$

After the solution of (30) and (31), it follows:

$$xR_k'(x)+2R_k(x)=-\left(\beta_k^2-2\right)R_k(1)x^{-\beta_k^2}. \quad (38)$$

Using solutions (29) and (33), we will have:

$$xW_k'(x)+2W_k(x)=$$
$$=D_k'(x)x^{-\beta_k^2+1}-\left(\beta_k^2-2\right)D_k(x)x^{-\beta_k^2}. \quad (39)$$

Substituting equations (38) and (39) into (37), we get:

$$F_k(x)=-x^2A_k(x)-\left(\beta_k^2-2\right)x^{-\beta_k^2}\left[D_k(x)+R_k(1)\right]=$$
$$=\frac{4x^2C'(x)}{\lambda_k J_1(\lambda_k)}-\left(\beta_k^2-2\right)x^{-\beta_k^2}\left[\begin{array}{l}\int_1^x\frac{4C'(t)}{\lambda_k J_1(\lambda_k)}t^{\beta_k^2+1}\mathrm{d}t+\\+W_k(1)+R_k(1)\end{array}\right]=$$
$$=-\left(\beta_k^2-2\right)C_k x^{-\beta_k^2}+\frac{4}{\lambda_k J_1(\lambda_k)}\times$$
$$\times\left[x^2C'(x)-\left(\beta_k^2-2\right)x^{-\beta_k^2}\int_1^x C'(t)t^{\beta_k^2+1}\mathrm{d}t\right]. \quad (40)$$

We choose an arbitrary function $C(x)$ so that to satisfy the condition

$$x^2C'(x)-\left(\beta_k^2-2\right)x^{-\beta_k^2}\int_1^x C'(t)t^{\beta_k^2+1}\mathrm{d}t=0, \quad (41)$$

then the function $F_k(x)$ will take the form

$$F_k(x)=-\left(\beta_k^2-2\right)C_k x^{-\beta_k^2}. \quad (42)$$

Solving the equation (41), we will get:

$$C'(x)=x^{-4}. \quad (43)$$

In accordance with solutions (25) and (43), the value of the coefficients $D_k(x)$, according to equations (34) will be:

$$D_k(x)=-\int_1^x t^{\beta_k^2+1}B_k(t)\mathrm{d}t=$$
$$=-\frac{4}{\lambda_k J_1(\lambda_k)}\int_1^x t^{\beta_k^2-3}\mathrm{d}t=-\frac{4\left(x^{\beta_k^2-2}-1\right)}{\lambda_k\left(\beta_k^2-2\right)J_1(\lambda_k)}. \quad (44)$$

Substituting the values of the coefficients C_k and $D_k(x)$ from equations (22) and (44) into equations (31) and (33), we calculate the values of the function $R_k(x)$ and $W_k(x)$. According to the calculated values of these functions, from equations (17) we obtain the pattern of speed change $u(x, \Psi)$:

$$u(x,\psi)=4\sum_{k=1}^{\infty}\left[\frac{2Ax^{-\beta_k^2}}{\lambda_k^3}+\frac{x^{-2}-x^{-\beta_k^2}}{\lambda_k\left(\beta_k^2-2\right)}\right]\frac{J_0(\lambda_k\psi)}{J_1(\lambda_k)}. \quad (45)$$

Substituting the calculated values of the function $F_k(x)$ from the equation (42) and the coefficients C_k from (22), substituting into equation (36), to calculate the value of the velocity component $\upsilon(x, \Psi)$, we obtain the equation

$$-\frac{1}{\psi}\frac{d\text{v}}{d\psi}=-\sum_{k=1}^{\infty}\frac{8\alpha A\left(\beta_k^2-2\right)}{\lambda_k^3 J_1(\lambda_k)}x^{-\beta_k^2}J_0(\lambda_k\psi). \quad (46)$$

After integrating the last equation, we finally obtain the value of the velocity component $\upsilon(x, \Psi)$

$$\upsilon(x,\psi)=8\alpha A\sum_{k=1}^{\infty}\frac{\left(\beta_k^2-2\right)x^{-\beta_k^2}}{\lambda_k^4 J_1(\lambda_k)}\times \times\left[\psi J_1(\lambda_k\psi)-J_1(\lambda_k\psi)\right]. \tag{47}$$

Calculating the patterns of pressure change from (11), we get the equation:

$$\bar{p}(x,\psi)= =\frac{2\nu}{U_0 r_0}\sum_{k=1}^{\infty}\left[\begin{array}{c}\dfrac{8x^{-\beta_k^2-1}}{\lambda_k^3}-\\ -\dfrac{x^{-\beta_k^2-1}-x^{-3}}{\lambda_k^2\left(\beta_k^2-2\right)}\end{array}\right]\frac{J_0(\lambda_k\psi)}{J_1(\lambda_k)}+C(x). \tag{48}$$

According to (43), the value of the function $C(x)$ will be:

$$C(x)=\frac{1-x^{-3}}{3}+C(1),$$

where the value $C(1)$ according to (48) will be:

$$C(1)=\bar{p}_0-\frac{2\nu}{U_0 r_0}\sum_{k=1}^{\infty}\frac{8}{\lambda_\kappa^3 J_1(\lambda_k)}. \tag{49}$$

Taking into account (49), we finally obtain the pattern of pressure change along the length of the conical diffuser:

$$\bar{p}(x,\psi)-\bar{p}_0=\frac{1-x^{-3}}{3}+\frac{2\nu}{U_0 r_0}\times \times\sum_{k=1}^{\infty}\frac{8}{\lambda_k^3}\left[\begin{array}{c}\left(\dfrac{x^{-\beta_k^2-1}J_0(\lambda_k\psi)-1}{J_1(\lambda_k)}\right)-\\ -\dfrac{\left(x^{-\beta_k^2-1}-x^{-3}\right)}{\lambda_k^2\left(\beta_k^2-2\right)}\dfrac{J_0(\lambda_k\psi)}{J_1(\lambda_k)}\end{array}\right]. \tag{50}$$

The formulas for determining the radial (45) and transverse (47) velocities and the pressure distribution throughout the diffuser length (50) are derived from the results of solving the boundary value problem.

5. 2. Calculating the separation point coordinate based on the diffuser opening angle and the Reynolds number

Flow separation occurs at points where shear stresses go to zero. A formula for calculating the shear stress on the wall of a fixed channel is derived. Due to the gradient of velocity and viscosity, shear stresses are formed between the layers of the liquid, which is determined by the formula [4]

$$\tau_{r,\phi}=\mu\left(\frac{1}{r}\cdot\frac{\partial\upsilon_r}{\partial\phi}+\frac{\partial\upsilon_\phi}{\partial r}-\frac{\upsilon_\phi}{r}\right). \tag{51}$$

In view of the negligible transverse velocity component υ_ϕ, compared to the derivative of υ_r to the angle of ϕ, the shear stress by the angle ϕ on the diffuser wall $\phi=\alpha$ will be determined by the formula

$$\tau=\frac{\mu}{r}\cdot\left(\frac{\partial\upsilon_r}{\partial\phi}\right)_{\phi=\alpha},$$

or dimensionless form

$$\bar{\tau}=\frac{\nu}{\alpha U_0 r_0}\cdot\frac{1}{x}\cdot\left.\frac{\partial u}{\partial\psi}\right|_{\psi=1}. \tag{52}$$

Substituting the expression for the radial velocity (45) into (52), we obtain the formula for determining the dimensionless shear stress between the fluid layers:

$$\bar{\tau}=-\frac{\nu}{\alpha U_0 r_0}\cdot 4\sum_{k=1}^{\infty}\left[\begin{array}{c}\dfrac{2Ax^{-\beta_k^2-1}}{\lambda_k^2}+\\ +\dfrac{x^{-3}-x^{-\beta_k^2-1}}{\left(\beta_k^2-2\right)}\end{array}\right]\frac{J_1(\lambda_k\psi)}{J_1(\lambda_k)}. \tag{53}$$

The pattern of the change in shear stresses on the fixed channel wall will be obtained from (53) under the condition $\Psi=1$, and we will have:

$$\left(\bar{\tau}\right)_{\psi=1}=-\frac{\nu}{\alpha U_0 r_0}\cdot 4\sum_{k=1}^{\infty}\left[\begin{array}{c}\dfrac{2Ax^{-\beta_k^2-1}}{\lambda_k^2}+\\ +\dfrac{x^{-3}-x^{-\beta_k^2-1}}{\left(\beta_k^2-2\right)}\end{array}\right]. \tag{54}$$

Based on the expression obtained, we get the place of flow separation from the diffuser wall from the condition that separation occurs at the place where the shear stresses become zero:

$$\frac{2Ax^{-\beta_k^2}}{\lambda_k^2}+\frac{x^{-2}-x^{-\beta_k^2}}{\left(\beta_k^2-2\right)}=0. \tag{55}$$

According to the proposed formula (55), the separation point coordinates are determined.

5. 3. Graphs of changes in the hydrodynamic parameters of a viscous fluid in conical diffusers

Numerical calculations were carried out and graphs were plotted to identify changes in the hydrodynamic parameters of a viscous flow in a conical diffuser. Based on the solutions obtained, we study the nature of the flow features in a conical diffuser. From the obtained equations for the distribution of velocities $u(x, \Psi)$ and $\upsilon(x, \Psi)$, it follows that for $x\to\infty$, $u(\infty, \Psi)\to 0$ and $\upsilon(x, \Psi)\to 0$. These conditions are fully consistent with the condition of constant flow.

The graphs were plotted in order to visualize the patterns of changes in the radial velocity $u(x, \Psi)$ (45) along the cross-section and along the length of a conical diffuser, as well as the shear stress on the wall of a fixed channel, depending on the opening angle $\alpha=20°$, $10°$, $5°$ and the Reynolds number 20 to 110 Re=20, 40, 60, 80, 100, 110. Fig. 2–9 show the indicated graphs for cases $\alpha=10°$ and $5°$ at Re=60 and Re=110.

Numerical calculations were carried out at a constant value $A=2.0$.

The calculated results and graphs clearly show the dynamics of current processes and the impact of the opening angle and Reynolds number on flow structural changes.

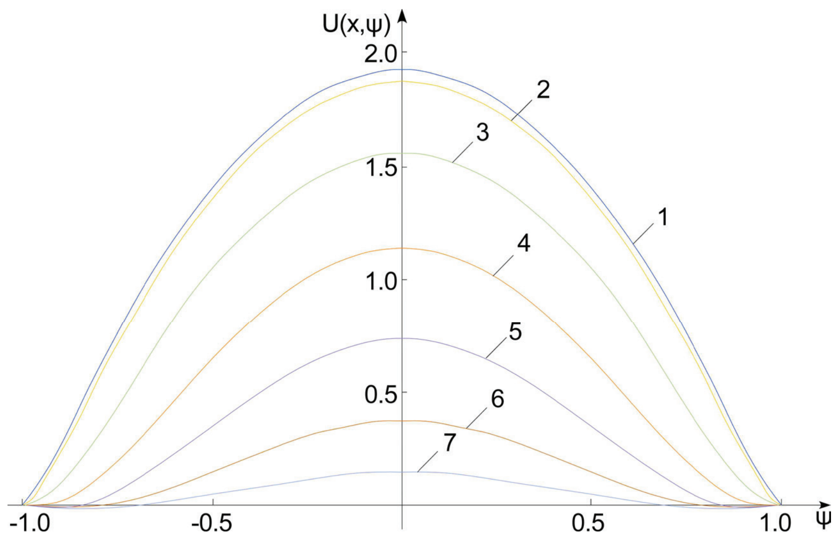

Fig. 2. Graphs of changes in the radial velocity according to (45) $u(x, \Psi)$ in a conical diffuser along the cross-section at an opening angle α=10° and the Reynolds number Re=60 at: 1 – x=1.03; 2 – x=1.05; 3 – x=1.2; 4 – x=1.5; 5 – x=2.0; 6 – x=3.0; 7 – x=5.0

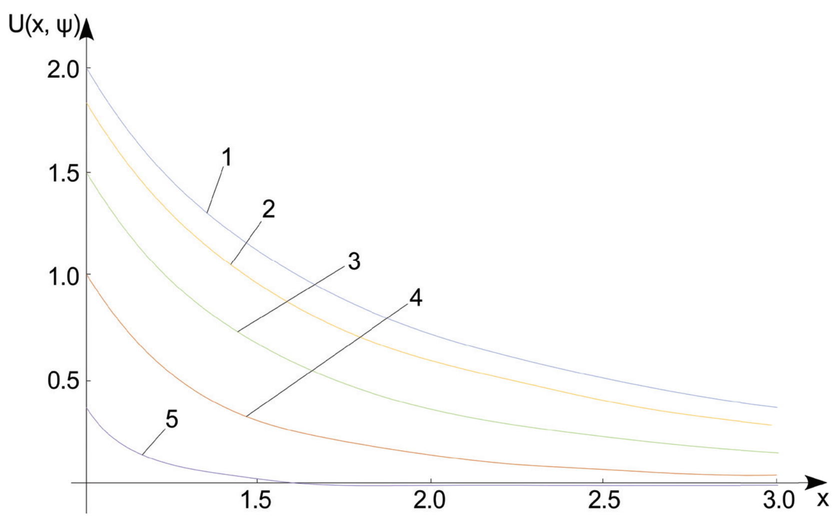

Fig. 3. Graphs of changes in the radial velocity according to (45) $u(x, \Psi)$ in a conical diffuser along the cross-section at an opening angle α=10° and the Reynolds number Re=60 at: 1 – Ψ=0.1; 2 – Ψ=0.3; 3 – Ψ=0.5; 4 – Ψ=0.7; 5 – Ψ=0.9

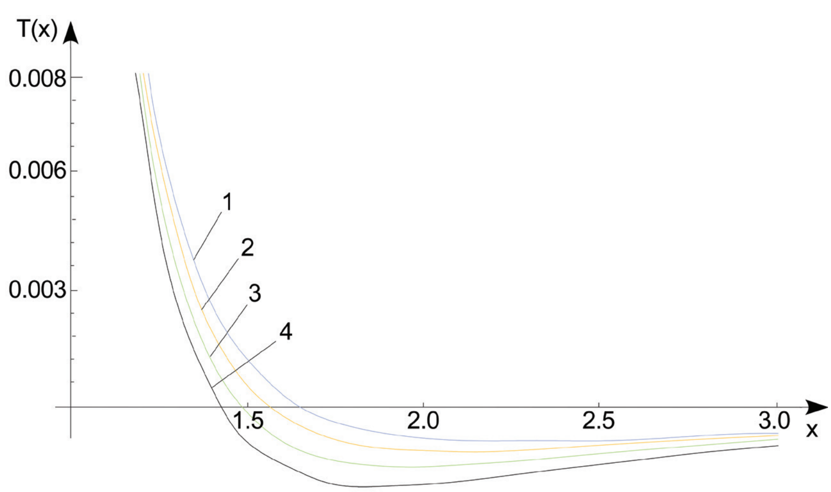

Fig. 4. Graphs of changes in shear stresses according to (53) in a conical diffuser at an opening angle α=10° and Reynolds numbers: 1 – Re=40; 2 – Re=60; 3 – Re=70; 4 – Re=80

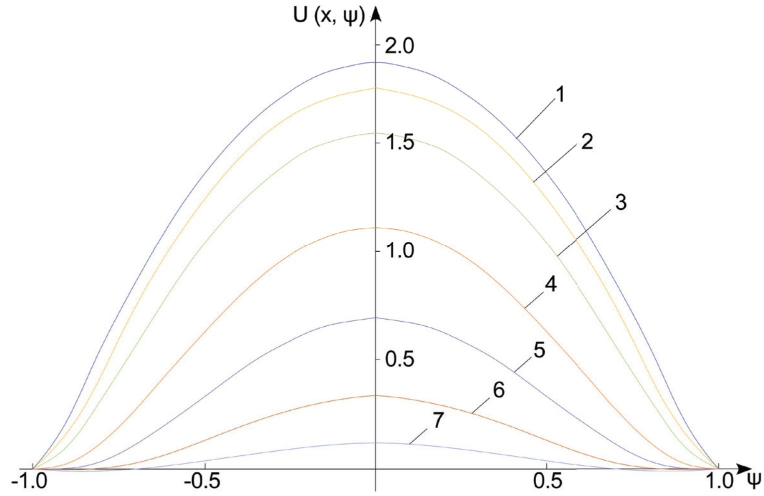

Fig. 5. Graphs of changes in the radial velocity according to (45) $u(x, \Psi)$ in a conical diffuser along the cross-section at an opening angle α=5° and the Reynolds number Re=110 at: 1 – x=1.05; 2 – x=1.1; 3 – x=1.2; 4 – x=1.5; 5 – x=2.0; 6 – x=3.0; 7 – x=5.0

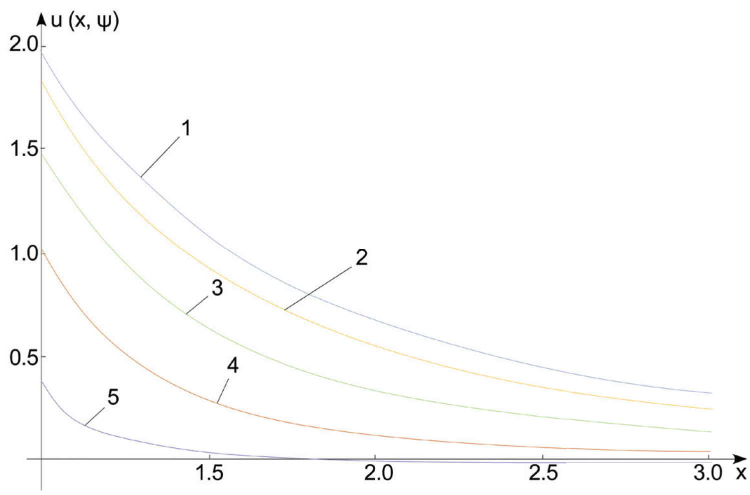

Fig. 6. Graphs of changes in the radial velocity according to (45) $u(x, \Psi)$ in a conical diffuser along at an opening angle α=5° and the Reynolds number Re=110 at: 1 – Ψ=0.1; 2 – Ψ=0.3; 3 – Ψ=0.5; 4 – Ψ=0.7; 5 – Ψ=0.9

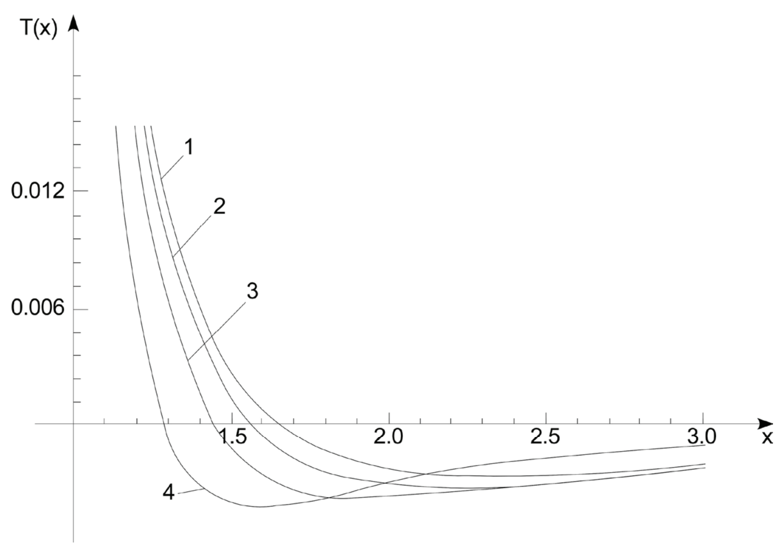

Fig. 7. Graphs of changes in the shear stresses according to (53) in a flat diffuser at an opening angle α=5° and the Reynolds number: 1 – Re=10; 2 – Re=20; 3 – Re=30; 4 – Re=40

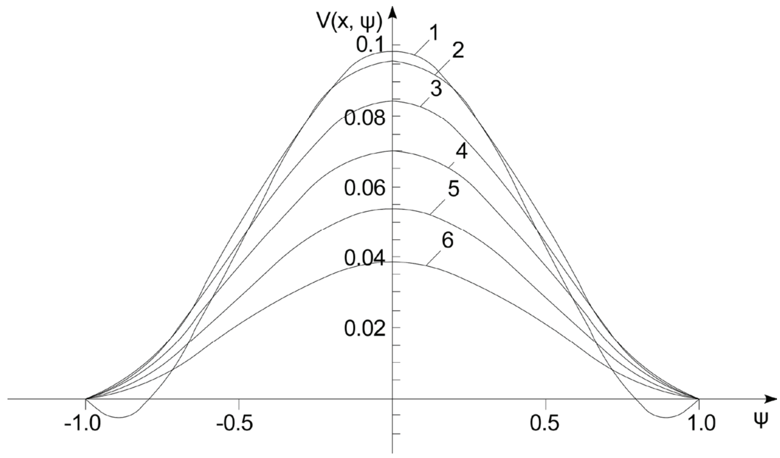

Fig. 8. Graphs of the change in the velocity component according to (47) $\upsilon(x, \Psi)$ at Re=50, α=10°: 1 – x=1.002; 2 – x=1.2; 3 – x=1.5; 4 – x=2.5; 5 – x=3.0; 6 – x=5.0

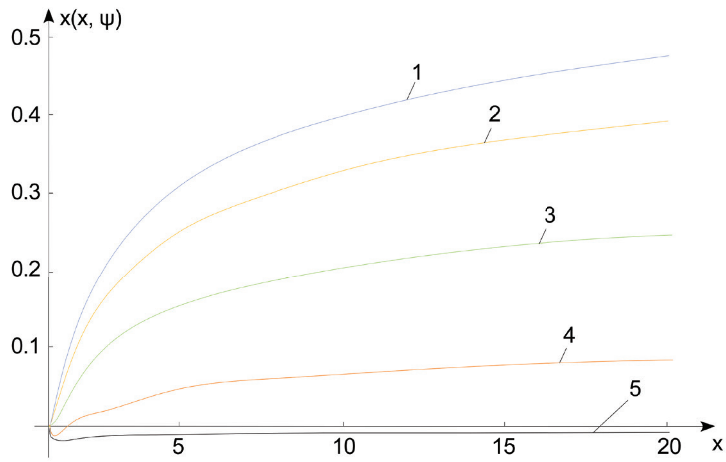

Fig. 9. Graphs of the change in the velocity component according to (47) $\upsilon(x, \Psi)$ at Re=90, α=10°: 1 – Ψ=0.1; 2 – Ψ=0.3; 3 – Ψ=0.5; 4 – Ψ=0.7; 5 – Ψ=0.9

5. 4. Identifying the parameters for the diffuser's reliable and stable operation

The conditions for continuous flow are obtained, a formula is derived, and a graph is constructed for determining the diffuser length from the condition of its reliable operation.

The separation point is a special point for the shear stress function. The coordinate value x in (55) is presented in an indefinite form. The decision of the uncertainty is revealed according to L'Hopital's rule [6]. As a result, it turns out

$$\sum_{k=1}^{\infty} \lim_{\beta_k^2 \to 2} \left[\frac{2Ax^{-\beta_k^2}}{\lambda_k^2} + \frac{x^{-2} - x^{-\beta_k^2}}{\left(\beta_k^2 - 2\right)} \right] (-1)^{k+1} = 0, \quad (56)$$

whence follows the condition

$$x^{-2} \left(\frac{A}{\lambda_k^2} - \ln x \right) = 0, \quad (57)$$

As $\lambda_k^2 = \frac{1}{a^2} = \alpha \mathrm{Re}$, we can get:

$$x = \exp \frac{A}{\alpha \mathrm{Re}}. \quad (58)$$

For a visual representation of the place of separation, a graph of the function (58) x from αRe was plotted when A=2, which is shown in Fig. 10.

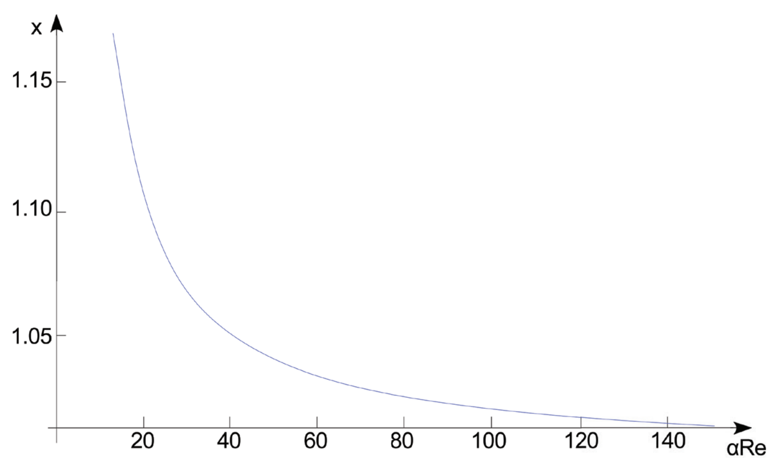

Fig. 10. Graph of (58)

According to the constructed graph, the diffuser length is calculated at which the viscous fluid moves in a stationary state.

6. Discussion of the results on the development of viscous fluid flow in a conical diffuser

Formulas for the distribution of radial (45) and transverse (47) velocities, as well as pressure along the length of the diffuser (50), and tangential stresses on the wall of a fixed channel (54) were obtained using the results of solving the boundary value problem for a parabolic distribution of velocities in the initial sections of the diffuser.

The studies were carried out with a parabolic distribution of velocities in the conical diffuser inlet section, which more accurately corresponds to reality than the uniform distribution considered in [3, 4].

The graphs constructed using computer calculations by formulas (45) and (54) demonstrate the development of the process in a conical diffuser, which is also confirmed by the results of experimental studies in [6, 7].

The analysis of numerical calculation results (58) and the obtained graphs (Fig. 2–7) revealed that the coordinates of the separation points were determined depending on the opening angle and Reynolds number. The viscous fluid flow to the separation point is considered stationary and strictly axisymmetric, as shown in the graphs (Fig. 2, 5). The flow is disrupted after the separation point, and the solutions obtained do not provide accurate results. They can, however, be used for qualitative studies. The sign of the shear stress and radial velocity change at the separation points. It is clear from the graph that the coordinates of the separation point coincide with the data defined by the graphs $u(x, \Psi)$ and $\tau(x, \Psi)$. In addition, it can be seen that the conditions for continuous flow in a conical diffuser at small opening angles are possible at significantly higher Reynolds numbers. With an increase in the opening angle, the area of single-mode stationary flow is sharply reduced (Fig. 4, 7), resulting in stationary regime disruption. Multimode flow begins, accompanied by pulsation disturbances and unstable operation of the diffuser, the results of which are unreliable. The main goal of diffuser design is to develop a stable operating regime, which can be accomplished by selecting the correct dimensions.

The approximate equations of motion for a viscous fluid were integrated in cylindrical coordinates, which reduces the accuracy of the integration. However, it is quite appropriate to use integration findings in engineering calculations. The initial distribution of velocities in the inlet portions of the diffuser's parabola's refinement coefficient is the only issue.

Based on the problem's relevance, further development is associated with the conditional clarification of the initial distribution of velocities and the related constructive changes in the diffuser's inlet section.

7. Conclusions

1. It has been possible to identify patterns of changes in hydrodynamic parameters in a channel with a parabolic distribution of flows in the inlet section, which is accurate, by determining the characteristics of the movement of a viscous fluid running in conical diffusers, formulating a boundary value problem, and developing a method for solving the Navier-Stokes approximation equations.

The pressure along the diffuser length, shear stresses on the wall of a fixed channel, and radial and transverse velocities are all calculated by a universal technique that uses dimensionless parameters.

2. Graphs of change in the dimensionless hydrodynamic parameters of the flow and shear stress on the channel wall have been plotted for dimensionless parameters of the diffuser. Due to the universality of the obtained graphs, it is possible to determine and analyze the influence of the geometric parameters of the diffuser on the nature of their change.

3. The conditions for reliable and uninterrupted operation of a conical diffuser depending on the opening angle and the Reynolds number at which the regime transition from symmetrical to asymmetric is carried out are established.

4. Based on the findings of the studies obtained, it is possible to correctly design a conical diffuser, that is, to select the angle of the opening and its length under the condition of non-separated movement, which can guarantee its reliable operation.

Conflict of interest

The authors declare that they have no conflict of interest in relation to this research, whether financial, personal, authorship or otherwise, that could affect the research and its results presented in this paper.

References

1. Jeffery, G. B. (1915). The two-dimensional steady motion of a viscous fluid. The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science, 29 (172), 455–465. doi: https://doi.org/10.1080/14786440408635327
2. Hamel, G. (1917). Spiralförmige Bewegungen zäher Flüssigkeiten.. Jahresbericht der Deutschen Mathematiker-Vereinigung, 25, 34–60. Available at: https://gdz.sub.uni-goettingen.de/id/PPN37721857X_0025?tify={%22pages%22:[41],%22panX%22:0.487,%22panY%22:0.784,%22view%22:%22info%22,%22zoom%22:0.348}
3. Targ, S. M. (1951). Osnovnye zadachi teorii laminarnykh techeniy. Moscow: Gostekhizdat, 420.
4. Slezkin, N. A. (1955). Dinamika vyazkoy neszhimaemoy zhidkosti. Moscow: Gostekhizdat, 519.
5. Tikhonov, A. N., Samarskiy, A. G. (1977). Uravneniya matematicheskoy fiziki. Moscow: Nauka, 735.
6. Akulenko, L. D., Kumakshev, S. A. (2008). Bifurcation of multimode flows of a viscous fluid in a plane diverging channel. Journal of Applied Mathematics and Mechanics, 72 (3), 296–302. doi: https://doi.org/10.1016/j.jappmathmech.2008.07.007
7. Kumakshev, S. A. (2020). Flat diffuser: Steady state flow of a viscous incompressible fluid. Engineering Journal: Science and Innovation, 7 (103). doi: https://doi.org/10.18698/2308-6033-2020-7-1993
8. Volkov, E., Fedyushkin, A. (2019). Symmetry of the flow of Newtonian and non-Newtonian fluid in a flat diffuser and confusor. Physical-Chemical Kinetics in Gas Dynamics, 20 (2), 1–19. doi: https://doi.org/10.33257/phchgd.20.2.791

9. Fedyushkin, A. I. (2016). The transition flows of a viscous incompressible fluid in a plane diffuser from symmetric to asymmetric and to non-stationary regimes. Physical-Chemical Kinetics in Gas Dynamics, 17 (3). Available at: http://chemphys.edu.ru/media/published/%D0%A1%D1%82%D0%B0%D1%82%D1%8C%D1%8F__%D0%A4%D0%B5%D0%B4%D1%8E%D1%88%D0%BA%D0%B8%D0%BD_%D0%90%D0%A4%D0%9C-10_2016_corr.pdf
10. El-Behery, S. M., Hamed, M. H. (2011). A comparative study of turbulence models performance for separating flow in a planar asymmetric diffuser. Computers & Fluids, 44 (1), 248–257. doi: https://doi.org/10.1016/j.compfluid.2011.01.009
11. Gerasimenko, V. P., Tkachuk, A. S., Ytsishin, A. A. (2012). About polars of straight-wall diffusers. Power and heat engineering processes and equipment, 8, 137–142. Available at: http://library.kpi.kharkov.ua/files/Vestniki/2012_8.pdf
12. Haines, P. E., Hewitt, R. E., Hazel, A. L. (2011). The Jeffery–Hamel similarity solution and its relation to flow in a diverging channel. Journal of Fluid Mechanics, 687, 404–430. doi: https://doi.org/10.1017/jfm.2011.362
13. Sarukhanyan, A., Vartanyan, A., Vermishyan, G., Tokmajyan, V. (2020). The Study of Hydrodynamic Processes Occurring on Transition of Sudden Expanding of Hydraulic Section of Plane – Parallel Full Pipe Flow. TEM Journal, 1494–1501. doi: https://doi.org/10.18421/tem94-23