ENERGY-SAVING TECHNOLOGIES AND EQUIPMENT

REFINEMENT OF THE MATHEMATICAL MODEL OF ELECTRICAL ENERGY MEASUREMENT UNCERTAINTY IN REDUCED LOAD MODE

Kateryna Vasylets
Postgraduate Student*
Volodymyr Kvasnikov
Doctor of Technical Sciences, Professor,
Merit Metrologist of Ukraine, Head of Department*
Sviatoslav Vasylets
Corresponding author
Doctor of Technical Sciences, Professor
Department of Automation, Electrical Engineering
and Computer-Integrated Technologies
National University of Water and
Environmental Engineering
Soborna str., 11, Rivne, Ukraine, 33028
E-mail: svyat.vasilets@gmail.com
*Department of Computerized
Electrotechnical Systems and Technologies
National Aviation University
Liubomyra Huzara ave., 1, Kyiv, Ukraine, 03058

The object of the study is a three-phase commercial electricity metering unit for 380 V electrical grids. The uncertainty of electricity measurement in the reduced load mode is estimated by the relative deviation of the active energy, measured by the metering unit, from the actual value. The specified deviation is considered as the value of relative deviations on measuring channels, weighted by phase currents. The method of estimating the uncertainty of electricity measurement by one channel of the metering unit is based on the approach to estimating non-random uncertainty using the fuzzy set theory. The parameters of membership functions for the relative deviation of the metering unit readings are estimated at fixed levels of the channel current. Approximation of such functions for different current levels allows you to obtain a set of boundaries of the L-R type fuzzy function corresponding to a set of confidence levels. This allows determining the impact of the load phase current on the measurement uncertainty if the amount of empirical data is limited. The mathematical model for estimating the uncertainty of electricity measurement at reduced load using a fuzzy function was refined. The proposed model differs from the known ones by taking into account the influence of load values for each phase of the metering unit on the measurement uncertainty indicators. The method for determining the membership function and the marginal confidence level, which characterize the uncertainty of energy metering by the metering unit, is proposed. The mathematical modeling results are confirmed as adequate to the experimental data. The proposed model for estimating the measurement uncertainty allows estimating the level of underestimation and clarifying financial calculations between the seller and the buyer of electricity

Keywords: metering unit, electricity meter, measurement uncertainty, fuzzy function, current transformer

1. Introduction

The efficiency of the electrical grid operation is determined by the level of electricity losses. Known studies make it possible to estimate technological losses [1]. At the same time, non-technological losses, including theft, fraud, non-payment of bills, deficiencies in electricity metering, are difficult to take into account, although in some countries their value can reach 15–18 % [2]. The first three of the listed causes of losses can be minimized by organizational measures, which are being worked on by regional energy companies. Increasing the efficiency of metering units is determined by the metrological characteristics of measuring equipment and its operation regimes. The accuracy class determines the measurement error only in the standardized operation mode of the metering unit. There are common cases in the practice of the metering unit operation in the reduced load mode when the current is a few percent of the nominal level. The reasons for the occurrence of such a regime are overestimated primary currents of measur-

ing transformers as part of metering units, low consumption during downtimes of the technological equipment, a decrease in production volumes, etc. The main sources of measurement uncertainty, in the reduced load mode, include errors of measuring current transformers operating in the zone of non-normalized, according to [3], relative error. Also, the accuracy of measurement decreases due to small currents of secondary circuits of current transformers, the values of which approach the sensitivity limit of the meter.

The lack of methods for estimating non-technological losses of electricity in distribution networks due to shortcomings of the measuring equipment functioning results in lower metering efficiency. Energy supply companies have no arguments to convince consumers with low consumption to update the equipment of metering units, in particular, to replace measuring current transformers. A scientifically based estimation of the electricity measurement uncertainty at a reduced load will be an effective tool for stimulating consumers to ensure the normal operation of metering units. So, the

importance of reducing the financial losses of energy supply companies due to underestimation of electricity when the measuring equipment of metering units is functioning in the reduced load mode determines the relevance of the revealed scientific problem.

2. Literature review and problem statement

Evaluation of the uncertainty of measurement results can be carried out a priori and a posteriori. In the first case (type B evaluation), all possible sources of uncertainty are taken into account, the maximum possible values of disturbances are considered. In particular, in [4] a significant influence of the parameters of measuring transformers on the metering unit operation accuracy was determined, especially when the power factor of load changes. It was also experimentally found that the accuracy of electricity metering is reduced due to the non-sinusoidal nature of load [5], especially in the case of LED lighting [6]. A priori uncertainty estimates may significantly exceed actual values. Standard uncertainties of type B are expressed as an interval of the admissible distribution law for a given confidence level [7].

A posteriori uncertainty assessment of measurement results is based on the general provisions of the Dempster-Shafer evidence theory. Meanwhile, the uncertainty of measurement results can be estimated using the methods of probability theory and mathematical statistics, or the possibility theory [8].

The methods of probability theory and mathematical statistics (type A evaluation of uncertainty) are basic for uncertainty assessment in metrological practice [9]. The estimate is given in the form of standard, combined standard or expanded uncertainties. Statistical methods make it possible to estimate the uncertainty caused by random and systematic factors based on the results of repeated observations. In particular, the work [10] proposed a method of statistical type A evaluation of the maximum uncertainty of an electricity meter using test signals. However, this approach does not take into account the long-term functioning of the meter in the reduced load mode.

If the systematic component of uncertainty prevails or cannot be excluded, which occurs in many practical cases, especially if such a component is unknown, the effectiveness of statistical methods decreases [11]. There are strict requirements for the number of measurements to guarantee reliable uncertainty evaluation. However, a sample of the required volume cannot always be obtained during measurements for technical, financial or organizational reasons. Also, the shortcomings of the statistical approach include the significant complication of finding the combined uncertainty with an increase in the number of factors. Such uncertainty cannot be evaluated when it is impossible to describe it with a continuous function of the relationship between the output and input measured values. A method of calculating the measurement uncertainty by combining type A and B evaluation is known [12]. Nevertheless, this approach is not without the listed disadvantages of statistical methods.

The application of the possibility theory provisions involving fuzzy sets [13] makes it possible to overcome the weaknesses of the statistical approach when processing measurement data, especially when the amount of experimental material is limited. The appropriate mathematical apparatus, which is a generalization of interval arithmetic, allows you

to correctly describe the systematic component of measurement uncertainty.

Thus, most studies of the uncertainty of electricity measurement by meters use statistical methods for analyzing experimental data. This approach is justified when studying the functioning of the metering unit in the normalized load range. At the same time, the study of the uncertainty of electricity measurement in the reduced load mode requires significantly more time to conduct each of the experiments, which reduces the number of experimental points. As literature analysis shows, the non-random measurement uncertainty can be estimated using the fuzzy set theory. This will increase the accuracy of commercial electricity metering in the reduced load mode. The application of the results of electricity measurement uncertainty obtained through mathematical modeling will increase the correctness of financial calculations between consumers and suppliers of electricity.

3. The aim and objectives of the study

The aim of the study is to increase the accuracy of electricity measurement by the metering unit in the reduced load mode based on mathematical modeling of measurement uncertainty. This enables to improve financial settlements between energy supply companies and electricity consumers.

To achieve the aim, the following objectives must be accomplished:

– to substantiate the indicator that characterizes the uncertainty of electric energy measurement when the metering unit is operating in the reduced load mode;

– to develop a methodology for estimating the uncertainty of electricity measurement for one channel of the metering unit under the condition of reduced load when the current level changes;

– to improve the mathematical model for estimating, using a fuzzy interval, the uncertainty of electricity measurement during the operation of a three-phase metering unit at a reduced load, taking into account the phase current values;

– to carry out an empirical evaluation of the parameter values of the mathematical model for the metering unit of a specific configuration;

– to assess the adequacy of the mathematical model with experimental data.

4. Materials and methods

The object of the study is a three-phase commercial electricity metering unit for 380 V electrical grids. Such a unit consists of a three-phase electricity meter PI1 and measuring current transformers TA_A, TA_B, TA_C of electromagnetic type, Fig. 1.

The subject of the study is the process of measuring the consumed electricity during user operation in the reduced load mode. To estimate the measurement uncertainty of the specified value, it is necessary to have an actual value of the active energy consumed for a certain time. This value should be the most accurate, under experimental conditions, approximation to the true value of the active energy consumed. To measure the actual value, it is suggested to use a direct connected meter PI2, the current windings of which are connected in series to the primary windings of the current transformers of the metering unit under investigation, Fig. 1.
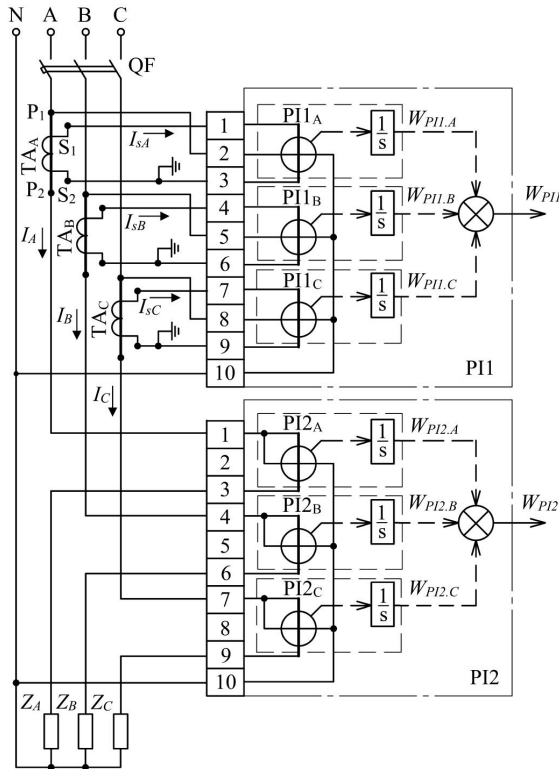
Fig. 1. Wiring diagram of the metering unit
consisting of the meter PI1 and measuring current
transformers TA$_A$–TA$_C$, and the additional direct
connected meter PI2, with the indication
of the measuring circuits of the meters

To analyze the uncertainty of electricity measurement by the metering unit, as a measuring converter, the following assumptions are made:

1. In the reduced load mode, the current of each phase is in the range from 0 to $2I^*_{min}$. Meanwhile, I^*_{min} – the minimum relative primary current of the current transformer for which, according to [3], the relative error is normalized. In particular, for measuring current transformers of the accuracy class 0.5S, $I^*_{min} = 1\%$, that is, the specified mode occurs at a phase current of up to 2 %.

2. The case of active load of each phase is considered. This is explained by the operation of only lighting, technical means of signaling, video surveillance, etc. during the downtime of the main equipment.

3. The effect of changes in the load and connecting wires resistance due to temperature changes on the values of phase currents is not taken into account.

4. The root mean square values of phase voltages are considered as the realization of a random stationary process. The probability of finding the root mean square value of the phase voltage in the permissible, according to [14], range (0.9÷1.1) p.u. is assumed to be no lower than 0.95 for an arbitrary moment.

5. It is assumed that during operation in the reduced load mode, the random change of phase currents is stationary and is caused solely by the action of unaccounted disturbances. This allows us to consider the root mean square values of phase currents unchanged during measurements.

6. A linear dependence of the active energy, measured by any measuring channel of the meter, on the measurement

duration is assumed. This follows from the assumption of constant voltage and current during the measurement.

The method of uncertainty estimation of the PI1 readings deviation from PI2 for one measuring channel is based on the approach to estimating non-random uncertainty using the fuzzy set theory [15]. The influence of the measuring channel current on the boundary of the fuzzy interval, which characterizes the measurement uncertainty, is supposed to be estimated by the L-R type fuzzy function [16] for a fixed confidence level. The mathematical model for estimating the uncertainty of electricity measurement at reduced load is proposed to be represented by a fuzzy function of phase currents.

Experimental studies were carried out in laboratory 509 of the National University of Water and Environmental Engineering. The wiring diagram of the laboratory facility corresponds to Fig. 1. Digital electricity meters of the following types were used (Fig. 2): transformer connected PI1 – NIK2307 ART T.1600.M2.21 (Ukraine); direct connected PI2 – NIK2307 ARP3 T.1600.M2.21 (Ukraine). The accuracy class of both meters when measuring active energy is 0.5S. Measuring current transformers T-0.66-600/5 (Ukraine) of accuracy class 0.5S were used to connect PI1. Incandescent lamps with a power of 100 W, 200 W, 300 W, 500 W were used as a load. They were connected in different numbers using a microprocessor control board. This enables to form the resistive load of each phase from 0 to 2,500 W in steps of 100 W.

During the processing of experimental data, the following numerical methods are used: the Nelder-Mead simplex method for minimizing a function with several variables; the method of finding the roots of a polynomial based on calculating the eigenvalues of the associated matrix; the least squares method for approximation by a nonlinear curve, which involves the confidence region method. The MATLAB system (USA) is used for numerical calculations.
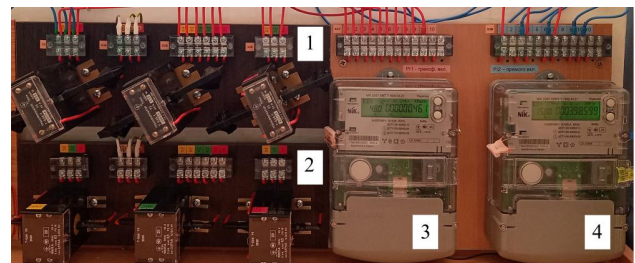

Fig. 2. Electricity metering unit in the laboratory:
1, 2 — measuring current transformers 600/5 and 100/5,
respectively; 3 — transformer connected digital electricity
meter PI1 NIK2307 ART T.1600.M2.21; 4 — direct connected
digital meter PI2 NIK2307 ARP3 T.1600.M2.21

Assessing the adequacy of the mathematical model with experimental data is carried out according to the actual value of the confidence level. This value is found in accordance with the membership function of the metering unit. The function is calculated for specific phase currents according to the experimental value of the relative deviation of the meter readings. The hypothesis of the adequacy of the mathematical modeling results with empirical data is accepted if the actual value exceeds the marginal confidence level.

5. Results of the study of electricity measurement uncertainty at reduced load

5. 1. Substantiation of the indicator for estimating electricity measurement uncertainty in the reduced load mode

The meter PI1 includes three measuring channels PI1ζ corresponding to the phases $\zeta=\{A,B,C\}$ of the power grid. Each of the channels measures the active energy $W_{PI1.\zeta}$ as a time integral of the active power consumed by the load Z_ζ. The dependence $I_{s\zeta}=I_{s\zeta}(I_\zeta)$ of the secondary current $I_{s\zeta}$ on the primary current I_ζ is a static characteristic of the measuring current transformer TAζ. Generally, the static transformation function for the ζ measuring channel of the metering unit is:

$$W_{PI1.\zeta} = W_{PI1.\zeta}\left[t, I_{s\zeta}\left[I_\zeta(t)\right], U_\zeta(t), \cos\varphi_\zeta(t)\right] = \\ = \int_0^t I_{s\zeta}\left[I_\zeta(t)\right] \cdot U_\zeta(t) \cdot \cos\varphi_\zeta(t)\mathrm{d}t. \tag{1}$$

Taking into account the accepted assumptions, the static transformation function (1) takes the form:

$$W_{PI1.\zeta} = W_{PI1.\zeta}\left(t, I_\zeta\right). \tag{2}$$

If the consumer operates in the reduced load mode during the time interval $\Delta t = t_2 - t_1$, then the active energy measured by the ζ channel is equal to:

$$W_{PI1.\zeta}\left(\Delta t, I_\zeta\right) = W_{PI1.\zeta}\left(t_2, I_\zeta\right) - W_{PI1.\zeta}\left(t_1, I_\zeta\right). \tag{3}$$

Suppose that during Δt, the current flowed only through the ζ channel, and the currents of the other two channels were equal to zero. Denote the readings of the meter PI1 at moments $t_{1(2)}$ as $W_{PI1.\zeta.mr}(t_{1(2)})$. Then the value of the static transformation function at moments $t_{1(2)}$ is:

$$W_{PI1.\zeta}\left(t_{1(2)}, I_\zeta\right) = k_{TA\zeta}\left(I_\zeta\right) \cdot W_{PI1.\zeta.mr}\left(t_{1(2)}\right), \tag{4}$$

where $k_{TA\zeta}\left(I_\zeta\right) = I_\zeta / I_{s\zeta}\left(I_\zeta\right)$ – the TAζ ratio.

The active energy on the ζ channel of the metering unit from (3) is:

$$W_{PI1.\zeta}\left(\Delta t, I_\zeta\right) = k_{TA\zeta}\left(I_\zeta\right) \cdot \left[W_{PI1.\zeta.mr}\left(t_2\right) - W_{PI1.\zeta.mr}\left(t_1\right)\right]. \tag{5}$$

The active energy measured by the three-phase metering unit is equal to:

$$W_{PI1}\left(\Delta t, I_A, I_B, I_C\right) = \sum_{\zeta=\{A,B,C\}} W_{PI1.\zeta}\left(\Delta t, I_\zeta\right). \tag{6}$$

The properties of the measuring circuits of the digital meter for each phase, as well as the parameters of the measuring current transformers, have a significant impact on the dependence $W_{PI1}(\Delta t, I_A, I_B, I_C)$.

The uncertainty of electricity measurement on the ζ measuring channel in the reduced load mode can be estimated by the relative deviation δW_ζ of the active energy, measured for the time interval Δt, between the readings of meters PI1 and PI2. At the same time, it is considered that the current of the other two channels is equal to zero. The absolute deviation of the meter readings is:

$$\Delta W_\zeta\left(\Delta t, I_\zeta\right) = W_{PI1.\zeta}\left(\Delta t, I_\zeta\right) - W_{PI2.\zeta}\left(\Delta t, I_\zeta\right), \tag{7}$$

where $W_{PI2.\zeta}(\Delta t, I_\zeta)$ – the active energy calculated on the ζ measuring channel of the direct connected meter equal to:

$$W_{PI2.\zeta}\left(\Delta t, I_\zeta\right) = W_{PI2.\zeta.mr}\left(t_2\right) - W_{PI2.\zeta.mr}\left(t_1\right). \tag{8}$$

Then the relative deviation can be defined as:

$$\delta W_\zeta\left(\Delta t, I_\zeta\right) = \frac{\Delta W_\zeta\left(\Delta t, I_\zeta\right)}{W_{PI2.\zeta}\left(\Delta t, I_\zeta\right)} = \frac{W_{PI1.\zeta}\left(\Delta t, I_\zeta\right)}{W_{PI2.\zeta}\left(\Delta t, I_\zeta\right)} - 1. \tag{9}$$

Assumption No. 6 allows us to exclude Δt from the arguments of function (9). Then, given (5) and (8), from (9) we have:

$$\delta W_\zeta\left(I_\zeta\right) = \frac{I_\zeta}{I_{s\zeta}\left(I_\zeta\right)} \cdot \frac{W_{PI1.\zeta.mr}\left(t_2\right) - W_{PI1.\zeta.mr}\left(t_1\right)}{W_{PI2.\zeta.mr}\left(t_2\right) - W_{PI2.\zeta.mr}\left(t_1\right)} - 1. \tag{10}$$

The measuring channels of the transformer and direct connected meters operates mutually independently. Errors accompanying measurements can be considered random. Therefore, the relative deviation of the transformer connected meter PI1 readings from the direct connected meter PI2 readings for three measuring channels is:

$$\delta W\left(I_A, I_B, I_C\right) = \frac{\sum\limits_{\zeta=\{A,B,C\}} \Delta W_\zeta\left(\Delta t, I_\zeta\right)}{\left|\sum\limits_{\zeta=\{A,B,C\}} W_{PI2.\zeta}\left(\Delta t, I_\zeta\right)\right|}. \tag{11}$$

Since the case of energy generation on the load side is not considered, then $W_{PI2.\zeta}>0$, so the denominator in expression (11) will always be positive. After transformations, the following dependence can be obtained from expression (11):

$$\delta W\left(I_A, I_B, I_C\right) = \frac{\sum\limits_{\zeta} I_\zeta \cdot \delta W_\zeta\left(I_\zeta\right)}{\sum\limits_{\zeta} I_\zeta}. \tag{12}$$

When $\delta W>0$, the readings of the transformer connected meter exceed the actual consumption of active energy. There is an underestimation of electricity when $\delta W<0$. To evaluate the dependence of δW on the values of phase currents in the reduced load mode according to (12), it is necessary to establish the dependences $\delta W_\zeta(I_\zeta)$ for each of the measuring channels of the metering unit.

5. 2. Substantiation of the methodology for estimating the electricity measurement uncertainty for one channel of the metering unit

5. 2. 1. Measurement uncertainty at a fixed level of the channel current

To estimate the uncertainty of measurement results at a fixed channel current, an assumption of the normal nature of the fuzzy set corresponding to these results is accepted. Then, for the membership function $\mu(\delta W)$ of the set $\{\delta W\}$ of measurement results, there exists such a value of $\delta W \in \{\delta W\}$ at which $\mu(\delta W)=1$. The membership function $\mu(\delta W)$ is considered as a set of left $\mu_1(\delta W)$ and right $\mu_2(\delta W)$ branches separated by the true value of the measured para meter δW_0:

$$\mu(\delta W) = \begin{cases} \mu_1(\delta W), \text{ as } \delta W \leq \delta W_0; \\ \mu_2(\delta W), \text{ as } \delta W > \delta W_0. \end{cases} \tag{13}$$

The sample values of δW_i, $i = \overline{1,n}$, obtained experimentally, are sorted by increasing: $\delta W_k^* < \delta W_{k+1}^*$, $k = \overline{1,(n-1)}$ [15]. The lengths of intervals $\Delta_k = \delta W_{k+1}^* - \delta W_k^*$ between adjacent values δW_k^* and δW_{k+1}^* are calculated. The largest $\Delta_{\max} = \max[\Delta_k]$ and the smallest $\Delta_{\min} = \min[\Delta_k]$ intervals are determined. Assume that the width of the interval Δ_k is inversely proportional to the number of corresponding values. Then the frequencies of the measured values in each interval are:

$$m_k = 1 - \frac{\Delta_k - \Delta_{\max}}{\Delta_{\max}}, \ k = \overline{1,(n-1)}. \tag{14}$$

Since at $\Delta_k = \Delta_{\min}$ the frequency is $m_k = 1$, the corresponding value of the measured parameter δW_υ is taken as the closest to the true value. The value δW_υ divides the vector δW^* into two parts corresponding to the branches of the membership function: the left branch includes the elements $\delta W_1^*,...,\delta W_\upsilon^*$; the right branch – the elements $\delta W_\upsilon^*,...,\delta W_n^*$, Fig. 3.
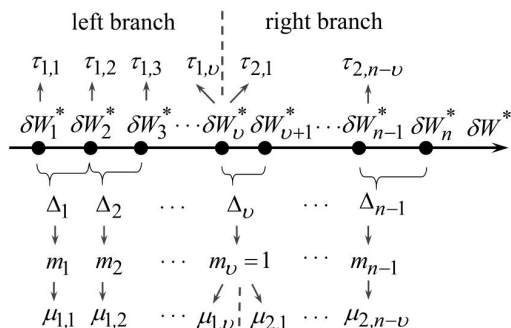

Fig. 3. Assignment of the measured values of the relative deviation δW, %, by branches of the membership function

Using the frequencies m_k for each branch, the sample values of the membership function are calculated [15]:

$$\mu_{1,k} = \frac{m_k - m_{\min 1}}{m_{\max 1} - m_{\min 1}}, \tag{15}$$

where $m_{\min 1} = \min\{m_1,...,m_\upsilon\}$, $m_{\max 1} = \max\{m_1,...,m_\upsilon\}$, $k = \overline{1,\upsilon}$;

$$\mu_{2,k} = \frac{m_{k+\upsilon-1} - m_{\min 2}}{m_{\max 2} - m_{\min 2}}, \tag{16}$$

while

$$m_{\min 2} = \min\{m_\upsilon,...,m_{n-1}\},$$

$$m_{\max 2} = \max\{m_\upsilon,...,m_{n-1}\}, \ k = \overline{1,(n-\upsilon)}.$$

The transition from absolute δW_k^* to relative τ_k values of the measured parameter is carried out as follows:

$$\tau_{1,k} = \frac{\left|\delta W_k^* - \delta W_\upsilon^*\right|}{\delta W_{\max}^* - \delta W_{\min}^*}, \ k = \overline{1,\upsilon}; \tag{17}$$

$$\tau_{2,k} = \frac{\left|\delta W_{k+\upsilon-1}^* - \delta W_\upsilon^*\right|}{\delta W_{\max}^* - \delta W_{\min}^*}, \ k = \overline{1,(n-\upsilon)}. \tag{18}$$

The obtained experimental points in [15] are proposed to be approximated by polynomials using the maximum norm method. For the left branch, the polynomial has the form (Fig. 4):

$$f_1(\tau_1) = 1 + \sum_{l=1}^{L} a_l \tau_1^l, \tag{19}$$

and for the right branch (Fig. 5):

$$f_2(\tau_2) = 1 + \sum_{l=1}^{L} b_l \tau_2^l. \tag{20}$$

The relative value of the fuzzy interval $U_{F_\lambda} = \xi_1 + \xi_2$, describing the uncertainty of measurement results, is defined as the sum of the relative values of the subranges ξ_1 and ξ_2. The value ξ_1 corresponds to the root of the function $f_1(\xi_1) = \lambda^*$ (Fig. 4), the value ξ_2 is determined from the expression $f_2(\xi_2) = \lambda^*$ (Fig. 5), where λ^* is the confidence level.
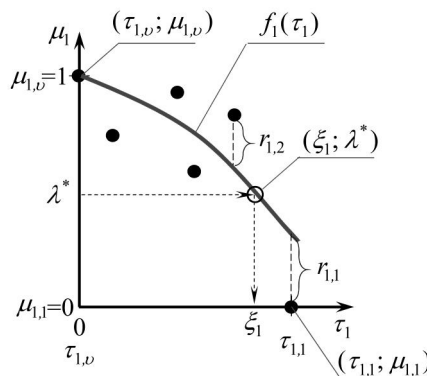

Fig. 4. Approximating polynomial for the left branch of the membership function μ_1 of the relative values τ_1 of the measured parameter and determining the left relative boundary of the fuzzy interval
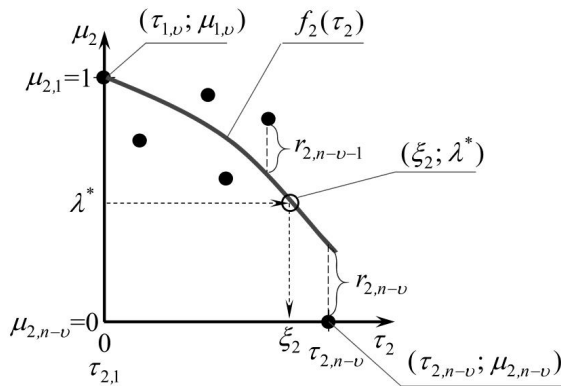

Fig. 5. Polynomial approximation of the right branch of the membership function μ_2 for the relative values τ_2 of the measured parameter and determining the relative value of the right boundary of the fuzzy interval

The boundaries of the fuzzy interval in units of the measured parameter are:

$$\delta W_L = \delta W_\upsilon - \xi_1(\delta W_{\max} - \delta W_{\min}), \tag{21}$$

$$\delta W_R = \delta W_\upsilon + \xi_2(\delta W_{\max} - \delta W_{\min}). \tag{22}$$

Dependencies (21) and (22) make it possible to estimate the uncertainty of electricity measurement by the metering unit at a fixed channel current. Such an estimate is provided by the left δW_L and right δW_R boundaries of the fuzzy interval, which at λ^* includes the true value of the measured quantity.

5. 2. 2. Estimating the influence of the channel current on the boundaries of the fuzzy interval for the measurement result

Suppose that for the ζ channel at a fixed current $I_{\zeta\gamma}$ (γ is the number of the current level), sample values of the relative deviations of the meter readings $\delta W_{\zeta\gamma i}\left(I_{\zeta\gamma}\right)$ are obtained in accordance with (10), where i is the number of the sample value. At the same time, the currents of the other two channels were zero. Let's establish the set of confidence levels $\left\{\lambda_j^*\right\}$, while $\lambda_j^* > \lambda_{j+1}^*$, $j = \overline{1,\Lambda}$, where Λ is the total number of confidence levels under consideration. Using the above method for each current level $I_{\zeta\gamma}$, we can find: the relative deviation of the meter readings $\delta W_{\upsilon\zeta}$ closest to the true value; L-R boundaries of the fuzzy interval for relative deviations of the meter readings at a given confidence level λ_j^*:

$$\widetilde{\delta W}_{\zeta\gamma} = \left[\delta W_{L\zeta\gamma}\big|_{\lambda_j^*}; \delta W_{R\zeta\gamma}\big|_{\lambda_j^*}\right]. \tag{23}$$

For a graphical presentation of the obtained fuzzy boundaries (23) and the $\delta W_{\upsilon\zeta}$ value, it is suggested to use a fuzzy box plot, which is formed similarly to a box-and-whisker plot for sample statistical characteristics of a random variable [17]. In this case, it is proposed to represent the right and left boundaries of the fuzzy parameter for the marginal confidence levels by equilateral trapezoids. Meanwhile, the smaller base of each trapezoid corresponds the value of the fuzzy parameter at λ_1^*, and the larger base – at λ_Λ^*, Fig. 6. The centers of the smaller trapezoid bases are connected by a vertical line corresponding to the value of the influence factor (load current). The horizontal line located between the smaller trapezoid bases corresponds to the value closest to the true one.
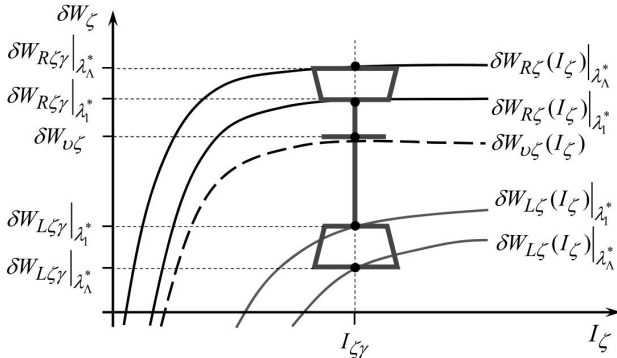

Fig. 6. The set of boundaries of the fuzzy function characterizing the metering uncertainty for the ζ channel as a dependence of the relative deviation δW_ζ on the channel current I_ζ for confidence levels λ_j^* and the proposed fuzzy box plot at a specific current value

The obtained L-R boundaries of fuzzy intervals for each of the γ current levels at the confidence level λ_j^* can be approximated by the dependencies $\delta W_{L\zeta}\left(I_\zeta\right)\big|_{\lambda_j^*}$, $\delta W_{R\zeta}\left(I_\zeta\right)\big|_{\lambda_j^*}$. Such dependencies represent the boundaries of the fuzzy function that in case of λ_j^* describes the metering accuracy for the ζ measuring channel (Fig. 6):

$$\widetilde{\delta W}_\zeta\left(I_\zeta\right) = \left[\delta W_{L\zeta}\left(I_\zeta\right)\big|_{\lambda_j^*}; \delta W_{R\zeta}\left(I_\zeta\right)\big|_{\lambda_j^*}\right]. \tag{24}$$

The values of $\delta W_{\upsilon\zeta}$ closest to the true ones can also be approximated, resulting in the dependence $\delta W_{\upsilon\zeta}(I_\zeta)$.

Suppose that for the approximation of the left and right boundaries of the sets of fuzzy functions, the dependencies F are used. The parameters of these dependencies form the following sets: for the left boundaries $\{L_{\zeta j}\}$, for the right boundaries $\{R_{\zeta j}\}$, namely:

$$\delta W_{L\zeta}\left(I_\zeta\right)\big|_{\lambda_j^*} = F\left[I_\zeta, \left\{L_{\zeta j}\right\}\right], \tag{25}$$

$$\delta W_{R\zeta}\left(I_\zeta\right)\big|_{\lambda_j^*} = F\left[I_\zeta, \left\{R_{\zeta j}\right\}\right]. \tag{26}$$

Similarly, a set of parameters $\{\Upsilon_{\zeta j}\}$ of the dependence F can be determined, which approximates the values of deviations closest to the true ones:

$$\delta W_{\upsilon\zeta}\left(I_\zeta\right) = F\left[I_\zeta, \left\{\Upsilon_\zeta\right\}\right]. \tag{27}$$

Thus, estimation of the ζ channel current influence on the uncertainty of measurement results can be represented by the dependence F, which approximates the empirical points with the smallest error, and by sets of its parameters. The set $\{\Upsilon_{\zeta j}\}$ determines, in accordance with (27), the values of deviations of the meter readings of the metering unit that are closest to the true values. The sets $\{L_{\zeta j}\}$ and $\{R_{\zeta j}\}$ characterize, according to (25) and (26), respectively, the left and right boundaries of the fuzzy function at a certain confidence level.

5. 3. Mathematical model of the uncertainty of electricity measurement by a three-phase metering unit at a reduced load

Representation of the characteristics of measuring channels by fuzzy functions (24) makes it possible, in accordance with the dependence (12), to obtain a fuzzy function, which characterizes the uncertainty of electricity measurement by a three-phase metering unit for given values of the phase currents I_A, I_B, I_C:

$$\widetilde{\delta W}\left(I_A, I_B, I_C\right) = \frac{\sum_\zeta I_\zeta \cdot \widetilde{\delta W}_\zeta\left(I_\zeta\right)}{\sum_\zeta I_\zeta}. \tag{28}$$

The left L (right R) boundary of such a fuzzy function for a set of confidence levels $\left\{\lambda_j^*\right\}$ is defined as follows:

$$\delta W_{L(R)}\left(I_A, I_B, I_C\right)\big|_{\lambda_j^*} = \frac{\sum_\zeta I_\zeta \cdot \delta W_{L(R)\zeta}\left(I_\zeta\right)\big|_{\lambda_j^*}}{\sum_\zeta I_\zeta}. \tag{29}$$

The relative deviation of the meter readings closest to the true value when current flows through three measuring channels in the reduced load mode is:

$$\delta W_\upsilon\left(I_A, I_B, I_C\right) = \frac{\sum_\zeta I_\zeta \cdot \delta W_{\upsilon\zeta}\left(I_\zeta\right)}{\sum_\zeta I_\zeta}. \tag{30}$$

The values of the boundaries of the fuzzy function $\widetilde{\delta W}\left(I_A, I_B, I_C\right)$ for a set of confidence levels, calculated according to (29), and the values of $\delta W_\upsilon(I_A, I_B, I_C)$ make it possible to construct (by approximation) the membership function $\mu_{abc}(\delta W)$ for $\delta W(I_A, I_B, I_C)$. This function corresponds to the specified values of the currents of measuring channels, Fig. 7, and includes the left $\mu_{abcL}(\delta W)$ and right $\mu_{abcR}(\delta W)$ branches:

$$\mu_{abc}(\delta W) = \begin{cases} \mu_{abc_L}(\delta W), \text{ as } \delta W \le \delta W_\upsilon; \\ \mu_{abc_R}(\delta W), \text{ as } \delta W > \delta W_\upsilon. \end{cases} \quad (31)$$

To evaluate the accuracy of electricity measurement by a particular metering unit, it is necessary to specify, according to experimental data, the marginal confidence level λ_b^* for the measuring equipment. So, it is proposed to conduct a number of tests on the operation of the metering unit in the reduced load mode. Such tests are to be planned as a full factorial experiment, when phase currents are considered as independent factors. The parameter δW_e acts as an objective function in each test. Using the obtained value, according to the membership function (Fig. 7), it is possible to calculate the actual sample value of the confidence level λ_e^*:

$$\lambda_e^* = \mu_{abc}(\delta W_e). \quad (32)$$

The sample λ_e^* obtained in this way is tested for random outliers. After rejecting them, it is possible to test the hypothesis of a normal distribution of the sample values using one of the known statistical tests. If the hypothesis of a normal distribution of empirical sample values of confidence levels is not rejected (at the accepted significance level), it is possible to calculate sample values of mean $m\left[\lambda_e^*\right]$ and standard deviation $s\left[\lambda_e^*\right]$. Then, to find the marginal confidence level, it is suggested to use the dependence that includes empirical values with a probability of 0.95, namely:

$$\lambda_b^* = m[\lambda_e^*] - 2\sigma[\lambda_e^*]. \quad (33)$$

Taking into account (33), from (28) we can obtain:

$$\widetilde{\delta W}(I_A, I_B, I_C) =$$

$$= \left[\frac{\sum_\zeta I_\zeta \cdot \delta W_{L\zeta}(I_\zeta)\big|_{\lambda_b^*}}{\sum_\zeta I_\zeta} ; \frac{\sum_\zeta I_\zeta \cdot \delta W_{R\zeta}(I_\zeta)\big|_{\lambda_b^*}}{\sum_\zeta I_\zeta} \right]. \quad (34)$$

Thus, the accuracy of the electricity metering unit in the reduced load mode at specific values of phase currents can be characterized by a fuzzy interval (34).
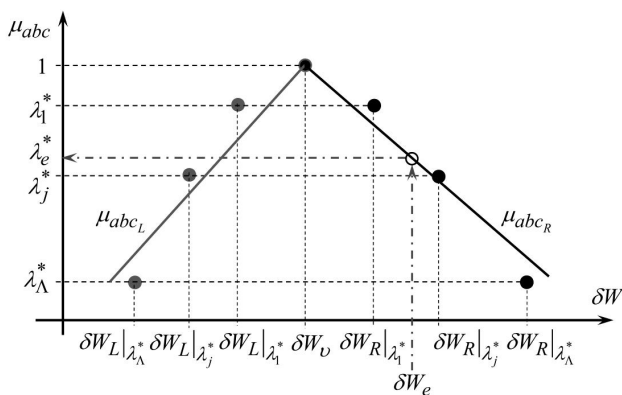

Fig. 7. General view of the membership function $\mu_{abc}(\delta W)$ for the relative deviation $\delta W(I_A, I_B, I_C)$ of the meter readings, as a fuzzy quantity, at the specified phase currents with the indicated empirical value δW_e and the corresponding actual confidence level λ_e^*

5. 4. Empirical evaluation of the parameter values of the mathematical model for estimating the electricity measurement uncertainty

To evaluate the uncertainty of deviation of the readings of the transformer connected meter from the direct connected meter, 67 tests were carried out on the measuring channel of phase A, 66 tests – phase B, 67 tests – phase C. The duration of each test was from 2 to 3 hours. During each test, the load of the studied channel was kept constant from the interval $I_\zeta^* = 0 \div 2\,\%$, the other two phases were not loaded. Tests on different phases and load levels were conducted randomly. It was not possible to maintain a stable load level for different tests conducted on different days due to the influence of disturbances (in particular, fluctuations in the supply voltage, different room temperatures). Therefore, actual current levels were recorded in the protocols. During further analysis, the load current change interval of each phase was divided into 7 ranges: the 1st range of $0 \div 0.2\,\%$ corresponded to the insensitivity zone of the metering unit, further ranges had a width of 0.3 %. Within the range $\gamma = 1, ..., 7$, the equivalent current $I_{\zeta\gamma}$ was determined as the expected value of actual currents. In each test, based on the meter readings, the value of relative deviation $\delta W_{\zeta\gamma i}(I_{\zeta\gamma})$ was determined according to (10).

The boundaries (21), (22) of fuzzy intervals (23), including the true value, were estimated for each current range γ of each channel ζ. The estimation was carried out for $\Lambda = 13$ confidence levels: $\lambda_1^* = 0.8, \lambda_2^* = 0.75, ..., \lambda_{13}^* = 0.2$. Straight lines were used to approximate the experimental points of the membership function branches according to (19) and (20). In particular, for the range $\gamma = 4$ of current values in phase B (Fig. 8): the left branch of the membership function was approximated by the dependence $f_1(\tau_1) = 1 - 1.47 \cdot \tau_1$, the right branch by $f_2(\tau_2) = 1 - 5.36 \cdot \tau_2$; the value closest to the true value was $\delta W_{\upsilon B} = -0.4\,\%$; at the confidence level $\lambda_9^* = 0.4$, the boundaries of the fuzzy interval were $\delta W_{LB4}\big|_{0.4} = -2.87\,\%$, $\delta W_{RB4}\big|_{0.4} = 0.28\,\%$.
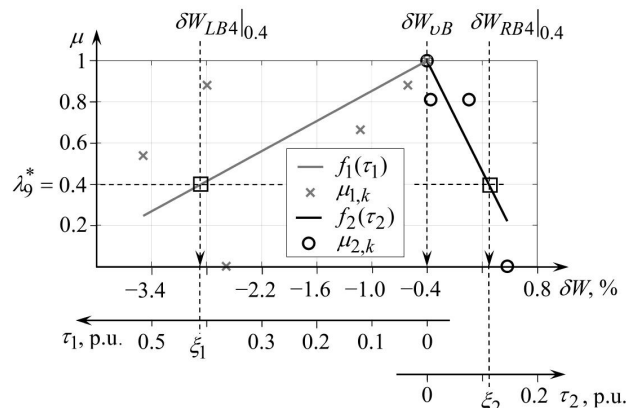

Fig. 8. Approximation of the branches of the membership function μ for the relative deviations δW, %, by straight lines along the experimental points for the range $\gamma = 4$ of phase B current values, determination of the value closest to the true one and the boundaries of the fuzzy interval for the measured value at the confidence level $\lambda_9^* = 0.4$; additional axes τ_1, τ_2 (p.u.) are shown for the relative values of the measured quantity corresponding to the left and right branches of μ

To approximate the boundaries (25), (26) of the fuzzy functions (24), which characterize the dependence of the measuring uncertainty on the channel current value, at each

of the determined confidence levels $\lambda_1^*,...,\lambda_{13}^*$, the following dependencies were used:

$$F\left(x,\{K\}\right)=K^{(1)}\cdot\exp\left[-x/K^{(3)}\right]+$$
$$+K^{(2)}\cdot\exp\left[-x/K^{(4)}\right]+K^{(5)}, \tag{35}$$

where $\{K\}=\{K^{(1)},...,K^{(5)}\}$ – set of parameters.

In particular, for phase B at the confidence level $\lambda_1^*=0.8$, the sets of parameters of approximating dependencies (35) for the left and right boundaries of the fuzzy function have taken the following values, respectively:

$$\{L_{B1}\}=\begin{Bmatrix}-1.78\cdot10^4;-1.00\cdot10^2;\\2.49\cdot10^{-2};9.73\cdot10^3;9.87\cdot10^1\end{Bmatrix};$$

$$\{R_{B1}\}=\begin{Bmatrix}-2.56\cdot10^4;-9.84\cdot10^{-1};\\2.33\cdot10^{-2};-1.09\cdot10^4;9.91\cdot10^{-1}\end{Bmatrix}.$$

The boundaries of the fuzzy function describing the measurement uncertainty on the phase B channel at reduced load are plotted in Fig. 9.

The ordinate axis in Fig. 9 was scaled using the «safe log». This transformation $y=\text{sign}\left(\delta W\right)\cdot\lg\left(1+|\delta W|\right)$ allows using negative numbers as an argument [18]. To illustrate the boundaries of fuzzy intervals, which characterize the measurement uncertainty at the averaged current value of each of the ranges, the proposed fuzzy box plots were used.
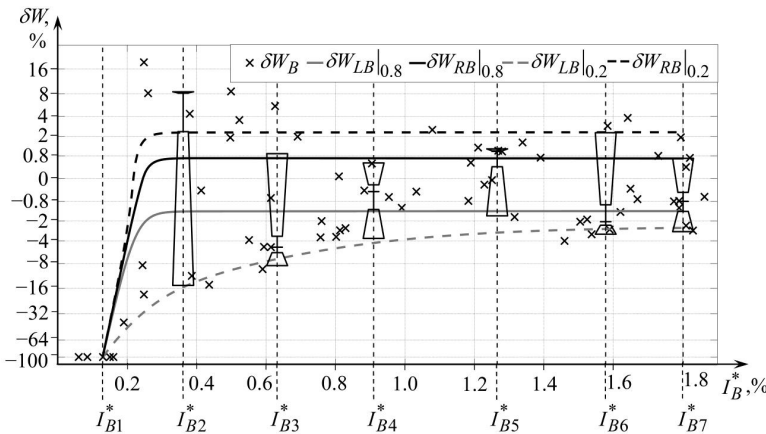

Fig. 9. Experimental points δW_B, % divided into 7 ranges by current I_B^*, %, of phase B with fuzzy box plots and limits of the fuzzy function at confidence levels $\lambda_1^*=0.8$ and $\lambda_{13}^*=0.2$, which approximate the corresponding fuzzy intervals for the measured parameter

5. 5. Assessment of the adequacy of the mathematical model with experimental data

The obtained numerical estimates of the fuzzy functions (24) parameters for each channel make it possible to estimate the measurement uncertainty of the laboratory metering unit as the boundaries of the fuzzy interval (34). The adequacy of the mathematical model with experimental data is suggested to be evaluated by the actual value of the confidence level λ_e^* determined by (32). The actual value λ_e^* exceeding the limit level of 0.4, which is typical for samples with a volume of up to several hundred elements, is taken as the adequacy criterion [15].

In particular, the experimental value of the relative deviation of the meter readings at currents $I_A^*=1.02\%$, $I_B^*=0.18\%$, $I_C^*=0.22\%$ was $\delta W_e=-5.6\%$. According to (30), $\delta W_\upsilon=-2.54\%$ was calculated. The points corresponding to the left branch of the membership function μ_{abc} were approximated by a third-degree polynomial (Fig. 10):

$$\mu_{abc_L}=1.60\cdot10^{-3}\cdot\delta W^3+4.30\cdot10^{-2}\cdot\delta W^2+$$
$$+3.94\cdot10^{-1}\cdot\delta W+1.75. \tag{36}$$

The points related to the right branch (31) were approximated by a straight line:

$$\mu_{abc_R}=-2.28\cdot10^{-1}\cdot\delta W+4.20\cdot10^{-1}. \tag{37}$$

The actual confidence level was:

$$\lambda_e^*=\mu_{abc_L}\left(\delta W_e\right)=\mu_{abc_L}\left(-5.6\right)=0.61.$$

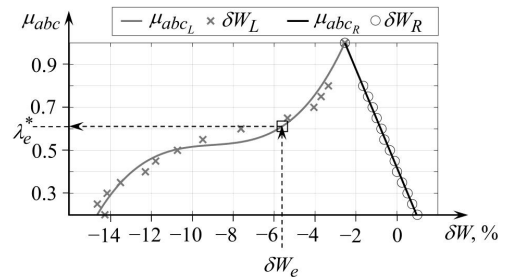The test results are listed in Table 1 (row 1).

Fig. 10. Approximation of the branches of the empirical membership function μ_{abc} for the relative deviation δW, %, of the meter readings of the laboratory metering unit at $I_A^*=1.020\%$, $I_B^*=0.182\%$, $I_C^*=0.220\%$; the left branch is approximated by a third-degree polynomial, the right — by a first-degree polynomial

Table 1

Evaluation of the adequacy of the mathematical model of electrical energy measurement uncertainty in reduced load mode with experimental data

Test number	Test conditions: currents of measuring channels of the metering unit			Obtained by the mathematical model: fuzzy interval parameters at a confidence level of 0.4			Experimental deviation	Actual confidence level
	I_A^*, %	I_B^*, %	I_C^*, %	$\delta W_L\big|_{0,4}$, %	δW_υ, %	$\delta W_R\big|_{0,4}$, %	δW_e, %	λ_e^*
1	1.020	0.182	0.220	−12.29	−2.54	0.01	−5.6	0.61
2	0.746	0.186	0.217	−14.58	−2.73	−0.14	−7.1	0.57
3	0.610	0.183	0.218	−16.51	−3.14	−0.52	−9.0	0.54
4	0.603	0.593	0.589	−3.07	−0.50	2.25	−0.7	0.97
5	0.534	0.524	0.454	−3.62	−0.51	2.20	−1.0	0.90
6	0.465	0.188	0.218	−19.23	−3.10	−0.46	−8.4	0.59
7	0.460	0.380	0.384	−5.55	−0.55	2.21	−2.7	0.62
8	0.224	0.220	0.147	−46.61	−17.67	−17.47	−24.9	0.78

Similarly, 7 more tests on the operation of the metering unit in the reduced load mode with asymmetric channel currents were conducted and the results were processed. Meanwhile, the values of channel currents in each test were chosen randomly (Table 1).

The analysis of Table 1 enables to establish that the actual value of the confidence level was $\lambda_e^* \geq 0.54$ in all tests. According to the formulated criterion, this gives grounds to accept the hypothesis about the adequacy of the results of mathematical modeling with empirical data.

6. Discussion of the results of electricity metering uncertainty study for reduced load mode

The obtained expression (12) makes it possible to estimate the uncertainty of electric energy measurement as a function of load phase currents. At the same time, the relative deviation δW in the reduced load mode corresponds to the weighted average, by phase currents, relative deviations of the readings of the transformer connected meter from the direct connected meter for each measuring channel. This approach, in contrast to [4], makes it possible to take into account the influence of load asymmetry on the accuracy of electricity metering.

The method of estimating the electricity measurement uncertainty by one channel using a fuzzy function (24), compared to the known approach [7], has the following advantage. The metrological characteristics of each measuring channel of the metering unit are taken into account when asymmetric currents flow, which increases the measuring accuracy in the reduced load mode.

The mathematical model of the electricity measurement uncertainty, which for specific values of the phase currents is given by the fuzzy function (28) with the membership function (31), takes into account the characteristics of each measuring channel. The advantage of this approach, compared to [10], lies in taking into account the influence of the operating mode of each load phase on the measuring uncertainty. Refinement of the marginal confidence level for measuring equipment of a specific configuration in accordance with (33) makes it possible to estimate the boundaries of the fuzzy interval of measurement results in terms of (34). Accepting the left boundary of such an interval as the amount of electricity underestimation in the most unfavorable conditions opens the way to the practical application of the proposed model.

It was found for the laboratory metering unit that straight lines are acceptable for approximating the branches of the membership function (13) for relative deviations δW, Fig. 8. This is explained by the limited number (approximately 5–6) of experimental points corresponding to each branch. Approximation of the fuzzy functions (24) boundaries (25), (26) is carried out using the function (35), which is the sum of two exponents. The choice of such dependence is explained by the significant nonlinearity of δW in the range of currents up to 0.5 %, Fig. 9. The specified dependence approaches the linear one at higher values of currents. Analysis of graphs in Fig. 9 allows us to establish that for phase B of the laboratory metering unit, the current level of 0.2 % is the limit of sensitivity. Power consumption at lower current levels cannot be measured. The level of underestimation in the most unfavorable conditions can be from −32 % to −8 % for currents from 0.2 % to 0.8 %. As the current value increases, the largest underestimation decreases, reaching 3 % at a current of 2 %.

A similar situation is observed for other measuring channels. The underestimation for the channel increases when the phase current decreases (Fig. 9) can be explained by the error rising of the current transformer and by the accuracy decreasing of the analog-to-digital signal conversion by the digital meter circuit.

Approximation of the experimental points corresponding to the left and right branches of the membership function (31) was carried out using polynomials of the third (36) and first (37) degrees, respectively. The empirical membership function was plotted (Fig. 11) for specific phase currents. At other currents, the numerical characteristics of the function will change. However, it was found that polynomials of the specified degrees for the branches of the membership function are characterized by the smallest approximation errors. On the basis of experimental data, it was found that for different values of phase currents, there is a tendency towards asymmetry of the membership function. The length of the δW interval corresponding to the left branch significantly (approximately 3–4 times) prevails over the interval of the right branch (Fig. 10). This corresponds to the shift of the channels characteristics (Fig. 9) into the negative half-plane. This feature is explained by a decrease in the sensitivity of the measuring equipment of the metering unit in the reduced load mode. The consequence of this is a predominant underestimation of electricity in the reduced load mode. This circumstance is significant from the point of view of financial losses of energy supply companies.

The refined mathematical model of the electricity measurement uncertainty at reduced load provides an analytical basis for assessing the amount of electricity underestimation in real conditions. The application of this approach allows the energy supply company to move from general appeals to consumers regarding the normalization of the operation mode of metering units to reasoned dialogue. Implementation of the necessary technical (for example, replacement of measuring current transformers, selection of metrological equipment of a higher accuracy class, etc.) or other measures will increase the accuracy of electricity measurement.

The developed mathematical model solves the issue of reducing non-technological losses of electricity caused by deficiencies in the functioning of metering units. Estimation of the uncertainty of electricity metering with a fuzzy interval (34), the boundaries of which are obtained by applying the proposed mathematical model, increases the measurement accuracy due to the following. The energy supply company clarifies monthly electricity consumption, calculated from the meter readings of the metering unit taking into account the unaccounted energy during the reduced load mode. Accordingly, the payment amount is specified. In the case of unacceptable, from the energy supplier's point of view, level of underestimated energy, the measuring equipment is replaced, which excludes the operation of the latter in an undefined mode.

Application of the proposed mathematical model for estimating the uncertainty of electricity measurement at reduced load is limited to the use in low-voltage power grids. In high-voltage grids, the meter is connected, in addition to current transformers, also using measuring voltage transformers. Such a case was not considered during the research.

The main drawback of the proposed approach to estimating the uncertainty of electricity measurement in the reduced load mode is the need for a preliminary assessment of the measuring channel characteristics of the metering unit. This

increases the time to obtain the final result and requires some organizational effort.

Planning, conducting and analyzing the results of a full factorial experiment to estimate the limit confidence level (33) for a specific electricity metering unit are planned to be carried out in further research.

7. Conclusions

1. It is proposed to use the relative deviation of the active energy measured by the metering unit from the actual value as an indicator characterizing the uncertainty of electricity measurement in the reduced load mode. In contrast to known studies, such an indicator is considered as a function of the measuring channel currents corresponding to the phases of the power grid. This enables to take into account the impact of load asymmetry on metering uncertainty.

2. The method of estimating the uncertainty of electricity measurement over one channel of the metering unit by a set of boundaries of the L-R fuzzy function for a set of confidence levels is substantiated. The intersections of the specified set of fuzzy function limits for fixed channel current values correspond to the membership function, obtained by approximating experimental data for the relative deviation of the measured energy. A feature of the method is the possibility to determine the impact of the load phase current on the measurement uncertainty with a limited amount of empirical data.

3. The mathematical model for estimating the uncertainty of electricity measurement at reduced load was refined, which involves estimating the parameters of the membership function for the relative deviations of the meter readings of the metering unit from the actual value. The model differs from the known models by taking into account the influence of load values over each phase of the metering unit on the measurement uncertainty.

4. The sensitivity limit of the laboratory metering unit, consisting of a digital meter of the NIK2307 ART type and measuring current transformers 600/5 of the 0.5S accuracy class, was estimated at the level of 0.2 %. For currents up to 0.8 %, the level of electricity underestimation under the most unfavorable conditions can reach 32 %, for currents up to 2 % – it can be up to 3 %.

5. The adequacy of mathematical modeling results is confirmed by experimental data. Comparing the analytically obtained membership function for the relative deviations of the metering unit readings with the empirically obtained value of such a deviation made it possible to determine the actual value of the confidence level. This value was not less than 0.54 in randomized tests. The obtained result is satisfactory at the limit value of 0.4 of the adequacy criterion.

Conflict of interest

The authors declare that they have no conflict of interest in relation to this research, whether financial, personal, authorship or otherwise, that could affect the research and its results presented in this paper.

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