У статті представлено загальні основи методу найменших квадратів (МНК) для випадків векторних та матричних спостережень. Також наведено деякі приклади, що демонструють перевагу застосування МНК для прогнозування показників у макроекономіці та телемедійному бізнесі. Запропоновано покроковий алгоритм застосування МНК до матричних спостережень з можливістю лінійного та нелінійного масштабування даних

Ключові слова: псевдоінверсія Мура-Пенроуза, регресія, метод найменших квадратів, макроекономіка, прогноз, економетрика

В статье представлены общие основы метода наименьших квадратов (МНК) для случая векторных и матричных наблюдений. Также предложены примеры, которые демонстрируют преимущества использования МНК для прогнозирования показателей в макроэкономике и телемедийном бизнесе. Предложен пошаговый алгоритм использования МНК для матричных наблюдений с возможностью линейного и нелинейного масштабирования данных

Ключевые слова: псевдоинверсия Мура-Пенроуза, регрессия, метод наименьших квадратов, макроэкономика, прогноз, эконометрика

УДК 51-77+519.854

MATRIXES LEAST SQUARES METHOD: EXAMPLES OF ITS APPLICATION IN MACRO-ECONOMICS AND TV-MEDIA BUSINESS

V. Donchenko
Professor*
E-mail: voldon@unicyb.kiev.ua
I. Nazaraga
Ph.D.*
E-mail: inna_na@ukr.net
O. Tarasova
Postgraduate student*
E-mail: olga_ta@bigmir.net
*Department of system analysis and decision making theory
Taras Shevchenko National University of Kyiv
st. Vladimirskaja, 60, Kiev, Ukraine, 01601

1. Introduction

The Least Squares Method (LSM) is reliable and prevalent means to solve prediction problems in applied research and in econometrics particularly. It is used in the case when the function is represented by its observations. Commonly used statistical form of LSM is called Regression Analysis (RA). It is necessary to say, that RA is only statistical shape for representing the link between the components in observations. So using RA terminology of LSM for solution of function estimating problem, and correspondingly, – prediction problem, is only the form for problem discussing.

2. Literature review

It is opportune to note, that the LSM is equivalent to Maximum Likelihood Method for classic normal regression. This method is widely used in econometric problems [1]. The development of technical capabilities of LSM for solution of optimization and predictive application tasks is proposed, in particular, in [2–4]. Some examples of the least squares method for macroeconomic models parameters identification are given in [5, 6]. The practical examples of LSM usage in business problems are proposed in [7].

Linear regression (LA) within RA has the advantage of having a closed form solution of parameter estimation problem and prediction problem. Real valued functions of vector argument are the object of investigation in RA in general and in LA in particular. Such suppositions are due to technical capabilities of technique for solving optimization problems in LSM. This technique is in the essence an investigation of extremum necessary conditions. This remark is entirely true for yet another widely used assumption, namely, full column rank assumption for appropriate matrix, which ensure uniqueness of parameter estimation. It's interesting that another technique: Moore – Penrose pseudo inverse (M-Ppi) [8–15] provides a comprehensive study and solution of parameter estimation problem.

And the remark in conclusion. Obvious advantage of matrixes LSM, besides the explicit closed estimation form, is the fact that matrixes observations preserve relationships between the characteristics of phenomenon under consideration. Examples of matrix least square method in macroeconomic and business problems with different types of relations between input and output data and different degree data discretization are given in [8, 15].

3. Formulation of the problem and the purpose of the research

Below in the article the results developing M-Ppi technique are presented. These ones enable operation with matrices as with real valued vectors and in optimization problem of LSM. And, as the consequence, the results enable designing of LSM for observations with matrix components. It is interesting to note, that such results would require the development of a full arsenal M-Ppi conception for objects in matrix Euclidean spaces. But in the case under consideration manage to use M-Ppi results for Euclidean spaces of real valued vectors to solve the problem of LSM estimation for matrixes observations. Correspondent results are also represented below as well as illustration of its applications for predicting in macroeconomics of Ukraine and in estimating of TV audience.

The purpose of our research is the development of a basis for prognostic research of indicators with different nature, whose observations have matrix form. Task of our research is to develop an algorithm for parameters prediction on the basis of the matrix least squares method with linear and nonlinear scaling of data and its testing on the macroeconomics and TV-media business data.

4. Theoretical foundation: the matrix least squares method

The LSM in its classic version – this is a way to "restore" the numeric functions $y=f(x,\theta), x\in X, \theta\in\Theta$ from parametric function family, when this function is represented by this or that collection observations $(x,y), x\in X, y\in R^1$. «Restored function» $\hat{y}=\hat{f}(x,\theta)=f(x,\hat{\theta})$ is defined by choosing appropriate $\hat{\theta}\in\Theta$ (estimation of parameter).

Technology of pseudoinverse by Moore - Penrose allows comprehensively obtain the solution of the matrix optimization problem. Detailed information about matrix least squares is given in the papers [8, 15]. The most important results of these articles in the form of theorems (which are necessary for the construction of general prediction algorithm for case of matrix data observations) we present here.

Theorem 1. For any $m\times n$ matrix **A**

$$\operatorname*{Arg\,min}_{X\in R^{n\times p}} \|Y-AX\|_{tr}^2 = X^+Y+(E_n-A^+A)R^{n\times n} =$$
$$=\left\{Z: Z=X^+Y+(E_n-A^+A)V, V\in R^{n\times n}\right\}. \quad (1)$$

The solutions of matrix optimization problem (1) coincide with the set of all solutions of matrix algebraic equations relatively **X**:

$$AX=Y, A-m\times n, Y-m\times p, X-n\times p,$$

when such solutions exist.

"Matrixes" case for observations *(x, y)* means, that both its components:, *x, y* - are simultaneously the matrixes correspondingly under supposition that relation between them determined by the components a $m\times n$ matrix **A**.

Theorem 2. Let the collection of matrix pairs $(X_i,Y_i): X_i\in R^{n\times p}, Y_i\in R^{m\times p}, i=\overline{1,N}$ are an observations of linear operator, defined by $m\times n$ - matrix $A: R^{n\times p}\to R^{m\times p}$.

Then the set of LSM estimation of the operator *A*, is determined by the set of optimization problem solutions

$$\operatorname*{Arg\,min}_{A\in R^{m\times n}} \Im(A)$$

with

$$\Im(A)=\left\{\sum_{i=1}^{N}(Y_i-AX_i, Y_i-AX_i)_{tr}^2, \text{ matrix observations}\right.,$$

is equivalent to optimization problem of the best quadratic approximation of the right hand part of algebraic equation $X^TA^T=Y^T$ by it left hand part respectively matrix A^T with matrices X, Y defined by the components of the observations accordingly to the relations:

$$X=\left\{(X_1\vdots\ldots\vdots X_N)-\text{matrix observation}\right., \quad (2)$$

$$Y=\left\{(Y_1\vdots\ldots\vdots Y_N)-\text{matrix observation}\right.. \quad (3)$$

Proof. Indeed, It is easy to verify, that simultaneous equations: matrixes $Y_i=AX_i, i=\overline{1,N}$, – in the observations model, are equivalent to matrix equations correspondingly: $(Y_1\vdots\ldots\vdots Y_N)=(AX_1\vdots\ldots\vdots AX_N)=A(X_1\vdots\ldots\vdots X_N)$, which follows from the definition of matrix algebra operations.

Thus, in the notation (2), (3) observation models for both types of observations are represented by matrix equation $AX=Y$ with known matrixes **X, Y** and unknown matrix **A**. Besides $\operatorname*{Arg\,min}_{A\in R^{m\times n}}\Im(A)=\operatorname*{Arg\,min}_{A\in R^{m\times n}}\|AX-Y\|_{tr}^2$.

So, equivalently

$$\operatorname*{Arg\,min}_{A\in R^{m\times n}}\Im(A)=\operatorname*{Arg\,min}_{A\in R^{m\times n}}\|AX-Y\|_{tr}^2=$$
$$=\operatorname*{Arg\,min}_{A^T\in R^{n\times m}}\|Y^T-X^TA^T\|_{tr}^2, \quad (4)$$

which proves the theorem.

Theorem 3. The set of all solutions for LSM – estimation of the linear operator by its matrixes observations is given by the relation:

$$\operatorname*{Arg\,min}_{A\in R^{m\times n}}\Im(A), \Im(A)=$$
$$=\left\{A: YX^+ + V(E_n-XX^+), V\in R^{m\times n}\right\}. \quad (5)$$

Proof. The proof follows directly from theorem 1, relation (1), that describes the solution of matrix algebraic equations through obvious changes in notation and subsequent transposition using commutative property for M-Ppi.

4. 1. The general prediction algorithm

Step 1. Scaling the observations' data. We scale the data vector observations for each of the parameters of the chosen type of transformation in the model (linear or nonlinear) [14].

Step 2. Construction of observations. We define observations as a pair of matrix values (x_i, y_i) of our indicators that cover adjacent periods.

Step 3. Construction of the block matrices With the first and the second components of the matrix observations we construct block matrixes X, Y.

Step 4. Calculation the matrix $\hat{A}$ by the formula $\hat{A}=YX^+$.

Step 5. Calculation of the prediction $\hat{y}^*$ by the formula $\hat{y}^*=\hat{A}x$ and reverse scale transformation – "liberation" of the scale $\hat{y}$. The quality assessment of prediction is calculated according to the criterion $APE=\left|\frac{y-\hat{y}}{y}\right|$.

Remarks. Step 1 shall not apply to models that do not involve scaling of observational data.

5. Examples of matrix LSM application

5. 1. Economic indicators prediction

The regression methods most often used to predict of economic indicators in the normal way. In this example, the theory of matrixes LSM was applied and the statistical data of the State Statistics Service of Ukraine and the data of the Ministry of Economic Development and Trade of Ukraine from official web-sites [16, 17] were used.

In particular, table 1 presents the value of gross domestic product (GDP), wages of employees (WE), final consumption expenditure (FCE), exports of goods and services (E) and absolute value of imports of goods and services (I) for the 2007 – 3 quarter 2013 years (quarterly discrete data at current prices).

Table 1

The value of 5 indicators for 1q2007–3q2013 years (at current prices; mln. UAH)

Period	GDP	WE	FCE	E	I
1q2007	139444	69078	112494	67513	76022
2q2007	166869	82021	130245	79664	85992
3q2007	199535	91922	140935	88491	93895
4q2007	214883	108915	174907	87537	108464
1q2008	191459	100492	161565	88516	110802
2q2008	236033	116441	182154	116640	135800
3q2008	276451	121522	194262	132177	144433
4q2008	244113	132009	220921	107526	129553
1q2009	189028	99206	172426	86994	92892
2q2009	214103	111616	188041	95390	96846
3q2009	250306	114251	196074	114962	116057
4q2009	259908	126270	216285	126218	133065
1q2010	217286	114062	194511	112105	114550
2q2010	256754	133690	216027	134553	131242
3q2010	301251	139108	232397	145563	156102
4q2010	307278	153791	271295	157144	179050
1q2011	261878	135831	236580	156545	173046
2q2011	314620	155367	268688	179626	187916
3q2011	376019	158186	285548	184258	202131
4q2011	364083	178727	314385	187524	215935
1q2012	293493	158145	272970	165810	186323
2q2012	349212	180432	311851	181413	215091
3q2012	387620	179944	328173	188467	214364
4q2012	378564	199638	356607	181657	219616
1q2013	301598	165337	291388	162250	180530
2q2013	351896	186968	338712	165391	182768
3q2013	392030	184728	347511	180186	220055

Let's use the general prediction algorithm for the indicators prediction for period 2013 on the basis of 2007–2012 years.

Step 1. We applied the linear transformation, see [14] to the interval [1, 100] for values from 2007 to 2012 years (Table 1, 1q2007–4q2012). Thus, we form scalable data tables (Table 2)

Step 2. Observation (x_i, y_i) is a pair of scaled values matrices for the main characteristics that correspond to the adjacent years (2007, 2008) (2008, 2009), (2009, 2010), .., (2011, 2012).

Step 3. We formed the matrix *X* with transformed values of 2007–2011 years, matrix *Y* with transformed values of 2008–2012 years and the matrix x with transformed values of 2012 year.

Table 2

The scaled values of 5 indicators for 1q2007–4q2012 years

Period	GDP	WE	FCE	E	I
1q2007	1	1	1	1	1
2q2007	11,94	10,81	8,2	10,95	7,87
3q2007	24,97	18,32	12,53	18,17	13,32
4q2007	31,09	31,21	26,31	17,39	23,37
1q2008	21,75	24,82	20,9	18,19	24,98
2q2008	39,53	36,91	29,25	41,21	42,21
3q2008	55,65	40,77	34,16	53,93	48,17
4q2008	42,75	48,72	44,97	33,75	37,91
1q2009	20,78	23,85	25,31	16,95	12,63
2q2009	30,78	33,26	31,64	23,82	15,36
3q2009	45,22	35,25	34,9	39,84	28,6
4q2009	49,05	44,37	43,09	49,05	40,33
1q2010	32,05	35,11	34,26	37,5	27,56
2q2010	47,8	49,99	42,99	55,87	39,07
3q2010	65,55	54,1	49,63	64,88	56,21
4q2010	67,95	65,24	65,4	74,36	72,03
1q2011	49,84	51,62	51,32	73,87	67,89
2q2011	70,88	66,43	64,34	92,76	78,14
3q2011	95,37	68,57	71,18	96,55	87,95
4q2011	90,61	84,14	82,88	99,23	97,46
1q2012	62,45	68,54	66,08	81,46	77,05
2q2012	84,68	85,44	81,85	94,23	96,88
3q2012	100	85,07	88,47	100	96,38
4q2012	96,39	100	100	94,43	100

Step 4. Matrix $\hat{A}$ was obtained from the equation $Y = AX$.

$$\hat{A} = \begin{pmatrix} 1{,}0174 & 0{,}6841 & 0{,}1288 & 0{,}7869 & -1{,}4224 \\ 0{,}0886 & 0{,}9660 & 0{,}5378 & 0{,}6506 & -1{,}0184 \\ 0{,}1527 & 0{,}5307 & 0{,}8024 & 0{,}5300 & -0{,}8009 \\ 0{,}1112 & 0{,}9251 & 0{,}6680 & 1{,}9534 & -2{,}4026 \\ -0{,}0297 & 0{,}1351 & 1{,}1983 & 1{,}6549 & -1{,}7416 \end{pmatrix}$$

Step 5. From the equation $\hat{\mathbf{y}}^* = \hat{\mathbf{A}}\mathbf{x}$ we calculated matrix predictive indicators (with scaled values).

$$\hat{y}^* = \begin{pmatrix} 73{,}4464 & 91{,}4904 & 112{,}9338 & 111{,}4238 \\ 81{,}8147 & 96{,}7002 & 105{,}5271 & 118{,}5205 \\ 80{,}4066 & 96{,}3071 & 107{,}2233 & 117{,}9960 \\ 88{,}4923 & 94{,}4245 & 112{,}6892 & 114{,}2184 \\ 87{,}2130 & 94{,}3246 & 112{,}1799 & 112{,}5927 \end{pmatrix}$$

We used the inverse transformation for matrix $\hat{\mathbf{y}}^*$. The matrix of predictive indicators $\hat{\mathbf{y}}$ is:

$$\hat{y} = \begin{pmatrix} 321054{,}65 & 366287{,}84 & 420042{,}79 & 416257{,}45 \\ 175655{,}44 & 195286{,}26 & 206927{,}13 & 224062{,}61 \\ 308293{,}84 & 347501{,}14 & 374418{,}17 & 400981{,}41 \\ 174407{,}37 & 181655{,}07 & 203970{,}18 & 205838{,}48 \\ 201069{,}11 & 211384{,}19 & 237282{,}27 & 237881{,}04 \end{pmatrix}$$

For matrices $\hat{\mathbf{y}}$ and actual values of indicators 2013 year $\mathbf{y}$ (Table 1, 1q2013–4q2013) we calculated the 3 columns of matrix APE:

$$\begin{pmatrix} 6,45\% & 4,09\% & 6,98\% \\ 6,24\% & 4,45\% & 12,02\% \\ 5,80\% & 2,59\% & 7,74\% \\ 7,49\% & 9,83\% & 13,20\% \\ 11,38\% & 15,66\% & 7,83\% \end{pmatrix}$$

In accordance with the submitted values matrix APE, maximum value of error prediction *GDP* in 2013 (for 3 quarters) was 6,98 %, *WE* – 12,02 %, *FCE* – 7,74 %, *E* – 13,20 %, *I* – 15,66 %. Excess error 10 % for some mentioned quarterly indicators can be explained by the fact that for prediction data of the years of crisis 2008–2009.

In general, comparing the results with the values of the relevant indicators the consensus prediction [16], it can be argued about the competitiveness of the proposed article approach for forecasting macroeconomic indicators.

5. 2. TV-media business indexes prediction

The TV market players use forecasts of TV-audience indicators for effective TV-products and commercial breaks media planning. TV audience indexes forecasting means having to "predict" redistribution of "TV market structure" through redistribution of channels' audience indexes. These indicators are used for media planning: Total TV rating (T – overall rating television; the average percentage of people from the target group who watched television at a certain time period); Audience share (S - the percentage of viewers who watched the TV channel from the total number); Channel rating (R – indicator defines a size of viewers' audience of TV channel; an average percentage of TV channel viewers from the total number of people which belong to the target group; during the calculating it is taken into attention a length of TV viewing by each viewer); Advertisement rating (Ra – this is an average percentage of the advertisement's viewers); Break-factor (K – index that characterizes the relative decrease of the commercial breaks audience to the total audience of the channel) [18].

The basis for solving the TV audience prediction problem is the data base of Ukrainian regular TV audience research 2003–2013 (Source – official site GfK Ukraine Media [19]). Data for our research have the monthly discretization. We do not show a complete data table in this article because of its large size. The values for the five audience indexes (T, S, R, Ra, K) are shown in Table 3.

Table 3

The values of five TV-media business indexes for 2013 year

Period	T	S	R	Ra	K
Jan	18,83	8,72	1,64	1,25	0,76
Feb	17,91	7,97	1,42	1,06	0,74
Mar	18,29	8,86	1,62	1,20	0,74
Apr	15,30	9,61	1,47	1,15	0,79
May	14,40	9,49	1,37	1,05	0,77
Jun	13,67	8,87	1,21	0,91	0,75
Jul	13,19	8,39	1,11	0,83	0,75
Aug	13,55	8,34	1,13	0,83	0,74
Sep	15,49	8,95	1,39	1,04	0,75
Oct	16,12	8,23	1,33	0,99	0,74
Nov	16,68	7,90	1,32	0,95	0,72
Dec	17,38	8,05	1,40	0,98	0,70

In this article we consider the problem of predicting five monthly indexes for 2013 year (T, S, R, Ra, K) with the use of retrospective data from 2003–2012 years with monthly discretization. For prediction we use the pseudo inverse and matrix least-squares method. Observations are determined by a pair of "previous-next" year interval.

The prediction problem for TV-media indexes is realized with 6 models that differ by the method of scaling for data observations:

- Model 1. The data observations are not scaled;
- Model 2. The data observations are scaled for each vector of 5 TV indexes by the linear transformation $y=az+b$ to the interval [0.01,1];
- Model 3–6. The data observations are scaled for one vector of 5 TV indexes with minimal range by the non-linear transformation from Table 4 and for the other four TV indexes by a linear transformation $y=az+b$ to the interval [0.01, 1]. In our case the index K has the minimum range. Nonlinear approach for data scaling was proposed by Donchenko V., Krivonos Yu., Krak Yu [14].

Table 4

Models 1–6: description, transformation

Model	description	transformation
Model1	with non-scaled approach	--------------
Model2	with linear-scaling only approach	$f(z)=az+b$
Model3	with linear and non-linear scaling approach	$f(z)=\frac{1}{az+b}$
Model4	with linear and non-linear scaling approach	$f(z)=\frac{a}{z}+b$
odel5	with linear and non-linear scaling approach	$f(z)=az^b$
Model6	with linear and non-linear scaling approach	$z=\mathrm{Arth}(az+b)$

In our case the argument corresponding to prediction matrix year 2012. That means that the forecast is made for year 2013. This choice of observations and the forecast period is determined by the ability to compare prediction and actual true value.

We applied a general prediction algorithm of the five indexes which previously scaled observational data for each of the seven models. The dimension of the matrix $\hat{A}$ for all models 1–6 is 5×5. Let's compare the results by the criterion of accuracy *APE*. Average APE criterion values for all 6 models are presented in Table 5.

According to the results of predictive models the average accuracy for all five indicators gets in an acceptable interval of 10 %. But in the Model 3 non-linear scaling for values of index K and linear scaling for another T, S, R and Ra allowed significantly improving the accuracy of prediction for all five indexes. Model 5 gives the best accuracy for index K and can be used for its prediction. However, the results of model 3 can be considered quite satisfactory for all five indicators.

Table 5

Average APE criterion values

Model	T	S	R	Ra	K
Model 1	6,0 %	6,1 %	9,4 %	7,4 %	4,5 %
Model 2	6,0 %	6,1 %	9,5 %	7,5 %	4,5 %
Model 3	1,4 %	3,4 %	3,4 %	5,6 %	4,1 %
Model 4	6,0 %	6,1 %	9,6 %	7,5 %	4,6 %
Model 5	6,0 %	6,2 %	9,8 %	7,5 %	3,7 %
Model 6	6,1 %	6,2 %	9,7 %	7,5 %	4,5 %

6. Conclusion

In the article case of matrix of observations for the arguments and values of the renewable function of the linear relationship between the components of observation has been considered. The general principles of the forecasting estimation method are presented for indexes with different nature, this method based on the least squares matrix. Among the advantages of the proposed method is the availability of explicit expressions for formula estimates saving structure of relations between the characteristics.

On the basis of this method step-by-step algorithm is proposed and described with linear and nonlinear scaling options of data observation. Based on the matrixes least squares method, approach to prediction of indicators was proposed. Testing approach with the use of statistical data of the economic and media indicators was made. Results of prediction with available statistics was compared. The proposed approach for finding predictive values indicators is competitive.

References

1. Magnus, Y. R. Ekonometrika. Nachalnyi kurs [Text] : tutorial / Y. R. Magnus, P. K. Katyshev, A. A. Peresetskij; 8-th ed., cor. — M.: Delo, 2007. — 504 p.
2. Seraya, O. V. Linear regression analysis of a small sample of fuzzy input data [Text] / O. V. Seraya, D. A. Demin // Journal of Automation and Information Sciences. – 2012. – Vol. 44, Issue 7. – P. 34–48. doi:10.1615/jautomatinfscien.v44.i7.40
3. Seraya, O. V. Estimation of representative truncated orthogonal subplans of complete factor experiment plan [Text] / O. V. Seraya, D. A. Demin // System Research and Information Technologies. — 2010. — Vol. 3. — P. 84–88.
4. Demin, D. A. Artificial orthogonalization in searching of optimal control of technological processes under uncertainty conditions [Text] / D. A. Demin // Eastern-European Journal of Enterprise Technologies. – 2013. – Vol. 5, Issue 9 (65). – P. 45–53. – Available at : http://journals.uran.ua/eejet/article/view/18452/ — 31.07.2014. — Title from the screen.
5. Nazaraga, I. M. Povedinkova model ta model portfelia aktyviv vyznachennia obminnogo kursu v umovah ekonomiky Ukrainy [Text] : teh. nauky zb. nauk. prats/ I. M. Nazaraga // Matematychne ta kompiuterne modeliuvannia. — 2010. – Vol. 3. – P. 160–168.
6. Kharazishvili, Yu. M. Investitsii: pidhid do prognozuvannia [Text] / Yu. M. Kharazishvili, I. M. Nazaraga // Actual problems of economics. — 2012. – Vol. 9 (135). – P. 213–222.
7. Slutskin, L. N. Kurs MBA po prognozirovaniiu v biznese [Text] / L. N. Slutskin. — M. : Alpina Biznes Buk, 2006. — 280 p.
8. Donchenko, V. S. Vectors and matrixes least square method: foundation and application examples [Text] / V. S. Donchenko, I. M. Nazaraga, O. V. Tarasova // International Journal "Information Theories & Applications". – 2013. – Vol. 20, No 4. – P. 311–322.
9. Moore, E. H. On the reciprocal of the general algebraic matrix [Text] / E. H. Moore // Bulletin of the American Mathematical Society. — 1920. – Vol. 26. – P. 394–395.
10. Penrose, R. A generalized inverse for matrices [Text] / R. Penrose // Proceedings of the Cambridge Philosophical Society. — 1955. – Vol. 51. – P. 406–413.
11. Kyrychenko, M. F. Zadacha terminalnoho sposterezhennia dynamichnoi systemy: mnozhynnist rozviazkiv ta optymizatsiia [Text] / M. F. Kyrychenko, V. S. Donchenko // Journal of Numerical and Applied Mathematics. — 2005. – Vol. 5. – P. 63–78.
12. Albert, A. Regression and the Moore-Penrose pseudoinverce [Text] / A. Albert. — M. : Nauka, 1977. — 305 p.
13. Donchenko, V. Evklidovy prostranstva chislovykh vektorov I matrits: konstruktivnye metody opisaniia bazovykh struktur i ikh ispolzovanie [Text] / V. Donchenko // International Journal "Information technologies & Knowledge". — 2011. – Vol. 5, Issue 3. – P. 203–216.
14. Donchenko, V. Recurrent procedure in solving the grouping information problem in applied mathematics [Text] / V. Donchenko, Yu. Krivonos, Yu. Krak // International Journal "Information Models and Analyses". — 2012. – Vol. 1. – P. 62–77.
15. Donchenko, V. Matrixes least squares method and examples of its application [Text] / V. Donchenko, I. Nazaraga, O. Tarasova // International Journal "Information Technologies & Knowledge". — 2013. – Vol. 7, Issue 4. – P. 325–336.
16. Official web-site of the Ministry of Economic Development and Trade of Ukraine [Electronic resource] / The Ministry of Economic Development and Trade of Ukraine. — Available at : http://www.me.gov.ua/ — 30.03.2014. — Title from the screen.
17. Official web-site of the State Statistics Service of Ukraine [Electronic resource] / The State Statistics Service of Ukraine. — Available at : \WWW/ URL: http://www.ukrstat.gov.ua/ — 30.03.2014. — Title from the screen.
18. Swann, P. An econometric analysis of television viewing and the welfare economics of introducing an additional channel in the UK [Text] / P. Swann, M. Tavakoli // Information and Economics Policy. — 1994. – Vol. 6, Issue 1. – P. 25–51. doi:10.1016/0167-6245(94)90035-3
19. Official site of GfK Ukraine Media [Electronic resource] / GfK Ukraine Media. — Available at : http://www.gfk.ua/ — 31.12.2013. — Title from the screen.