

The qualitative nature and quantitative parameters of motion instability of a two-fraction granular filler of a rotating drum were determined.

The factors of motion instability and key parameters of the oscillating system were identified and their influence on the self-excitation of pulsed self-oscillations was estimated.

Two continuous and one periodic steady-state modes of filler motion were found. Periodic self-oscillations due to the development of instability during the transition from continuous circulation mode to the wall layer mode were revealed. As factors of motion instability, filler dilatancy and damping effect of fine fraction particles on the pulsed interaction of coarse fraction particles were taken.

It turned out that the main key parameter of the oscillating system is the drum speed, which determines a change in dilatancy. The increase in instability is realized as a reduction of the bifurcation values of speed and dilatancy. Other key parameters are the content of the fine fraction in the filler κ_{ff} and the filling degree of the chamber κ_{fj} , the growth of which increases the self-oscillating instability.

The features of the oscillatory system are the relaxation type, discontinuous nature of self-oscillations and hard self-excitation mode under bistability. The discontinuous character and oscillatory hysteresis increase with decreasing κ_{ff} and κ_{fj} .

The limit values of the dynamic motion parameters corresponding to the conditions of self-excitation of self-oscillations in the absence and presence of fine fraction were determined: 0.96–1.11 and 0.218–0.382 for the bifurcation value of relative speed, 0.745–0.855 and 0.24–0.322 for the bifurcation value of dilatancy.

The effects found make it possible to substantiate the parameters of the self-oscillating process of processing polygranular materials in drum-type machines

Keywords: rotating drum, two-fraction granular filler, motion stability, self-oscillation, bifurcation speed, dilatancy

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REVEALING THE MECHANISM OF STABILITY LOSS OF A TWO-FRACTION GRANULAR FLOW IN A ROTATING DRUM

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1. Introduction

Drum-type machines are widely used in many industries for processing granular materials. However, the main drawback of such equipment is low mechanical efficiency due to high specific energy consumption [1]. This is caused by the low intensity of relative motion during the circulation of the processed medium in the rotating drum chamber. A significant part of the filler is passive, does not undergo deformation and does not participate in the processing. The active moving part is only 30–45%. To activate filler circulation, by creating a forced pulsation motion by periodic portioned pushing, intra-chamber energy exchange devices in the form of protruding elements are used. However, due to high filler abrasiveness, such lifters wear out quickly, lose functionality and have low reliability. The problem of reducing the energy

intensity of the working processes of drum machines, primarily the most energy-intensive type of them – drum mills, remains relevant [2].

A new direction for increasing low energy efficiency is self-oscillating processing in drum machines of traditional design with a smooth chamber surface. Self-excitation of self-oscillations allows you to set into periodic pulsating motion and activate the passive part of the intra-chamber filler and significantly intensify the interaction of the particles of the processed medium with the working bodies and the environment.

Video recording of the self-oscillating motion of the polygranular filler of a high-speed drum was first made in [3] (Fig. 1). The relative size of grinding bodies in the laboratory ball mill chamber was 0.067, and the relative particle size of the material being ground was 0.01. The chamber rotated at a close-to-critical speed.

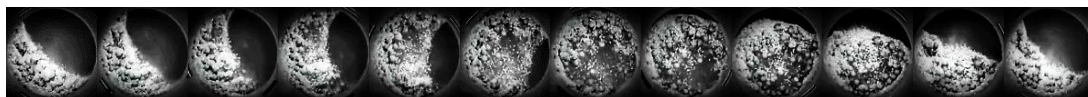


Fig. 1. Sequential images of the self-oscillating motion of the laboratory ball mill filler for one oscillation period [3]

2. Literature review and problem statement

In [4–7], the effectiveness of the innovative self-oscillating grinding process in a drum mill was evaluated. Quantitative estimation of the dynamic parameters of the filler impact action and the energy and technological parameters of grinding for one value of the filling degree of the chamber was carried out in [4]. In [5], the effect of the filling degree on the efficiency of self-oscillating grinding for one value of the content of ground material particles in the filler is considered. The effect of the material content on the motion modes of grinding bodies and the efficiency of self-oscillating grinding for one value of the chamber filler was studied in [6]. In [7], the influence of a simultaneous change in the chamber filling degree and ground material content on the grinding process was studied. However, conditions for self-oscillations of the rotating drum granular filler remain unclear. This significantly limits the industrial application of self-oscillating processing of granular materials in drum machines.

In [8], images of motion at the beginning of self-excitation of self-oscillations were obtained using video recording. A single-fraction filler (Fig. 2, *a, b*) containing only coarse fraction particles and a two-fraction filler (Fig. 2, *c, d*) containing coarse and fine fraction particles were used. The relative particle size of the coarse fraction in the chamber was 0.0104, and the relative particle size of the fine fraction was $0.13 \cdot 10^{-3}$. The filling degree of the chamber was 0.25–0.5. However, conditions for self-excitation of self-oscillations were not determined.

In fact, the processes of generation and preservation of self-oscillations of the polygranular filler of the rotating drum chamber are caused by the development of oscillatory instability and determined by the properties of a specific mechanism of motion stability loss. The implementation of such a mechanism is achieved only if a number of key parameters of the oscillatory system acquire and maintain values in rather narrow bifurcation ranges.

Therefore, the problem of determining the quality characteristics of the manifestation and quantitative values of the functioning parameters of the mechanism of motion stability loss during self-excitation of self-oscillations of the drum machine filler seems quite relevant.

Motion modes of the granular filler of the rotating cylindrical chamber significantly affect the technological processes and energy consumption of the drive of drum-type machines [9]. Modeling the fluid dynamics of such modes is of interest in the study of various rotary systems [10]. However, the results obtained in the considered works do not take into account possible manifestations of motion instability.

The behavior of the granular filler of a rotating drum has a pronounced unstable character [11]. Such instability is manifested in the formation of clusters in the medium and deformation of the free surface. Clusters are groupings in the form of particle segregation by size and emergence of cluster-like structures. The most well-known process is radial segregation [12]. It appears as a dense slow-moving wave formation in the central part of the shear flow of the granular medium and loose moving layers around such a quasi-solid cluster. Fairly known is the process of axial segregation of the medium [13] in the formation of clusters spread along the axis of the chamber. Cluster-like structures appear as patterns and striped groupings. Patterned cluster groupings [14] occur in a slow-moving granular flow due to the loss of stability and formation of disordered pulsating vortices. Radially symmetric striped formations [15] in the form of shear striped clusters arise in dense shear granular flows due to fluctuations in the medium density upon loss of stability. The free surface of the filler is deformed by periodic progressive collapse during slow rotation of the drum and pulsations during fast rotation. However, the possibility of practical application of pulsating motion of the granular filler was not highlighted.

The applied relevance of the problem of predicting the working processes of drum machines has recently attracted increased research attention to describing the possible unstable behavior of the processed filler. Interest is also increased by the need to overcome difficulties due to characteristic features of the problem under consideration. They are related to the complexity of flow geometry due to large deformation of the free surface, dilatancy, polydispersity of the medium structure, and mobility of the solid wall.

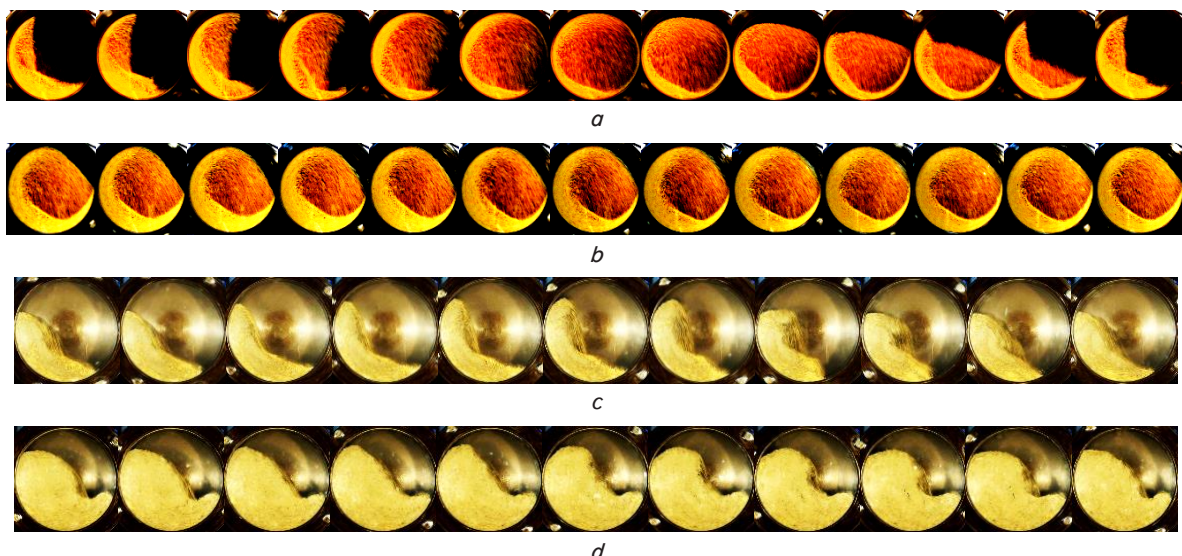


Fig. 2. Sequential patterns of motion at the beginning of self-excitation of self-oscillations of the granular filler of a rotating drum for one oscillation period (according to [8]): *a* – single-fraction filler at a chamber filling degree of 0.25; *b* – single-fraction filler at a filling degree of 0.5; *c* – two-fraction filler with a filling degree of 0.25; *d* – two-fraction filler with a filling degree of 0.5

A specific effect of granular filler dilatancy on motion stability due to an increase in volume during sputtering in the rotating chamber was studied. Using the visualization method [16], the determining effect of dilatancy of the non-cohesive monogranular filler on stability loss during collapse was found. A simplified dimensional plastic model for the qualitative prediction of unstable behavior of the medium is proposed. In [17], the effect of dilatancy on the stability of the gravity flow was visually evaluated and the significant damping effect of sputtering on the interaction of the non-cohesive particles of a disordered shape was shown. In [18], tomographic analysis by speckle-visual spectroscopy was used to determine the dilatancy effect of non-cohesive monofiller on the flow stability during the collapse of the free surface. An experimental study of the dilatancy effect on the loss of stability of the progressive collapse of a suspension during slow drum rotation was carried out in [19]. An increase in instability with increasing dilatancy due to a decrease in the frictional interaction of particles and fluidization of the medium was revealed. The obtained data proved a certain qualitative effect of shear dilatancy on the motion stability of the granular filler of the rotating drum. However, the results apply only to individual parts of the monogranular filler, mainly under slow rotation.

The effect of fine fraction particles on the motion stability of the polygranular intra-chamber filler of the rotating drum was considered. By visualization in [20], a significant effect of the fine fraction on the loss of motion stability of the two-fraction filler with particle size segregation was experimentally revealed. In [21], it is numerically shown that an increase in the content of ground material significantly reduces the impact of the drum mill chamber filler due to the damping of the interaction of grinding bodies. In [22], a strong influence of the fine fraction on the reduction of motion stability due to fluidization of the filler was experimentally found. In [23], visualization revealed a decrease in motion stability during a progressive collapse of the free filler surface with an increase in the content and dispersion of the fine fraction. In [24], it is visually shown that as the fine fraction content increases, the average speed and temperature of the filler grow. In [25], a significant fluidization effect of fine particles on motion modes of the polygranular filler of the rotating chamber was revealed. The data obtained in these works proved a certain destabilizing qualitative effect of the content of fine particles on the intra-chamber filler motion. However, the results apply only to the traditional processing of polygranular media with a simple steady-state motion of the medium in a low-speed drum. The possible effect of fine particles on self-excitation of self-oscillations of the polygranular filler remained unconsidered.

The manifestation of hysteresis due to the loss of stability during the transition of different motion modes of the granular filler of the rotating drum was studied. In [26], an overview of nonlinear transitions of various motion modes of the non-cohesive granular filler is given, mainly when applied to the drum mill filler. It is shown that such transitions are realized in the form of frictional hysteresis caused by complex, far from fully studied, rheological properties of the intra-chamber medium.

The manifestation of hysteresis consisting in the transition of the angles of inclination of the free surface of the filler of the slowly rotating chamber before and after the collapse was considered. Numerical and experimental analysis of such a frictional hysteresis transition was carried out [27]. The

work [28] shows that such frictional transitions are caused by the occurrence of solid, fluidized, and quasi-gaseous rheological states in separate filler zones. The significant influence of particle adhesion on rheological hysteresis during the transition of inclination angles of the free surface of the cohesive granular medium was investigated in [29]. It was found that an increase in cohesion increases the inclination angles and thickness of the collapsing layer and enhances the dissipation of the filler kinetic energy.

Frictional hysteresis was also identified in the transition of the movement of non-cohesive granular material from an unstructured state to a structured one. In [30], at a low filling degree of the chamber, high-speed hysteresis mutual transition of motion modes in the form of a uniform wall state with radially symmetric rings was revealed.

A number of works studied hysteresis transitions of motion modes with the periodic collapse of the free surface and continuous flow at a considerable drum speed. Such a hysteresis transition between the intermittent transient and stationary steady-state modes of the granular flow was first experimentally recorded in [31]. In [32, 33], it was experimentally shown that an increase in particle adhesion increases hysteresis due to the phenomenon of granular flow clustering. In [34, 35], frictional hysteresis was revealed using a numerical model describing partial fluidization of a non-cohesive granular medium. Experimental results [36] proved the loss of motion stability as a factor in the hysteresis transition of modes. In [37], on the basis of a large array of experimental data, the bifurcation nature of the transition between transient and steady modes of the filler was revealed. Experimental analysis of differences in such hysteresis in dry granular materials and suspensions is given in [38]. Hysteresis in suspensions is shown to be of a frictional rheological nature and caused by interparticle friction. In [39, 40], hysteresis transition of motion modes of the cohesive granular filler was investigated numerically by DEM with experimental testing, taking into account cohesion. It was found that with increasing cohesion, the range of periodic collapse avalanches decreases and the transition to the steady continuous mode is accelerated.

Hysteresis transitions of the circulation and wall layer modes during the rapid rotation of the drum for non-cohesive [41] and cohesive [42] granular fillers were also found experimentally. However, these works ignore the possibility of self-excitation of self-oscillations of the filler during such motion mode transitions.

The results obtained are only qualitative in nature and do not allow predicting quantitative characteristics of unstable transitions of granular filler motion modes.

Attempts were made to create models of self-excitation and motion instability of the granular filler of the rotating drum. Experimental studies of multidirectional stability under periodic plastic deformation and destruction of the free surface of the granular filler during slow rotation of the drum were performed in [43]. Empirical conditions for the motion instability of the filler elements caused by low stress disturbances were obtained. In [44], a strict relationship was experimentally found between the range of particle velocity fluctuations and spatial and temporal characteristics of the self-oscillating process, in particular, stresses. The task of building a strict model describing the self-oscillation dynamics during self-excitation of the filler collapses was set. The spatiotemporal stochastic dynamics of self-oscillating compaction and loosening processes during self-excitation of the periodic collapse of the granular filler was studied experimentally and analytically in [45].

Based on the measurement and estimation of the value of the generalized Lyapunov vector, a simplified condition for motion stability of the system under consideration was obtained. In [46], a strict correlation between general dynamic properties and the instability of the system was experimentally and analytically revealed. To assess the motion stability of the filler, it is suggested to use the Lyapunov exponent. The concept of rotational stability was used in [47] to explain the axial segregation of the polygranular filler of the rotating drum. The core of the radially separated granular mixture is considered as a dissipative rotating solid. In [48], a theoretical model explaining the transition between continuous and discrete avalanche motion modes of the filler as supercritical Hopf bifurcation is presented. However, the proposed concepts for efficient solution of the problem of determining conditions for motion stability of the intra-chamber filler cannot be considered universal applied models. They do not quantitatively predict the unstable behavior of the polygranular medium, taking into account wide variations of dynamic and rheological parameters of the system.

At the same time, the most promising in terms of technological application is the self-oscillating [49] pulsating mode of the filler, which is self-excited upon loss of stability during rapid rotation of the drum. However, the examples of self-oscillating systems considered in the work are limited mainly to electrical and radio-technical devices.

In [50], an analytical method of forecasting qualitative conditions and stability factors of steady motion of the machine unit of the drum mill was developed. Such instability is manifested as the impossibility of free rotation of the filled drum at a given speed without its stabilization. This may cause the speed to deviate arbitrarily from the initial value, mainly to increase. It was shown that the factors of the instability effect are variations in the rigidity of dependences of the axial moment of inertia and the moment of filler resistance on speed. However, under forced speed stabilization, for example by automatic control of the drum drive, self-excitation of the filler self-oscillations occurs in the form of pulsations in the cross-section of the chamber, which makes it difficult to stabilize the speed. The obtained motion factors of the machine unit can only be considered as external disturbances of the oscillating system of the filled drum. The loading structure of the working chamber is ignored. Therefore, the model proposed in [50] does not explain the reasons for the motion instability of the intramill filler.

In [51], analytical modeling of the shear polygranular flow was performed and wave processes were formalized. It was found that the unstable behavior of the filler of the rotating drum chamber occurs mainly in a non-free fall zone. In such a zone, there is an inertial flow mode with low filling density and high velocity displacement and interaction of coarse particles through continuous collision. It was found that the main factor in the motion instability of the filler is the growth of dilatancy of the granular medium. An additional factor is the damping effect of fine particles on the pulsed interaction of coarse fraction particles. However, stability loss conditions obtained in [51] are only qualitative and do not quantitatively characterize the motion instability. Such results do not allow predicting the self-oscillating behavior of the filler.

So, the data of analytical modeling and experiments made it possible to determine the factors of motion stability of the granular filler of the rotating drum chamber. However, such factors are only qualitative conditions for stability loss, which does not allow quantitative assessment and prediction of unstable behavior.

The quantitative effect of the dynamic parameters of the filler on motion stability was not determined. This is due to the high complexity of the hardware experimental study of the behavior of the polygranular intra-chamber medium. The lack of such data is especially negatively manifested in the case of possible innovative self-oscillating processes of processing polygranular materials in drum-type machines.

3. The aim and objectives of the study

The aim of the work is to find out the effect of the key parameters on the mechanism of the loss of motion stability of the granular filler of the rotating drum chamber. This will make it possible to predict the efficiency of self-oscillating processes of processing polygranular materials in drum-type machines with variations in the characteristics of the oscillating system of the filled drum.

To achieve the aim, the following objectives were set:

- to identify characteristics of the qualitative nature of the mechanism of the loss of motion stability of the two-fraction granular filler of the rotating drum chamber;
- to estimate the values of key parameters of the oscillating system corresponding to the conditions of self-excitation of self-oscillations of the intra-chamber filler.

4. Research materials and methods

4.1. Object of research

Rational parameters of the self-oscillating process of processing polygranular materials in drum-type machines with the self-excitation of the periodic pulsating motion mode of the filler in the rotating chamber were studied.

4.2. Subject of research

Qualitative and quantitative characteristics of the mechanism of motion stability loss during self-excitation of self-oscillations of the polygranular filler of the rotating drum were investigated. Characteristic stationary continuous and steady-state periodic motion modes of the granular filler in the rotating chamber were found. The instability of motion modes was revealed as a condition for implementation. The nature of growing disturbances, the law and speed of their growth, and the physical mechanism for the development of self-oscillatory instability were analyzed. The effect of changes in the dynamic parameters of the two-fraction dispersion medium on the bifurcation processes of generation, establishment, and transformation of the modes was investigated.

4.3. The main research hypothesis

It was believed that the motion mode of the polygranular filler of the rotating drum chamber with pulsation deformation in the form of periodic collapse of the free surface is self-oscillating. Self-excitation of the filler self-oscillations due to loss of flow stability was assumed. Redistribution and dynamic interaction of granular particles in the chamber were considered as factors of motion instability. The structure of the polygranular filler and the characteristics of the rotating motion of the drum were adopted as key parameters of the oscillating system.

4.4. Assumptions and simplifications adopted in the study

It was assumed that the factors of filler motion instability are dilatancy and particle interaction. It was believed that

the interaction between coarse particles occurs in the form of solids collision, and their interaction with fine particles has a damping character. The drum chamber was assumed long, that is, the influence of the end walls on filler motion was neglected.

As a simplification, a two-fraction structure of the polygranular filler was adopted. Coarse and fine filler fractions with constant relative dimensions in the chamber were used. The shape of coarse fraction particles was considered spherical. Discrete values of the chamber filling degree in the range of 0.25–0.5 were taken. Only extreme cases of the two-fraction polygranular filler structure were examined. The first case corresponded to the absence of a fine fraction, the second – the complete filling of the gaps between coarse fraction particles with fine fraction particles at rest.

4. 5. Research method

The parameters of the motion modes were determined by numerical modeling based on experimental visualization of the filler behavior through the transparent end wall of the drum. Transient modes were studied with a continuous slow change in the drum speed. Steady-state modes were determined by recording and further processing of stationary patterns of filler motion in the cross-section of the chamber.

4. 6. Experiment planning

The case of a two-fraction intra-chamber filler of a rotating drum was considered. The coarse fraction simulated the grinding bodies of the drum mill, the fine fraction – the ground material. The content of the drum chamber filler was estimated by the volume degree of filling with the coarse fraction $\kappa_{cf} = w_{cf} / (\pi R^2 L)$, where w_{cf} is the volume of coarse fraction particles at free rest, R is the drum chamber radius, and L is the chamber length. The content of the fine fraction in the filler was estimated by the volume degree of filling the gaps between the elements of the coarse fraction $\kappa_{ff} = w_{ff} / (0.4 \pi R^2 L)$, where w_{ff} is the volume of fine fraction particles at free rest, 0.4 is the approximate value of the volume fraction of the gaps between spherical same-size coarse fraction particles at rest.

The discrete values of the volume filling degree of the chamber with the coarse fraction at free rest varied in the traditional range $\kappa_{cf} = 0.25 - 0.5$ for drum mills with a step of 0.05. The values of the volume degree of filling of the gaps between spherical coarse fraction particles with fine fraction particles at rest were $\kappa_{ff} = 0$ and 1. The condition $\kappa_{ff} = 0$ roughly corresponds to ultrafine and fine grinding, and $\kappa_{ff} = 1$ to coarse grinding in the drum mill.

4. 7. Experimental equipment

An experimental setup was used (Fig. 3) containing a drum with a horizontal axis of rotation and a split cylindrical chamber. One end wall of the chamber was made transparent to visualize filler motion. The drum drive made it possible to smoothly change the speed.

Drum speed was measured by a stroboscopic tachometer. The value of the stationary speed in steady-state and transient modes of the chamber filler was constantly checked to ensure the correctness of measurements. When using error propagation analysis, the speed measurement error was approximately $\pm 3\%$. The evaluation was carried out by measuring the stationary speed 5 times for one filler motion mode.

The particle size of granular materials of the filler fractions was measured by a laser-type analyzer.

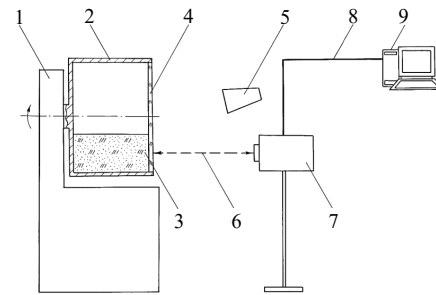


Fig. 3. Scheme of the experimental setup:

- 1 – drive and measuring unit; 2 – drum with a split chamber;
- 3 – granular filler; 4 – transparent end wall; 5 – illuminator;
- 6 – video recording of filler motion patterns;
- 7 – video camera; 8 – data transmission channel;
- 9 – computer for visualizing motion patterns

For dosing portions of the granular filler fractions of the drum chamber, laboratory beakers were used. The volume of the filler portion was determined at rest, without compaction when filling the measuring chamber.

The video recording rate when visualizing the filler behavior was 24 frames per second.

4. 8. Granular materials under study

Non-cohesive granular material with spherical particles of absolute $d_{lf} = d_{lf} / (2R) = 0.0104$ was used as the material of the filler coarse fraction. The bulk density of the coarse material at rest was $\rho_{lf} = 900 \text{ kg/m}^3$. M500 grade cement with particles of absolute d_{ff} and relative size $\psi_{dff} = d_{ff} / (2R) = (0.0236 - 0.236) \cdot 10^{-3}$ was used as the fine fraction material. The bulk density of the fine fraction material at rest was $\rho_{ff} = 1,200 \text{ kg/m}^3$.

4. 9. Calculation parameters

Positive shear dilatancy ν characterizes an increase in the volume of the granular filler of the chamber due to the movement of particles in the direction normal to the shear one. The current dilatancy value is $\nu = w / (\kappa_{cf} \pi R^2 L)$, where w is the current value of the volume of the entire filler in motion. The minimum and maximum dilatancy values for the period of self-oscillations are $\nu_{min} = w_{min} / (\kappa_{cf} \pi R^2 L)$ and $\nu_{max} = w_{max} / (\kappa_{cf} \pi R^2 L)$, where w_{min} and w_{max} are the minimum and maximum values of the filler volume for the period of self-oscillations in motion.

Bifurcation value of dilatancy:

$$\nu_{bif} = \nu_{semin}, \tag{1}$$

where ν_{semin} is the minimum dilatancy value for the oscillation period at the beginning of self-excitation of the filler self-oscillations.

An increase in the bifurcation value of dilatancy:

$$\Delta \nu_{bif} = \nu_{bif} - 1 = \nu_{semin} - 1. \tag{2}$$

Extreme value of dilatancy:

$$\nu_{ext} = \nu_{semax}, \tag{3}$$

where ν_{semax} is the maximum dilatancy value for the oscillation period at the beginning of self-excitation of self-oscillations.

An increase in the extreme value of dilatancy:

$$\Delta v_{ext} = v_{ext} - 1 = v_{semax} - 1. \quad (4)$$

The range of self-oscillations $R_v = v_{max} - v_{min}$ corresponds to the difference between the maximum and minimum values of filler dilatancy. The relative range of self-oscillations $\Psi_{Rv} = [2(v_{max} - v_{min})] / (v_{max} + v_{min})$ corresponds to the ratio of the range to the average dilatancy value for one self-oscillation period $(v_{max} + v_{min})/2$.

The limit value of the range at the beginning of self-excitation of self-oscillations:

$$R_{vlim} = v_{semax} - v_{semin}. \quad (5)$$

The limit value of the relative range at the beginning of self-excitation of self-oscillations:

$$\Psi_{Rvlim} = \frac{2(v_{semax} - v_{semin})}{v_{semax} + v_{semin}}. \quad (6)$$

The current value of the relative drum speed $\Psi_{\omega} = \omega / \omega_{cr}$ corresponds to the ratio of the current value of angular velocity ω to the critical speed value $\omega_{cr} = \sqrt{g/R}$, where g is the gravitational acceleration.

Bifurcation value of the relative speed:

$$\Psi_{\omega bif} = \frac{\omega_{bif}}{\omega_{cr}}, \quad (7)$$

where ω_{bif} is the bifurcation value of angular velocity.

The discontinuous nature of relaxation self-oscillations of the rotating drum chamber filler is determined by the dissipative properties of the oscillating system. The quality factor of the self-oscillating system and the relaxation degree of self-oscillations numerically characterize the ratio of nonlinear dissipative and reactive forces and processes in the system.

The quality factor of the oscillating system corresponds to the ratio of the energy stored in the system to the energy lost in the system for one oscillation period. The quality factor of the considered self-oscillations of the rotating drum filler:

$$QF = \frac{E_k}{E_p}, \quad (8)$$

where $E_k = I_{minv} \omega^2 / 2$ – kinetic energy of the filler accumulated for one oscillation period; $E_p = m \Delta h_c g$ – kinetic energy of the filler spent for one oscillation period; $I_{minv} = I_{pminv} L \rho_{minv}$ – moment of inertia of the filler upon reaching the minimum dilatancy value for one oscillation period; I_{pminv} – polar moment of inertia of the filler in the cross-section of the drum chamber for the minimum dilatancy value; $m = m_{ff} + m_{cf}$ – filler mass; $m_{ff} = \omega_{ff} \rho_{ff}$ and $m_{cf} = \omega_{cf} \rho_{cf}$ – mass of coarse and fine filler fractions; $\rho_{minv} = m / \omega_{min}$ – filler density for the minimum dilatancy value; $\Delta h_c = h_{cmax} - h_{cmin}$ – vertical displacement of the center of mass of the filler for one oscillation period; h_{cmax} and h_{cmin} – maximum and minimum values of the vertical coordinate of the center of mass of the filler for one oscillation period.

The degree of relaxation corresponds to the ratio of the duration of the slow stage of energy accumulation to the duration of the fast stage of energy relaxation for one self-oscillation period. The degree of relaxation of the considered self-oscillations:

$$DR = \frac{24 - z_r}{z_r}, \quad (9)$$

where z_r is the number of consecutive motion patterns in the chamber cross-section for the relaxation stage obtained by video recording at a frequency of 24 frames per second.

Linear frequency of self-oscillations:

$$v = \frac{24}{z}, \quad (10)$$

where z is the number of consecutive motion patterns in the chamber cross-section for one oscillation period.

Relative critical circular frequency of self-oscillations:

$$\Psi_{vcr} = \frac{\omega_v}{\omega_{cr}}, \quad (11)$$

where $\omega_v = 2\pi v$ is the circular frequency at the beginning of self-excitation of self-oscillations. The value Ψ_{vcr} characterizes the ratio of the circular frequency at the beginning of self-excitation of self-oscillations to the critical value of angular velocity.

Relative bifurcation circular frequency of self-oscillations:

$$\Psi_{vbif} = \frac{\omega_v}{\omega_{bif}}. \quad (12)$$

The value Ψ_{vbif} characterizes the number of filler oscillations for one revolution of the drum during self-excitation of self-oscillations.

The values of the inertial parameters of the filler were determined on the basis of visual analysis of motion patterns.

Expression for filler dilatancy:

$$v = \frac{\sum_{i=1}^n S_i}{\kappa_{ff} \pi R^2}, \quad (13)$$

where S_i is the area of the surface element i of the cross-section of the drum chamber containing the filler; n is the number of selected surface elements in the chamber cross-section.

Expression for the polar moment of inertia of the filler:

$$I_p = \sum_{i=1}^n S_i K_{fdi} r_i^2, \quad (14)$$

where $K_{fdi} = 0-1$ is the empirical filler density factor of the surface element i of the chamber cross-section; r_i is the radial coordinate of the center of the surface element i relative to the center of the chamber cross-section.

Expression for vertical displacement of the center of mass of the filler for one oscillation period:

$$h_c = \frac{\sum_{i=1}^n h_{ci} S_i K_{fdi}}{\sum_{i=1}^n S_i K_{fdi}}, \quad (15)$$

where h_{ci} is the radial coordinate of the center of the surface element i relative to the horizontal reference axis.

5. Results of the studies of the motion stability loss mechanism

5.1. Results of the study of the qualitative nature of the motion stability loss mechanism

By visualizing the flow patterns [8], all the main motion modes of the self-oscillating system were experimentally

determined [49, 52]. Under certain conditions, there can be two continuous and one periodic steady-state motion mode of the granular filler of the chamber of the stationary rotating drum.

The low-speed circulation mode [42] occurs in the speed range, the upper limit of which slightly exceeds the critical speed value ω_{cr} . The high-speed mode as a wall layer is formed at a speed exceeding ω_{cr} .

In the mutual transition of the continuous modes of circulation motion and the wall layer, the periodic self-oscillating mode may occur [4–7]. If the chamber speed increases, instability of the continuous circulation mode develops, which leads to periodic self-oscillations of the filler [8] (Fig. 2). With a further increase in speed, self-oscillations decrease due to an increase in the proportion of the wall layer and a decrease in filler dispersion. If the chamber speed decreases, the reverse transformation of the modes occurs due to the development of the corresponding motion instabilities.

When the speed increases from a state of rest to the formation of a wall layer, the dilatancy value changes from zero, passes through the maximum and reaches a zero value. The increase in dilatancy is caused by an increase in filler dispersion (Fig. 2), and the decrease is caused by the gradual formation of the wall layer. The reverse decrease in speed is characterized by a similar dependence of the change in the dilatancy value. Therefore, along with dilatancy, speed can also be attributed to the key parameters of the oscillating system, which are factors of self-excitation of self-oscillations. Based on the causal relationship of the filler motion modes, speed is the primary parameter, and dilatancy is the secondary parameter of the system.

Of the highest applied interest is the condition of stability during self-excitation of self-oscillations of the filler [49, 52]. Visualization made it possible to reveal a significant nonlinearity of the instability mechanism during mutual transitions of the circulation and self-oscillating modes of the filler during self-excitation and disappearance of self-oscillations. This nonlinearity is due to dispersion peculiarities of the medium and boundary conditions of the considered distributed oscillatory system. It was found that the instability mechanism is largely determined by the fractional structure of the polygranular filler and the filling degree of the chamber. With a low fine fraction content, the self-excitation mode of self-oscillations has a hard character (Fig. 2, *a, b*). Under a low filling degree of the chamber, the hard self-excitation mode becomes pronounced (Fig. 2, *a*). On the other hand, when the fine fraction content is commensurate with the volume of cavities between coarse fraction particles, the self-oscillation mode becomes soft (Fig. 2, *c, d*). If the chamber is significantly filled, the soft self-excitation mode becomes clearly pronounced (Fig. 2, *d*). The hard self-excitation mode corresponds to a significantly non-harmonic form of self-oscillations of the filler, and the soft mode – quasi-harmonic form.

In the hard self-excitation mode of self-oscillations, bistability occurs [49, 52]. Bifurcation values of speed and dilatancy have two values – upper and lower. Provided that the speed and dilatancy of the upper bifurcation values Ψ_{obifUP} and ν_{bifUP} are reached, there is an abrupt occurrence of steady self-oscillations with the upper limit value in the range $\Psi_{RolimUP}$ (Fig. 2, *a, b*). With a further increase in speed, until the wall layer is formed, self-oscillations persist. In the case of a reverse decrease in speed and dilatancy, the range of self-oscillations slightly decreases. As the speed and dilatancy decrease below the upper bifurcation values Ψ_{obifUP} and ν_{bifUP} , self-oscillations persist and their range continuously

decreases below the upper limit value $\Psi_{RolimUP}$. There is a pulling effect [49, 52] characterized by the loss of self-oscillations, with the parameters of the oscillating system below the excitation point (Ψ_{obifUP} and ν_{bifUP}). If the speed and dilatancy further decrease until the lower bifurcation values $\Psi_{obifLOW}$ and ν_{bifUP} are reached, the range reaches the lower limit value $\Psi_{RolimLOW}$. At the same time, self-oscillations are abruptly lost and a steady-state continuous circulation mode of the filler occurs. The ratio between the lower and upper bifurcation values of relative speed in the hard self-excitation mode is approximately $\Psi_{obifLOW} = (0.9 - 0.95)\Psi_{obifUP}$.

The soft self-excitation mode of self-oscillations is characterized by monostability [49, 52]. The upper and lower bifurcation values of relative speed $\Psi_{obifUP} \approx \Psi_{obifLOW} \approx \Psi_{obif}$ and dilatancy $\nu_{bifUP} \approx \nu_{bifLOW} \approx \nu_{bif}$ coincide. When the speed and dilatancy pass through the bifurcation values Ψ_{obifUP} and ν_{bifUP} , smooth self-excitation of self-oscillations occurs (Fig. 2, *c, d*). With a further increase in speed and dilatancy, the range of self-oscillations, starting from zero, will grow continuously and rather slowly, without jumps. In the case of a reverse decrease in speed and dilatancy, the range of self-oscillations decreases gradually and continuously, without jumps, reaching zero. When the speed and dilatancy of the bifurcation values Ψ_{obifUP} and ν_{bifUP} are reached, self-oscillations gradually disappear and the circulation mode of the filler appears.

Characteristic components of clearly delimited stages of the period of steady-state self-oscillations of the filler were identified. The stages were determined based on the analysis of the energy balance condition of the energy inflow into the system from an external source and dissipative energy loss in the oscillating system [49, 52]. Each oscillation period is a discontinuous relaxation cycle having two stages. During one period, the dilatancy value changes from the minimum, passes through the maximum and reaches the minimum value (Fig. 2). Two clearly demarcated stages of transformation of the oscillating system state due to the complete energy exchange over the oscillation period were revealed – slow and fast. Mutual transitions between such states have a bifurcation character [49, 52].

The post-relaxation stage of dilatancy change from the minimum to the maximum value is slow. It is associated with a smooth rise of the center of the filler mass along with the chamber surface. At this stage, there is a relatively slow accumulation of potential energy to a certain critical state. At the same time, there is a loss of a steady equilibrium state of the system.

The relaxation stage of dilatancy change from the maximum to the minimum value is fast. It is associated with a sharp lowering of the center of the filler mass due to collapse. At this stage, a relatively fast abrupt dissipative discharge occurs with significant dissipation of mechanical energy not involved in the subsequent exchange. At the same time, a steady quasi-equilibrium state is restored.

The significantly non-harmonic form of self-oscillations under the hard self-excitation mode (Fig. 2, *a*) is characterized by a large difference in the duration of post-relaxation and relaxation stages of the relaxation cycle of filler motion. On the other hand, the quasi-harmonic form of self-oscillations under the soft self-excitation mode (Fig. 2, *d*) is characterized by the proximity of the duration of the cycle stages.

5. 2. Results of the study of quantitative conditions of self-excitation of self-oscillations

The graphs of the results of the experimental determination of changes in the parameters of the self-oscillating system are shown in Fig. 4–11. In this case, $\kappa_f = 0.25, 0.3, 0.35, 0.4$,

0.45 and 0.5, and the degree of filling the gaps between coarse fraction elements with fine fraction particles is $\kappa_{ff}=0$ and 1.

The graphs of the increase in the bifurcation value of filler dilatancy Δv_{bif} depending on the degree of chamber filling with the coarse fraction κ_{if} are shown in Fig. 4. The values of Δv_{bif} were determined by expression (2) taking into account (1) and (13).

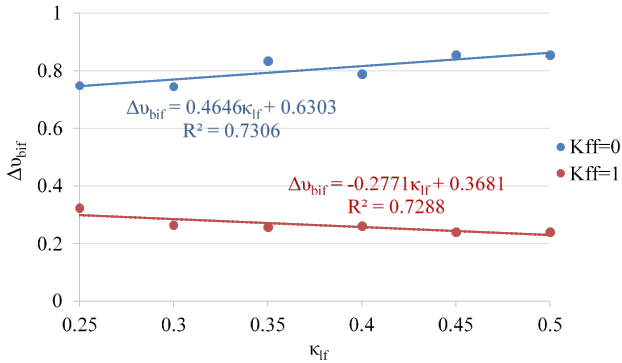


Fig. 4. Experimental dependence of the increase in the bifurcation dilatancy value Δv_{bif} on κ_{if}

The graphs of changes in the bifurcation value of the relative drum speed ψ_{obif} depending on κ_{if} for $\kappa_{ff}=0$ and 1 are shown in Fig. 5. The values of ψ_{obif} were determined by expression (7).

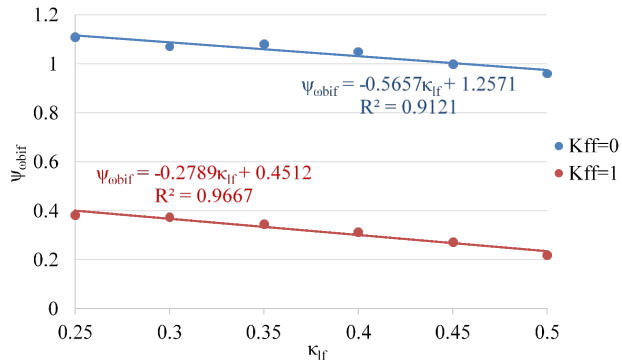


Fig. 5. Experimental dependence of the bifurcation value of the relative speed ψ_{obif} on κ_{if}

The graphs of changes in the increase in the extreme value of filler dilatancy Δv_{ext} depending on κ_{if} for $\kappa_{ff}=0$ and 1 are shown in Fig. 6. The values of Δv_{ext} were determined by expression (4) taking into account (3) and (13).

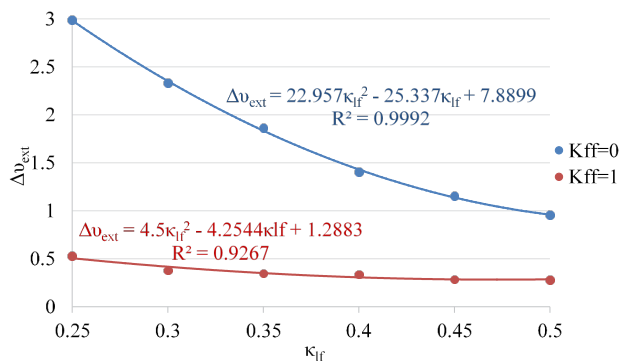


Fig. 6. Experimental dependence of the increase in the extreme dilatancy value Δv_{ext} on κ_{if}

The graphs of changes in the limit value of the relative amplitude at the beginning of self-excitation of self-oscillations ψ_{Rolim} depending on κ_{if} for $\kappa_{ff}=0$ and 1 are shown in Fig. 7. The values of ψ_{Rolim} were determined by expression (6) taking into account (5) and (13).

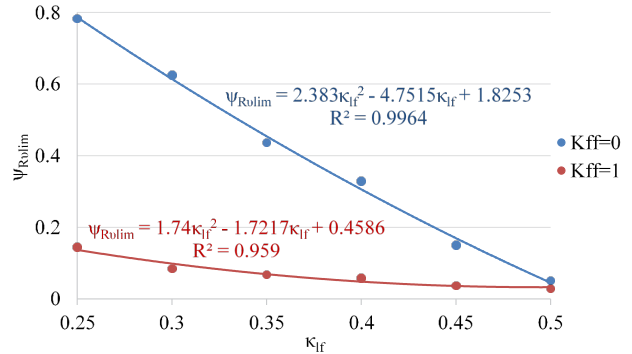


Fig. 7. Experimental dependence of the limit value of the relative amplitude at the beginning of self-excitation of self-oscillations ψ_{Rolim} on κ_{if}

The graphs of changes in the quality factor of the self-oscillating system QF depending on κ_{if} for $\kappa_{ff}=0$ and 1 are shown in Fig. 8. The QF values were determined by expression (8) taking into account (14) and (15).

The graphs of changes in the degree of relaxation of self-oscillations DR depending on κ_{if} for $\kappa_{ff}=0$ and 1 are shown in Fig. 9. The DR values were determined by expression (9).

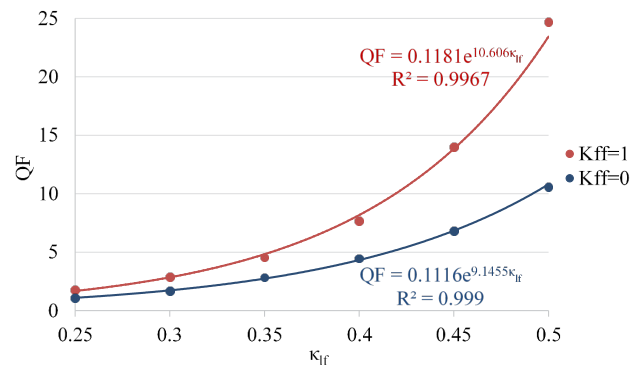


Fig. 8. Experimental dependence of the quality factor of the self-oscillating system QF on κ_{if}

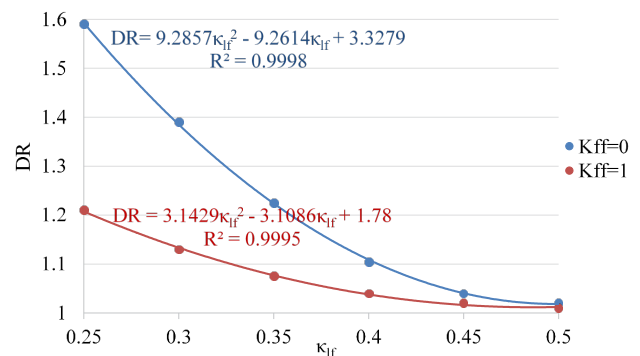


Fig. 9. Experimental dependence of the degree of relaxation of self-oscillations DR on κ_{if}

The graphs of changes in the relative critical circular frequency of self-oscillations ψ_{vcr} depending on κ_{if} for $\kappa_{ff}=0$

and 1 are shown in Fig. 10. The values of ψ_{vcr} were determined by expression (11) taking into account (10).

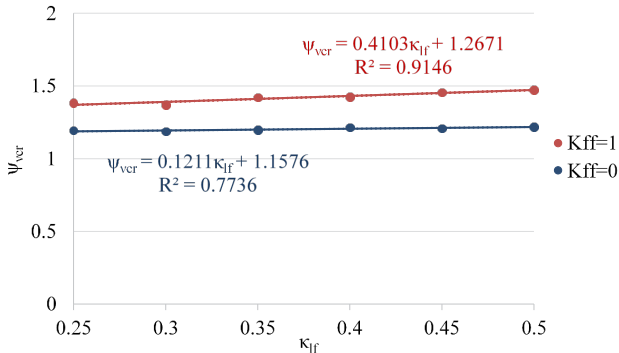


Fig. 10. Experimental dependence of the relative critical circular frequency of self-oscillations ψ_{vcr} on κ_{ff}

The graphs of changes in the relative bifurcation circular frequency of self-oscillations ψ_{vbif} depending on κ_{ff} for $\kappa_{ff}=0$ and 1 are shown in Fig. 11. The values of ψ_{vbif} were determined by expression (12) taking into account (10).

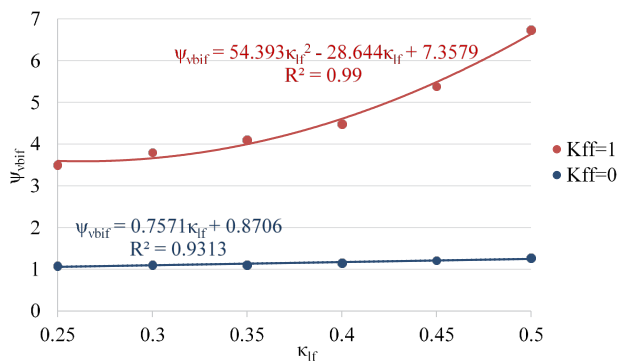


Fig. 11. Experimental dependence of the relative bifurcation circular frequency of self-oscillations ψ_{vbif} on κ_{ff}

The obtained experimental dependences of the numerical values of the inertial and frequency parameters of the self-oscillating system characterize the quantitative effect of the filler structure on motion stability.

6. Discussion of the results of the study of motion instability

The results of experimental flow visualization and numerical data made it possible to qualitatively and quantitatively assess the effect of the factors of motion instability of the polygranular intra-chamber filler. The effect of the structure of the two-fraction granular medium on the motion stability was empirically determined. The main instability factor was an increase in the granular filler dilatancy, and an additional factor – damping effect of fine fraction particles on the pulsed interaction of coarse fraction particles.

A significant influence of the fine fraction content on the bifurcation value of filler dilatancy at the beginning of self-excitation of self-oscillations was found (Fig. 4). As κ_{ff} increases, the increase in the bifurcation value of dilatancy Δv_{bif} decreases by 2.33–3.56 times, from the maximum to the minimum value. The maximum value of Δv_{bif} , in the

absence of fine fraction $\kappa_{ff}=0$, is 0.749–0.855. The minimum value of Δv_{bif} , when the gaps between coarse fraction particles are completely filled with fine fraction particles $\kappa_{ff}=1$, is 0.24–0.322. The decrease in Δv_{bif} also depends on the filling degree of the chamber and increases with increasing κ_{ff} .

This testifies to a decrease in the filler motion stability due to the influence of the fine fraction, which is manifested as a decrease in the critical dilatancy value at the beginning of stability loss. This is due to the dynamic effect of damping enhancement during the interaction of solid particles of the coarse fraction through collision under the action of soft particles of the fine fraction. This effect increases with increasing filling degree of the chamber.

A similar significant effect of the fine fraction content on the bifurcation value of the relative drum speed during self-excitation of self-oscillations ψ_{obif} was also revealed (Fig. 5). With an increase in κ_{ff} , the value of ψ_{obif} decreases by a factor of 2.91–4.4: from 0.96–1.11, in the absence of a fine fraction $\kappa_{ff}=0$, to 0.218–0.382, when the gaps are completely filled $\kappa_{ff}=1$. The value of ψ_{obif} at any content of the fine fraction κ_{ff} depends slightly on the filling degree of the chamber and increases somewhat with an increase in κ_{ff} . The observed decrease in ψ_{obif} , by 2.91–4.4 times (Fig. 5), is quite close to the detected decrease in the increase of Δv_{bif} , by 2.33–3.56 times (Fig. 4).

This also indicates a decrease in the filler motion stability due to the influence of the fine fraction manifested as a decrease in the critical speed value at the beginning of self-excitation. Such a decrease in speed leads to a decrease in the critical value of the fraction of the filler dispersion zone and the dilatancy value necessary for the loss of stability. This effect also depends on the chamber filling degree and is somewhat enhanced with an increase in κ_{ff} .

A significant effect of the fine fraction content and chamber filling degree on the extreme dilatancy value and self-oscillation range under loss of stability was revealed (Fig. 6, 7).

The value of the increase in the extreme dilatancy value Δv_{ext} decreases by 3.44–5.66 times: from 0.953–2.99, at $\kappa_{ff}=0$, to 0.277–0.528, at $\kappa_{ff}=1$ (Fig. 6). The decrease in Δv_{ext} significantly weakens with increasing κ_{ff} : from 5.66 times, at $\kappa_{ff}=0.25$, to 3.66 times, at $\kappa_{ff}=0.5$.

Similarly, the limit value of the relative range at the beginning of self-excitation of self-oscillations ψ_{Rvlim} decreases by 1.75–5.42 times: from 0.0515–0.783, at $\kappa_{ff}=0$, to 0.0294–0.145, at $\kappa_{ff}=1$ (Fig. 7). The decrease in ψ_{Rvlim} is also significantly weakened with an increase in κ_{ff} : from 5.42, at $\kappa_{ff}=0.25$, to 1.75, at $\kappa_{ff}=0.5$.

This testifies to a decrease in the filler motion stability due to the influence of the fine fraction manifested as a decrease in the extreme dilatancy value and self-oscillation range under loss of stability. Such a decrease in motion stability significantly increases with increasing chamber filling degree. This is due to the reduction of the critical value of the fraction of the filler dispersion zone and the dilatancy value necessary for the loss of stability.

A significant effect of the fine fraction content and chamber filling degree on the quality factor of the self-oscillating system and relaxation of self-oscillations upon loss of stability was found (Fig. 8, 9).

The quality factor of the self-oscillating system QF increases by 1.62–2.34 times: from 1.09–10.6, at $\kappa_{ff}=0$, to 1.77–24.7, at $\kappa_{ff}=1$ (Fig. 8). The growth of QF significantly increases with increasing κ_{ff} : from 1.62, at $\kappa_{ff}=0.25$, to 2.34, at $\kappa_{ff}=0.5$.

On the other hand, the degree of relaxation of self-oscillations DR decreases by a factor of 1.01–1.31: from 1.02–1.59, at $\kappa_{ff}=0$, to 1.01–1.21, at $\kappa_{ff}=1$ (Fig. 9). The decrease in DR significantly increases with decreasing κ_{ff} : from 1.01, at $\kappa_{ff}=0.5$, to 1.31, at $\kappa_{ff}=0.25$.

This indicates the weakening of the discontinuous nature of relaxation self-oscillations of the filler with an increase in the fine fraction content and chamber filling degree, accompanied by the loss of stability. This is due to a decrease in energy losses in the oscillating system caused by a decrease in the extreme dilatancy value and the limit value of the relative range of self-oscillations during self-excitation.

The influence of the fine fraction content and chamber filling degree on the frequency of self-oscillations during self-excitation was revealed (Fig. 10, 11).

The value of the relative critical circular frequency of self-oscillations ψ_{vcr} increases slightly by 1.15–1.21 times: from 1.19–1.22, at $\kappa_{ff}=0$, to 1.37–1.47, at $\kappa_{ff}=1$ (Fig. 10). The growth of ψ_{vcr} is weakly dependent on the chamber filling degree and somewhat increases with an increase in κ_{ff} .

At the same time, the value of the relative bifurcation circular frequency of self-oscillations ψ_{vbif} increases significantly by 3.26–5.29 times: from 1.07–1.27, at $\kappa_{ff}=0$, to 3.5–6.72, at $\kappa_{ff}=1$ (Fig. 11). The growth of ψ_{vbif} also depends weakly on the chamber filling degree and somewhat increases with an increase in κ_{ff} .

This proves that the self-oscillation range of the filler is determined only by the properties of the oscillating system and is largely independent of the external influence of a stable energy source. In particular, the change in the critical circular frequency of self-oscillations ψ_{vcr} , for $\kappa_{ff}=0.25$ –0.5 and $\kappa_{ff}=0$ –1, is limited to a rather narrow range of 1.19–1.47 (Fig. 10). In this case, the values of the number of oscillations per revolution of the drum during self-excitation ψ_{vbif} acquire a rather wide range of 1.07–6.72 (Fig. 11).

Therefore, the mechanism of the loss of motion stability of the two-fraction filler of the rotating drum was clarified. The speed, as the main, and fine fraction content and chamber filling degree, as other key dynamic parameters of the oscillating system, were identified. Numerical bifurcation values of the key parameters and their influence on the dilatancy, range and frequency of self-oscillations were determined. The obtained qualitative and quantitative characteristics of the stability loss mechanism allow predicting the conditions of self-excitation of filler pulsations with the given parameters for self-oscillating processes of processing polygranular materials.

It turned out that to generate the beginning of self-excitation of self-oscillations of the filler, the initial relative drum speed ψ_{oi} must reach the bifurcation value $\psi_{oi}=\psi_{obif}$. The initial speed for the single-fraction monogranular filler ($\kappa_{ff}=0$) is approximately $\psi_{oi0}=0.9$ –1.1 (Fig. 5, $\kappa_{ff}=0$). The initial speed for the two-fraction polygranular filler at a high degree of filling the gaps between coarse fraction particles ($\kappa_{ff}\gg 1$) with fine fraction particles is $\psi_{oi>1}=0.2$ –0.4 (Fig. 5, $\kappa_{ff}=1$). The initial speed for the two-fraction filler at an intermediate degree of filling the gaps between coarse fraction particles with fine fraction particles ($\kappa_{ff}=0$ –1) is $\psi_{oi0-1}=0.4$ –0.9 (Fig. 5).

At the same time, to reliably reproduce the self-oscillating mode of filler motion with a significant range, it is necessary to maintain the current relative speed ψ_{oc} in a rational range relative to the initial value ψ_{oi} . The need to change the current speed value ψ_{oc} is caused by possible variations in

the operating parameters of the drum machine. The limits of this range are determined by the hardness degree of the self-excitation mode of self-oscillations under bistability (Fig. 2).

In particular, the range of the current speed values for the single-fraction monogranular filler ψ_{oc0} ($\kappa_{ff}=0$) is approximately from ψ_{oi0} to $(0.9$ – $0.95)\psi_{oi0}$ (ψ_{oc0} decreases to 5–10 % relative to ψ_{oi0}). This range corresponds to the hard self-excitation mode of self-oscillations (Fig. 2, *a*).

The current speed range for the two-fraction polygranular filler at a high degree of filling the gaps between coarse fraction particles $\psi_{oc>1}$ ($\kappa_{ff}\gg 1$) with fine fraction particles is from $\psi_{oi>1}$ to $(1.3$ – $1.4)\psi_{oi>1}$ ($\psi_{oc>1}$ increases to 30–40 % relative to $\psi_{oi>1}$) (Fig. 5, $\kappa_{ff}=1$). This range corresponds to the soft self-excitation mode (Fig. 2, *d*).

The current speed range for the two-fraction filler at an intermediate degree of filling the gaps between coarse fraction particles with fine fraction particles ψ_{oc0-1} ($\kappa_{ff}=0$ –1) is from $0.95\psi_{oi0-1}$ to $1.3\psi_{oi0-1}$ (ψ_{oc0-1} ranges from –5 % to +30 % relative to ψ_{oi0-1}) (Fig. 5). The range corresponds to the intermediate, between hard and soft, self-excitation mode (Fig. 2, *b, c*).

The applicability of the limit values of the dynamic motion characteristics corresponding to the conditions of self-excitation of self-oscillations of the two-fraction granular filler of the rotating drum is limited by discrete values of the initial parameters. Such values of the degree of chamber filling with coarse fraction particles at free rest were $\kappa_{ff}=0.25$ –0.5 with a step of 0.5. The discrete values of the degree of filling the gaps between spherical particles of the coarse fraction with fine fraction particles at rest were $\kappa_{ff}=0$ and 1. The relative particle size of the coarse fraction in the chamber was 0.0104, and the fine fraction – $0.13\cdot 10^{-3}$.

The shortcomings of the applied approach to assessing the impact of self-oscillations include the failure to take into account geometric criteria for the similarity of the considered system with a multiphase medium of variable structure.

In the future, it is advisable to find out the qualitative and quantitative effect of intermediate values of the characteristics of the filler structure on the dynamic and technological process parameters. In particular, the effect of the filling degree of the chamber with variations in the content of fine fraction particles in the gaps between coarse fraction particles. This will make it possible to determine rational conditions for self-excitation of filler pulsations during the self-oscillating process of processing various polygranular materials in drum-type machines.

7. Conclusions

1. The mechanism of stability loss of the granular flow in a rotating drum consists in the establishment of a periodic steady self-oscillating motion mode due to the loss of stability during the transition of two continuous modes. Such continuous modes are circulation and wall layer mode.

The mechanism of stability loss is caused by oscillatory instability factors. The main instability factor is the growth of dilatancy of the granular filler. An additional factor of instability of the polygranular filler is the damping effect of fine fraction particles on the pulsed interaction of coarse fraction particles.

The stability loss mechanism is related to the key parameters of the oscillating system. The main key parameter is the drum speed, which causes a change in filler dilatancy.

Other key parameters are the content of the fine fraction in the filler κ_{ff} and chamber filling degree κ_f , the growth of which increases self-oscillating instability. The increase in instability is realized in the reduction of the bifurcation values of speed and filler dilatancy.

The features of the stability loss mechanism are due to the type of oscillating system. The rotating filled drum is a self-oscillating relaxation-type system, where stationary oscillations having a pronounced discontinuous character are established. Self-excitation of self-oscillations occurs in the hard mode under bistability. The discontinuous nature of oscillations and oscillatory hysteresis increase with decreasing κ_{ff} and κ_f . The increase in energy losses in the oscillating system is due to an increase in the extreme dilatancy value and the range of self-oscillations over the period.

2. The limit values of the dynamic motion parameters corresponding to the conditions of self-excitation of self-oscillations of the two-fraction granular filler of the rotating drum chamber during the implementation of the stability loss mechanism were determined. The first value corresponds

to the absence of the fine fraction in the filler, the second – the complete filling of the gaps between coarse fraction particles with fine fraction particles. These are 0.745–0.855 and 0.24–0.322 for the bifurcation value of filler dilatancy, respectively. For the bifurcation value of the relative drum speed – 0.96–1.11 and 0.218–0.382. For the limit value of the relative self-oscillation range of the filler at the beginning of self-excitation – 0.0515–0.783 and 0.0294–0.145. For the relative circular frequency of self-oscillations at the beginning of self-excitation – 1.19–1.21 and 1.38–1.47. For the number of self-oscillations per revolution of the drum during self-excitation – 1.07–1.27 and 3.5–6.72.

Conflict of interest

The authors declare that they have no conflict of interest in relation to this research, whether financial, personal, authorship or otherwise, that could affect the research and its results presented in this paper.

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