

*This paper considers the physical processes in the structure of the material for a heat-emitting fuel element (FE) shell, caused by various damaging defects, on its outer and inner surfaces, and affecting the change in the geometric parameters of a nuclear reactor's FE.*

*The task to improve the model of damage to an FE shell is being solved, taking into consideration structural and phase changes in the material of the shell with damaging defects on the outer and inner surfaces, in order to establish the actual criterion for assessing the FE hermeticity degree.*

*It is proposed to study the structure of the shell material with damaging defects (macropores and microcracks), which is a porous heterogeneous structure with fractal properties of self-similarity and scalability, to use the apparatus of fractal geometry.*

*A physical model of the FE shell has been built and proposed, in the form of a geometric cylinder-shaped figure, which makes it possible to investigate the fractal properties of the structure of the material of the damaged shell and their influence on a change in the geometric parameters of FE*

*An improved model of damage to the FE shell was derived, which makes it possible to take into consideration fractal increases in the geometric parameters of FE, for the established values of the fractal dimensionality.*

*Experimental studies of the FE shell, using the skin effect, confirmed the theoretical results and showed the validity of the choice of practical use of the fractal dimensionality parameter as an effective criterion for assessing the hermeticity degree of an FE shell. It has been experimentally established that the value of the fractal dimensionality of 2.68 corresponds to the maximum degree of damage to the shell for a leaky FE*

**Keywords:** fuel element shell, shell damage model, fractal structure, fractal dimensionality

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# BUILDING A MODEL OF DAMAGE TO THE FRACTAL STRUCTURE OF THE SHELL OF THE FUEL ELEMENT OF A NUCLEAR REACTOR

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## 1. Introduction

One of the real issues for creating safe operating conditions for power nuclear reactors is to enable the reliable operation of fuel elements (FEs) under stationary and transient operating modes and in emergency [1, 2].

The reliability of fuel elements is determined by their ability to hold the fission products of nuclear fuel inside the shell, not exceeding the level of geometric forming (elongation, narrowing, stretching, bulge, etc.), leading to a deterioration in their cooling capacity [3–5].

The most important parameters affecting the resource characteristics and safety of the fuel element include the degree of damage to the shell as the main barrier preventing the release of radioactive fission products into the heat carrier and the environment [6].

Consequently, it is necessary to develop a database on damaging defects of fuel element shells, as well as to improve

existing and devise new models and methods for systems that monitor the tightness of the shell, for their detection, and identification.

The characteristics of the damaging defects of the shell (location, type, dimensions) make it possible to find out the probable cause of their occurrence: violation of the FE manufacturing technology; design flaws of fuel element bundles (FEB); deviations from the normal operating modes, etc.

Therefore, the detection and identification of damage to the fuel element shell during operation is one of the relevant tasks of FE post-reactor studies in the protective chambers of NPP research centers.

In shell tightness monitoring systems (STMS), the following destructive and non-destructive methods of control are used [7–11]: capillary, radiographic, radio wave, mass spectrometric, acoustic emission, ultrasonic, magnetic, eddy current flaw detection, etc.

Analysis of known models and methods in FE STMS revealed their characteristic shortcomings, namely [7–11]:

- restrictions and assumptions for changing the geometric parameters of FE are introduced and structural-phase changes in the entire volume of the FE shell material are not taken into consideration;

- empirical correlations and unreliable extrapolations are used to estimate the uncertainty of the calculation results.

Hence, it follows that the study and modeling of physical processes on the outer and inner surface of the FE shell, in which damage and destruction of the structure of the material occurs, is a relevant task when conducting post-reactor control at nuclear power plants.

At the same time, in the models of damage to the FE shell, when modeling the physical processes occurring in the structure of the FE material, it is necessary to take into consideration both the geometric parameters of FE and their increments, taking into consideration the damaging defects.

The results of scientific research on this topic are important for determining the increments of the geometric parameters of FE and are necessary in practice to determine the actual criterion for assessing the degree of tightness (depressurization) of the FE shell.

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## 2. Literature review and problem statement

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Paper [12] reports results of a study into the surface relief of the material, in the structure of which, under the influence of external factors and damaging defects, local inhomogeneities, micropores, macrocracks are formed. It is shown that such a structure is characterized by fractal properties of self-similarity, scalability, invariance, and can be investigated on the basis of the theory of fractals. However, the issues related to determining the degree of damage to the relief of the surface of the material remained unresolved. The reason for this may be the difficulties associated with determining the criterion for assessing the degree of damage to the relief due to a change in the area of the real perceiving surface. One of the ways to overcome these difficulties may be to find the dependence of the criterion for assessing the degree of damage to the relief of the material on the quantitative assessment of the fractal structure of the real perceiving surface – the value of fractal dimensionality. All this makes it possible to assert that it is expedient to conduct studies of physical processes in the structure of the surface layer of the relief of the material, to establish the dependence of the criterion for assessing the state of the surface on the fractal dimensionality.

Paper [13] describes the models of interaction of nuclear fuel tablets (NFT) with the FE shell, for calculating the mechanical interaction of FE fuel with its shell in a fast neutron nuclear reactor. The results of test calculations of mechanical stresses and deformations in the FE shell, when solving problems, for various geometric parameters of FE are given. The interaction of NFT with the shell, which is characteristic of sharp rises in heat dissipation power, can lead to the emergence of mechanical stress concentrators in the FE shell, significantly changing its shape and volume.

In work [14], a model of the mechanical behavior of FE shells is considered, describing the deformation of a multi-layer structure, taking into consideration the dependence of the physicochemical properties of the shell on temperature, oxygen concentration, deformation rate, etc. The cited work develops a software module designed to describe the melting

of materials (analysis of geometry changes). The disadvantages of the model include the fact that only individual physical phenomena occurring on the outer surface of the FE shell are simulated, which does not give an idea of the change in the properties and phase state of the structure of the shell material.

In works [15, 16], the applicability of the built-in model of deformation of the FE shell of the calculation code RELAP5/MOD3.2 for VVER-1000 fuel with zirconium alloy shells (Zr+1 % Nb) is investigated. The applicability of the model is checked for the degree of blockage of the hot channel after swelling and rupture of the FE shell when heated in the temperature ranges of 600..1200 °C and pressure drops from 1 to 12 MPa. It is shown that the data of the built-in model can be used in estimating the destruction of the shell of VVER-1000 FE from zirconium alloy (Zr+1 % Nb) only in a certain limited area of parameters. The influence of model parameters on the maximum shell temperature at the maximum design accident was evaluated.

In work [17], it is shown that local geometric violations in NFT are caused by manufacturing defects, which in some cases can lead to surface stress of the shell and cause its destruction. A model for the process of global thermomechanical behavior of fuel rods, including the evolution of thermal and mechanical properties of fuel during fission, the formation and release of fission gas, is proposed. The model takes into consideration local defects that can be modeled by directly including them in the three-dimensional model of the FE shell. This makes it possible to use the full set of physical parameters used in the analysis of fuel characteristics, which must be included to calculate the local defect. The simulation results can be used with the appropriate failure criterion to determine the increased risk of leakage, due to cracking of the shell surface.

Paper [18] considers a mathematical model that makes it possible to determine the estimated activity of control radionuclides in each reactor unit, at any time after the violation of the tightness of the fuel element. In this model, for TMS systems, methods for controlling the tightness of FE shells, according to the activity of gas in the gas volumes of nuclear power plants, were proposed. It is shown that the processes of modeling the damage to the shell were carried out with limitations and assumptions regarding the geometric parameters of FE, which significantly affect the accuracy of calculations in known models. Consequently, this approach introduces significant errors in the calculations of the FE depressurization criteria and makes it impossible to fully apply analytical classical expressions for modeling the processes of damage and destruction of the FE shell.

In works [19, 20], it is shown that surfaces with damaging defects have a heterogeneous and porous structure, which has fractal properties of self-similarity and scalability and can be characterized by a quantitative value – fractal dimensionality. Therefore, in the cited work, to study the fractal structure of the material of the FE shell and calculate the real geometric parameters of FE, it is proposed to use the computing apparatus of fractal geometry.

Thus, the problem of monitoring dimensional changes in the geometric parameters of FE for the VVER-type reactor requires constant attention and study since it can be one of the decisive factors leading to the destruction of the fuel element shell.

Improvement and construction of models for the methods of a VVER FE TMS system, taking into consideration structural-phase changes in the volume of shell material and increments of geometric parameters of FE, for post-reactor

research, is an important scientific and practical task. Resolving this issue will help solve the problem associated with determining the non-hermetic fuel elements without removing fuel from FEB during its reloading in a nuclear reactor at a nuclear power plant.

### 3. The aim and objectives of the study

The purpose of this study is to develop a model of shell damage to improve the accuracy of the assessment of the degree of depressurization of the fuel element, taking into consideration the influence of damaging defects in the structure of the shell material on the change in the FE geometric parameter. This makes it possible to determine the real location and size of the damaging defect on the outer and inner surface of the shell and track the dynamics of its growth until a through crack forms in the shell of an unpressurized fuel element.

To accomplish the aim, the following tasks have been set:

- to build a physical model of the FE shell;
- to conduct a study of the fractal structure of a physical model with a damaged shell of a fuel element using the apparatus of fractal geometry;
- to improve the analytical expressions of the FE shell damage model, based on accounting for the fractal increments of the FE geometric parameters;
- to process experimental data on the study of the fractal structure of the material of the damaged shell, to assess the criterion of FE depressurization.

### 4. The study materials and methods

To conduct theoretical studies into the process of damage to the FE shell, as an object of research, physical processes in the structure of the shell material caused by various damaging defects and affecting the change in the geometric parameters of FE are considered.

FE is the main structural element of the core of the nuclear power plant of the VVER type, which is a rod consisting of a zirconium shell (alloy E110: Zr+1 % Nb), a column of tablets of uranium dioxide inside the tube (Fig. 1).

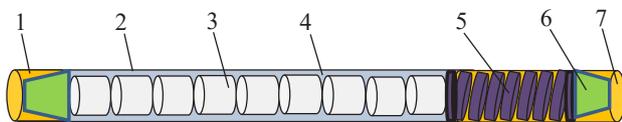


Fig. 1. Diagram of the design of the fuel element of the VVER-1000 reactor: 1 – upper plug; 2 – shell, alloy Zr+1 % Nb; 3 – tablet, uranium dioxide; 4 – gas gap; 5 – retainer, stainless steel; 6 – bushing; 7 – bottom plug

The main design characteristics and geometric parameters of NR TMS FE of the VVER-1000 type, which are taken into consideration in the calculation of damage to the FE shell include:

- the external (9.15 mm) and internal (7.73 mm) diameter of the FE shell;
- the thickness (0.69 mm) and geometric length of the shell (3837 mm);
- a gas gap between the inner surface of the shell and the nuclear fuel tablet.

As is known [3], in the presence of damaging defects on the outer and inner surface of the material of the FE shell, determining the parameter of damage to the FE shell is carried out using expression (1):

$$\omega(\tau) = \frac{A(\tau)}{A_0}, \tag{1}$$

where  $\omega(\tau)$  is the parameter of damage to the structure of the FE shell material;  $A_0$  – specific energy dissipation; it characterizes the non-damaged structure of the FE shell material;  $A(\tau)$  – specific dissipation of energy; it characterizes the intensity of damage to the structure of the FE shell material over time  $\tau$  and is determined from expression (2):

$$A(\tau) = \int_0^\tau W(\tau) dr, \tag{2}$$

where  $W(\tau)$  is the specific scattering power: it characterizes the intensity of the creep process of the FE shell material at any given time and is determined from expression (3):

$$W(\tau) = G_e \cdot \rho_e, \tag{3}$$

where  $\rho_e$  is the rate of equivalent creep deformation of the FE shell material.

$G_e$  is the equivalent stress of the simplest one-dimensional model, assuming the isotropy of the FE shell material; it is determined from expression (4):

$$G_e = \sqrt{\frac{1}{2}[(G_\theta - G_r)^2 + (G_r - G_z)^2 + (G_z - G_\theta)^2]}, \tag{4}$$

where  $G_\theta$ ,  $G_r$ ,  $G_z$  are the stresses in the coordinate directions  $\theta$ ,  $r$ ,  $z$ .

The equivalent deformation of the creep of the fuel element shell for all considered loading modes of the VVER-1000 type nuclear reactor gradually increases over time. Therefore, the rate of equivalent deformation of the creep of the shell material is calculated taking into consideration the radiation effects and, according to [21], is represented as a function of the parameters shown in expression (5):

$$\rho_e = K \cdot \Phi(G_e + B \cdot e^{C \sigma_e}) \cdot \exp(-10,000 / R \cdot T) \cdot t^{-1/2}, \tag{5}$$

where  $\rho_e$  is the rate of equivalent creep deformation of the FE shell material,  $s^{-1}$ ;  $\Phi$  is the flux density of fast neutrons,  $1/m^2 \cdot s$ ;  $T$  – temperature, K;  $R$  is the gas constant;  $G_e$  is the equivalent voltage, Pa;  $t$  is the time of deformation, s;  $K = 5.129 \cdot 10^{-29}$ ;  $B = 7.252 \cdot 10^2$ ;  $C = 4.967 \cdot 10^{-8}$  – dimensionless coefficients.

According to the law of neutron flux distribution along the length of FE, the flux density of fast neutrons  $\Phi(z)$  on the axial segment  $z$  ( $0 \leq z \leq H_{geom}$ ) is determined from expression (6):

$$\Phi(z) = \cos \left[ \frac{\pi}{H_{ef}} \left( z - \frac{H_{geom}}{2} \right) \right], \tag{6}$$

where  $H_{geom}$  is the geometric length (height) of the fuel element;  $z$  is the axial segment, that is the section of the FE length, for determining the flux density of fast neutrons  $\Phi(z)$ ;  $H_{ef} = H_{geom} + 2\delta_{ef}$  is the estimated length (height) of the fuel element, taking into consideration the value of the effective additive ( $\delta_{ef} \approx 10$  cm).

As practice shows, the calculations of the estimated length (height) of the fuel element ( $H_{ef}$ ), taking into consideration the value of the effective additive  $\delta_{ef}$ , according to expression (6), are very approximate. In addition, this generally leads to errors in the calculations of the main accepted mode characteristics during the operation of the VVER-1000 nuclear reactor, such as flux density of fast neutrons; linear specific energy release, etc.

Therefore, in the models of damage to the FE shell, it is necessary to take into consideration the real increment (elongation) of the height of the fuel element, due to stretching, swelling, and creep of the shell, under the influence of damaging defects. At the same time, structural-phase changes occur in the shell material, with the formation of local inhomogeneities, micropores, and macrocracks, and, consequently, such a structure, according to [22–24], has fractal properties.

Thus, taking into consideration the above, when calculating the real length of the FE, it is proposed in this work to take into consideration its fractal increment, according to expression (7):

$$H_f = H_{geom} + \Delta H_f, \quad (7)$$

where  $H_f$  is the fractal (real) length of the FE;  $\Delta H_f$  is the fractal increment of the FE length.

Then, based on the above, expression (6) takes the form of (8):

$$\begin{aligned} \Phi(z) &= \cos \left[ \frac{\pi}{H_{geom} + \Delta H_f} \left( z - \frac{H_{geom}}{2} \right) \right] = \\ &= \cos \left[ \frac{\pi}{H_f} \left( z - \frac{H_{geom}}{2} \right) \right]. \end{aligned} \quad (8)$$

Substituting intermediate expressions (3) to (8) into formula (1) and converting it to analytical form (9), we get an improved mathematical model for determining the damage parameter of the FE shell, taking into consideration the fractal length:

$$\begin{aligned} \omega(\tau) &= \frac{1}{A_0} \times \\ &\times \int_0^\tau \left\{ G_e \cdot K \cdot \cos \left[ \frac{\pi}{f} \left( z - \frac{H_{geom}}{2} \right) \right] \times \right. \\ &\left. \times (G_e + B \cdot e^{C\sigma_e}) \cdot \exp(-10,000 / R \cdot T) \cdot t^{-1/2} \right\} \cdot dr. \end{aligned} \quad (9)$$

To find the fractal FE length  $H_f$ , it is proposed to use the apparatus of fractal geometry in the current work, on the example of a physical model of the FE shell in the form of a geometric shape of a hollow cylinder.

Analytical expression (9) can be taken as conditions for the destruction of the FE shell and for determining the depressurization criterion. Thus, FE can be considered sealed if the following conditions of damage and destruction of the shell are met (10):

$$\omega(\tau) = \frac{A(\tau)}{A_0} \leq 1, \quad (10)$$

at  $H_f/H_{geom} \leq 1$ .

Hence, FE is leaky, subject to damage and destruction of the shell (11):

$$\omega(\tau) = \frac{A(\tau)}{A_0} > 1, \quad (11)$$

at  $H_f/H_{geom} > 1$ .

## 5. Results of studying the structure of the material of the damaged shell of a fuel element

### 5.1. Construction of a physical model of the shell of a fuel element in the form of a geometric shape of a hollow cylinder

The physical model of the FE shell, which is as close as possible to the research, which is a geometric shape in the form of a hollow cylinder (Fig. 2), is considered. At the same time, it is taken into consideration that in the physical model the fractality of the nanolayers of the five-layer structure of the FE shell material is characteristic of both the external and internal surfaces. In addition, all the geometric parameters of the hollow cylinder are taken into consideration when calculating the fractal dimensionalities of both the outer and inner surfaces of the FE shell.

To devise a methodology for calculating the values of fractal dimensionalities, the above geometric parameters of the physical model of the FE shell in the form of a geometric shape of a hollow cylinder were chosen, as shown in Fig. 2.

To study the physical model of the FE shell in the form of a geometric shape of a hollow cylinder, a skin effect was applied. The mechanism of occurrence of the skin effect in the proposed physical model in the form of a geometric shape of a hollow cylinder (Fig. 2) is as follows.

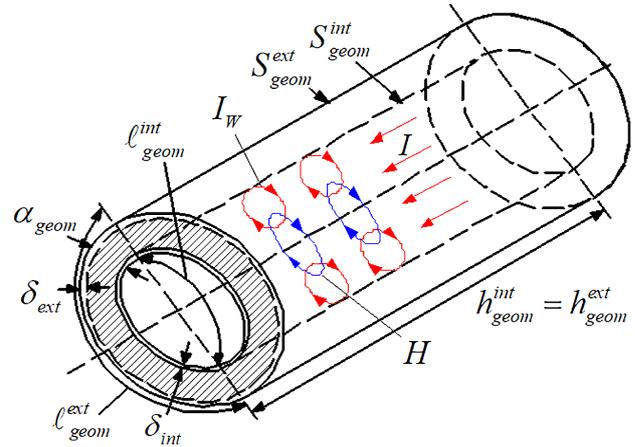


Fig. 2. Physical model of the shell of a fuel element in the form of a geometric shape of a hollow cylinder:  $h_{geom}^{int}$  – longitudinal geometric length of the outer surface of a hollow cylinder;  $h_{geom}^{ext}$  – longitudinal geometric length of the inner surface of a hollow cylinder;  $\rho_{geom}^{ext}$  – geometric circumference of the outer surface of the hollow cylinder;  $\rho_{geom}^{int}$  – geometric circumference of the inner surface of a hollow cylinder;  $\alpha_{geom}$  – geometric wall thickness of a hollow cylinder;  $S_{geom}^{ext}$  – geometric area of the outer surface of the hollow cylinder;  $S_{geom}^{int}$  – geometric area of the inner surface of the hollow cylinder;  $\delta_{ext}$  – outer depth (thickness) of the skin layer;  $\delta_{int}$  – inner depth (thickness) of the skin layer

The alternating current in the conductor generates an alternating vortex magnetic field, the lines of force of which are perpendicular to the axis of the conductor (Fig. 2). Due to electromagnetic induction, the alternating magnetic field  $H$  generates a vortex electric field that causes the flow of the Foucault vortex current. At the same time, on the surface of the conductor, the vortex current  $I_W$  is directed in the direction of the conductor's current, and inside the conductor – the opposite. This phenomenon reduces the current in the core of the conductor and increases the surface current. Consideration of the distribution of the current density in the hollow cylindrical conductor in the cross-section (Fig. 2) has showed that for alternating current, the current density decreases exponentially from the surface deep into the conductor.

The thickness of the skin layer  $\delta_{skin}$  is defined as the depth from the surface, at which the current density decreases to 37 % of the value on the surface, that is the thickness of the shell. This thickness depends on the frequency of the current and the electrical and magnetic properties of the conductor. The current density is maximum at the surface of the conductor (Fig. 2). When moving away from the surface, it decreases exponentially and at a depth of  $e/1$  becomes less by  $e$  times (by about 70 %), that is, this depth is the thickness of the skin layer [25]. It follows that the skin effect is a promising means of measuring the increments of the geometric parameters of the FE shell model in the form of a geometric shape of a hollow cylinder.

If we assume that an alternating electric potential with an amplitude  $U$  and a frequency of  $\omega$  is applied along the hollow cylinder, then the corresponding depth of the skin layer  $\delta_{skin}$ , according to [25], is determined from expression (12):

$$\delta_{skin} = \sqrt{\frac{2}{\mu \cdot \mu_0 \cdot \sigma \cdot \omega}} \cdot \frac{1}{\sqrt{\omega}}, \tag{12}$$

where  $\delta_{skin}$  is the depth of the skin layer;  $\mu$  is the magnetic permeability;  $\mu_0$  is the magnetic constant.

**5. 2. Studying the fractal structure of a physical model with a damaged shell of a fuel element**

According to the theory of fractals [22–24], for an arbitrary fractal outer and inner surface of a hollow cylinder, the following inequalities (13) and (14) must be met:

$$S_f^{ext} \neq h_{geom}^{ext} \cdot \ell_{geom}^{ext}, \tag{13}$$

$$S_f^{int} \neq h_{geom}^{int} \cdot \ell_{geom}^{int}. \tag{14}$$

Therefore, based on expressions (13) and (14), the following inequality (15) must be observed for the total fractal surface area of the hollow cylinder  $\Sigma S_{geom}$ :

$$\Sigma S_f \neq S_f^{ext} + S_f^{int} \neq (h_{geom}^{ext} \cdot \ell_{geom}^{ext}) + (h_{geom}^{int} \cdot \ell_{geom}^{int}). \tag{15}$$

According to the theory of fractals [22–24], the value of the length of a hollow cylinder, which differs from the geometric length by the amount of some geometric increment, is the fractal value of this length.

Consequently, we have fractal values of longitudinal ( $h_f^{ext}, h_f^{int}$ ) and transverse ( $\ell_f^{ext}, \ell_f^{int}$ ) lengths, which generate fractal dimensionalities ( $d_{h_f^{ext}}, d_{h_f^{int}}, d_{\ell_f^{ext}}, d_{\ell_f^{int}}, d_{S_f^{ext}}, d_{S_f^{int}}$ ) on the outer ( $S_f^{ext}$ ) and inner ( $S_f^{int}$ ) fractal surfaces of the hollow cylinder. Therefore, they can be specified by the expressions (16), (17):

$$d_{h_f^{ext}} = \frac{\ln(h_f^{ext} / \delta_{ext})}{\ln(1 / \delta_{ext})}, \tag{16}$$

$$d_{h_f^{int}} = \frac{\ln(h_f^{int} / \delta_{int})}{\ln(1 / \delta_{int})}, \tag{17}$$

$$h_f^{ext} = \frac{1}{\delta_{ext}^{d_{S_f^{ext}}-1}}, \tag{18}$$

$$h_f^{int} = \frac{1}{\delta_{int}^{d_{S_f^{int}}-1}}, \tag{19}$$

$$d_{\ell_f^{ext}} = \frac{\ln(\ell_f^{ext} / \delta_{ext})}{\ln(1 / \delta_{ext})}, \tag{20}$$

$$d_{\ell_f^{int}} = \frac{\ln(\ell_f^{int} / \delta_{int})}{\ln(1 / \delta_{int})}, \tag{21}$$

$$\ell_f^{ext} = \frac{1}{\delta_{ext}^{d_{S_f^{ext}}-1}}, \tag{22}$$

$$\ell_f^{int} = \frac{1}{\delta_{int}^{d_{S_f^{int}}-1}}, \tag{23}$$

$$d_{S_f^{ext}} = \frac{\ln(S_f^{ext} / \delta_{ext}^2)}{\ln(1 / \delta_{ext})}, \tag{24}$$

$$d_{S_f^{int}} = \frac{\ln(S_f^{int} / \delta_{int}^2)}{\ln(1 / \delta_{int})}, \tag{25}$$

$$S_f^{ext} = \frac{1}{\delta_{ext}^{d_{S_f^{ext}}-2}}, \tag{26}$$

$$S_f^{int} = \frac{1}{\delta_{int}^{d_{S_f^{int}}-2}}, \tag{27}$$

where  $\delta_{ext}$  is the depth of the skin layer on the outer surface of the hollow cylinder shell;  $\delta_{int}$  is the depth of the skin layer on the inner surface of the hollow cylinder shell (Fig. 2).

Substituting expressions (26) and (27) into expression (15), the analytical expression (28) was obtained to calculate the total fractal area of  $\Sigma S_f$  for the inner and outer surface of the physical model of the hollow cylinder:

$$\Sigma S_f = \frac{1}{\delta_{ext}^{d_{S_f^{ext}}-2}} + \frac{1}{\delta_{int}^{d_{S_f^{int}}-2}}. \tag{28}$$

If we abstract from the consideration of the nature of fractal dimensionalities on the outer and inner surfaces of the physical model of a hollow cylinder, then in the most general case the following inequalities hold:

1.  $1 \leq d_{h_f^{ext}}, d_{h_f^{int}}, d_{\ell_f^{ext}}, d_{\ell_f^{int}} < 2$  (equality is valid for smooth differentiable curves);
2.  $2 \leq d_{S_f^{ext}}, d_{S_f^{int}} < 3$  (equality is valid for smooth differentiable surfaces).

As is known [25], the energy dissipated by the material of the hollow cylinder shell material is determined from expression (29):

$$P_\Sigma = P_{ext} + P_{int}, \tag{29}$$

where  $P_\Sigma$  is the energy dissipated by the material of the hollow cylinder shell;  $P_{ext}$  – the energy dissipated by the outer

surface of the hollow cylinder; it is determined from expression (30);  $P_{int}$  is the energy dissipated by the inner surface of a hollow cylinder; it is defined by expression (31):

$$P_{ext} = \int j_{ext}(r_{ext}) \cdot E_{ext}(r_{ext}) \cdot d_{ext}^3 \cdot r_{ext}, \quad (30)$$

$$P_{int} = \int j_{int}(r_{int}) \cdot E_{int}(r_{int}) \cdot d_{int}^3 \cdot r_{int}, \quad (31)$$

where  $j_{ext}(r_{ext})$  is the current density on the outer surface of the sample;  $j_{int}(r_{int})$  is the current density on the inner surface of the sample;  $E_{ext}(r_{ext})$  is an electric field at the point  $r_{int}$ , on the outer surface of the sample;  $E_{int}(r_{int})$  is the electric field at the point  $r_{ext}$ , on the inner surface of the sample;  $r_{ext}$  is the outer radius of the shell of the hollow cylinder sample;  $r_{int}$  is the inner radius of the shell of the hollow cylinder sample.

According to Ohm's law, the current density on the surface of a conductor is defined as  $j(r) = \sigma \cdot E(r)$ , where  $\sigma$  is electrical conductivity, which makes it possible to write relations (30) and (31) in the form of expressions (32) and (33):

$$\begin{aligned} P_{ext} &= \sigma_{ext} \int E_{ext}^2 \cdot d_{ext}^3 \cdot r_{ext} = \sigma_{ext} \cdot \langle E_{ext}^2 \rangle_{skin} \int d_{ext}^3 \cdot r_{ext} = \\ &= \sigma_{ext} \cdot \langle E_{ext}^2 \rangle_{skin} \cdot S_{ext} \cdot \delta_{ext}; \end{aligned} \quad (32)$$

$$\begin{aligned} P_{int} &= \sigma_{int} \int E_{int}^2 \cdot d_{int}^3 \cdot r_{int} = \sigma_{int} \cdot \langle E_{int}^2 \rangle_{skin} \int d_{int}^3 \cdot r_{int} = \\ &= \sigma_{int} \cdot \langle E_{int}^2 \rangle_{skin} \cdot S_{int} \cdot \delta_{int}, \end{aligned} \quad (33)$$

where «skin» means the volume:

$$V \equiv \int_{skin} d^3 \cdot r = S \cdot \delta$$

of a layer of matter in which the electric field is substantially different from zero since the electric field decreases exponentially as it moves deeper into the material;  $\sigma_{ext}$  – electrical conductivity on the outer surface of the conductor;  $\sigma_{int}$  – electrical conductivity on the inner surface of the conductor.

Then, the total value of the energy dissipated by the substance of the hollow cylinder shell material  $P_{\Sigma}$  is determined from expression (34):

$$P_{\Sigma} = \sigma_{ext} \cdot \langle E_{ext}^2 \rangle_{skin} \cdot S_{ext} \cdot \delta_{ext} + \sigma_{int} \cdot \langle E_{int}^2 \rangle_{skin} \cdot S_{int} \cdot \delta_{int}. \quad (34)$$

If we reproduce the experiment with a fixed density of the electromagnetic field, then, from ratio (34), expressions (35) and (36) for the values of  $P_{ext}$  and  $P_{int}$  are obtained:

$$P_{ext} \approx S_{ext} \delta_{ext} = \delta_{ext}^{3-d_{S_{ext}}}, \quad (35)$$

$$P_{int} \approx S_{int} \delta_{int} = \delta_{int}^{3-d_{S_{int}}}. \quad (36)$$

Note that equality in expressions (35) and (36) follows from ratios (24) and (25). Using ratio (12), we transformed (35), (36) to the form of (37), (38):

$$P_{ext} = \frac{1}{\omega^{\frac{3-d_{S_{ext}}}{2}}}, \quad (37)$$

$$P_{int} = \frac{1}{\omega^{\frac{3-d_{S_{int}}}{2}}}. \quad (38)$$

At  $d_{S_{ext}} \equiv d_{S_{int}} \equiv 2$ , these ratios reproduce the standard value of the total energy  $P_{\Sigma} = 1/\sqrt{\omega}$ , which is dissipated by the material of the hollow cylinder shell material.

It should be noted that, from ratios (35) and (36), dependences for the external  $V_{ext} = \delta_{ext}^{3-d_{S_{ext}}}$  and internal  $V_{int} = \delta_{int}^{3-d_{S_{int}}}$  volume follow, which, when the characteristic length changes, reproduce (24) and (25).

After performing an experiment with the passage of an electric current, the following expressions (39) and (40) were obtained for the values  $E_{ext}^2$  and  $E_{int}^2$ :

$$\langle E_{ext}^2 \rangle_{skin} = V_{ext}^2 / (h_f^{ext})^2, \quad (39)$$

$$\langle E_{int}^2 \rangle_{skin} = V_{int}^2 / (h_f^{int})^2. \quad (40)$$

Then, taking into consideration ratios (32) and (33), we have:

$$P_{ext} = \frac{\sigma_{ext} \cdot V_{ext}^2 \cdot S_{ext} \cdot \delta_{ext}}{(h_f^{ext})^2}, \quad (41)$$

$$P_{int} = \frac{\sigma_{int} \cdot V_{int}^2 \cdot S_{int} \cdot \delta_{int}}{(h_f^{int})^2}. \quad (42)$$

Comparing ratios (41) and (42) with the formula  $P = V^2/R$  (where  $R$  is the electrical resistance), we derive expressions (43) and (44):

$$R_{ext} = \frac{h_f^{ext^2}}{\sigma_{ext} \cdot S_{ext} \cdot \delta_{ext}}, \quad (43)$$

$$R_{int} = \frac{h_f^{int^2}}{\sigma_{int} \cdot S_{int} \cdot \delta_{int}}. \quad (44)$$

From ratios (18) to (27), we find the value of the external  $R_{ext}$  (45) and the internal  $R_{int}$  electrical resistance (46), in accordance with expressions (43) and (44):

$$R_{ext} \sim \frac{\delta_{ext}^{d_{S_{ext}}-2d_{h_f^{ext}}-1}}{\sigma_{ext}}, \quad (45)$$

$$R_{int} \sim \frac{\delta_{int}^{d_{S_{int}}-2d_{h_f^{int}}-1}}{\sigma_{int}}, \quad (46)$$

or, taking into consideration ratio (12), we obtain the value of the external  $R_{ext}$  (47) and the internal  $R_{int}$  (48) electrical resistance depending on the applied frequency  $\omega$ , in accordance with expressions (45) and (46):

$$R_{ext} \approx \omega^{\frac{1+2d_{h_f^{ext}}-d_{S_{ext}}}{2}}, \quad (47)$$

$$R_{int} \approx \omega^{\frac{1+2d_{h_f^{int}}-d_{S_{int}}}{2}}. \quad (48)$$

At  $d_{S_{ext}} = d_{S_{int}} = 2$  and  $d_{h_f^{ext}} = d_{h_f^{int}} = 1$  this ratio becomes standard  $R \approx \sqrt{\omega}$ .

Summarizing and applying experimental data to plots in coordinates  $(\ln P_{ext}, \ln \omega; \ln P_{int}, \ln \omega)$  and  $(\ln R_{ext}, \ln \omega; \ln R_{int}, \ln \omega)$ , we obtain straight lines with angular coefficients:  $(d_S-3)/2$  and  $(1+2d_h-d_S)$ . This makes it possible to directly measure  $d_S$  and  $d_h$  (that is  $d_{S_{ext}}, d_{S_{int}}, d_{h_f^{ext}}, d_{h_f^{int}}$ ).

In the mentioned particular case, namely, at  $d_{S_f} = d_{h_f} + d_{c_f}$ , from relations (37), (38), (45), (46), we obtain expressions (49) to (52):

$$P_{ext} \approx \frac{1}{\omega^{\frac{3-d_{h_f^{ext}}-d_{c_f^{ext}}}{2}}}, \tag{49}$$

$$P_{int} \approx \frac{1}{\omega^{\frac{3-d_{h_f^{int}}-d_{c_f^{int}}}{2}}}, \tag{50}$$

$$R_{ext} \approx \omega^{\frac{1+d_{h_f^{ext}}-d_{c_f^{ext}}}{2}}, \tag{51}$$

$$R_{int} \approx \omega^{\frac{1+d_{h_f^{int}}-d_{c_f^{int}}}{2}}. \tag{52}$$

In another particular case, namely when  $d_{S_f} \neq d_{h_f} + d_{c_f}$  from relations (37), (38), (45), (46), the value of the quantities  $P_{ext}$ ,  $P_{int}$ ,  $R_{ext}$ ,  $R_{int}$  are determined by expressions (53) to (56), respectively:

$$P_{ext} \approx \frac{1}{\omega^{\frac{2-d_{h_f^{ext}}}{2}}}, \tag{53}$$

$$P_{int} \approx \frac{1}{\omega^{\frac{2-d_{h_f^{int}}}{2}}}, \tag{54}$$

$$R_{ext} \approx \omega^{\frac{d_{h_f^{ext}}}{2}}, \tag{55}$$

$$R_{int} \approx \omega^{\frac{d_{h_f^{int}}}{2}}. \tag{56}$$

In this case, a single experiment, e.g., measurement  $R_{\Sigma} = R_{ext} + R_{int}$ , as a function of the frequency  $\omega$ , makes it possible to determine the fractal dimensionalities of the lengths on the outer  $d_{h_f^{ext}}$  and  $d_{h_f^{int}}$  inner surfaces of the hollow cylinder.

**5.3. Improving the shell damage model taking into consideration fractal increments of geometric parameters of the fuel element**

On the basis of the above justifications, analytical expressions were proposed that determine an improved model of damage to the FE shell, based on taking into consideration the fractal properties of the structure of the FE shell material.

Substituting the value of the fractal length value from expression (18) to expression (9), we obtain analytical expression (57) for an improved model of damage to the outer surface of the FE shell based on the use of a fractal geometry apparatus:

$$\omega_{ext}(\tau) = \frac{1}{\omega} \cdot \int_0^{\tau} \left\{ \begin{aligned} & G_e \cdot K \times \\ & \times \cos \left[ \frac{\pi}{\delta_{ext}^{\frac{d_{h_f^{ext}}-1}{2}}} \left( z - \frac{H_{geom}}{2} \right) \right] \times \\ & \times (G_e + B \cdot e^{C\sigma_e}) \times \\ & \times \exp(-10,000 / R \cdot T) \cdot t^{-1/2} \end{aligned} \right\} \cdot dr. \tag{57}$$

From expression (57), it follows that the damage to the inner surface of the shell depends on the change in the fractal

dimensionality  $d_{h_f^{ext}}$  and the depth of the skin layer  $\delta_{ext}$  on the inner surface of the FE shell.

Similarly, substituting the value of the fractal length from expression (19) into expression (9), we obtain the analytical expression (58) for determining the damage to the inner surface of the fuel element shell:

$$\omega_{int}(\tau) = \frac{1}{\omega} \cdot \int_0^{\tau} \left\{ \begin{aligned} & G_e \cdot K \times \\ & \times \cos \left[ \frac{\pi}{\delta_{int}^{\frac{d_{h_f^{int}}-1}{2}}} \left( z - \frac{H_{geom}}{2} \right) \right] \times \\ & \times (G_e + B \cdot e^{C\sigma_e}) \times \\ & \times \exp(-10,000 / R \cdot T) \cdot t^{-1/2} \end{aligned} \right\} \cdot dr. \tag{58}$$

From expression (58), it follows that the damage to the inner surface of the shell depends on the change in the fractal dimensionality  $d_{h_f^{int}}$  and the depth of the skin layer  $\delta_{int}$  on the inner surface of the FE shell.

The fractal dimensionalities  $d_{h_f^{int}}$  and  $d_{h_f^{ext}}$  are determined from expressions (57) and (58) by building plots (Fig. 3).

**5.4. Results of experimental studies of the fractal structure of the material of the damaged shell of the fuel element**

In experiments with the passage of electric current and a fixed density of electromagnetic radiation in the cavity, the spatial distribution of the electromagnetic current evolves with a frequency of  $\omega$ . It is this evolution that allows the field to fix the fractal dimensionality of the structure of the material of the damaged shell of the fuel element.

To this end, the process of spatial distribution of the electromagnetic field outside and inside the structure of the material of the outer  $S_f^{ext}$  and inner  $S_f^{int}$  surfaces of the hollow cylinder shell was considered. It is shown that in an experiment with a fixed density of electromagnetic radiation, it is characterized by a certain specificity in the distribution of the values  $P_{\Sigma}$ ,  $R_{\Sigma}$ , when an electric current passes through in a skin layer with a certain frequency  $\omega$ .

At the same time, the scattering of energy on the surface of the nanoparticles (depending on the frequency  $\omega$  is accompanied by the effects of dissipation into the external environment; the absorption  $P_{\Sigma}$  in the cavity of the structure (resonant absorption of energy). It should be noted that the nature of the distribution of the total electrical resistance  $R_{\Sigma}$ , for the fractal structure in the skin layer, also depends on the applied frequency  $\omega$  [25]. Fig. 3 shows plots of the dependence of cylinder impedance  $R_{\Sigma} = R_{ext} + R_{int}$  on the frequency  $\omega$  for different values of fractal dimensionality  $d_{h_f}$  (Table 1) of the cylinder surface in double logarithmic scale are shown.

It is assumed that the outer and inner surfaces have the same fractal dimensionality  $d_{h_f} / 2$ . Straight line 1 in the plot in Fig. 3 corresponds to a smooth geometric surface with fractal dimensionality  $d_{h_f} = 1$  and describes the classical dependence  $R \sim \sqrt{\omega}$ . With an increase in the fractal dimensionality  $d_{h_f}$ , the angle of inclination of straight lines 2 to 6 in the plot in Fig. 3 increases.

Fig. 4 shows plots of the dependence of the full power allocated on the surfaces of the hollow cylinder  $P_{\Sigma}$  on the frequency  $\omega$ , for different values of the fractal dimensionality  $d_{S_f}$  of the surface area of the cylinder in a double logarithmic scale. It is assumed that the outer and inner surfaces

have the same fractal dimensionality of the area equal to  $d_{s_f} = (d_{s_{ext}} + d_{s_{int}}) / 2$ . The angular coefficient of the lines, according to (41) and (42), is equal to  $(d_{s_f} - 3) / 2$ . Straight line 1 on the plot in Fig. 4 corresponds to a smooth geometric surface with dimensionality  $d_{s_f} = 2$  and describes the classical dependence  $P \sim 1/\sqrt{\omega}$ . With an increase in fractal dimensionality, the angle of inclination of straight lines 2 to 6, as shown on the plot in Fig. 4, decreases.

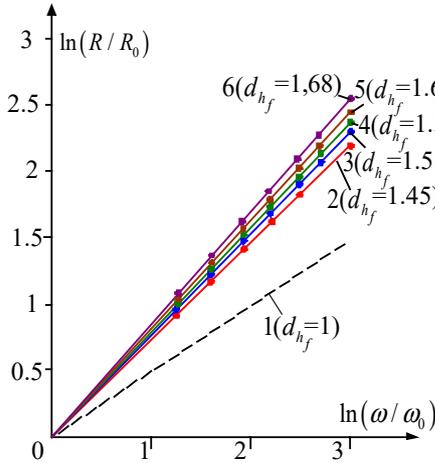


Fig. 3. Plots of fractal dimensionality change over the surface area of a hollow cylinder as a function of impedance ( $R_{\Sigma} = R_{ext} + R_{int}$ ) and frequency  $\omega$

Thus, the plots, in Fig. 3, 4, built on the basis of experimental data (Tables 1, 2), demonstrate the dynamics of the

shell damage process along the entire length of the fuel element, on the measured axial segments ( $z_1 \div z_5$ ).

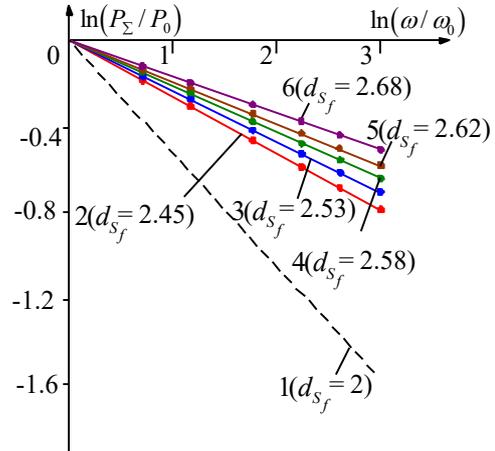


Fig. 4. Plots of the change in fractal dimensionality over the surface area of the hollow cylinder as a function of the total power allocated ( $P_{\Sigma} = P_{ext} + P_{int}$ ) and the frequency  $\omega$

It is obtained that at the maximum elongation of the fuel element, there is a change in the value of the fractal dimensionality:  $d_{h_f}$  from 1.45 to 1.68;  $d_{s_f}$  from 2.45 to 2.68. As can be seen from Fig. 5, the maximum damage to the structure of the FE shell material is observed in the active segment  $z_5$  according to the FE height  $h_f$  from 2.1 to 2.6 m. At the same time, there is a change in the value of the fractal dimensionality:  $d_{h_f}$  from 1.62 to 1.68;  $d_{s_f}$  from 2.62 to 2.68.

Table 1

Results of measurements of fractal dimensionality on axial segments of the shell surface according to the height of the fuel element, depending on the applied frequency and electrical resistance (initial frequency:  $\nu_0 = 0.5$  GHz)

Experimental parameter		Axial segment z									
		1		2		3		4		5	
$\frac{\omega}{\omega_0}$	$\ln \frac{\omega}{\omega_0}$	$\frac{R}{R_0}$	$\ln \frac{R}{R_0}$								
1	0	1	0	1	0	1	0	1	0	1	0
3	1.10	2.22	0.80	2.32	0.84	2.38	0.87	2.44	0.89	2.51	0.92
5	1.61	3.22	1.17	3.42	1.23	3.56	1.27	3.67	1.30	3.86	1.35
7	1.95	4.10	1.41	4.44	1.49	4.66	1.54	4.85	1.58	5.16	1.64
10	2.30	5.31	1.67	5.81	1.76	6.17	1.82	6.42	1.86	6.89	1.93
15	2.71	7.09	1.96	7.92	2.07	8.50	2.14	9.03	2.20	9.78	2.28
20	3.00	8.84	2.18	9.97	2.30	10.70	2.37	11.36	2.43	12.43	2.52
Fractal dimensionality		$d_{h_f} = 1.45$		$d_{h_f} = 1.53$		$d_{h_f} = 1.58$		$d_{h_f} = 1.62$		$d_{h_f} = 1.68$	

Table 2

Results of fractal dimensionality measurements on axial segments of the shell surface according to the height of the heat-generating element, depending on the applied frequency and power (initial frequency:  $\nu_0 = 0.5$  GHz)

Experimental parameter		Axial segment z									
		1		2		3		4		5	
$\frac{\omega}{\omega_0}$	$\ln \frac{\omega}{\omega_0}$	$\frac{P}{P_0}$	$\ln \frac{P}{P_0}$								
1	0.00	1.00	-0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
3	-1.10	0.73	-0.30	0.77	-0.25	0.79	-0.23	0.81	-0.21	0.48	-0.17
5	-1.61	0.64	-0.44	0.68	-0.37	0.71	-0.33	0.73	-0.31	0.34	-0.25
7	-1.95	0.58	-0.53	0.63	-0.45	0.66	-0.41	0.69	-0.37	0.27	-0.31
10	-2.30	0.53	-0.63	0.58	-0.54	0.61	-0.48	0.64	-0.43	0.21	0.36
15	-2.71	0.47	-0.74	0.52	-0.63	0.56	-0.56	0.59	-0.51	0.16	0.43
20	-3.00	0.43	-0.82	0.49	-0.70	0.53	-0.63	0.56	-0.57	0.13	0.48
Fractal dimensionality		$d_{s_f} = 2.45$		$d_{s_f} = 2.53$		$d_{s_f} = 2.58$		$d_{s_f} = 2.62$		$d_{s_f} = 2.68$	

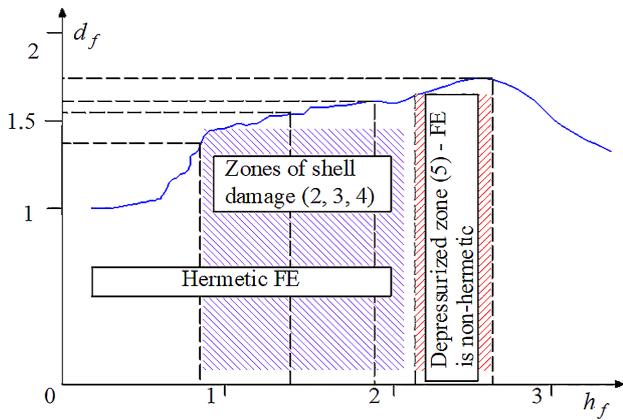


Fig. 5. Change in the value of the fractal dimensionality along the length (height) of the fuel element

Based on our results, as an assessment of the criterion for the degree of depressurization of the studied FE, when controlling the damage to the shell, the value of the fractal dimensionality of the real geometric parameters of the FE is selected, while the following condition must be met:

- at  $d_{h_j} < 1.45 \dots 1.62$ ;  $d_{s_j} < 2.45 \dots 2.62$  – the FE shell is damaged but hermetic;
- at  $d_{h_j} > 1.62$ ;  $d_{s_j} > 2.62$  – the FE shell is depressurized.

**6. Discussion of the results of the development of an improved model of damage to the shell of the fuel element**

A physical model of the FE shell in the form of a geometric shape of a hollow cylinder (Fig. 2) has been built, which allows simulation modeling of structural changes on the external (internal) surface of the damaged shell that affect the geometry of the FE. Based on the physical model of the FE shell, which has fractal properties, a fractal assessment of the outer and inner surface of the shell material was carried out using the skin effect.

Our study into the fractal structure of the physical model of the damaged shell, using the apparatus of fractal geometry, showed that the change in the geometric parameters of FE depends on structural-phase changes in the shell material under the influence of damaging defects. To quantify the state of the structure of the FE shell, it is proposed to use the value of the fractal dimensionality of the area of the outer and inner surface of the shell along the entire length of the FE, in accordance with expressions (16), (17), (24), (25). For the direct measurement of the value of the fractal dimensionality, it was proposed to use on the basis of the application of the theory of the skin effect of the dependence of electrical resistance (power) on the applied frequency in expressions (53) to (56). Plots (Fig. 3, 4) were constructed in the coordinates ( $\ln P, \ln \omega$ ;  $\ln R, \ln \omega$ ) and straight lines with angular coefficients were obtained:  $(d_s - 3)/2$  and  $(1 + 2d_h - d_s)$ , which made it possible to directly measure  $d_s$  and  $d_h$ .

Expressions (57), (58) demonstrate that the improvement of the shell damage model was carried out on the basis of taking into consideration the fractal increments of the geometric parameters of FE derived from formulas (18), (19), (26), (27), which take into consideration structural-phase changes at different depths of the skin layer. A special feature of the constructed model of damage to the FE shell is that it makes it possible to comprehensively determine the degree of

damage to the shell, depending on the temperature, pressure, neutron flux rate, radiation exposure when changing the geometry of the fuel element.

The processing of experimental data (Tables 1, 2) on the study of the fractal structure of the damaged shell showed that the increments of the geometric parameters (area, height) of the FE obey a degree dependence on the value of the fractal dimensionality. It was experimentally established that at the maximum elongation of the fuel element there was a change in the value of the fractal dimensionality from 1.45 to 2.68. At the same time, as can be seen from the plot in Fig. 5, the dynamics of achieving the maximum damage to the structure of the FE shell material in the active segment  $z_5$  in terms of the height of the fuel element from 2.1 to 2.6 m were observed.

When implementing the results of this study in practice, it is necessary to take into consideration the characteristics of the specific material (for example, steel, aluminum, zirconium) from which the FE shell is made, for which different operating limits (for example, temperature, pressure) are established.

As a disadvantage, it should be noted that the data on the geometry of the micro- and macrostructure in the volume of the FE shell material require the performance of complex labor-intensive computational operations. Therefore, with further research, to determine the value of the fractal dimensionality in the structure of the FE shell, we proposed using a special computing module of the TMS system for interfacing with the APCS software of the NPP power unit.

As the prospects of this study, a method for determining an integrated criterion for assessing the degree of tightness of the shell, taking into consideration the change in the volume of the pore space of the structure (filled with helium), for TMS systems on a working nuclear reactor, is considered.

**7. Conclusions**

1. A physical model of the FE shell in the form of a geometric shape of a hollow cylinder was developed, which made it possible to bring the study of the structural state and fractal properties of the shell material as close as possible when exposed to surface damaging defects. The model has made it possible to use the mechanism of the skin effect to study changes in the thickness (depth) of the FE shell with a different dependence of electrical resistance (power) on the applied frequency.

2. When studying the fractal structure of the physical model of the shell, the dependence of changes in geometric parameters on the structural-phase state in the volume of the material, in the presence of damaging defects on the outer and inner surface of the FE shell, was established. The value of the fractal dimensionality was determined and found as a quantitative measure linking the change in the geometry of FE with the fractal properties of the structure of the damaged shell. Based on the application of the skin effect theory, analytical expressions were derived to determine the value of the fractal dimensionality of the area of the damaged surface along the entire length of the fuel element. It is shown that in the studied fractal structure of the physical model of the FE shell, the real external and internal area of the damaged surface and the height of the FE shell obeys a degree dependence on the value of the fractal dimensionality. It is found that the power dependence is a constant value for

no-fractal (smooth) structures of the FE shell material and is determined by computational experiment. For fractal structures, the power dependence changes with an increase in the number of local inhomogeneities, micropores, and macrocracks formed in the structure of the shell material as a result of exposure to damaging defects and is determined during a full-scale experiment.

3. A mathematical model of damage to the FE shell has been improved, which, when calculating, takes into consideration the distribution of damage on the outer and inner surface of the shell according to the height of the FE, based on the application of the fractal geometry method. A deformation criterion has been developed for calculating the damage to the shell, taking into consideration structural-phase changes in the fractal structure of the shell material, by calculating the fractal dimensionality on the axial segments according to the height of the FE. A feature of our model is determining the location, type, and size of the damaging defect in the axial sections (segments) of the surface by the height of the fuel element, which makes it possible to determine the leaky FE without removing the nuclear reactor from FEB.

The model can be used to track the process of damage to the shell, by controlling the dynamics of the development of micro and macrocracks and their escalation into a through

crack ( $>50 \mu\text{m}$ ), which leads to depressurization of the fuel element.

4. The choice of practical use of the fractal dimensionality parameter as a criterion for assessing the tightness (depressurization) of the FE shell has been experimentally substantiated. As a result of the experiment, it was found that when the value of the fractal dimensionality changes in the height of the FE from 1.45 to 1.62 and on the surface area from 2.45 to 2.62, the FE shell is damaged but sealed. With the values of the fractal dimensionality greater than 1.62 and 2.62, respectively, for the height and surface area, the FE is depressurized.

The adequacy of the obtained improved model of shell damage was confirmed, based on an experiment on a physical model of the shell using the skin effect, which confirmed, by appropriate calculations, the results of our theoretical studies.

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#### Conflict of interest

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The authors declare that they have no conflict of interest in relation to this research, whether financial, personal, authorship or otherwise, that could affect the research and its results presented in this paper.

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