

*The object of the research is a measuring current transformer of the electromagnetic type, which is used as part of the electricity metering unit. The current transformers functioning in the mode of reduced primary current is accompanied by significant errors. The existence of such a regime for a long time due to downtime of production equipment leads to a significant underestimation of electricity. This leads to unjustified financial losses for energy supply companies as electricity in many countries has become more expensive. The static characteristic of the measuring current transformer at a reduced load of the metering unit is described by a linear statistical model. The parameters of the model are estimated on the basis of empirical data using methods of covariance analysis. The adequacy of the model is confirmed by analysis of regression residuals. The obtained statistical model of the static characteristic, unlike the known ones, is characterized by universality, as it describes current transformers with an arbitrary transformation ratio within the studied accuracy class of 0.5S. The uncertainty of the current error is estimated using the prediction intervals for the transformers secondary currents as a function of the primary currents, calculated on the basis of the obtained model. The prediction intervals for sample values of the current error were obtained through the application of the proposed approach for uncertainty estimation of such an error at a reduced load of the metering unit. It was found that the current error can reach -20 %. Taking into account the indicated intervals for measuring current transformers at a reduced load of the metering unit during billing will allow energy supply companies to increase the accuracy of commercial electricity metering*

**Keywords:** *current transformer, static characteristic, current error, reduced load, regression line*

# DETERMINING THE STATIC CHARACTERISTIC OF A MEASURING CURRENT TRANSFORMER AT A REDUCED LOAD OF THE METERING UNIT

**Kateryna Vasylets**  
Postgraduate Student\*

**Volodymyr Kvasnikov**

Doctor of Technical Sciences, Professor,  
Head of Department, Merit Metrologist of Ukraine\*

**Sviatoslav Vasylets**

Corresponding author

Doctor of Technical Science, Professor  
Department of Automation, Electrical Engineering and  
Computer-Integrated Technologies  
National University of Water and  
Environmental Engineering  
Soborna str., 11, Rivne, Ukraine, 33028  
E-mail: svyat.vasilets@gmail.com

\*Department of Computerized  
Electrical Systems and Technologies  
National Aviation University

Liubomyra Huzara ave., 1, Kyiv, Ukraine, 03058

Received date 15.07.2022

Accepted date 27.09.2022

Published date 31.10.2022

**How to Cite:** Vasylets, K., Kvasnikov, V., Vasylets, S. (2022). Determining the static characteristic of a measuring current transformer at a reduced load of the metering unit. *Eastern-European Journal of Enterprise Technologies*, 5 (8 (119)), 13–20. doi: <https://doi.org/10.15587/1729-4061.2022.265068>

## 1. Introduction

The global market of measuring current transformers for electricity meters is forecasted at USD 295.9 million in 2022 and will grow by 2028 [1]. As part of commercial electricity metering units, such devices perform two main functions. The protective function consists in the galvanic separation of primary power circuits from secondary low-voltage circuits. The measurement function provides current scaling, which allows universal electricity meters operation in grids with different load levels. Permissible measurement errors, characterized by the accuracy class of the current transformer, are provided only in the normalized range of primary currents [2]. The rated primary current of such a transformer is recommended to be chosen 10–40 % higher than the calculated operating current [3]. However, the actual primary current may be significantly less than the calculated value, especially during planned or emergency shutdowns of the main equipment, at night, etc. This increases the errors of

measuring current transformers above the standard values, which reduces the accuracy of metering and can lead to unjustified financial costs for both suppliers and consumers of electricity.

The meter readings and the measuring transformers ratios are taken into account when calculating the electricity consumed for a certain period of time. At the same time, the latter values are considered constant, i.e. the linearity of the static characteristic of each of the measuring current transformers is assumed. However, in the reduced load mode, when the consumer current is a few percent of the rated primary current of the metering unit, this assumption leads to unacceptable errors. Energy, underestimated for such reasons, refers to non-technological losses. Considering the electricity cost increase in the EU (in the first quarter of 2022 in Spain and Portugal – by 411 %, in Greece – by 343 %, in France – by 336 % [4]), such losses can cause significant economic harm. The listed factors determine the expediency of research aimed at clarifying the static characteristics of

the measuring current transformer during the functioning of the metering unit in reduced load mode.

Thus, the need to increase the profitability of energy supply companies by clarifying bills for consumed electricity determines the relevance of the disclosed scientific issue.

## 2. Literature review and problem statement

The significant influence of the current error of measuring current transformers on the accuracy of electricity metering by digital meters was confirmed in [5]. Such errors arise due to the distortion of the current transformer static characteristic  $I_s(I_p)$  under non-standard operating conditions, which are caused by the following factors. Many studies point to higher harmonics in the current curve as a significant factor, which can cause errors of up to 3.4 % [6]. However, there is an opposite opinion. In particular, in [7], by reproducing the real current curves through the transformer and estimating the total error, it was found that the non-sinusoidal current has an insignificant effect on the measurement accuracy (the error did not exceed the allowable one for the accuracy class). Temperature and magnetic properties of the core are also indicated as essential factors [8]. The electricity metering errors of more than 9 % occur when the actual current is double the rated primary current of the measuring transformers as found in [9].

The accuracy of the measurement, especially in the area of low primary currents (up to 10 % of the rated value) of the measuring transformer, is significantly affected by the core material. In particular, the current error is from -3 % to -5.5 % when using traditional cold rolled steel. The utilization of nanocrystalline materials or their combination with steel reduces the current error to -1 % on average [10]. In order to minimize the error during the operation of the measuring current transformer in non-standard conditions, it is proposed to produce the current transformer using the multiple winding technique [11].

It was found in [12] that the static conversion function for the measuring channel of the metering unit at reduced load is determined, among other factors, by the static characteristics of the measuring current transformers. However, the specified characteristics were not determined, only the transformation ratios were taken into account when processing the experimental data, which somewhat reduced the accuracy of the obtained results.

Several error compensation systems of measuring current transformers are known, which differ in the ways of control signal generation. For this, in particular, it is proposed in [13] to use an artificial neural network, which was trained in different operating modes of the current transformer. There is also a proposal to use correction factors [14]. In addition, the control signal can provide compensation for the constant component of the measured current [15].

The proposed technical solutions for compensating the error that occurs during the operation of measuring current transformers are focused on transformer operation in the primary overcurrent mode. Existing solutions cannot provide complete compensation for static characteristic distortion in the mode of reduced primary current. This makes it impossible to solve the problem of maintaining the necessary accuracy of commercial electricity metering at a reduced level of power load. All this allows us to state that it is reasonable to clarify the static characteristic of the measuring current transformer at a reduced load of the metering unit as

a theoretical basis for the functioning of current error compensation systems in the specified mode.

## 3. The aim and objectives of the study

The aim of the study is to increase the accuracy of commercial electricity metering in case of reduced load of the metering unit by determining the static characteristic of measuring current transformers. This will enable energy supply organizations to increase the accuracy of billing for consumed electricity.

To achieve the aim, the following objectives were set:

- to determine the influence of the measuring current transformer transformation ratio within the specified accuracy class on the character of the dependence of the secondary current on the primary current in the reduced load mode of the metering unit;
- to estimate the parameters of the statistical model that links the primary and secondary currents of measuring transformers in the area of reduced loads of the metering unit;
- to specify the uncertainty of current measurement by a transformer of electromagnetic type at a reduced primary current.

## 4. Materials and methods

The object of the study is a measuring current transformer of the electromagnetic type as part of the electricity metering unit. The primary windings (terminals P<sub>1</sub>, P<sub>2</sub>) of the measuring current transformers are connected in the break of the phase wires, and the secondary windings (S<sub>1</sub>, S<sub>2</sub>) – to the current circuits of the electricity meter P11, Fig. 1.

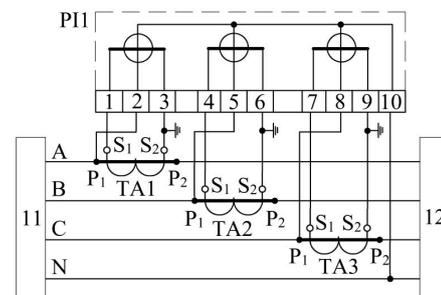


Fig. 1. Circuit diagram of the measuring current transformers TA1–TA3, as part of the electricity metering unit, connection: 1–10 – electricity meter terminals; 11 – 380 V electrical grid; 12 – load

From the point of view of measurement accuracy, the current transformer is characterized by the following parameters [2]: ratio error (current error), phase displacement, composite error, turns ratio error, etc. In particular, the current error  $\epsilon$  is determined as follows:

$$\epsilon = \frac{k_r \cdot I_s - I_p}{I_p} \cdot 100\%, \tag{1}$$

where  $I_p, I_s$  – the actual root mean square values, respectively, of the primary and secondary winding currents;  $k_r = I_{pr}/I_{sr}$  – rated transformation ratio;  $I_{pr}, I_{sr}$  – rated primary and secondary, respectively, current of the measuring transformer.

The largest current error  $\epsilon$  in the range of rated primary current values and rated loads of the measuring transformer secondary circuits determines the accuracy class of the latter. For measuring transformers of accuracy classes 0.1, 0.2, 0.5 and 1.0, the current error is normalized starting from the primary current relative value of 5 %, at which it should be, respectively: 0.4 %, 0.75 %, 1.5 %, 3.0 % [16]. For accuracy classes 0.2S and 0.5S, the current error is normalized from  $I_p=1$  %, and at this current  $\epsilon$  is 0.75 % and 1.5 %, respectively.

The subject of the study is the static characteristic of the measuring current transformer, which establishes the relationship between the root mean square values of the secondary and primary currents, at a reduced load of the metering unit.

The static characteristic for an ideal current transformer is defined as:

$$I_s(I_p) = I_p / k. \quad (2)$$

Considering expression (2) in dependence (1), it can be found that for an ideal current transformer, the current error is zero:  $\epsilon=0$ . For real current transformers with a primary current from the normalized range, the use of dependence (2) leads to measurement errors, according to (1), which are acceptable for a given accuracy class.

The main hypothesis of the study consists in the possibility of representing the static characteristic of the measuring current transformer at a reduced load of the metering unit by the following statistical model:

$$\hat{I}_s^*(I_p^*) = \hat{\mu} + \tau + \hat{\beta} \cdot (I_p^* - \bar{I}_p^*), \quad (3)$$

where  $\hat{I}_s^*$  – statistical estimate of the secondary current relative value of the current transformer;  $I_p^*$  – primary current relative value of the current transformer;  $\bar{I}_p^*$  – average relative value of the primary current in the reduced load mode of the metering unit;  $\hat{\mu}$  – estimate of the secondary currents average value of the measuring transformer;  $\tau$  – parameter corresponding to the influence of the measuring transformer ratio;  $\hat{\beta}$  – estimate of the linear regression coefficient.

At the same time, it is assumed that the parameter estimates of model (3) correspond to current transformers with different transformation ratios within the same accuracy class. The possibility to compare measuring devices with different transformation ratios is ensured by using the relative values of the primary  $I_p^*$ , p.u., and secondary  $I_s^*$ , p.u., currents in model (3), which are calculated as follows:

$$I_p^* = I_p / I_{pr}, \quad (4)$$

$$I_s^* = I_s / I_{sr}. \quad (5)$$

The statistical evaluation of the model (3) parameters is carried out under the following assumptions:

1) the primary current value of the current transformer does not depend on its transformation ratio within the specified accuracy class;

2) the transformation ratio of the  $i$ -th measuring transformer under study does not affect the static characteristic within the accuracy class, corresponding to the fulfillment of the condition  $\Sigma\tau_i=0$ ;

3) the coefficient of linear regression is not zero  $\beta \neq 0$ , i.e. it is assumed that there is a linear relationship between the root mean square values of the primary and secondary

currents of the measuring transformer in the reduced load mode of the metering unit.

Experimental studies were carried out using three measuring current transformers of type T-0.66-300/5 (Ukraine) and three – of type T-0.66-600/5 (Ukraine). The laboratory unit (Fig. 2) is equipped with a digital meter PI1 of type NIK2307 ART T.1600.M2.21 (manufacturer – Ukraine). The connection scheme corresponds to Fig. 1. The design of the laboratory metering unit provides the possibility to change panels with current transformers. The ability of the digital meter PI1 to measure and display phase currents was used to record secondary currents. Recording of primary currents of measuring transformers was carried out using an additional digital meter PI2 of type NIK2307 ARP3 T.1600.M2.21 (Ukraine). The current windings of such a meter were connected in series to the primary windings of measuring current transformers. All devices are of accuracy class 0.5S. This approach makes it possible to obtain the primary and secondary currents of measuring transformers, which are the primary experimental data, when they function as part of the metering unit.

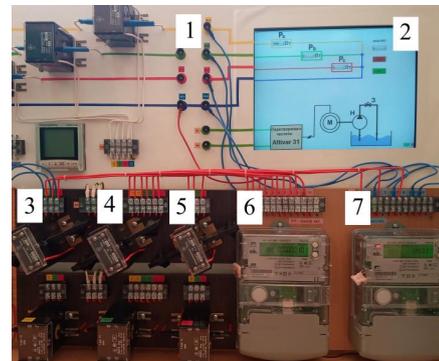


Fig. 2. Laboratory equipment for studying the static characteristic of the measuring current transformer at a reduced load of the metering unit: 1 – three-phase adjustable load terminals; 2 – load control screen; 3, 4, 5 – measuring current transformers T-0.66-600/5 of A, B, C phases, respectively, of the metering unit; 6, 7 – electricity meters PI1, PI2, respectively

Measurements were carried out in a 380 V electrical grid. The load of each phase was resistive and was formed using incandescent lamps of different numbers and power. Load control was carried out using a microprocessor board and a mnemonic scheme on screen. The initial load of each phase was 100 W. During the measurements, the load of one of the phases increased by 100 W or 200 W, reaching a value of 2,700 W for three phases in the last measurement.

For the statistical evaluation of the model (3) parameters based on experimental data, it is assumed to use the analysis of covariance (ANCOVA) methods, combining the analysis of variance (ANOVA) and regression analysis methods [17]. In particular, analysis of variance makes it possible to determine whether the parameters of individual measuring current transformers significantly affect the character of the sought dependence. Regression analysis allows estimating the parameters of the statistical model (3), which describes the static characteristic of the measuring current transformer at a reduced load. The appropriateness of the selected regression model is checked using residual analysis. The use of test statistics (in particular, Student's t-test, F-test,

Kolmogorov-Smirnov test, Durbin-Watson test) is envisaged for testing statistical hypotheses. The asymptotic significance (p-value) is also evaluated.

**5. Results of the study of the static characteristic of the measuring current transformer**

**5.1. The influence of the transformation ratio on the static characteristic of the current transformer at a reduced load of the metering unit**

The experiments were carried out for three current transformers T-0.66-300/5, the transformation ratio of which is  $k_r=60$ , and for three T-0.66-600/5 with  $k_r=120$ . Empirical points, corresponding to the dependence  $I_s^*(I_p^*)$ , were obtained as a result of the measurements, Fig. 3.

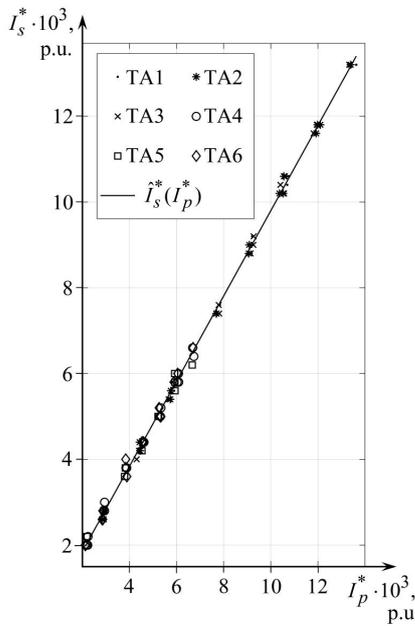


Fig. 3. Empirical points of dependence of the secondary current  $I_s^*$ , p.u., on the primary current  $I_p^*$ , p.u., for current transformers TA1–TA3 (type T-0.66-300/5) and TA4–TA6 (type T-0.66-600/5), and the regression line  $\tilde{I}_s^*(I_p^*)$  corresponding to the static characteristic

As an independent variable (covariate), the relative value of the primary current  $I_p^*$  is considered, which during the measurements took  $n=17$  values from  $2.122 \cdot 10^{-3}$  p.u. to  $1.363 \cdot 10^{-2}$  p.u. The dependent value (response) is the relative value of the secondary current  $I_s^*$ , which was in the range from  $2 \cdot 10^{-3}$  p.u. to  $1.32 \cdot 10^{-2}$  p.u. For each of  $m=6$  measuring current transformers for  $j=1, n$  measurements, the relation between the secondary and the primary currents according to (3) is characterized by uncertainty  $\xi_{ij}$ . The normal and independent distribution  $(0, \Sigma^2)$  of uncertainties  $\xi_{ij}$  is assumed. Then for each experimental point [17]:

$$I_{sij}^* = \mu + \tau_i + \beta(I_{pij}^* - \bar{I}_p^*) + \xi_{ij}, \tag{6}$$

where  $i = \overline{1, m}$  – treatment number (measuring current transformer); the value  $\bar{I}_p^*$  is calculated as:

$$\bar{I}_p^* = \frac{1}{m \cdot n} \sum_{i=1}^m \sum_{j=1}^n I_{pij}^*. \tag{7}$$

The first stage of the covariance analysis involves performing a one-way analysis of variance to test the following hypothesis. The null hypothesis H0: the transformation ratio does not have a statistically significant effect on the static characteristic of the current transformer at a reduced load of the metering unit, which corresponds to the fulfillment of the condition  $\tau_i=0$ . Alternative hypothesis H1: the influence of the transformation ratio on the static characteristic is statistically significant,  $\tau_i \neq 0$ . The F-test statistic is calculated taking into account the empirical value of the residual variance  $D_{ME}$  as:

$$F_0 = \frac{(D'_{DE} - D_{DE}) / (m - 1)}{D_{ME}}, \tag{8}$$

moreover:

$$D'_{DE} = D_{ss} - (D_{ps})^2 / D_{pp}; \tag{9}$$

$$D_{DE} = E_{ss} - (E_{ps})^2 / E_{pp}; \tag{10}$$

$$D_{ME} = \frac{D_{DE}}{m(n-1)-1}; \tag{11}$$

$$D_{ss} = \sum_{i=1}^m \sum_{j=1}^n (I_{sij}^* - \bar{I}_s^*)^2; \tag{12}$$

$$\bar{I}_s^* = \frac{1}{m \cdot n} \sum_{i=1}^m \sum_{j=1}^n I_{sij}^*; \tag{13}$$

$$D_{pp} = \sum_{i=1}^m \sum_{j=1}^n (I_{pij}^* - \bar{I}_p^*)^2; \tag{14}$$

$$D_{ps} = \sum_{i=1}^m \sum_{j=1}^n (I_{pij}^* - \bar{I}_p^*)(I_{sij}^* - \bar{I}_s^*); \tag{15}$$

$$E_{ss} = \sum_{i=1}^m \sum_{j=1}^n \left( I_{sij}^* - \frac{1}{n} \sum_{j=1}^n I_{sij}^* \right)^2; \tag{16}$$

$$E_{pp} = \sum_{i=1}^m \sum_{j=1}^n \left( I_{pij}^* - \frac{1}{n} \sum_{j=1}^n I_{pij}^* \right)^2; \tag{17}$$

$$E_{ps} = \sum_{i=1}^m \sum_{j=1}^n \left( I_{pij}^* - \frac{1}{n} \sum_{j=1}^n I_{pij}^* \right) \left( I_{sij}^* - \frac{1}{n} \sum_{j=1}^n I_{sij}^* \right). \tag{18}$$

Based on the results of measurements on laboratory equipment in accordance with (7), (9)–(18), the following were calculated:  $\bar{I}_s^* = 5.965 \cdot 10^{-3}$ ,  $\bar{I}_p^* = 6.143 \cdot 10^{-3}$ ,  $D_{ss} = 1.021 \cdot 10^{-3}$ ,  $D_{pp} = 1.038 \cdot 10^{-3}$ ,  $D_{ps} = 1.029 \cdot 10^{-3}$ ,  $E_{ss} = 6.821 \cdot 10^{-4}$ ,  $E_{pp} = 6.901 \cdot 10^{-4}$ ,  $E_{ps} = 6.854 \cdot 10^{-4}$ ,  $D'_{DE} = 1.353 \cdot 10^{-6}$ ,  $D_{DE} = 1.340 \cdot 10^{-6}$ ,  $D_{ME} = 1.410 \cdot 10^{-8}$ . According to (8), the value of test statistic  $F_0 = 0.186$  was calculated. The critical point of the Fisher distribution  $F_c(\alpha; m-1; m(n-1)-1)$  at the significance level  $\alpha = 0.05$  is:  $F_c(0.05; 5; 95) = 2.310$ . Since  $F_0 < F_c$ , the null hypothesis cannot be rejected at the accepted level of significance, so there are reasons to assume that  $\tau_i = 0$ . This result is confirmed by comparing  $p \gg \alpha$  asymptotic significance for the specified degrees of freedom ( $p = 0.453$ ) and the level of significance.

**5.2. Evaluation of the statistical model parameters of the measuring current transformer static characteristic**

The estimate  $\hat{\mu}$  of the average value of the measuring transformer secondary currents can be determined as  $\hat{\mu} = \bar{I}_s^*$

according to (13), i.e.  $\hat{\mu} = 5.965 \cdot 10^{-3}$ . The estimate  $\hat{\beta}$  of the linear regression coefficient can be found after testing the hypothesis H0:  $\beta=0$ . Alternative hypothesis H1:  $\beta \neq 0$ . The value of F-statistic is calculated as:

$$F_0 = \frac{(E_{ps})^2 / E_{pp}}{D_{ME}}. \quad (19)$$

The critical point  $F_c(\alpha; 1; m(n-1)-1)$  at the significance level  $\alpha=0.05$  is:  $F_c(0.05; 1; 95)=3.941$ . The value of the criterion  $F_0=4.828 \cdot 10^4$  calculated by (19) significantly exceeds the critical point ( $F_0 \gg F_c$ ), so the null hypothesis H0 is rejected. This gives reason to accept the alternative hypothesis H1:  $\beta \neq 0$ .

To calculate the estimate of the linear regression coefficient, the expression can be used:

$$\hat{\beta} = \frac{E_{ps}}{E_{pp}}. \quad (20)$$

For the experiment conditions, according to (20),  $\hat{\beta} = 9.932 \cdot 10^{-1}$  was calculated.

Then from (3), taking into account the obtained parameter estimates, the statistical model corresponding to the static characteristic of the 0.5S accuracy class measuring current transformer at a reduced load of the metering unit is:

$$\hat{I}_s^*(I_p^*) = \hat{\mu}' + \hat{\beta} \cdot I_p^*, \quad (21)$$

where  $\hat{\mu}' = \hat{\mu} - \hat{\beta} \cdot \bar{I}_p^* = -1.369 \cdot 10^{-4}$ .

The regression line according to (21) is plotted in Fig. 3. The adequacy of the covariance model to the experimental data can be checked by examining the regression residuals:

$$e_{ij} = I_{sij}^* - \hat{I}_{sij}^*. \quad (22)$$

The regression residuals for all current transformers versus the relative values of the primary currents are plotted in Fig. 4. Separately for each of the current transformers, the regression residuals are presented in Fig. 5. The residuals  $e_{ij}$  must be independent normally distributed random variables with zero mean and the first-order autocorrelation must be eliminated [17]. The Kolmogorov-Smirnov goodness of fit test is used for checking for normality of the regression residuals distribution. Null hypothesis H0: the sample belongs to the normal distribution. Alternative hypothesis: H1: the sample does not belong to the normal distribution. The test statistic  $K_0=5.310 \cdot 10^{-2}$  is smaller than the critical value  $K_c=1.327 \cdot 10^{-1}$ , i.e.  $K_0 < K_c$ , at the significance level  $\alpha=0.05$ , which does not give grounds for rejecting H0. The fact that the sample belongs to the normal distribution law is also confirmed by the asymptotic significance  $p=0.921$  exceeding the value of  $\alpha: p > \alpha$ .

Testing of the hypothesis H0:  $m[e]=0$  concerning the zero mean of the regression residuals with the alternative hypothesis H1:  $m[e] \neq 0$  was carried out using the Student's t-test. The t-test statistic ( $-2.556 \cdot 10^{-14}$ ) is significantly smaller than the critical value of the test  $t(1-\alpha; m \cdot n - 1) = t(0.95; 102-1)=1.66$  at the 0.95 confidence level, which does not give evidence against the null hypothesis H0.

Testing of the first-order autocorrelation of regression residuals was carried out according to the Durbin-Watson test.

The main hypothesis H0 is tested: the random deviations of the regression residuals are independent. At the same time, the alternative hypothesis H1: there is autocorrelation between the regression residuals. The calculated test statistic value is  $D=2.115$ . The test critical values for the number of observations  $m \cdot n=102$  at the significance level  $\alpha=0.05$  are: lower  $d_L=1.657$ , upper  $d_U=1.696$ . Since  $D > 2$ , the value  $(4-D)=1.885$  is compared with  $d_L$  and  $d_U$ . The inequality  $(4-D) > d_U$  holds, so there is no reason to reject H0. Accordingly, the autocorrelation between the regression residuals is not statistically significant.

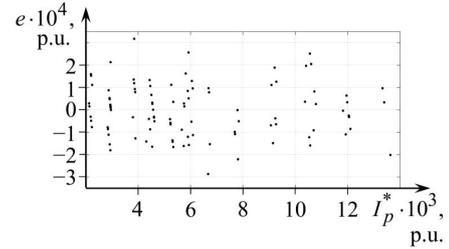


Fig. 4. Regression residuals  $e$ , p.u., versus the value of covariate  $I_p^*$ , p.u.

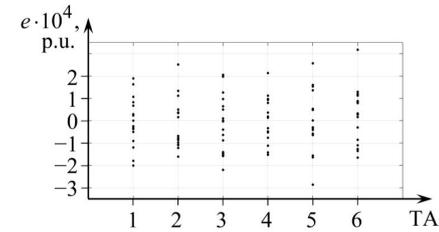


Fig. 5. Regression residuals  $e$ , p.u., for current transformers TA1-TA6

To evaluate the deviations of the response values calculated by the empirical regression from the true values given by the theoretical regression, confidence intervals for the regression were constructed, Fig. 6. With the probability  $(1-\alpha)=0.95$ , the theoretical regression lies in the range:

$$C_{1,2}(I_p^*) = \hat{I}_s^*(I_p^*) \pm t_{\alpha/2} \cdot s \cdot \sqrt{\frac{1}{m \cdot n} + \frac{(I_p^* - \bar{I}_p^*)^2}{D_{pp}}}, \quad (23)$$

where  $t_{\alpha/2}$  – the point of Student's distribution for the significance level  $\alpha/2$  with  $m \cdot n - l - 1$  degrees of freedom;  $l$  – the approximate polynomial degree;  $s$  – the standard deviation of the observations results  $I_s^*$  versus the regression values:

$$s = \sqrt{\frac{1}{m \cdot n - l - 1} \sum_{i=1}^m \sum_{j=1}^n e_{ij}^2}. \quad (24)$$

To estimate the prediction interval  $P_{1,2}(I_p^*)$ , which with probability  $(1-\alpha)=0.95$  covers the sample response values, Fig. 6, the following dependency was used:

$$P_{1,2}(I_p^*) = \hat{I}_s^*(I_p^*) \pm t_{\alpha/2} \cdot s \cdot \sqrt{1 + \frac{1}{m \cdot n} + \frac{(I_p^* - \bar{I}_p^*)^2}{D_{pp}}}. \quad (25)$$

Thus, estimates of the parameters of the statistical model (21) linking the primary and secondary currents of the

measuring transformers in the area of reduced loads of the metering unit were obtained. A fragment of the graphical representation of such a statistical model is shown in Fig. 6, illustrating the position of the regression line towards the experimental points. The graph also enables us to determine the location of the confidence intervals for the regression line and the prediction intervals for the sample values of the secondary currents.

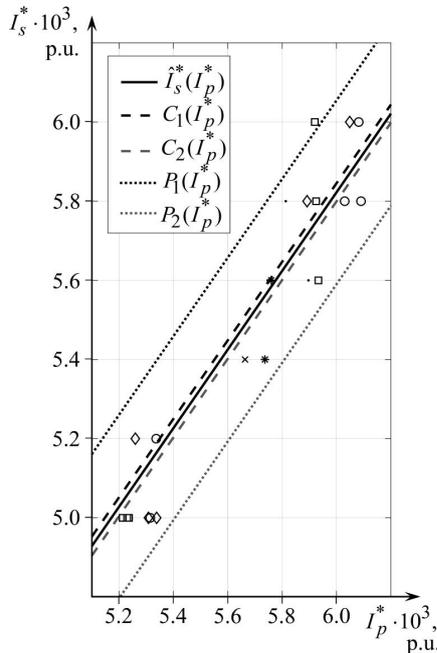


Fig. 6. Fragment of the regression line  $\hat{I}_s^*(I_p^*)$ , p.u., confidence intervals  $C_1(I_p^*)$ ,  $C_2(I_p^*)$ , p.u., for the regression line and prediction intervals  $P_1(I_p^*)$ ,  $P_2(I_p^*)$ , p.u., for the secondary currents of the current transformer versus the relative values of the primary currents  $I_p^*$ , p.u., from the interval  $I_p^* \in [5.1 \cdot 10^{-3}; 6.2 \cdot 10^{-3}]$ ; designations of experimental points correspond to those adopted in Fig. 3

### 5. 3. Uncertainty of current measurement by an electromagnetic transformer at reduced primary current

Based on the obtained statistical model (21) of the static characteristic of the measuring current transformer of accuracy class 0.5S, it is possible to obtain an expression for determining the current error. Using relative current values in expression (1), in accordance with (4), (5), the regression dependence of the current error on the relative value of the primary current can be obtained (Fig. 7):

$$\hat{\varepsilon}(I_p^*) = \left[ \frac{\hat{I}_s^*(I_p^*)}{I_p^*} - 1 \right] \cdot 100\% = \left( \frac{\hat{\mu}'}{I_p^*} + \hat{\beta}' \right) \cdot 100\%, \quad (26)$$

where  $\hat{\beta}' = \hat{\beta} - 1 = -6.764 \cdot 10^{-3}$ .

The uncertainty of the regression dependence  $\hat{\varepsilon}(I_p^*)$  can be estimated by confidence intervals, which are calculated using (23) as follows:

$$\varepsilon_{C1,2}(I_p^*) = \left[ \frac{C_{1,2}(I_p^*)}{I_p^*} - 1 \right] \cdot 100\%. \quad (27)$$

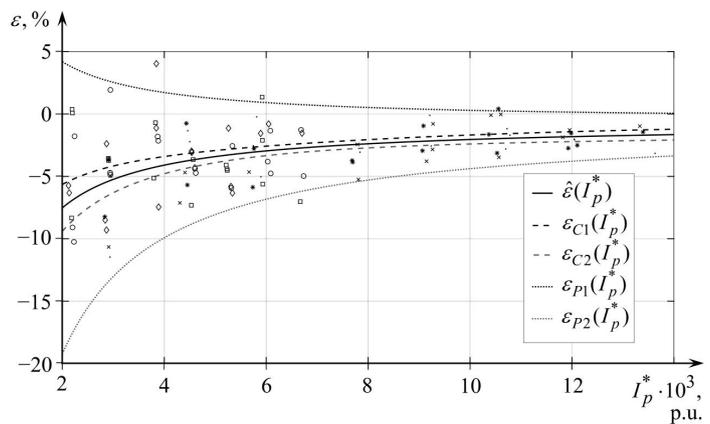


Fig. 7. Regression dependence of the current error  $\hat{\varepsilon}$ , %, of measuring current transformers of the accuracy class 0.5S, confidence intervals  $\varepsilon_{C1}$ ,  $\varepsilon_{C2}$  for the regression line, prediction intervals  $\varepsilon_{P1}$ ,  $\varepsilon_{P2}$  for values of the current error (at the 0.95 confidence level) versus the primary current  $I_p^*$ , p.u.; empirical sample current errors are plotted, the correspondence of which to TA1–TA6 is marked similarly to Fig. 3

The uncertainty of the sample empirical values of the current error, for a given confidence level, can be estimated by prediction intervals bounded by lines:

$$\varepsilon_{P1,2}(I_p^*) = \left[ \frac{P_{1,2}(I_p^*)}{I_p^*} - 1 \right] \cdot 100\%. \quad (28)$$

Confidence and prediction intervals for current errors, the bounds of which are estimated by dependencies, respectively, (27) and (28), are plotted in Fig. 7.

Thus, the uncertainty of current measurement by the electromagnetic transformer at a reduced primary current is estimated using the intervals of the current error change. The uncertainty of estimating the theoretical value of the current error at a given value of the primary current is estimated by the bounds (27) of the confidence interval. Bounds (28) of prediction intervals provide the uncertainty estimates of sample experimental values.

### 6. Discussion of the results of studying the static characteristic of the measuring current transformer

The conducted analysis of variance for experimental data, in accordance with (7), (9)–(18), for measuring current transformers 300/5 and 600/5 of the 0.5S accuracy class gave grounds for not rejecting the null hypothesis  $H_0: \tau_i = 0$  at  $\alpha = 0.05$ . This result confirms the correctness of assumption No. 2. This gives reason to consider the parameter  $\tau$ , which determines the influence of the transformation ratio on the static characteristic, to be zero in model (3). That is, the influence of the transformation ratio of the measuring current transformer within the specified accuracy class on the dependence of the secondary current on the primary current in the reduced load mode is not statistically significant. This result is also confirmed by the insignificant variance of experimental points around the regression line, Fig. 3. The possibility of neglecting the specified influence determines the universality of the proposed static characteristic model within the accuracy class. From a technical point of view, this property is explained by the similarity of the parameters

of measuring current transformers magnetic systems for low-voltage electricity metering units. Since the secondary rated current of most measuring transformers is 5A, model (3) is correct for low-voltage devices of the 0.5S accuracy class with any primary rated current.

The F-test application gave reason to accept the alternative hypothesis  $H1: \beta \neq 0$  at a significance level of 0.05, since the F-statistic value calculated by (19) exceeds the critical point by 4 orders of magnitude. This confirms the correctness of assumption No. 3. It also gives reason to assert the existence of a statistically significant relationship between the current values of the primary and secondary currents of the measuring transformers in the metering unit at a reduced load of the latter. This result is explained by the distance of the low primary currents zone from the saturation zone according to the current-voltage characteristic of the current transformer. The application of the regression analysis apparatus made it possible to calculate the numerical values of the static characteristic parameters estimates. The latter, after simplification, is represented by linear regression (21), which is plotted in Fig. 3. The study of the regression residuals (22) made it possible to determine the adequacy of the regression line with the experimental data. This is confirmed by the fulfillment of the main conditions that are put forward for the regression residuals. Firstly, the hypothesis that the sample belongs to a normal distribution was not rejected at the 0.05 significance level according to Kolmogorov-Smirnov test (Fig. 4). Secondly, the hypothesis about the null mean of the regression residuals was not rejected at the 0.95 confidence level according to the Student's t-test. Thirdly, using the Durbin-Watson test, it was found that there are no grounds for rejecting the hypothesis about the independence of random deviations of the regression residuals. Fig. 5 illustrates the latter circumstance and allows determining that the regression residuals do not depend on the type of current transformer TA1–TA6. It was found that the width of the confidence intervals, constructed in accordance with (23), for the empirical regression line does not exceed  $\pm 6.083 \cdot 10^{-5}$  p.u., Fig. 6. The insignificant value (by 2 orders of magnitude less than the estimate of the secondary current) of the confidence intervals indicates that the empirical regression is sufficiently close to the theoretical one. This determines the possibility of using dependence (21) with the obtained parameter estimates as a static characteristic of a measuring current transformer of the 0.5S accuracy class at a reduced load of the metering unit in low-voltage power grids. The width of the forecasting intervals for the response sample values, obtained according to (25), does not exceed  $\pm 2.389 \cdot 10^{-4}$  p.u., Fig. 6. This makes it possible to predict the interval of finding the measured secondary current values of the measuring transformer as part of the metering unit.

Using the obtained static characteristic (21) of the measuring current transformer, the expression (26) was obtained, which makes it possible to estimate the current error at a given value of the primary current. For the studied range of the primary current (from  $2 \cdot 10^{-3}$  p.u. to  $1.4 \cdot 10^{-2}$  p.u.), the estimate of the current error varies from  $-7.5\%$  to  $-1.7\%$ , respectively (Fig. 7). The width of the confidence intervals of the regression line for the current error, estimated according to (27), with a confidence probability of 0.95 does not exceed  $\pm 1.9\%$ . This characterizes the uncertainty of the regression dependence of the current error on the primary current value. The uncertainty of individual estimates of the current error is determined by prediction intervals (28), the width

of which varies from  $\pm 11.7\%$  to  $\pm 1.7\%$  when the primary current changes in the studied range. This determines the possible range of changes in the current error for each of the metering unit transformers from  $23.4\%$  to  $3.4\%$  when the latter operates in the reduced load mode, Fig. 7. A significant change in the width of the current error prediction interval is explained by the inversely proportional dependence (28) of the latter on the primary current.

Determining the universal static characteristic (21) of measuring current transformers within a given accuracy class, compared to training an artificial neural network [13] for each specific device, reduces the time spent on improving the metering unit. The uncertainty of current measurement by the measuring transformer is estimated, in contrast to the experimental graphs in [5], by the regression dependence (26) for the current error with confidence intervals (27). The advantage of this approach is the ability to analytically determine the range of changes in the current error for a given value of the primary current. Estimation of the bounds of prediction intervals (28) for sample values of the current error, in contrast to the application of correction factors [14], provides information about the actual level of underestimation of electricity by a commercial metering unit.

The proposed statistical model (21) of the static characteristic of the measuring current transformer solves the issue of maintaining the necessary accuracy of commercial electricity metering at a reduced power load level. The model (21), characterized by confidence intervals (23), can be directly used in the software of the metering unit to adjust the meter readings during periods of reduced primary current. Improvement of the accuracy of commercial metering is achieved by billing the consumed electricity considering the distortion of the static characteristic of measuring transformers depending on each phase current level. The need for such an adjustment, in case of doubts among electricity consumers, is substantiated by significant values and considerable range of prediction intervals for current errors (Fig. 7) of measuring transformers.

The application of the proposed static characteristic of measuring current transformers is limited to devices of the 0.5S accuracy class. Characteristics for transformers of other accuracy classes were not obtained during the study. However, the proposed approach using covariance analysis can be applied to current transformers of other common accuracy classes.

The main drawback of the given method of determining the static characteristic of the measuring current transformer at a reduced load of the metering unit is ignoring the impact of temperature, grid voltage fluctuations and other factors on the level of the secondary current.

It is planned to combine the obtained statistical model of the current transformer static characteristic and the mathematical model [12], proposed by the authors, of the uncertainty of electricity measurement at reduced load in future research. This implementation of the integrated model in the software part of an automated commercial electricity metering system will reduce non-technological losses by increasing metering accuracy.

---

## 7. Conclusions

---

1. The statistical insignificance of the influence of the measuring current transformer ratio for a specific accuracy

class on the static characteristic in relative units was determined. This, in contrast to known approaches, determines the universality of the obtained statistical model of the static characteristic.

2. The existence of a statistically significant linear relationship between the primary and secondary currents of measuring current transformers at a reduced load of the metering unit was confirmed. The adequacy of estimates of linear regression parameters to experimental data at a significance level of 0.05 was confirmed by analysis of regression residuals. Sufficient closeness of the empirical regression to the theoretical one is confirmed by the insignificant width of the confidence intervals ( $\pm 6.083 \cdot 10^{-5}$  p.u.) for the experimental characteristic, which is 2 orders of magnitude smaller than the estimates of the secondary current of the measuring transformer.

3. The sample estimates uncertainty of the current transformer current error of the 0.5S accuracy class changes from  $\pm 11.7\%$  to  $\pm 1.7\%$  when the primary current increases, respectively, from  $2 \cdot 10^{-3}$  p.u. to  $1.4 \cdot 10^{-2}$  p.u. in the reduced load mode of the metering unit. This confirms the expediency of increasing the accuracy of commercial electricity metering in low-voltage grids by using the obtained static characteristic as part of the digital metering unit software.

---

#### Conflict of interest

---

The authors declare that they have no conflict of interest in relation to this research, whether financial, personal, authorship or otherwise, that could affect the research and its results presented in this paper.

---

#### References

1. Global AC Current Transformers (CT) for Electrical Meters Market Research Report 2022 (2022). Market Reports World. Available at: <https://www.marketreportsworld.com/global-ac-current-transformers-ct-for-electrical-meters-market-21024831>
2. IEC 61869-2:2012. Instrument transformers – Part 2: Additional requirements for current transformers. Available at: <https://webstore.iec.ch/publication/6050>
3. Instrument transformers. Application guide (2015). 1HSM 9543 40-00en. ABB. Available at: <https://library.e.abb.com/public/94c2ba5a2f381077c1257df000504e0c/1HSM%209543%2040-00en%20IT%20Application%20Guide%20Ed4.pdf>
4. Quarterly report on European electricity markets (2022). Market Observatory for Energy, 15 (1). Available at: [https://ec.europa.eu/info/sites/default/files/energy\\_climate\\_change\\_environment/quarterly\\_report\\_on\\_european\\_electricity\\_markets\\_q1\\_2022.pdf](https://ec.europa.eu/info/sites/default/files/energy_climate_change_environment/quarterly_report_on_european_electricity_markets_q1_2022.pdf)
5. Soinski, M., Pluta, W., Zurek, S., owski, A. K. (2014). Metrological attributes of current transformers in electrical energy meters. *International Journal of Applied Electromagnetics and Mechanics*, 44 (3-4), 279–284. doi: <https://doi.org/10.3233/jae-141790>
6. Kaczmarek, M. (2012). Method of current transformer metrological properties estimation for transformation of distorted currents. 2012 IEEE International Power Modulator and High Voltage Conference (IPMHVC). doi: <https://doi.org/10.1109/ipmhvc.2012.6518847>
7. Mingotti, A., Peretto, L., Bartolomei, L., Cavaliere, D., Tinarelli, R. (2020). Are Inductive Current Transformers Performance Really Affected by Actual Distorted Network Conditions? An Experimental Case Study. *Sensors*, 20 (3), 927. doi: <https://doi.org/10.3390/s20030927>
8. Ghaderi, A., Mingotti, A., Peretto, L., Tinarelli, R. (2019). Inductive current transformer core parameters behaviour vs. temperature under different working conditions. 23rd IMEKO TC4 International Symposium Electrical & Electronic Measurements Promote Industry 4.0. Xi'an, 107–112. Available at: <https://www.imeko.org/publications/tc4-2019/IMEKO-TC4-2019-024.pdf>
9. Puzovic, S., Koprivica, B., Milovanovic, A., Djekic, M. (2014). Analysis of measurement error in direct and transformer-operated measurement systems for electric energy and maximum power measurement. *Facta Universitatis - Series: Electronics and Energetics*, 27 (3), 389–398. doi: <https://doi.org/10.2298/fuee1403389p>
10. Lesniewska, E., Rajchert, R. (2019). Behaviour of measuring current transformers with cores composed from different magnetic materials at non-rated loads and overcurrents. *IET Science, Measurement & Technology*, 13 (7), 944–948. doi: <https://doi.org/10.1049/iet-smt.2018.5176>
11. Lee, S.-J., Jung, H.-K., Chung, T.-K., Lee, Y.-S., Ro, J.-S. (2019). Ratio Error Reduction of a Current Transformer Using Multiple Winding Technique. *Journal of Electrical Engineering & Technology*, 14 (2), 645–651. doi: <https://doi.org/10.1007/s42835-018-00040-6>
12. Vasylets, K., Kvasnikov, V., Vasylets, S. (2022). Refinement of the mathematical model of electrical energy measurement uncertainty in reduced load mode. *Eastern-European Journal of Enterprise Technologies*, 4 (8 (118)), 6–16. doi: <https://doi.org/10.15587/1729-4061.2022.262260>
13. Ballal, M. S., Wath, M. G., Suryawanshi, H. M. (2020). Measurement Current Transformer Error Compensation by ANN Methodology. *Journal of The Institution of Engineers (India): Series B*, 101 (3), 261–271. doi: <https://doi.org/10.1007/s40031-020-00454-9>
14. Ballal, M. S., Wath, M. G., Venkatesh, B. (2018). Current Transformer Accuracy Improvement by Digital Compensation Technique. 2018 20th National Power Systems Conference (NPSC). doi: <https://doi.org/10.1109/npsc.2018.8771706>
15. Wath, M. G., Raut, P., Ballal, M. S. (2016). Error compensation method for current transformer. 2016 IEEE 1st International Conference on Power Electronics, Intelligent Control and Energy Systems (ICPEICES). doi: <https://doi.org/10.1109/icpeices.2016.7853244>
16. C57.13-2016 – IEEE Standard Requirements for Instrument Transformers. doi: <https://doi.org/10.1109/ieeestd.2016.7501435>
17. Montgomery, D. C. (2020). Design and analysis of experiments. John Wiley & Sons, 752. Available at: <https://www.wiley.com/en-ie/Design+and+Analysis+of+Experiments,+9th+Edition,+EMEA+Edition-p-9781119638421>