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DEVELOPMENT OF COMPUTATIONAL METHOD FOR MATCHED FILTRATION WITH ANALYTICAL PROFILE OF THE BLURRED DIGITAL IMAGE

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A computational method for matched filtration with analytical profile of the blurred digital image of the investigated objects on digital frames has been developed. Such «blurred» objects can be the result of an involuntary shift of a fixed camera, an incorrect choice of the mode of guiding the telescope (diurnal or object tracking) or a failure of the diurnal tracking.

This computational method is based on the analytical selection of the typical form of the object's image, as well as on the choice of special parameters for the transfer function of the matched filter for the blurred digital image, which makes it possible to evaluate the required parameters of the blurred digital image.

In addition, determining the number of Gaussians of the object's image makes it possible to perform the most accurate assessment of the initial approximation of the parameters of their shape. Thus, matched filtration makes it possible to highlight the investigated objects with a blurred image of a typical shape against the background of substrate noise. Using the computational method of matched filtration makes it possible to improve the segmentation of images of reference objects on the frame and reduce the number of false detections.

The developed computational method for matched filtration with analytical profile of the blurred digital image of the investigated objects on the frames was tested in practice as part of the research of the CoLiTec project. It was implemented in the intraframe processing unit of the Lemur software for the operational automated detection of new and observation of known objects with a weak brightness. Owing to the Lemur software using and the proposed computational method introduced into it, more than 500,000 measurements of the various investigated objects were successfully processed and identified

Keywords: matched filter, transfer function, OLS evaluation, Gaussian, image processing

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1. Introduction

Given the problem of asteroid-comet danger [1], automatic processing of the results of asteroid surveys is the leading direction for the use of the most modern methods of astrometry [2] and photometry [3]. The accumulation of archival big data [4], astronomical catalogs [5], and virtual observatories make it possible to collect, gain knowledge [6], and analyze historically accumulated data and measurements of the investigated Solar System's celestial objects (SSO) [7].

Such SSOs in general have a wide range of speeds of visible motion (near-zero speed [8], non-zero speed, extremely fast speed). Other objects that do not belong to the solar system (stars, galaxies) have zero speed of apparent motion.

However, it is the shooting conditions that affect how an object will look on the frames taken by the charge-coupled device (CCD) [9]. Such shooting conditions include the following: the mode of guiding the telescope, the side wind, and the weather in general, the mechanical failure of the diurnal tracking, and others. Under adverse conditions, images of individual objects or the entire CCD frame as a whole can be blurred. This circumstance significantly reduces the quality of detection of the investigated objects by known methods.

Therefore, it is relevant to devise a computational method for matched filtration with analytical profile of a blurred digital image of the investigated objects on CCD frames. This method will make it possible to more accurately assess the image parameters [2] of such «blurred» objects and identify them with those already known from the list of cataloged ones. In addition, the method to be devised will reduce the number of false detections and increase the conditional probability of correct detection (CPCD) of real objects [10].

2. Literature review and problem statement

A fully blurred CCD frame in this work is a digital image formed under the condition of significant synchronous movements of all objects of the CCD frame over the entire time of exposure. This blurring is considered only when this movement cannot be neglected categorically.

Typical conditions for the appearance of such a blurred CCD frame are an involuntary shift of a fixed CCD camera, gusts of wind, or an incorrect choice of the mode of guiding the telescope (diurnal or object tracking). In addition, the formation of a frame by a telescope without diurnal tracking or with its failure can lead to blurring the image.

The characteristic of the blurred frame, first of all, is the synchronous elongation of all images of objects as a measure of blurring. Synchronous elongation refers to the same directions and lengths of blurred images of all objects in a CCD frame.

In addition, images of such SSOs as asteroids, meteors, comets are blurred due to their natural motion. In this work, the blurring of the image of objects due to its natural movement is not considered.

However, blurred images of the investigated objects on CCD frames affect the quality and accuracy of various image processing and machine vision tasks [11]. Namely, they affect the detection of images of objects and the evaluation of their parameters [2], image segmentation [12], image pixelation [13], object recognition and their classification [14]. In addition, blurred images of objects affect detection of their movement and evaluation of the parameters of the trajectories of movement [8], and even a wavelet transformation (analysis) [15] of various signals and data.

In general, the detection of images of objects in the process of image processing and machine vision [16] is based on analytical or numerical methods. Some of them are based on image segmentation, where only those pixels that potentially belong to the investigated object are analyzed, and the intensity of which exceeds the specified limit value [12, 17]. Methods of addition of frames [18] can be used only when the image of the investigated object is visible with clear boundaries on all CCD frames in the series.

Other numerical methods use Hough, Radon transformations, or their modifications [19, 20] but they are not able to detect an image of an object with the required conditional probability of correct detection (CPCD) under conditions of blurring of the image itself. In addition, the evaluation of image parameters by processing methods [2, 13, 14] in the absence of clear boundaries of the object image does not correspond to even the lowest limit value for accuracy.

As practice shows, it is the agreed filter that has the best efficiency in relation to the CPCD with a fixed value of the conditional probability of false detection [21]. This is a linear filter that minimizes the signal-to-noise ratio (SNR) and, therefore, increases the probability of detecting the investigated objects, in the images of which random additive noise with a normal Gaussian distribution is embedded [22].

The problem statement involves the construction of the transfer function of the matched filter on the blurred images of all objects on the digital frame and in the matched filtration of the original digital frame.

A blurred digital frame A_{in} the size of $N_{CCDx} \times N_{CCDy}$ is selected as the initial data. The image of any object, for example, j-th, is actually present in the frame and is located in the area of intra-frame processing (AIFP). In this case, AIFP is a set of Ω_{Nobj} pixels in which a blurred image of the j-th object is assumed to be present. The number of pixels belonging to AIFP is considered to be N_{IPSj}.

Blurred images of objects, especially with a significant exposure time, have the shape of a hill stretched along the direction of their apparent motion. Images of such objects can be represented by a set of Gaussians. According to the model used in the work, the Gaussian centers lie on one straight line passing through the point of reference of the blurred image of the j-th object with coordinates $x_{\tau j}(\Theta_{\tau j}^{over})$ and $y_{\tau j}(\Theta_{\tau j}^{over})$ at an angle Ω_j to the abscissa axis in the coordinate system of the digital frame.

3. The aim and objectives of the study

The aim of this study is to perform matched filtration of the blurred digital image of various objects on CCD frames. Such «blurred» objects can be the result of an involuntary shift of a fixed CCD camera, or a failure of the diurnal tracking of the telescope, as well as its coma. Matched filtration makes it possible to highlight objects with a typical image against the background of noise. Typical shapes of objects with a blurred digital image on CCD frames, as well as their profile, can be set analytically.

To accomplish the aim, the following tasks have been set:

– to select the shape and parameters of the transfer function of the matched filter for the blurred digital frame, as well as evaluate the required image parameters;

– to determine the number of Gaussians of the image of the investigated object and estimate the initial approximation of the parameters of their shape;

– to devise a computational method for matched filtration with analytical profile of a blurred digital image.

4. The research materials and methods

The object of this research is blurred digital images of various objects on CCD frames.

Within the current research, the main hypothesis was put forward that the use of matched filtration as a preliminary preparatory method for processing the original blurred image will significantly increase the CPCD of object images. In addition, matched filtration will increase the accuracy of object parameter estimation when further performing the main tasks of image processing by known methods.

The obtained research results, as well as the devised computational method of matched filtration, were converted into program code using the C++ programming language. This code was integrated into the system unit of intraframe processing of the Lemur software for operational automated detection of new and observation of known objects with low brightness (Ukraine) [23] within the framework of the CoLiTec project [7].

As initial test data, we used data acquired from a variety of telescopes installed at various observatories in Ukraine and the world. Namely, the Mayaki Astronomical Observatory [24], the ISON-NM and ISON-Kislovodsk observatories [25], the Vihorlat Observatory [3], and the National Astronomical Research Institute of Thailand (NARIT) [26].

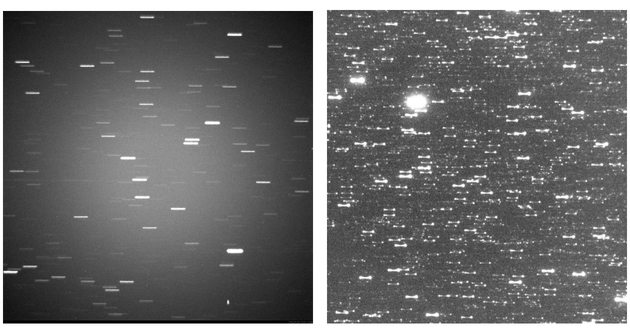
The observational conditions were specially selected in such a way that the obtained series of CCD frames contained blurred images of the investigated SSOs [7].

The devised computational method, integrated into the Lemur software, contributed to the successful processing and identification of more than 500,000 measurements of various objects on a number of series consisting of many CCD frames. Given this, the method of matched filtration confirmed its practical necessity within the framework of the main hypothesis put forward.

5. Results of the matched filtration research with analytical profile of the blurred digital image

5. 1. Selecting the form and parameters of the matched filter transfer function for a blurred digital frame

The original blurred CCD frames typically have a synchronous elongation of all images of objects (Fig. 1). To select the form and parameters of the matched filter transfer function, one must first select the blurred images of objects and determine the model brightness.



a *b*

Fig. 1. Blurred images of objects: *a* — without diurnal tracking; *b* — in the case of failure of diurnal tracking

The model brightness $A_{ik\tau j}(x_{ij}, y_{ij}, \Theta_{\tau j}^{over})$ of the *ik*-th pixel for the blurred image of the *j*-th object is determined by the following expression [19]:

$$A_{ik\tau j}\left(x_{ij}, y_{ij}, \Theta_{\tau j}^{over}\right) = C_j + \frac{A_{Gj}}{2\pi\sigma_{Gj}^2}\sum_{n=0}^{N_{Gj}-1}\exp\left\{-\frac{1}{2\sigma_{Gj}^2}\left[\begin{array}{l}\left(\begin{array}{l}x_{ij} - x_{\tau j}\left(\Theta_{\tau j}^{over}\right) - \\ -\ell_{nj}\cos\omega_j\end{array}\right)^2 + \\ +\left(\begin{array}{l}y_{ij} - y_{\tau j}\left(\Theta_{\tau j}^{over}\right) - \\ -\ell_{nj}\sin\omega_j\end{array}\right)^2\end{array}\right]\right\}, \quad (1)$$

where C_j is the average brightness of the background substrate of the blurred image of the *j*-th object; $\Theta_{\tau j}^{over}$ – vector of parameters of the blurred image of the *j*-th object; x_{ij}, y_{ij} – coordinates of the *ik*-th pixel of the blurred image of the *j*-th object; A_{Gj} is the model amplitude of Gaussians corresponding to the blurred image of the *j*-th object; σ_{Gj} is a parameter of the form of the Gaussians corresponding to the blurred image of the *j*-th object; the position ℓ_{nj} of the *n*-th Gaussian of N_{Gj} used to approximate the blurred image of the *j*-th objects; d_j is the length of the movement traveled by the *j*-th object during the exposure time, in pixels relative to the frame; $N_{Gj} = E\left[d_j/\sigma_{Gj}\right]+1$ is the number of Gaussians in the model of the blurred image of the *j*-th object; $E[\bullet]$ is the operation of selecting an integer part of a number.

$$\ell_{nj} = -\frac{d_j}{2} + n\frac{d_j}{N_{Gj}-1}. \quad (2)$$

It is also necessary to determine the spectrum of the typical image that is subject to matched filtration. It follows that the value of the *uv* harmonic S_{uvjn} of the discrete spectrum of the *n*-th Gaussian of the image of the *j*-th object is determined by the following expression [22]:

$$S_{uvjn} = A_{Gj}\exp\left[-2\pi^2\sigma_{Gj}^2\left(\begin{array}{l}\left(\frac{u}{N_{CCDx}}\right)^2 + \\ +\left(\frac{v}{N_{CCDy}}\right)^2\end{array}\right)\right]\times$$
$$\times\exp\left[i2\pi\left(\begin{array}{l}u\frac{x_{\tau j}\left(\Theta_{\tau j}^{over}\right)+\ell_{nj}\cos\omega_j}{N_{CCDx}} + \\ +v\frac{y_{\tau j}\left(\Theta_{\tau j}^{over}\right)+\ell_{nj}\sin\omega_j}{N_{CCDy}}\end{array}\right)\right]. \quad (3)$$

According to the linearity property of the discrete Fourier transform (DFT), the discrete spectrum of the image sum is equal to the sum of the discrete spectra of the images [27, 28]:

$$S_{uvj} = \sum_{n=0}^{N_{Gj}-1} S_{uvjn}. \quad (4)$$

Taking into consideration the obtained expressions (3) and (4), the value of the *uv*-th harmonic of the discrete spectrum of the digital blurred image of the *j*-th object S_{uvj} will take the following form [29]:

$$S_{uvj} = A_{Gj}\exp\left[-2\pi^2\sigma_{Gj}^2\left(\left(\frac{u}{N_{CCDx}}\right)^2 + \left(\frac{v}{N_{CCDy}}\right)^2\right)\right]\times$$
$$\times\sum_{n=0}^{N_{Gj}-1}\exp\left[i2\pi\left(\begin{array}{l}u\frac{x_{\tau j}\left(\Theta_{\tau j}^{over}\right)+\ell_{nj}\cos\omega_j}{N_{CCDx}} + \\ +v\frac{y_{\tau j}\left(\Theta_{\tau j}^{over}\right)+\ell_{nj}\sin\omega_j}{N_{CCDy}}\end{array}\right)\right], \quad (5)$$

where $u = \overline{0, N_{CCDx}-1}$, $v = \overline{0, N_{CCDy}-1}$ are the harmonic numbers of the discrete spectrum of the blurred image.

To select the form and parameters of the transfer function of the matched filter for a blurred digital frame, it is necessary to determine the transfer function of the matched filter, which is equal to the complex-conjugate spectrum S_{uvj}^* of the object image [30]:

$$S^*_{uvj}=A_{Gj}\exp\left[-2\pi^2\sigma^2_{Gj}\left(\left(\frac{u}{N_{CCDx}}\right)^2+\left(\frac{v}{N_{CCDy}}\right)^2\right)\right]\times$$
$$\times\sum_{n=0}^{N_{Gj}-1}\exp\left[-i2\pi\left(\begin{array}{l}u\dfrac{x_{\tau j}\left(\Theta^{over}_{\tau j}\right)+\ell_{nj}\cos\omega_j}{N_{CCDx}}+\\+v\dfrac{y_{\tau j}\left(\Theta^{over}_{\tau j}\right)+\ell_{nj}\sin\omega_j}{N_{CCDy}}\end{array}\right)\right]. \qquad (6)$$

To ensure the position of maximum response of the matched filter in the center of the blurred images of objects in the coordinate system (CS) of the digital frame, expression (6) sets zero values for the coordinates of the image binding center [31]:

$$x_{\tau j}\left(\Theta^{over}_{\tau j}\right)=0,\ y_{\tau j}\left(\Theta^{over}_{\tau j}\right)=0. \qquad (7)$$

To agree on the aperture (total) brightness of the pixels of the image of the j-th object on the source frame and on the frame after the matched filter, the complex-conjugate spectrum S^*_{uvj} is multiplied by its normalizing coefficient:

$$k_{norm}=\frac{1}{A_{Gj}N_{Gj}}. \qquad (8)$$

According to (6) to (8), the expression for calculating the transfer function H_{MF} of the matched filter for a blurred digital frame will take the following form:

$$H_{FMuv}=\frac{1}{N_{Gj}}\exp\left[-2\pi^2\sigma^2_{Gj}\left(\begin{array}{l}\left(\dfrac{u}{N_{CCDx}}\right)^2+\\+\left(\dfrac{v}{N_{CCDy}}\right)^2\end{array}\right)\right]\times$$
$$\times\sum_{n=0}^{N_{Gj}-1}\exp\left[-i2\pi\left(u\frac{\ell_{nj}\cos\omega_j}{N_{CCDx}}+v\frac{\ell_{nj}\sin\omega_j}{N_{CCDy}}\right)\right], \qquad (9)$$

where H_{MFuv} is the uv-th harmonic of the transfer function of the matched filter.

After calculating the H_{MF} transfer function of the matched filter for a blurred digital frame, one must evaluate the desired image parameters.

The parameters of blurred images of objects on the frame are evaluated by Q selected images of objects. To select object images to evaluate the required image parameters, a set of images is formed with the aperture (total) brightness of the pixels of the images of objects $A^*_{\Sigma j}$, satisfying the condition:

$$A^*_{\Sigma j}\geq(10\div 20)\sigma_{noise}, \qquad (10)$$

where σ_{noise} is an estimate of a standard deviation of the background brightness;

$$A^*_{\Sigma j}=\sum_{i,k=\Omega_{Nobj}}\left(A^*_{ikj}-C_j\right)$$

is the aperture (total) brightness of the pixels of the j-th image of the object.

This set is ordered in descending order of the aperture (total) brightness of the pixels of the images of objects. From the ordered set, 10–20 % of images of objects with the highest aperture (total) brightness of pixels are excluded. From the resulting set, the Q first images of the objects are selected.

In other words, the first image of an object in an ordered set selected to evaluate the blurring parameters has a number equal to $(0.1\div 0.2)Q_{\max}$, where $Q_{\max}$ is the total number of images of objects in the ordered set. The number of the last selected image of the object is $(0.1\div 0.2)Q_{\max}+Q$.

To assess the required image parameters, the Gaussian pixel model of the digital blurred image of the object was used in this work (1). The quality criterion for evaluating the parameters of a blurred image of the j-th object is the minimum of the sum of the squares of deviations between the experimental A^*_{ikj} and model $A_{ik\tau j}(\Theta^{over}_{\tau j})$ brightness of the pixels of the blurred image of the j-th object:

$$F_{\Delta A\tau}\left(\Theta^{over}_{\tau j}\right)=\sum_{i,k=\Omega_{Nobj}}\left(A^*_{ikj}-A_{ik\tau j}\left(\Theta^{over}_{\tau j}\right)\right)^2\underset{\Theta^{over}_{\tau j}}{\rightarrow}\min, \qquad (11)$$

where

$$\Theta^{over}_{\tau j}=\left(C_j,x_{\tau j}\left(\Theta^{over}_{\tau j}\right),y_{\tau j}\left(\Theta^{over}_{\tau j}\right),A_{Gj},\sigma_{Gj},\omega_j,d_j\right) \qquad (12)$$

is the vector of estimated parameters of the blurred image of the j-th object.

The estimated parameters of the blurred image of the j-th object are the position $x_{\tau j}(\Theta^{over}_{\tau j})$, $y_{\tau j}(\Theta^{over}_{\tau j})$, the model amplitude of the Gaussian A_{Gj}, the parameter of the shape σ_{Gj}, the average brightness value of the background substrate C_j, the angle between the direction of the blur of the object image and the abscissa axis Ω_j and the distance of movement of the object relative to frame d_j.

In turn, the length of movement d_j of the object relative to the frame (the path traveled by the object) during the exposure is determined by the expression:

$$d_j=\left(N_{Gj}-1\right)\sigma_{Gj}. \qquad (13)$$

To obtain the estimated parameters of the blurred image of the j-th object, the sum of the squares of deviations (11) is fed into the Levenberg-Marquardt algorithm (LMA) [2, 32].

5. 2. Determining the number of Gaussians of the investigated object's image and assessment of the initial approximation of the parameters of their form

The number of Gaussians of the image of the j-th object should be estimated separately from the LMA procedure [33]. The desired number of Gaussians N_{Gj} is selected from the range of values $\overline{N_{Gfirstj},N_{Gendj}}$. The initial $N_{Gfirstj}$ and resulting N_{Gendj} values of the range are determined by the following expressions:

$$N_{Gfirst}=\frac{d_{conj}-k_{first}\sigma_{Gj0}}{\sigma_{Gj0}}, \qquad (14)$$

$$N_{Gend}=\frac{d_{conj}-k_{end}\sigma_{Gj0}}{\sigma_{Gj0}}, \qquad (15)$$

where d_{conj} is the maximum size of the blurred image of the j-th object; σ_{Gj0} is the initial approximation of the Gaussian form parameter corresponding to the blurred image of the j-th object; k_{first}, k_{end} – coefficients that specify the minimum and maximum number of Gaussians.

During the studies and subsequent analysis, the values of the coefficients k_{first} and k_{end} were derived empirically and are equal to 7 and 2, respectively.

The maximum size d_{conj} of the image of the j-th object is considered to be equal to the maximum distance between the two pixels belonging to the outline of the image of the j-th object (Fig. 2).

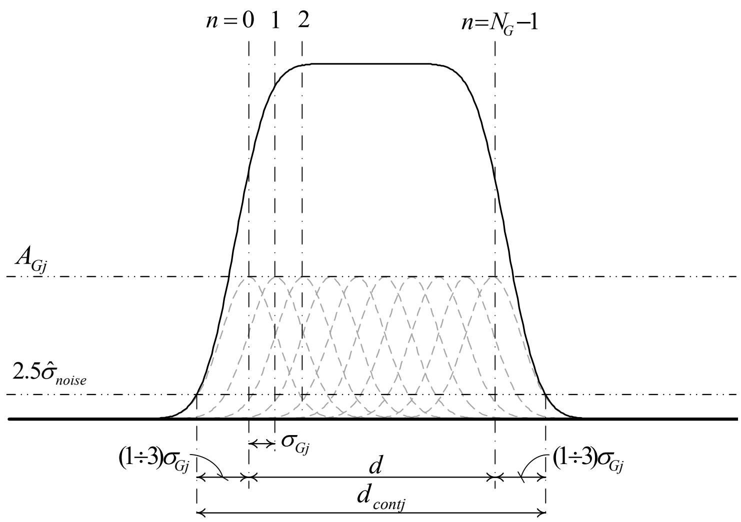


Fig. 2. Relationship of the length of movement d of the image of the j-th object relative to the frame with the maximum size of the contour d_{conj} of the image of the j-th object

For each number of Gaussians from a given range, a $\Theta_{\tau jn}^{over}$ vector of estimated parameters of the blurred image of the j-th object is determined. As the desired number of Gaussians N_{Gjmin} of the image of the j-th object, the one is chosen at which the sum of the squares of deviations $F_{\Delta A\tau}(\Theta_{\tau jn}^{over})$ (11) is minimal:

$$N_{Gj\min} = \arg\min_{N_{Gj}} F_{\Delta A\tau}\left(\Theta_{\tau jn}^{over}\right). \quad (16)$$

Next, it is necessary to evaluate the initial approximation of the Gaussian form parameters from the vector $\Theta_{\tau jn}^{over}$ of estimated parameters of the blurred image of the j-th object. To do this, the blurred image is divided along the semi-major axis into N_{segj} segments with a width of Δx_{seg} pixels (Fig. 3).

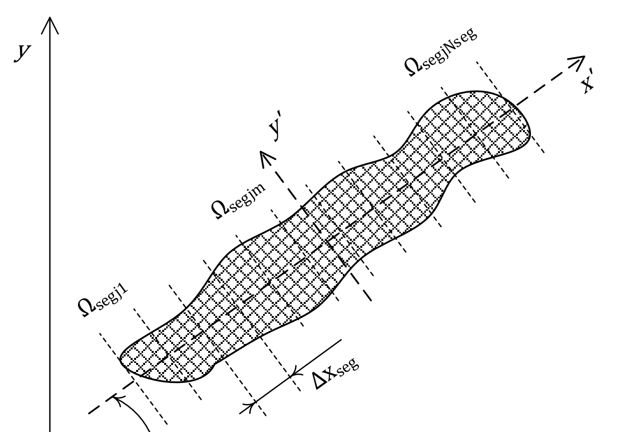


Fig. 3. Splitting an image of the j-th object into N_{seg} segments

Segmentation is carried out in the CS associated with the image of the object. The center of the CS coincides with the center and image of the object. The coordinates x_{cj}, y_{cj} of the center of the image are defined by the expressions:

$$x_{cj} = \frac{\sum_{i,k=\Omega_{Nobj}} x_{ij}\left(A_{ikj}^{*} - C_j\right)}{\sum_{i,k=\Omega_{Nobj}} \left(A_{ikj}^{*} - C_j\right)}, \quad (17)$$

$$y_{cj} = \frac{\sum_{i,k=\Omega_{Nobj}} y_{kj}\left(A_{ikj}^{*} - C_j\right)}{\sum_{i,k=\Omega_{Nobj}} \left(A_{ikj}^{*} - C_j\right)}. \quad (18)$$

The CS abscissa axe of the object image is rotated relative to the abscissa axis of the CS frame by an angle Ω_j:

$$\omega_j = \frac{1}{2}\arctan\frac{2m_{11}}{m_{20} - m_{02}}, \quad (19)$$

where

$$m_{20} = \sum_{i,k=\Omega_{Nobj}} \left(A_{ikj}^{*} - C_j\right)\left(x_{ij} - x_{cj}\right)^2, \quad (20)$$

$$m_{02} = \sum_{i,k=\Omega_{Nobj}} \left(A_{ikj}^{*} - C_j\right)\left(y_{kj} - y_{cj}\right)^2, \quad (21)$$

$$m_{11} = \sum_{i,k=\Omega_{Nobj}} \left(A_{ikj}^{*} - C_j\right)\left(x_{ij} - x_{cj}\right)\left(y_{kj} - y_{cj}\right) \quad (22)$$

are the second-order moments.

The number N_{seg} of segments is defined by the expression:

$$N_{segj} = \frac{d_{conj}}{\Delta x_{seg}}. \quad (23)$$

During the studies and subsequent analysis, the width of the segment Δx_{seg} was derived empirically and is equal to 3÷5 pixels.

The pixels of the blurred image of the j-th object are considered to belong to the m-th segment Ω_{segjm} if the condition is met:

$$m \cdot \Delta x_{seg} \le x_{ij} < (m+1) \cdot \Delta x_{seg}. \quad (24)$$

To obtain an estimate of the initial approximation of the Gaussian form parameter σ_{Gjm} in the m-th segment of the image of the j-th object, the sum of squares of deviations between the experimental A_{ikj}^{*} and model $A_{Sikmj}(\Theta_{\sigma m})$ brightness of the pixels of the m-th segment of the blurred image of the j-th object is added to the LMA procedure [33]:

$$F_{\Delta A\sigma}\left(\Theta_{\sigma m}\right) = \sum_{k=\Omega_{segjm}} \left(A_{ikj}^{*} - A_{Sikmj}\left(\Theta_{\sigma m}\right)\right)^2 \underset{\Theta_{\sigma m}}{\rightarrow} \min, \quad (25)$$

where $\Theta_{\sigma m}=(\sigma_{Gjm}, A_{Gjm}, y_{0jm})$ is the vector of the estimated parameters of the m-th segment of the blurred image of the j-th object;

$$A_{Sikmj}\left(\Theta_{\sigma m}\right) = A_{Gjm}\exp\left\{-\frac{1}{2\sigma_{Gjm}^2}\left[\left(y_{kj} - y_{0jm}\right)^2\right]\right\},$$

– model brightness of the pixels of the m-th segment of the blurred image of the j-th object; σ_{Gjm} – estimates of the initial approximation of the Gaussian form parameter in the m-th segment; A_{Gjm} – Gaussian model amplitude in the m-th segment; y_{0jm} is the coordinate of the reference center of the Gaussian in the m-th segment along the ordinate axis.

The estimation of the initial approximation of the Gaussian form parameter σ_{Gj0} of the blurred image of the j-th object is determined by the following expression:

$$\sigma_{Gj0} = \frac{1}{N_{segj}} \sum_{m=0}^{N_{segj}-1} \sigma_{Gjm}. \tag{26}$$

Thus, with the help of expression (26), the parameters of the blurring of the selected images of objects Q are evaluated.

5. 3. Computational method for matched filtration with analytical profile of a blurred digital image

The devised computational method of matched filtration with analytical profile of the blurred digital image of the investigated objects on CCD frames is the following sequence of actions:

1. Determine Q selected blurred images of objects to evaluate the blurring parameters according to condition (10).

2. For Q selected blurred images of objects, the initial approximation of the Gaussian form parameter σ_{Gj0} (26) is determined for the estimation of the blurring parameters.

3. For each blurred image of the object, the initial $N_{Gfirstj}$ and the final N_{Gendj} values of the range of changes in the number of Gaussians in the blurred image model of the j-th objects (14) and (15) are calculated.

4. Evaluate the vector $\Theta_{\tau j}^{over}$ of estimated parameters of the blurred image of the j-th object:

– for each number of Gaussians of the blurred image of the j-th object from a given range using the LMA procedure, the vector $\Theta_{\tau jn}^{over}$ of the estimated parameters is determined by minimizing the sum of the squares of deviations (11);

– select the optimal number of Gaussians N_{Gj} and the vector of estimated parameters $\Theta_{\tau j}^{over}$ for the blurred image of the j-th object in accordance with condition (16).

5. Calculate the average values of the parameters of the blurred image of the object on the digital frame from the Q selected images:

$$\sigma_G = \frac{1}{Q}\sum_{j=1}^{Q}\sigma_{Gj}, \tag{27}$$

$$d = \frac{1}{Q}\sum_{j=1}^{Q} d_j, \tag{28}$$

$$N_G = E\left[\frac{1}{Q}\sum_{j=1}^{Q} N_{Gj}\right]. \tag{29}$$

6. Determine the transfer function of the matched filter according to expression (9) from the calculated average values of the parameters σ_G, d, and N_G.

7. Determine a digital frame filtered by a matched filter as the inverse of DFT of its spectrum. The brightness of the ik-th pixel of the filtered digital frame is determined by the expression:

$$A_{out\ ik} = \sum_{u=0}^{N_{CCDx}-1}\left(\sum_{v=0}^{N_{CCDx}-1} S_{out\ uv} \exp\left[i2\pi\left(\frac{ux_{ij}}{N_{CCDx}} + \frac{vy_{kj}}{N_{CCDy}}\right)\right]\right), \tag{30}$$

where $S_{outuv}=S_{inuv}H_{Muv}$ is the uv-th harmonic of the filtered digital frame spectrum;

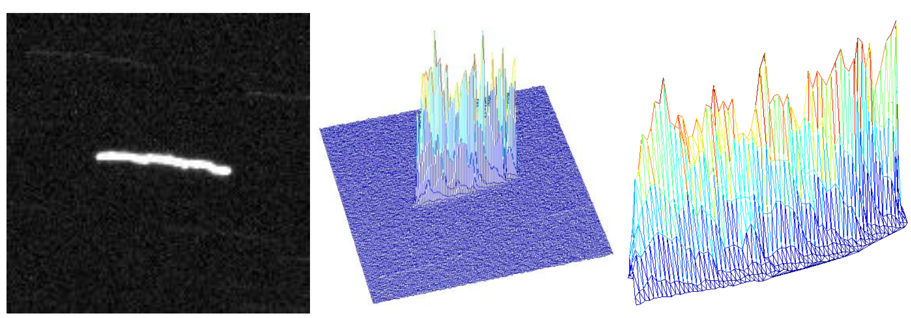
$$S_{in\ uv} = \sum_{u=0}^{N_{CCDx}-1}\left(\sum_{v=0}^{N_{CCDx}-1} A_{in\ ik} \exp\left[-i2\pi\left(\frac{ux_{ij}}{N_{CCDx}} + \frac{vy_{kj}}{N_{CCDy}}\right)\right]\right),$$

– uv-th harmonic of the digital frame spectrum; A_{inik} is the brightness of the ik-th pixel of the digital frame.

Below are examples of blurred initial and 3D images of the different investigated objects under conditions without diurnal tracking (Fig. 4) and in the case of failure of the diurnal tracking (Fig. 5).

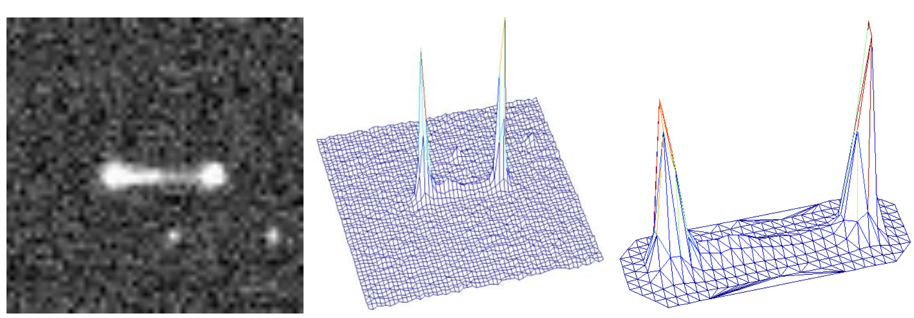
Fig. 6, 7 also show the corresponding results of processing the initial blurred images of the investigated objects using the computational method of matched filtration with analytical profile devised in this work.

The 3D images of blurred objects after applying the matched filtration in Fig. 6, 7 indicate the formation of a separate brightness peak, which indicates the success of the application of the devised computational method.



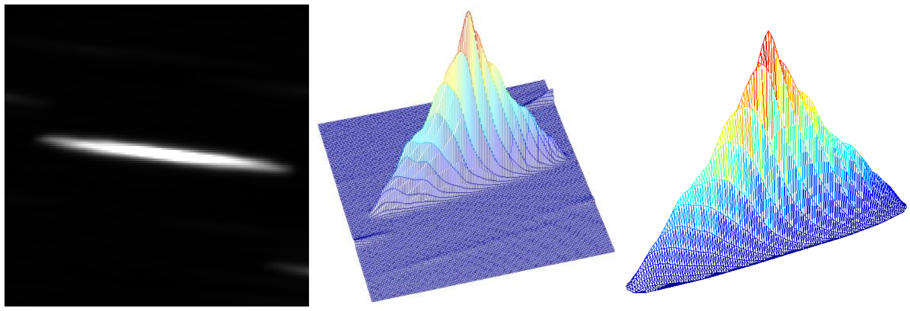
a *b* *c*

Fig. 4. Blurred image of an object without diurnal tracking: *a* – original image; *b* – 3D image of the CCD frame; *c* – 3D image of a blurred object



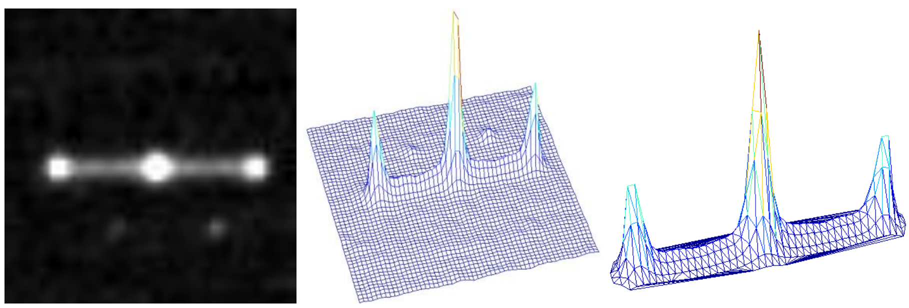
a *b* *c*

Fig. 5. Blurred image of the object when the diurnal tracking is failed: *a* – original image; *b* – 3D image of the CCD frame; *c* – 3D image of a blurred object



a *b* *c*

Fig. 6. The result of processing a blurred image of an object without diurnal tracking: *a* – original image; *b* – 3D image of the CCD frame; *c* – 3D image of a blurred object



a *b* *c*

Fig. 7. The result of processing the blurred image of the object when the diurnal tracking is failed: *a* – original image; *b* – 3D image of the CCD frame; *c* – 3D image of a blurred object

6. Discussion of results of investigating the matched filtration with analytical profile of the blurred digital image

Within the framework of this work, the possibility of processing blurred digital images of the investigated objects on a series of CCD frames was investigated. As a result of the studies, the typical conditions for the appearance of such blurred images were analyzed (Fig. 1). Existing methods of both preliminary and basic image processing and machine vision [11] were also analyzed. Basically, they are aimed at detecting objects [2], their movement, and assessing image and trajectory parameters [8]. Both classical and non-standard analytical and numerical methods of image processing were considered. However, the accuracy and quality of processing by such methods directly depended on the accuracy and quality of the original image of the investigated object in the CCD frame.

Therefore, within the framework of the CoLiTec project [34], studies were conducted on the application of the devised method of matched filtration with analytical profile. As our research showed, after applying the matched filtration, the image of the investigated object in 94 % of cases began to clearly show only one peak of brightness (Fig. 6, 7). This significantly affects the quality and accuracy of a number of tasks of data acquisition [35], image processing and machine vision [11] (detection of images of objects and assessment of their parameters [36], detection of the movement of objects, and assessment of the parameters of motion trajectories [19]). This indicator clearly indicates that our task has been successfully solved for the case of the analytically determined profile of the image of the investigated object.

Analysis of the results of our studies revealed an increase in the probability of identification of the investigated objects relative to those already known from the list of cataloged ones [37]. In addition, CPCD and the accuracy of estimating the image parameters increased by 15–20 % after the preliminary execution of the matched filtration as compared to the main processing method.

These results are primarily due to determining the transfer function of the matched filter (9) from the calculated average values of parameters (27) to (29). In addition, an important factor is the definition of the digital frame filtered by the matched filter as the reverse DFT of its spectrum (30).

The devised method was also successfully applied to the calculation and analysis of the orbital parameters of fragments formed after the collision of the first Indian anti-satellite (ASAT) rocket used against the Microsat-R satellite launched on January 24, 2019 [38]. The data that were tracked and provided by NASA were pre-processed by matched filtration.

Further, based on cloud computing, orbital parameters and pulse distribution were calculated 18 % more accurately for the fragments of the Microsat-R satellite [39], obtained after a collision with an ASAT rocket.

Thus, the devised computational method of matched filtration makes it possible to successfully select the investigated objects with a blurred image of a typical shape against the background of substrate noise. In addition, the use of the devised method makes it possible to improve the segmentation of images of reference objects on the CCD frame and reduce the number of false detections.

The application of the proposed computational method of matched filtration of a blurred digital image involves the task of setting an analytical model of an image of an object with known parameters. Therefore, the limitation of this study is the fact that for the whole variety of encountered images of objects on various digital frames, this task is difficult to solve. Due to unfavorable shooting conditions, the typical shape of the image of objects changes from frame-to-frame. Taking into consideration all random factors affecting the shape of the image of the object in the analytical model leads to its complication and an increase in the parameters of the model, which leads to large computational costs and a decrease in the accuracy of estimating the parameters of the model. The disadvantage of the study is precisely the fact that a typical analytical shape of the image was used.

Further research should be focused on the adaptation and application of the devised computational method of matched filtration for the case of an analytically unspecified profile of the blurred digital image of the investigated objects in CCD frames. Namely, when the shape of the image is determined not on the basis of standard analytical forms but on the basis of data obtained directly from CCD frames, which may contain various atypical shapes.

7. Conclusions

1. To devise a computational method for matched filtration of a blurred digital image, an analytical choice of a typical image shape was proposed. The form and special parameters of the transfer function of the matched filter for the blurred digital image of the investigated objects were also selected. Based on them, the H_{MF} transfer function of the matched filter was formed. This was followed by an OLS-evaluation of the required parameters of the blurred digital image.

2. Determining the number of Gaussians for the blurred image of the investigated object has made it possible to perform the most accurate assessment of the initial approximation of the parameters of their shape. During the studies and subsequent analysis, the values of the coefficients k_{first} and k_{end}, setting the minimum and maximum number of Gaussians, were derived empirically and are equal to 7 and 2, respectively. To estimate the initial approximation of the parameters of the blurred image, it is divided along the semi-major axis into N_{segj} segments with a width of Δx_{seg} pixels. Based on empirical studies, the width of the segment Δx_{seg} is 3÷5 pixels, which made it possible to estimate the initial approximation of the Gaussian form parameter σ_{Gj0} of the blurred image of the j-th object.

3. Owing to preliminary calculations, a computational method of matched filtration with analytical profile of the blurred digital image of the investigated objects on CCD frames was developed. The ultimate step of this method is to define the digital frame A_{out} filtered by a matched filter as the inverse of the DFT of its spectrum. The use of matched filtration for a blurred image of the investigated object in 94 % of cases makes it possible to clearly select only one peak of brightness, which significantly affects the quality and accuracy (an improvement of 15–20 %) of the subsequent execution of a few image processing and machine vision tasks.

Conflict of interest

The authors declare that they have no conflict of interest in relation to this research, whether financial, personal, authorship or otherwise, that could affect the research and its results presented in this paper.

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