*When solving many tasks related to mine workings, rock pressure management, development systems, support structures, the issues of strength and stability of rocks become relevant. Limitations and gaps are identified, emphasizing the need for further research and development of new methods for solving applied problems of elasticity theory.*

*It is of theoretical and practical interest to determine the influence of half-space geometry on the stressed state of the medium and to assess whether it would suffice, in this case, to confine oneself to radial stress when characterizing the stressed state. To build a mathematical model of the stressed state of the array, a complex variable function argument method was used. Based on the developed complex variable function argument method, the applied problem of mechanics on loading the wedge with a concentrated force in polar coordinates was solved.*

*A feature of the proposed approach is the introduction of tangential stresses with the need to meet boundary conditions along inclined faces. The introduction to the consideration of tangential stress shows that it cannot be neglected at a certain stage of the search for a solution. First of all, this is due to the half-space geometry, the angle at the apex, and the depth of the array. When changing the angle of the wedge, the interface surface changes fundamentally and can pass from a convex shape to a concave one. Simplification of the proposed expressions leads to a complete coincidence with the solutions by other authors obtained by the stress method, which indicates the reliability of the result reported here. This method may be advanced by complicating the half-space geometry, as well as loading, and by building a mathematical model for assessing the effect of tangent stresses on the strength and stability of soils*

*Keywords: soil arrays, soil mechanics, stressed state, argument functions, half-space*

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# SOLVING APPLIED PROBLEMS OF ELASTICITY THEORY IN GEOMECHANICS USING THE METHOD OF ARGUMENT FUNCTIONS OF A COMPLEX VARIABLE

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## 1. Introduction

When solving many tasks related to mine workings, rock pressure management, development systems, support structures, the issues of strength and stability of rocks become relevant.

The volume of geomechanics literature on these topics has increased rapidly over the past five years. This indicates a sharp increase in this field of research: determining the stressed-strained state of rocks based on research methods of the theory of elasticity, plasticity, destruction, dynamic theory of elasticity.

In the process of solving applied problems, difficulties often arise related to meeting boundary conditions. To obtain the desired result, the problem is simplified. Thus, in the problem of loading the wedge with a concentrated force solved by the method of stress functions, the boundary condition that there are no tangential stresses on the side surfaces of the wedge is met by disregarding it at all. At the same time, tangential stresses play a significant role in the mechanics of soils, especially in matters of stability and strength of arrays. In addition, the presence of tangential stresses in the solution is a determining element in assessing the adequacy of the result for normal stresses. There is a need for a theoretical and technical justification for the use of this factor to solve the problem of the theory of elasticity when loading a half-space of different geometries.

Research into development of new methods for solving applied problems of geomechanics is relevant.

## 2. Literature review and problem statement

Paper [1] reports the results of studies into the stressed state of the soils. The stability of outcrops of side rocks, the strength of the roof and soil are among the most important factors taken into account when choosing a development system, a technique for controlling mountain pressure, and support structures [2, 3]. It is noted that insufficient attention is paid to theoretical and practical developments that take into account the influence of half-space geometry on the distribution of internal loads, the effect of loads that cause the emergence of tangential stresses inside arrays.

Classical solutions to the elastic problem of loading a wedge with a concentrated force using the method of stress functions in Cartesian coordinates are known [4]. Paper [5] reports a solution to the problem of elasticity theory in Cartesian coordinates using a complex variable function argument method. The problem of elasticity theory using the complex variable function argument method in polar coordinates has been solved in [6].

In applied problems of soil mechanics, tangential stresses are given a leading place in determining the critical load on the soil, the exhaustive bearing capacity associated with the limit equilibrium [7].

The stressed state of the arrays can be estimated by lines of identical vertical stresses, horizontal and tangent, called isobars, spacers, and shifts, from which stresses can be determined depending on the width of the load [8].

The issues of anisotropy and heterogeneity of loading are considered in [9]. The stressed state in the soils is closely related to deformations in the soils [10]. In work [11], numerical modeling is performed using the method of two-dimensional discrete elements. The numerical model is calibrated to correspond to the results of the experiment on the dependence of stress-strain and permeability-deformation. The results of the simulation show that clusters of contact force chains occur near the shear band of the sample, and in the surrounding regions zones of sparse force circuits are found.

In work [12], in the study of the distribution of stresses and the concentration of deformation energy during the development of long faces near the dam, three-dimensional numerical modeling was carried out.

Although the researchers have proposed several hypotheses based, for example, on the stiffness of the system or the strength of the rock, the actual mechanisms are still debated. Until now, there is no universal acceptable method or criterion for predicting the stressed state, the concentration of deformation energy near the main geostructures; only mixed results have been achieved.

Work [13] considers the development of a closed solution to the stressed state around the wellbore drilled in anisotropic permeable rocks, taking into account the non-stationary action of the fluid flow. However, a simplified method of solving the equation of anisotropic hydraulic diffusion was adopted to this end.

In mechanics, problem statement equations in partial derivatives are often used, acceptable information about which is given in works [14, 15]. Their classification is presented, generalizing solutions to different types of equations are given, including one of the most common methods, the method of separating variables. It is a product of functions, each of which is a variable from one of the coordinates. It is often necessary to represent a solution by the product or sum of functions from several variables (as presented in D'Alembert's solutions). However, such solutions are of a particular simple nature, without a theoretical justification for their development and generalization.

In the mechanics of the continuous medium, numerical methods for solving problems have become widespread [16, 17]. One of them is the method of finite differences and the method based on the variational principles of mechanics, a finite-element method [18–20]. This is a modern trend in science that makes it possible to solve applied and fundamental problems of engineering and theoretical physics. However, there are difficulties with the possibility of using it, associated with the speed of the mathematical model to derive a result, the limited visibility of the method, and the development of great skills in organizing cycles and working out programs. In addition, each applied problem requires a lot of time to bring it to the capabilities of software.

New approaches to solving problems of continuum mechanics are proposed. The very fact of using differential relations in the theory of elasticity, but not applicable to these approaches, is shown in [21].

Paper [22] discusses an R-function that can be interpreted as an additional function that makes it possible to close a problem. It is a good example of how the introduction of additional functions, in the form of a function argument, can close the problem.

Study [23] proposes a method of the complex variable function but it is not applicable in this approach due to different boundary conditions. An example of solving a contact problem for an orthotropic half-space under given boundary conditions, but without the use of additional functions that can close the problem, is reported in [24]. The level of the problems under consideration is both fundamental and applied in nature [25, 26].

An option for overcoming the relevant difficulties at this stage, a more effective approach, may be to find not the solution itself but the conditions of its existence through the determining differential, integral relations. This approach is used in [27] when solving the dynamic problem of elasticity theory. The same approach is used in solving the problems of elasticity theory in Cartesian [28] and polar coordinates [29]. With the help of this approach, the closed problem of the theory of plasticity [30] was solved, which enhances the reliability of the method and its use in the problems of continuum mechanics.

Our literature review shows the approaches that can be used in solving the applied problem of the theory of elasticity; in this case, in geomechanics in relation to the angular half-space of different geometry. There is a contradiction: on the one hand, it is necessary to meet the boundary conditions on the side surface of the wedge (in the form of the absence of loading), and, on the other hand, to introduce into consideration the tangential stresses that determine the stability of the soil and its strength characteristics. In this regard, the complex variable function argument method is well suited. There are possibilities here in the form of Cauchy-Riemann relations, which correspond in the simplest version of the D'Alembert solution. There is a set of functions that can close the solution to the problem. It only remains, given this set, if there is a solution with tangential stresses, to determine the argument functions corresponding to the boundary conditions. With this approach, there is a real opportunity to diversify the influence of the geometry of the half-space on the stressed state of the soils, in which tangential stresses play a significant role. As a result of research and analysis, it becomes possible to evaluate, to show this influence not only from the point of view of the effect of normal but also tangential stresses in relation to polar coordinates.

## 3. The aim and objectives of the study

The purpose of this study is to determine the stressed state of ground massifs by the method of argument function of a complex variable, taking into account the influence of the geometry of the half-space in polar coordinates and the acting tangent stresses. This will make it possible to predict the result of the solution in relation to polar coordinates.

To accomplish the aim, the following tasks have been set:

– based on the method of argument functions, to devise and develop new approaches to solving the problems of elastic half-space for determining the stressed state of ground massifs;

– to demonstrate the reliability of the proposed approach in comparison with the solutions of the classical theory of elasticity and geomechanics;

– to construct mathematical models for calculating the stressed state of soil arrays of different shapes in polar coordinates, taking into account the action of tangent stresses;

– to determine the stressed state of arrays of different geometry of the surface half-space;

– to determine the influence of the geometry of the half-space on the force parameters of the ground massifs.

## 4. The study materials and methods

The object of this study is the stressed state of the angular semi-infinite space of the soils under the influence of concentrated force.

The hypothesis of the study assumes that the introduction of the tangential stress when meeting boundary conditions on the side faces will improve the accuracy of predicting the stressed state of the angular semi-infinite space of the soils described in polar coordinates.

Assumptions are made that the problem has been complicated by the introduction of tangential stresses in the absence of tangential stresses on the side faces.

For theoretical analysis, the approaches of the theory of elasticity are used, in which the methods of stress functions and methods of variational principles (a finite-element method) are most widely applied [2, 4–6]. The priority of the solution belongs to Mitchell:

$$\sigma_\rho = -k\cdot\frac{\cos\theta}{r},\quad \sigma_\theta = \tau_{r\theta} = 0. \tag{1}$$

The equality of zero of the last two stresses is dictated by the boundary conditions on the lateral surface of the wedge, although this is not always justified within space. Similar approaches are used in soil mechanics [7]. Separately, the flat problem of stress distribution under embankments, retaining walls is considered:

$$\sigma_z = \frac{P}{\pi}(\alpha + \sin\alpha\cdot\sin 2\beta),$$

$$\sigma_z = \frac{P}{\pi}(\alpha - \sin\alpha\cdot\sin 2\beta),$$

$$\tau = \frac{P}{\pi}(\sin\alpha\cdot\sin 2\beta).$$

Of interest is the distribution of stresses depending on natural weight, the magnitude of which increases according to a linear law and, at depth *z* from the surface, will be:

$$\sigma_z = \int_o^z \gamma(z)\,\mathrm{d}z,\quad \sigma_x = \sigma_y = \xi_o\cdot\sigma_z.$$

There are three phases of soil deformations: soil compaction; occurrence of shifts; bulging of the soil associated with the immersion of the loading surface. Noteworthy is the assessment of deformations - deformations of shifts and, consequently, tangential stresses. However, according to solution (1), there are no tangential stresses in the array.

The beginning of the occurrence of shifts at this point corresponds to the state of limiting equilibrium. From the resistance of soils to shear, it is known that the tangential stresses for loose soils are:

$$\tau_{sh} = \sigma\cdot\mathrm{tg}\phi,$$

for cohesive soils:

$$\tau_{sh} = \sigma\cdot\mathrm{tg}\phi + C,$$

where σ, φ is the normal stress and the angle of maximum friction.

Thus, in the applied problems of soil mechanics, when determining the critical load on the soil, tangential stresses are given a leading place.

With regard to the mechanics of soils, it is necessary to consider the problem of loading the angular elastic half-space with a concentrated force, Fig. 1.

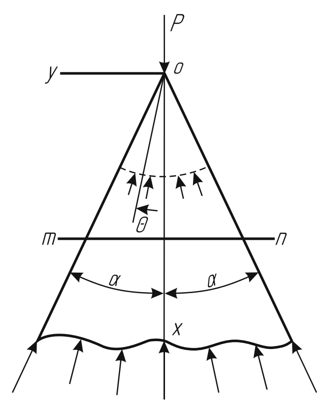



Fig. 1. Loading the elastic half-space with concentrated force

By adjusting the value of the angle α at the top, it is possible to achieve a change in the surface of the half-space from a wedge-like shape to a concave one, at the bottom of which a load *P* is applied. In this case, it becomes possible to accept the problem of loading the soil inside the array.

## 5. Results of investigating the use of the complex variable function argument method for solving problems of the theory of elasticity

**5. 1. Solving the problems of elastic half-space for determining the stressed state of soil arrays**

There is a known solution to the flat problem of continuum mechanics using the complex variable function argument method [27, 28]:

$$\sigma_\rho = \left[C_{\sigma1}\exp(\theta) - C_{\sigma2}\exp(-\theta)\right]\cos A\Phi + \sigma_0 + f(\phi),$$

$$\sigma_\phi = -\left[C_{\sigma1}\exp(\theta) - C_{\sigma2}\exp(-\theta)\right]\cos A\Phi + \sigma_0 + f(\rho), \quad (2)$$

$$\tau_{\rho\phi} = \left[C_{\sigma1}\exp(\theta) + C_{\sigma2}\exp(-\theta)\right]\sin A\Phi,$$

$$\sigma_0 = \pm\left[C_{\sigma1}\exp(\theta) - C_{\sigma2}\exp(-\theta)\right]\cos A\Phi,$$

at

$$\rho\theta_\rho = \mp A\Phi_\phi, \quad \theta_\phi = \pm\rho A\Phi_\rho,$$

$$\rho^2\theta_{\rho\rho} + \rho\theta_\rho + \theta_{\phi\phi} = 0, \quad \rho^2 A\Phi_{\rho\rho} + \rho A\Phi_\rho + A\Phi_{\phi\phi} = 0,$$

where AΦ, θ is the argument of the coordinate function; $\sigma_0$ is the average normal stress; $f(\varphi)$, $f(\rho)$, $C_{\sigma1}$, $C_{\sigma2}$ are the functions and integration constants that determine the stresses of different boundary conditions.

Works [4–6] consider solutions for loading the angular half-space (applying to the soils) with concentrated force (1), Fig. 1. The acceptance of the tangential stress as zero in solution (1) is caused by the need to meet the boundary conditions on the lateral surface of the wedge. At the same time, tangential stresses in soils affect their ultimate equilibrium, significantly changing the stressed state.

Solution (1) greatly simplifies the problem. The value of $k$ is determined by the equilibrium condition between the external and internal forces of the wedge:

$$\int_{-\alpha}^{\alpha} \sigma_\rho \rho d\theta \cos\theta = P.$$

After substituting the appropriate expression:

$$k = \frac{P}{\alpha + \frac{1}{2}\sin 2\alpha},$$

in (1), and integration, one can obtain:

$$\sigma_\rho = -\frac{P}{\alpha + \frac{1}{2}\sin 2\alpha}\cdot\frac{\cos\theta}{\rho}. \quad (3)$$

The simplified version in solution (3) makes it possible to use the result in applied problems. Basically, it correctly characterizes the distribution of stresses in the rock massif [7].

**5. 2. Comparing the results of solution with the classical theory of elasticity**

It is of interest to compare the results of solving problems by different methods, i.e., the method of stress functions (1) and the method of argument functions (2). Then it is possible to prove the legitimacy and reliability of the devised approach for solving applied, more complex problems. Let's show it.

Using the conditions for the existence of a solution to the problem, in the form of Cauchy-Riemann relations and Laplace equations, the argument functions AΦ and θ are determined. These are harmonic functions, of which there can be many. Consider some of them. As a result of solving the Laplace equations and meeting the Cauchy-Riemann relations, we have a number of dependences, including the coordinate functions below:

$$1.\ A\Phi_1 = \phi, \quad \theta_1 = -\ln\rho.$$

$$2.\ A\Phi_2 = AA_6\phi\ln\rho, \quad \theta_2 = -AA_6\left(\frac{\ln^2\rho}{2} - \frac{\phi^2}{2}\right), \quad (4)$$

where $AA_6$ are integration constants.

Consider the first option (4). Using expressions (1) and boundary conditions of the following form:

$$\phi = \alpha, \quad \rho = \rho_1, \quad A\Phi = A\Phi_1, \quad \theta = \theta_1, \quad \sigma_\phi = 0,$$

$$\sigma_\rho - \sigma_\phi = -2k_1, \quad f(\phi) = f(\rho) = 0, \quad C_{\sigma2} = 0, \quad (5)$$

we obtain expressions for stresses:

$$\sigma_\rho = -\frac{2k_1}{\cos\alpha}\exp\left(\ln\frac{\rho_1}{\rho}\right)\cos\phi = -\frac{2k_1}{\cos\alpha}\cdot\frac{\rho_1}{\rho}\cdot\cos\phi,$$

$$\sigma_\phi = 0, \quad (6)$$

$$\tau_{\rho\phi} = -\frac{k_1}{\cos\alpha}\exp\left(\ln\frac{\rho_1}{\rho}\right)\sin\phi = -\frac{k_1}{\cos\alpha}\frac{\rho_1}{\rho}\sin\phi,$$

where $A\Phi_1$, $\theta_1$ are the values of argument functions on the side surface of the wedge at $\rho=\rho_1$, $\rho_1$ is the fixed value of the current coordinate ρ relative to which the initial reference is made; $k_1$ is a fixed value of the difference of normal stresses on the lateral surface of the wedge at the minimum value of the radial coordinate.

One can see from (6) that at $\rho\to\infty$, $\sigma_\rho$ and $\tau_{\rho\varphi}\to 0$.

The value of the constant value $k_1$ is obtained from the equilibrium condition of the wedge, at $\rho=\rho_1$. Let's build an equilibrium equation for the upper cut off part of the wedge, as is done in [4–6]. After integration into parts, we can write:

$$k_1 = \frac{P\cos\alpha}{2\rho_1\left(\alpha + \frac{1}{2}\sin 2\alpha\right)}, \quad (7)$$

Substituting the value $k_1$ (7) in (6), we have:

$$\sigma_\rho = -\frac{P}{\left(\alpha + \frac{1}{2}\sin 2\alpha\right)}\frac{1}{\rho}\cos\phi,$$

$$\tau_{\rho\phi} = -\frac{P}{2\left(\alpha + \frac{1}{2}\sin 2\alpha\right)}\frac{1}{\rho}\sin\phi. \quad (8)$$

Comparing the normal stresses in formulas (3) and (8) we are convinced of their identity. However, the problem, due to tangential stresses, does not meet the boundary conditions (1). Let's continue the discussion and apply a more general approach that echoes work [28]. To achieve the result, we shall apply formulas (2), at $f(\varphi)=f(\rho)=0$. Next, we shall use boundary conditions (5), then:

$$\sigma_\rho = -\frac{2k_1}{\left\{1 + \exp\left[2(\theta_2 - \theta_1)\right]\right\}\cos A\Phi_1}\times$$

$$\times\exp(\theta - \theta_1)\left\{1 + \exp\left[2(\theta_2 - \theta)\right]\right\}\cos A\Phi,$$

$$\sigma_\phi = 0, \tag{9}$$

$$\tau_{\rho\phi} = -\frac{k_1}{\left\{1+\exp\left[2(\theta_2-\theta_1)\right]\right\}\cos A\Phi_1}\times$$
$$\times\exp(\theta-\theta_1)\left\{1-\exp\left[2(\theta_2-\theta)\right]\right\}\sin A\Phi.$$

If $\theta=\theta_2=\theta_1$, $A\Phi=A\Phi_1$ we obtain:

$$\sigma_\rho = -2k_1, \quad \sigma_\phi = 0, \quad \tau_{\rho\phi} = 0.$$

Substituting the first option (4) of the argument function for the exponent, we obtain:

$$\sigma_\rho = -\frac{2k_1}{\left[1+\left(\frac{\rho_1}{\rho_2}\right)^2\right]\cos A\Phi_1}\left(\frac{\rho_1}{\rho}\right)\left[1+\left(\frac{\rho}{\rho_2}\right)^2\right]\cos A\Phi,$$

$$\sigma_\phi = 0, \tag{10}$$

$$\tau_{\rho\phi} = -\frac{k_1}{\left[1+\left(\frac{\rho_1}{\rho_2}\right)^2\right]\cos A\Phi_1}\left(\frac{\rho_1}{\rho}\right)\left[1-\left(\frac{\rho}{\rho_2}\right)^2\right]\sin A\Phi.$$

If a given surface within a half-space is considered, then there is an equality $\rho=\rho_2$, therefore, (10) is transformed to the following form:

$$\sigma_\rho = -4\frac{k_1}{\left[1+\left(\frac{\rho_1}{\rho_2}\right)^2\right]\cos A\Phi_1}\left(\frac{\rho_1}{\rho_2}\right)\cos A\Phi,$$

$$\sigma_\phi = 0, \quad \tau_{\rho\phi} = 0. \tag{11}$$

When considering the surface $\rho_2=\rho_1$, we obtain an expression that can be used in the equilibrium condition and determine the value of $k_1$ that coincides with the value (7). Then (11) takes the form:

$$\sigma_\rho = -\frac{P}{\left(\alpha+\frac{1}{2}\sin 2\alpha\right)}\left(\frac{1}{\rho}\right)\cos\phi, \quad \sigma_\phi = 0, \quad \tau_{\rho\phi} = 0. \tag{12}$$

This completely corresponds to boundary conditions. The fundamental difference between the solution on argument functions is that the tangential stresses are not initially taken to be zero. In the proposed solution, expressions for determining the tangent stress (10) are obtained but the conditions for these functions under which the tangents meet the boundary conditions of the problem are shown.

**5. 3. Constructing a mathematical model of the stressed state of an array with a different geometry of the angular surface**

The absence of tangential stresses in the solution narrows the possibilities of the resulting solution in polar coordinates. To build a mathematical model of the stressed state of the array, we use the method of argument functions, and a more complex version (10), (option 2). This enhances the solution to the problem for a half-space in the presence of tangent stresses.

Consider the second variant of solution (10). The initial expressions in the solution to the problem are the dependences for the components of stress tensor (9), taking into account the argument functions shown.

In this case, the problem becomes more complicated because argument functions are no longer linear coordinate functions of one variable. This makes it possible to get additional argument changes to the argument functions on the side of the array and inside it. Let's use the same scheme for determining the operating parameters of formula (9) as in the previous case for expressions (10).

It is necessary to determine the constant values $\theta_1$, $\theta_2$ at the given values of the argument functions and boundary conditions. Boundary conditions are defined on the axis of symmetry of the wedge and on its lateral surface, i.e.:

$$\phi=0, \quad \rho=\rho_1, \quad A\Phi=A\Phi_1=0, \quad \theta=\theta_1,$$

$$\phi=\pm\alpha, \quad \rho=\rho_2, \quad A\Phi=A\Phi_2, \quad \theta=\theta_2. \tag{13}$$

Using boundary conditions of the following form

$$\phi=\pm\alpha, \quad \rho=\rho_2, \quad A\Phi_1=A\Phi_2=\alpha, \tag{14}$$

we define the constant $AA_6$. Using boundary conditions (13), (14) and substituting the obtained values into (9), we write:

$$\sigma_\rho = -\frac{2k_1}{1+\exp\left\{-\frac{1}{\ln\rho_2}\left[\left(\ln^2\rho_2-\ln^2\rho_1\right)-\alpha^2\right]\right\}}\times$$
$$\times\exp\left\{-\frac{1}{2\ln\rho_2}\left[\left(\ln^2\rho-\ln^2\rho_1\right)-\phi^2\right]\right\}\times$$
$$\times\left\langle 1+\exp\left\{-\frac{1}{\ln\rho_2}\left[\left(\ln^2\rho_2-\ln^2\rho\right)-\left(\alpha^2-\phi^2\right)\right]\right\}\right\rangle\times$$
$$\times\cos\left(\frac{1}{\ln\rho_2}\phi\ln\rho\right),$$

$$\sigma_\phi = 0, \tag{15}$$

$$\tau_{\rho\phi} = -\frac{k_1}{1+\exp\left\{-\frac{1}{\ln\rho_2}\left[\left(\ln^2\rho_2-\ln^2\rho_1\right)-\alpha^2\right]\right\}}\times$$
$$\times\exp\left\{-\frac{1}{2\ln\rho_2}\left[\left(\ln^2\rho-\ln^2\rho_1\right)-\phi^2\right]\right\}\times$$
$$\times\left\langle 1--\exp\left\{-\frac{1}{\ln\rho_2}\left[\left(\ln^2\rho_2-\ln^2\rho\right)-\left(\alpha^2-\phi^2\right)\right]\right\}\right\rangle\times$$
$$\times\sin\left(\frac{1}{\ln\rho_2}\phi\ln\rho\right). \tag{16}$$

Let's analyze to what extent the boundary conditions are met, i.e., provided that

$$\phi=0, \quad \rho=\rho_1=\rho_2, \quad \sigma_\rho=-2k_1, \quad \tau_{\rho\phi}=0,$$

on the lateral surface of the wedge, at $\varphi=\pm\alpha$, $\rho=\rho_2$:

$$\sigma_\rho = -4\frac{k_1}{1+\exp\left\{-\frac{1}{\ln\rho_2}\left[\left(\ln^2\rho_2-\ln^2\rho_1\right)-\alpha^2\right]\right\}}\times$$

$$\times\exp\left\{-\frac{1}{2\ln\rho_2}\left[\left(\ln^2\rho_2-\ln^2\rho_1\right)-\alpha^2\right]\right\}\cdot\cos(\alpha),$$

$$\sigma_\phi = 0,\quad \tau_{\rho\phi} = 0.$$

With the new argument functions, the boundary conditions are met in the same way as (11), (12).

Let's define the parameter $k_1$ for the new values of the argument function. To do this, we take a fixed value of the radial coordinate, $\rho=\rho_2=\rho_1$, then, from the equilibrium condition, we have:

$$2k_1 = \frac{P}{\rho_1\cdot I}. \tag{19}$$

Substituting (19) into (15) and (16), we obtain expressions that correspond to a certain extent to (12). In expressions (15), (16), we can proceed to relative stress values, for example:

$$\frac{\sigma_\rho}{2k_1} = -\frac{1}{1+\exp\left\{-\frac{1}{\ln\rho_2}\left[\left(\ln^2\rho_2-\ln^2\rho_1\right)-\alpha^2\right]\right\}}\times$$

$$\times\exp\left\{-\frac{1}{2\ln\rho_2}\left[\left(\ln^2\rho-\ln^2\rho_1\right)-\phi^2\right]\right\}\times$$

$$\times\left\langle 1+\exp\left\{-\frac{1}{\ln\rho_2}\left[\left(\ln^2\rho_2-\ln^2\rho\right)-\left(\alpha^2-\phi^2\right)\right]\right\}\right\rangle\times$$

$$\times\cos\left(\frac{1}{\ln\rho_2}\phi\ln\rho\right), \tag{20}$$

$$\sigma_\phi = 0,$$

$$\frac{\tau_{\rho\phi}}{k_1} = -\frac{1}{1+\exp\left\{-\frac{1}{\ln\rho_2}\left[\left(\ln^2\rho_2-\ln^2\rho_1\right)-\alpha^2\right]\right\}}\times$$

$$\times\exp\left\{-\frac{1}{2\ln\rho_2}\left[\left(\ln^2\rho-\ln^2\rho_1\right)-\phi^2\right]\right\}\times$$

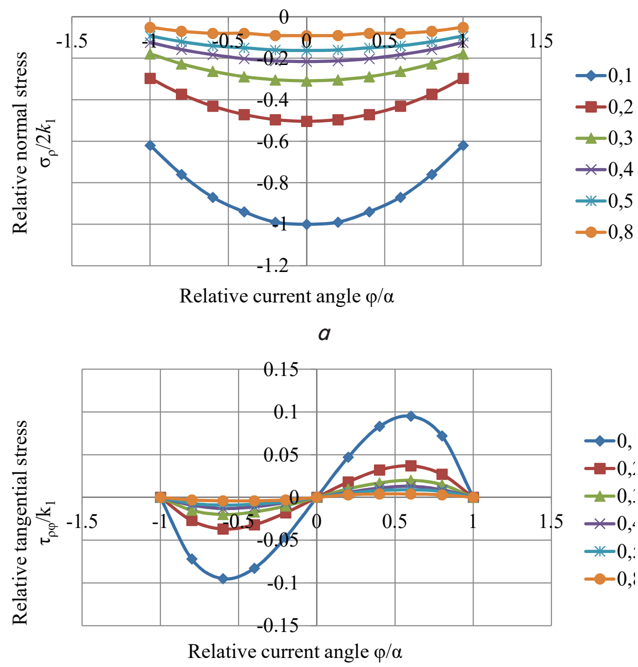
$$\times\left\langle 1-\exp\left\{-\frac{1}{\ln\rho_2}\left[\left(\ln^2\rho_2-\ln^2\rho\right)-\left(\alpha^2-\phi^2\right)\right]\right\}\right\rangle\times$$

$$\times\sin\left(\frac{1}{\ln\rho_2}\phi\ln\rho\right). \tag{21}$$

In calculations, it is more convenient to use relative values (20), (21), which simplifies the analysis. In the final formulas for determining the stressed state, we have the expression of tangential stresses. Comparing (2) and (3), we are convinced that a more complex dependence for tangential stresses in the second variant of the argument functions (10) has greater possibilities for taking into account boundary conditions.

**5. 4. Determining the stressed state of soils, a different geometry of the surface half-space**

To assess the stressed state of the array, expressions (20), (21) were used. The plots also indicate in relative quantities, where $\sigma_\rho/2k_1$ is the relative normal stress; $\tau_{\rho\varphi}/k_1$ is the relative tangential stress; $\varphi/\alpha$ – relative current angle; $\alpha/\pi$ is the relative angle at the apex of the wedge; $\rho/R$=−0.1...0.5; 0.8 are the relative values of the depression in the half-space. The following angle values are selected: α, (α/π); (π/6), (1/6); (π/4), (1/4); (π/3), (1/3); (π/2), (1/2); (2π/3), (2/3); (3π/4), (3/4); (5π/6), (5/6); π.Results of calculations according to formulas (20), (21), for angles α=π/3; π/2; 2π/3; π are shown in Fig. 2–5. For small angles, α=π/6, at the top of the wedge, there is a heterogeneity of the stressed state - minimal with the parameter $(\sigma_{max}-\sigma_{min})/2k_1$=0.082, which indicates, when reformatting, a slight increase in normal stresses to the middle of the wedge (−1.082). A tangent stress diagram is shown, which demonstrates the equality to zero of the tangential stresses on the side faces of the wedge. Their values at a given angle at the apex are insignificant and are within the relative values of 0.0002...0.012. Stresses in the half-space environment decrease when deepening into the array. With an increase in the angle of α to π/4, π/3, the unevenness of the stressed state of the medium increases, respectively, to 0.199 and 0.383, which leads to an increase in normal and tangential stresses. After reformatting, the maximum normal stresses take values of 1.199...–1.383. Tangential stresses are in the ranges, respectively, of 0.0007...0.035 and 0.003...0.095 (Fig. 2).




*b*

Fig. 2. Stress distribution depending on the angular coordinate and depth of development (α=π/3): *a* – distribution of normal stresses; *b* – tangent stress distribution

Noteworthy is the case illustrated in Fig. 3 (α=π/2). The geometry of the half-space division has changed fundamentally. At the same time, the normal stresses on the faces have zero values, the unevenness of the stressed state is maximum and reaches

$$\frac{\sigma_{max}-\sigma_{min}}{2k_1} = 1.$$

In this case, after reformatting, the stress on the axis takes a value equal to −2. Tangential stresses have increased and are within 0.0064...0,299.

In Fig. 4, 5, with the transition to the concave interface of the half-space, the stressed state of the elastic medium changes qualitatively and quantitatively.

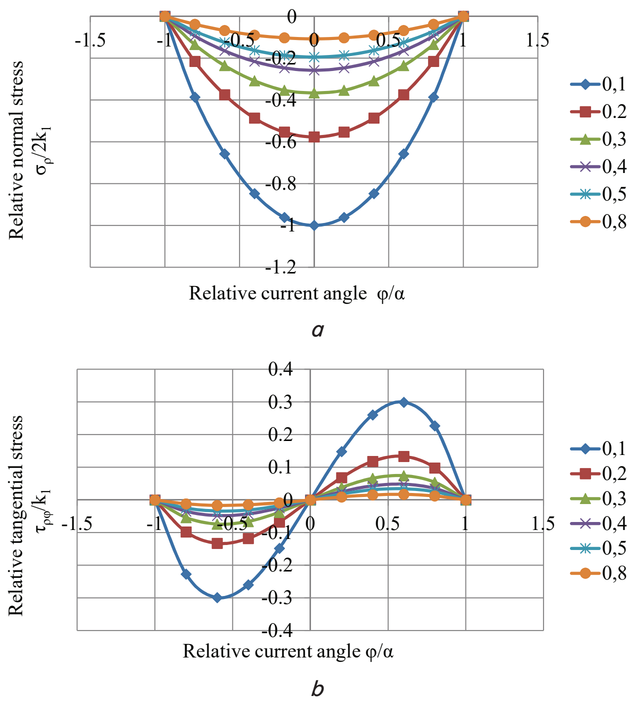



Fig. 3. Stress distribution depending on the angular coordinate and depth of development (α=π/2): *a* — distribution of normal stresses; *b* — tangent stress distribution

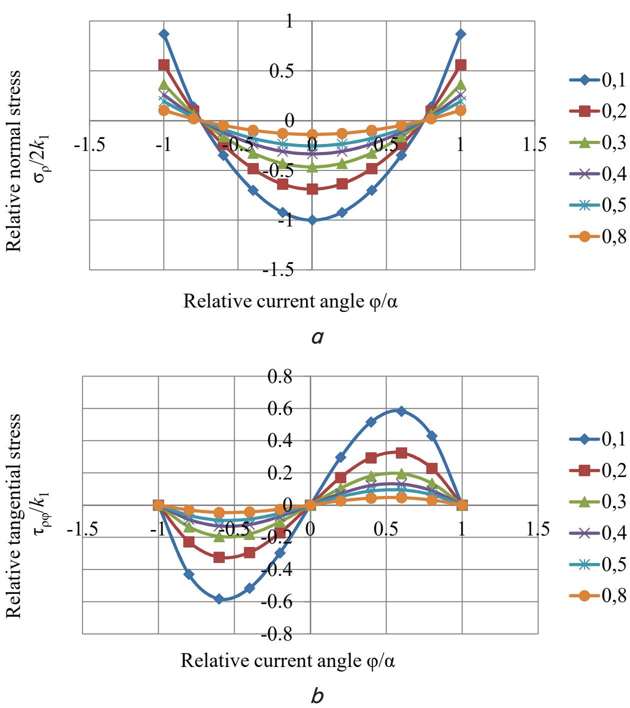



Fig. 4. Stress distribution depending on the angular coordinate and depth of development (α=2π/3): *a* — distribution of normal stresses; *b* — tangent stress distribution

The emergence of tensile stresses in layers of space exceeding the height of the position of the external force is observed. The change in the stressed state in the soils of the array led to an increase in tangential stresses in magnitude, within 0.033...0.583 and 0.051...0,713.

Fig. 5 shows the actually complete opening of the angles at the top of the wedge: we have a trench with a loaded bottom. There is an increase in the value, the zone of action of tensile stresses and the unevenness of the stressed state.

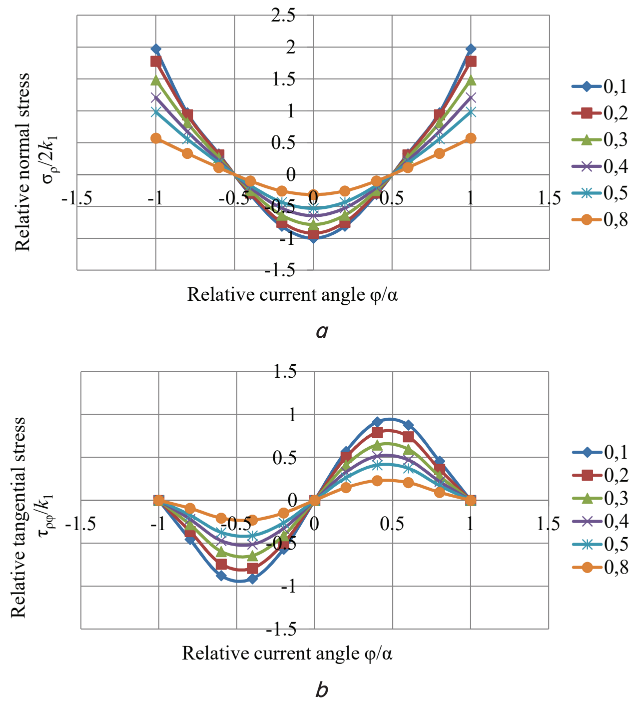



Fig. 5. Stress distribution depending on the angular coordinate and depth of development (α=π): *a* — distribution of normal stresses; *b* — tangent stress distribution

The maximum relative values of normal tensile stresses at α=5π/6 and α=π are 1.648 and 1.973.

Tangential stresses in both cases are maximum and are, respectively, within 0.044...0.756 and 0.067...0.913.

**5. 5. Influence of half-space geometry on the force parameters of soil massifs**

In all cases, there are general patterns of attenuation of the internal load with an increase in the depth of the array, which largely confirms the Saint-Venant principle. With a decrease in the magnitude of stresses, their unevenness decreases sharply, both in tensile zones and in compression zones. At the same time, the very nature of the force attenuation is different. Fig. 6 shows the change in the stressed state of the array from the angle at the apex and the depth of its occurrence.

The change in radial stress in the central zone at different values of the wedge angle is shown. Fig. 6 illustrates the influence on the stressed state of the half-space of the angle of the vertex of the wedge, that is, its geometry. Moreover, the effect is significant for normal and, most importantly, for tangential stresses. This is also evident from Fig. 2–5. Tangential stresses at small wedge angles are minimal, they can be neglected. This can explain the absence of tangential stresses in classical solutions to the problem for angular half-space by the method of stress functions [5]. However, a further increase in the angle leads to a significant increase in tangential stresses, especially for the internal location in the array of concentrated force. The change in relative tangential stress for a given range of angles is within 0.. (0.207...0.878). The change in relative normal stress for this range of angles is within (0.098...0.465)...(0.237...0.921). In the region of surface tensile stresses, normal and tangential stresses reach their maximum value. An increase in tangential stresses in the hanging layers of the half-space can adversely affect the exhaustive bearing capacity of the soils [7], which contributes to the emergence of lines of their maximum values, hence landslides.

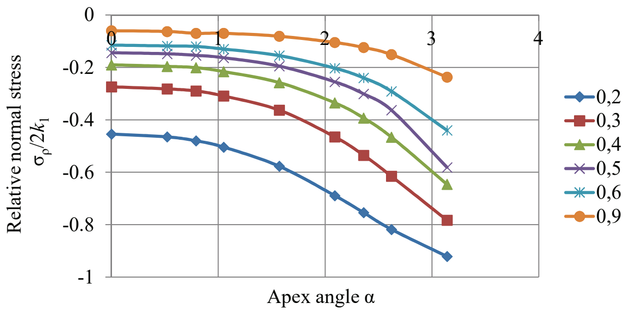



Fig. 6. Distribution of normal stresses depending on the angle at the apex and depth of development

## 6. Discussion of using the complex variable function argument method for solving problems of elasticity theory

Based on the developed complex variable function argument method, the applied problem of mechanics on loading the wedge with a concentrated force has been solved. New approaches have made it possible to use the result in the study of the stressed state of the half-space of different geometries. Such objects are formulas (15), (16), (20), (21), Fig. 2–6.

A feature of the proposed approach is the introduction of tangential stresses (16) to (21) into consideration with the need to meet boundary conditions along the inclined faces. Simplification of the proposed expressions leads to a complete coincidence of the result with the solutions by other authors [4–6], obtained by the stress method.

The problem of loading the wedge with concentrated force has been expanded. At the same time, the boundary conditions on the side surfaces are locally met. However, the loading of the half-space is more complex. For example, the action of the moment at the top of the wedge, or the joint action of the moment and the concentrated force. The proposed solution within the framework of the function argument method is limited to the application of the load in question since in other cases the stressed state of the half-space changes significantly.

The limitations of this study include the lack of assessment of the effect of tangential stresses in the angular zones of the half-space. In which, when loading soils along the trajectory of maximum tangential stresses, landslides (stability) occur.

This method may be advanced by complicating the geometry of half-space, loading, and the construction of a mathematical model for assessing the effect of tangent stresses on the strength and stability of soils.

## 7. Conclusions

1. A method has been proposed for solving problems of elastic half-space, a method of argument functions of a complex variable, for determining the stressed state of soil arrays with different geometry of half-space. The use of the inverse exponential function in the model has made it possible to abandon the simplification of the boundary conditions of the problem, to introduce tangential stresses into consideration with ensuring meeting them on the side faces.

2. The reliability of the proposed approach in comparison with the solutions of the classical theory of elasticity is shown. A particular solution to the proposed problem in comparison with the studies by other authors using the method of stress function showed their coincidence.

3. A mathematical model for calculating the stressed state of soil arrays of different shapes in polar coordinates, taking into account the current tangent stresses, has been built. This makes it possible to analyze and take into account the effect of tangent stresses on the loading parameters of soils, including their strength characteristics.

4. The stressed state of arrays of different geometries has been determined. When the angle of the wedge changes at the vertex relative to the point of application of the load, the stressed state of the half-space is characterized by a change in the sign (transition from compression to stress). The zone of stretching and action of the maximum tangential stresses is represented as the distance from the point of application of the force to the boundary of the half-space within the angle change at the apex from 90° to 180°.

5. It has been established that with a change in the geometry of the half-space, the stressed state of the soil massifs changes qualitatively and quantitatively. An analytical dependence makes it possible to identify the tensile stress zones that are most dangerous in the upper layers of the development, as well as a significant increase in tangent stresses in the hanging layers of the arrays within the angle change at the apex from 90° to 180°.

## Conflict of interests

The authors declare that they have no conflict of interest in relation to this research, whether financial, personal, authorship or otherwise, that could affect the research and its results presented in this paper.

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