UDC 539.374
DOI: 10.15587/1729-4061.2022.266166

IDENTIFICATION OF THE PATTERNS OF INFLUENCE THE NUMBER OF REINFORCING ELEMENTS AND THE INHOMOGENEITY PARAMETER OF THE SHELL MATERIAL ON FREQUENCIES OF A REINFORCED INHOMOGENEOUS ORTHOTROPIC SPHERICAL SHELL WITH A MEDIUM

Etibar Guliyev
PhD, Associate Professor*
Rashad Allahverdiyev
PhD, Associate Professor*
Qezale Kheyrabadi
Corresponding author
PhD, Associate Professor
Department of Mechanics
Azerbaijan State Oil and Industry University
Azadliq ave., 34, Baku, Azerbaijan, AZ1010
E-mail: qezale@mail.ru
*Department of Transport and Logistics
Azerbaijan University of Architecture and Construction
A. Sultanova str., 11, Baku, Azerbaijan, AZ1073

Spherical shells are used in many areas of the national economy. Spherical domes are widely used in the construction of various structures (technoparks, testing laboratories, entertainment complexes, reservoirs, etc.). They are also used in aircraft, ship structures, radar antennas and other structures. It is known that coatings have sufficient strength and durability even with a small thickness. However, to increase the working life of coatings, to ensure their long-term operation, as well as to increase their hardness, it is necessary to strengthen them on the surface or inside with rods. Sometimes it is possible to reduce the weight of the structure and save material consumption by strengthening it with. One of the advantages of these structures is that they give the maximum useful volume, being both load-bearing and enclosing structures. Checking the shells for stability is a priority task, since it is known that the shells, even with an insignificant thickness, have great strength and therefore their insufficient stability can be a criterion determining the bearing capacity. This article is devoted to identifying the regularities of the influence of the number of reinforcing elements and the inhomogeneity parameter of the shell material on the frequencies supported by an inhomogeneous orthotropic spherical shell with a medium. To solve the problem under consideration, the Hamilton-Ostrogradsky variation principle is applied. The frequency equation is constructed and implemented numerically. Such studies have not been considered for a reinforced spherical shell with a no uniform filler in thickness

Keywords: spherical shell, free oscillation, frequency, Legendre polynomial, spherical Bessel functions

Received date 21.07.2022
Accepted date 17.10.2022
Published date 30.10.2022

How to Cite: *Guliyev, E., Allahverdiyev, R., Kheyrabadi, Q. (2022). Identification of the patterns of influence the number of reinforcing elements and the inhomogeneity parameter of the shell material on frequencies of a reinforced inhomogeneous orthotropic spherical shell with a medium. Eastern-European Journal of Enterprise Technologies, 5 (7 (119)), 35–43. doi: https://doi.org/10.15587/1729-4061.2022.266166*

1. Introduction

Polymer, carbon, metal and organic composites and porous aluminum are widely used in various branches of technology. The main constant load on the shell is its own weight. To reduce this load, the use of lightweight porous materials with low bulk weight and other useful properties is promising, but they have low strength characteristics. To compensate for this disadvantage, technological heterogeneity is created, and to create heterogeneity in load-bearing structures, another material with high strength characteristics is introduced. As a result, technological heterogeneity appears in the design. In addition, to give greater rigidity, the thin-walled part of the shell is reinforced with ribs, which significantly increases its strength with a slight increase in the mass of the structure, even if the ribs have a small height. The use of polymer materials in engineering practice, in particular fiberglass, makes it mandatory to take into account the anisotropy of elastic properties in the study of low-frequency vibrations of shells. Therefore, there is a need to develop methods for calculating such inhomogeneous shells and to study the effect of inhomogeneity on the frequencies of their own oscillations. Algorithms are necessary for determining resonant frequencies that lead to the destruction of inhomogeneous shells.

In the future, it is assumed to solve this problem for a three-dimensional spherical shell, for reinforcements only in the meridional, as well as in parallel and meridional directions, for a visco-elastic medium.

Therefore, the study of vibrations of a parallel-reinforced spherical shell with an elastic filler, as the first step, opens the way to further study of vibrations of a three-dimensional spherical shell for reinforcements only in the meridional, as well as in parallel and meridional directions, filled viscoelastic medium, therefore, the considered problem is relevant.

2. Literature review and problem statement

In [1], an asymptotic analysis of the problem of free oscillations of an unsupported spherical shell with a viscoelastic filler was carried out. Work [2] is devoted to the study of vibrations of a longitudinally reinforced inhomogeneous orthotropic cylindrical shell with a flowing ideal fluid. The results of studies of vibrations of an inhomogeneous cylindrical shell supported by rings dynamically interacting with a moving ideal fluid are given in [3]. The paper [4] presents the results of studies of free vibrations of an inhomogeneous cylindrical shell supported by rings and dynamically in contact with a flowing liquid.

In the general regularities of the oscillation process of these shells in contact with the medium and liquid are studied based on an asymptotic analysis based on the smallness of the shell thickness, the nature of wave formation and the difference in the elastic properties of the shell materials, solid medium and liquid. Frequency equations are constructed for all the studied cases. Asymptotic formulas of natural frequencies of oscillations of the considered systems are obtained. Dynamic contact stresses between the shell-medium and the shell-liquid are investigated. Analytical formulas of the "bed" coefficients related to stresses on contact surfaces are derived. In all these questions, the heterogeneity of the coating material was not taken into account, and the solution, taking into account certain simplifications, was subjected to asymptotic analysis.

In [5] the problem of oscillations of inhomogeneous transversely reinforced cylindrical shells with an elastic medium was solved, and in [6] the problem of oscillation of inhomogeneous longitudinally reinforced cylindrical shells with a viscoelastic medium was investigated. There are practically no such studies for a reinforced spherical shell with a filler that is inhomogeneous in thickness.

The paper [7] presents a multi-level, mathematical model was used to estimate the stressed-strained state of a cylindrical reservoir with a defect in the wall shape in the form of a dent; the concentration of stresses in the defect zone was studied. The proper choice of the mathematical model was verified; it has been shown that the engineering assessment of the stressed-strained state of the wall of a cylindrical tank with the variable thickness could employ ratios for a cylindrical shell with a constant wall thickness. The spread of values is 2–10 %. This indicates the proper choice of the mathematical model, as well as the fact that it is possible, for an engineering assessment of the stressed-strained state of the wall of a cylindrical tank with variable thickness, to use the ratios for a cylindrical shell with a constant wall thickness. The stressed-strained state of the dent zone in the tank wall was numerically estimated, which proved the assumption of significant stress concentrations in the dent zone and indicated the determining effect on the concentration of stresses in the dent zone exerted by its geometric dimensions and its depth in particular.

The problem under consideration was solved by an approximate method, i. e. the tense-deformation state of a cylindrical coating of variable thickness was reduced to the tense-deformation state of a cylindrical coating of constant thickness, compared and confirmed by a numerical method in which the difference is very small. The article does not consider the possibility of manufacturing coatings from various materials (composite, anisotropic, heterogeneous, etc.).

The paper [8] presents paper determines the load on the load-bearing structure of a universal gondola car during the transportation of cargo with a temperature of 700 °C in it. It has been established that the maximum equivalent stresses, in this case, significantly exceed permissible ones. The maximum temperature of the cargo, at which the strength indicators of the carrying structure of the gondola do not exceed the permissible values, is 94 °C. At the same time, the temperature of the cargo transported in the cars by rail can be much higher. In this regard, in order to use gondola cars for the transportation of cargoes with high temperatures, it is possible to arrange them in heat-resistant containers of open type – flatcars. Therefore, in this study, a structure of the flatcar with convex walls has been proposed. Such configuration of the sidewalls makes it possible to increase the usable volume of the container by 8 % compared to the prototype. As a flatcar material, a composite with heat-resistant properties is used. In the article [8], the selected object was considered only as a structure, and its useful volume was selected in accordance with its shape. But it was noted that the structure is made only of composite material. Functional gradient (FG) materials included in a new class of composites resistant to high temperatures and pressure have not been mentioned anywhere, although serious research is being conducted in Ukraine.

This paper [9] has analyzed the use of fiberglass pipes in the body of the railroad embankment by a method of pushing them through the subgrade. A flat rod model has been improved for assessing the deformed state of the transport structure "embankment-fiberglass pipe" by a method of forces when replacing the cross-section of the pipe with a polygonal one. The analytical model accounts for the interaction between the pipe and soil of the railroad embankment. To this end, radial and tangential elastic ligaments are introduced into the estimation scheme, which make it possible to simulate elastic soil pressure, as well as friction forces that occur when the soil comes into contact with the pipe. It can be seen from the article that for the application of the method of forces in structural mechanics, the solution of the problem is simplified, i. e. the cross-section of the pipe is selected in the form of a polygon and the solution by the finite element method is completed. But it is not explained by which model the force of interaction between the soil and the pipe is chosen, this model resembles the classical or dynamic Pasternak model. Let's believe that for solving such engineering problems using the principle of variation would be more effective.

This paper [10] reports a study of the cement-concrete coating on bridges using FRP reinforcement. That has made it possible to design optimal structures by selecting the height for reinforcement arrangement in the layers of a roadbed in order to ensure strength characteristics. An engineering method for calculating a hard roadbed with composite reinforcement has been devised, which makes it possible to take into consideration its work both in a joint package of the structure with a slab and separately – when it exfoliates from the slab of the bridge's span structure. Underlying this research are effort-determining methods, estimation dependences from the theory of bending layered structures, as well as dependences from elasticity theory to assess the strength of materials for a roadbed. The consideration of shear strains when designing slabs has helped establish that the deflections according to the devises method were 1.4 times larger than those in the classical approach. The choice of composite reinforcement in this article ensures the durability of the road surface, since composite reinforcement coatings are resistant to dynamic loads.

In [11], a method for calculating the surface effect in piecewise homogeneous bodies with large deformations is proposed, based on the combined use of one-level applied and two-level frame theories. The applied theory is used for a micromechanical continuation of the solution of the problem, but the carcass theory is used in the final part of the loading path or directly at the final loading of the body. The implementation of the problem by the carcass theory for the body as a whole ends with solution of extreme problems using a highly gradient scheme for assemblies of structural blocks near boundary surfaces of the body. The rotation-caused development of configurations of cylinders reinforced with ring fibers is studied using this method and the model of a piecewise homogeneous medium. The results obtained by the carcass theory and the model of a piecewise homogeneous medium differed only slightly, confirming the high accuracy of the analysis using the two-level approach.

All this allows to assert that it is expedient to conduct a study on generalization of these models, i.e., a reinforced spherical shell filled with a solid medium is considered.

3. The aim and objectives of the study

The aim of the study is to find the natural or resonant frequencies of a reinforced inhomogeneous orthotropic spherical shell with a medium. At the request of practice, large spherical structures are required to install solar panels for the use of alternative energy sources. These constructs should also be able to store areas that can be used internally.

To achieve this aim, the following objectives are accomplished:

– build a physical and mathematical model of the considered design;

– build a functional describing the behavior of this structure during oscillatory processes;

– apply the Hamilton-Ostrogradsky variational principle, construct a frequency equation using Newton's method, solve a transcendental equation with respect to an unknown frequency and analyze the results obtained.

4. Materials and methods

In the presented article, a spherical shell in which there is a thickness inhomogeneity with a filler supports the selected object of research.

The vibrations of a spherical shell with a solid filler or a liquid without reinforcements and a reinforced spherical shell without a medium have been studied before us. In the proposed work, it is considered as a generalization of these models, i.e., a reinforced spherical shell filled with a solid medium is considered. The task is to study the natural oscillation frequencies of the marked structure. A similar technique can be used to solve other problems, namely, for various reinforcements and when the fillers are liquid. In the proposed article a frequency equation is constructed, which is a transcendental equation and implemented numerically for specific parameter values. From this frequency equation, it is possible to obtain the oscillation frequencies of the system for other values of the problem parameters.

The Hamilton-Ostrogradsky variation principle is used to construct the frequency equation. The Hamilton-Ostrogradsky variation principle is formulated as follows: the Hamilton action functional takes a stationary value on a real trajectory in the class of roundabout trajectories. To apply the Hamiltonian-Ostrogradsky variation principle, let's calculate the Lagrange function the difference between the kinetic energy and the potential energy of the system. Calculating the Hamilton action, the problem is reduced to finding a function in the presence of which the Hamilton action has a stationary value, i. e. the value at which the first variation of the action is zero. Based on this, let's make up the frequency equation.

Newton's method is used to find the roots of the obtained frequency equation. To apply Newton's method, let's first find a segment where the function takes values of different signs at the ends of the segment. Let's draw a tangent to the function at the point corresponding to the right end of the found segment. Let's find the intersection point of the tangent and the horizontal axis. Calculations are carried out until the specified accuracy is reached. To avoid calculating the derivative in which the formulas for calculating the roots of the equation are included, let's replace the derivative with an approximate value calculated from the two previous points.

5. The results of the study of free oscillations supported by an inhomogeneous spherical shell with a filler

5. 1. Construction of a physical and mathematical model, the considered design

A physic-mathematical model is constructed to study the free oscillations of a reinforced spherical shell with a no uniform thickness with a filler. To apply the Hamilton-Ostrogradsky variational principle and to account for heterogeneity in thickness of the spherical shell (Fig. 1) let's proceed from the three-dimensional functional [2–4, 12]:

$$U=\frac{1}{2}\int\int\int_{-\frac{h}{2}}^{\frac{h}{2}}\begin{pmatrix}\sigma_{11}\varepsilon_{11}+\sigma_{22}\varepsilon_{22}+\sigma_{12}\varepsilon_{12}+\\+\rho(z)\left(\frac{\partial u}{\partial t}\right)^2+\left(\frac{\partial v}{\partial t}\right)^2+\left(\frac{\partial w}{\partial t}\right)^2\end{pmatrix}\times$$
$$\times R\sin\psi dz d\varphi d\psi. \quad (1)$$

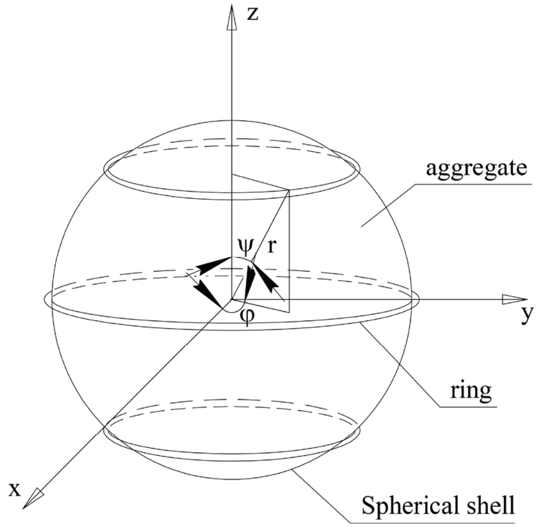

Fig. 1. Reinforced spherical shell with filler

In solving mechanical issues, the density of the material, unlike physics, is usually replaced by the elasticity module E, which is the mechanical indicator of the material, since all equations are included in the elasticity module, which is the mechanical dimension of the material [11]. In all cases, the Poisson coefficient is considered constant:

$$\sigma_{11}=b_{11}(z)\varepsilon_{11}+b_{12}(z)\varepsilon_{22},$$
$$\sigma_{22}=b_{12}(z)\varepsilon_{11}+b_{22}(z)\varepsilon_{22}, \quad (2)$$
$$\sigma_{12}=b_{66}(z)\varepsilon_{12}.$$

In spherical system of coordinates the strain sensor components are of the form [8]:

$$\varepsilon_{11}=\frac{\partial u_r}{\partial r},\quad \varepsilon_{22}=\frac{1}{r\sin\psi}\frac{\partial u_\phi}{\partial\phi}+\frac{1}{r}u_r+\frac{1}{\mathrm{tg}\psi}u_\psi,$$
$$\varepsilon_{33}=\frac{\partial u_\psi}{\partial\psi}+\frac{1}{r}u_r,\quad \varepsilon_{12}=\left(\frac{1}{r\sin\psi}\frac{\partial u_r}{\partial\phi}+\frac{\partial u_\psi}{\partial r}-\frac{1}{r}u_\phi\right). \quad (3)$$

Using expressions (2), (3) it is possible to express the energy functional of the considered reinforced inhomogeneous spherical shell by tense and relative deformations.

5. 2. Construction of a functional describing the behavior of this structure during oscillatory processes

To construct the frequency equation, a functional describing the behavior of this construction during oscillatory processes is first constructed.

Given that $E=E(z)$, $\rho=\rho(z)$, it is possible to write [5, 6] down the expressions of energy, that is, the functional of the shell and edges.

Taking into account $E=E(z)$, $\rho=\rho(z)$ it is possible to write [5, 6]:

$$V=\left[\begin{array}{l}\frac{1}{2}\int\int\left\{\begin{array}{l}\tilde{b}_{11}\tilde{\varepsilon}_{11}^2+2\tilde{b}_{12}\varepsilon_{11}\varepsilon_{22}+\\+\tilde{b}_{22}\tilde{\varepsilon}_{22}^2+\tilde{b}_{66}\varepsilon_{12}^2\end{array}\right\}+\\+\int\int\tilde{\rho}\left(\left(\frac{\partial u}{\partial t}\right)^2+\left(\frac{\partial\vartheta}{\partial t}\right)^2+\left(\frac{\partial w}{\partial t}\right)^2\right)\end{array}\right]dxdy, \quad (4)$$

here,

$$\tilde{b}_{11}=\int_{-\frac{h}{2}}^{\frac{h}{2}}b_{11}(z)\mathrm{d}z,\quad \tilde{b}_{12}=\int_{-\frac{h}{2}}^{\frac{h}{2}}b_{12}(z)\mathrm{d}z,\quad \tilde{b}_{22}=\int_{-\frac{h}{2}}^{\frac{h}{2}}b_{22}(z)\mathrm{d}z.$$

$$\tilde{b}_{66}=\int_{-\frac{h}{2}}^{\frac{h}{2}}b_{66}(z)\mathrm{d}z,\ b_{11}(z)=\frac{E_1(z)}{1-v_1v_2},\ b_{22}(z)=\frac{E_2(z)}{1-v_1v_2},$$

$$b_{12}(z)=\frac{v_2E_1(z)}{1-v_1v_2}=\frac{v_1E_2(z)}{1-v_1v_2},$$

$b_{66}(z)=G_{12}(z)=G(z)$ – are the main elasticity module of the orthotropic material, $\rho=\int_{-h}^{h}\rho(z)\mathrm{d}z$.

Let's assume that the elasticity constants $b_{11}(z)$; $b_{12}(z)$; $b_{22}(z)$; $b_{66}(z)$, $b_{11}(z)$... change by the law $b_{ij}=\tilde{b}_{ij}\left(1+\alpha\frac{z}{h}\right)$, i, j=1, 2 with respect to the coordinate z. Here α is a heterogeneity parameter, h is the thickness of the cylindrical shell, b_{ij} are elastic constants belonging to the homogeneous cylindrical shell.

Let's write the potential energy of the j-th ring [12]:

$$\Pi_j=\frac{R}{2}\int_0^{2\pi}\left(\begin{array}{l}\tilde{E}_jF_j\left(\frac{\partial\vartheta_j}{\partial y}-\frac{w_j}{R}\right)^2+\\+\tilde{E}_jJ_{xj}\left(\frac{\partial^2w_j}{\partial x^2}+\frac{w_j}{R}\right)^2+\\+\tilde{E}_jJ_{zj}\left(\frac{\partial^2u_i}{\partial y^2}-\frac{\varphi_{kpj}}{R}\right)^2+\\+\tilde{G}_jJ_{kpj}\left(\frac{\partial^2\varphi_{kpi}}{\partial y}-\frac{1}{R}\frac{\partial u_j}{\partial y}\right)^2\end{array}\right). \quad (5)$$

The kinetic energy of the j-th ring is in the following form [9]:

$$K_j=\rho_jF_j\int_0^{2\pi}\left[\begin{array}{l}\left(\frac{\partial u_j}{\partial t}\right)^2+\left(\frac{\partial\vartheta_j}{\partial t}\right)^2+\\+\left(\frac{\partial w_j}{\partial t}\right)^2+\frac{J_{kpj}}{F_j}\left(\frac{\partial\varphi_{kpj}}{\partial t}\right)^2\end{array}\right]\mathrm{d}\varphi. \quad (6)$$

In the expressions (2)–(6) expressions $u, v, w, \tilde{E}_j, F_j, I_{xj}, I_{(kp.j)}$ are the displacements of the cover in accordance with the signs, the geometric dimensions and the cross-section area of the ring j and the moments of inertia, $\boldsymbol{G_j}$ is the elasticity modulus of the j-th ring in shift, $u_j, v_{j,}, w_j$ – are displacements of the points of the j-th ring, ρ_j is the density of the material of the j-th ring, are the angles of rotation and twisting of the cross section of the j – rod, through the displacements of the shell are expressed as follows:

$$\varphi_j(y)=\varphi_2(x_j,y)=-\left(\frac{\partial w}{\partial y}+\frac{\vartheta}{r}\right)\bigg|_{x=x_j},$$

$$\varphi_{kpj}(y)=\varphi_1(x_j,y)=-\frac{\partial w}{\partial x}\bigg|_{x=x_j}.$$

The work done by the force effective on a spherical shell as viewed from the medium in the displacements of the shell is written as follows:

$$A_0=-\int_0^{2\pi}\int_0^{\pi}(q_ru+q_\varphi\vartheta+q_\psi w)\mathrm{d}\varphi\,\mathrm{d}\psi. \quad (7)$$

Here q_r, q_φ, q_ψ are stress vector components and are determined from the equations of motion of a filler in displacements [1, 5, 6]:

$$a_i^2 grad\ div\vec{s}-a_e^2 rot\ rot\vec{s}+\frac{\vec{b}}{\rho_s}=\frac{\partial^2\vec{s}}{\partial t^2}, \quad (8)$$

$$a_t=\sqrt{\frac{\lambda_s+2\mu_s}{\rho_s}},\quad a_e=\sqrt{\frac{\mu_s}{\rho_s}},$$

where λ_s and μ_s are Lame elasticity module, are dimensional velocities.

The expressions for projections of displacements of the filler have the form [12]:

$$s_r = a_0 Y^c,$$

$$s_\beta = a_1 \frac{\partial}{\partial\varphi} Y^c - a_2 \frac{k}{\sin\varphi} Y^s, \qquad (9)$$

$$s_\varphi = -a_2 \frac{\partial}{\partial\varphi} Y^c - a_1 \frac{k}{\sin\varphi} Y^s.$$

Here let's accept the following denotations

$$a_0 = \begin{bmatrix} \frac{1}{\mu_e}\left\{A_1 \frac{\partial}{\partial r}\left[j_n(\mu_e r)\right] + A_2 \frac{\partial}{\partial r}\left[n_n(\mu_e r)\right]\right\} + \\ + \frac{\tilde{\lambda}_n}{\mu_t r}\left[C_1 j_n(\mu_t r) + C_2 n_n(\mu_t r)\right] \end{bmatrix},$$

$$a_1 = \begin{bmatrix} \frac{1}{\mu_e r}\left[A_1 j_n(\mu_e r) + A_2 n_n n_n(\mu_e r)\right] + \\ + \frac{1}{\mu_t r}\left\{C_1 \frac{\partial}{\partial r}\left[r j_n(\mu_t r)\right] + C_2 \frac{\partial}{\partial r}\left[r n_n(\mu_t r)\right]\right\} \end{bmatrix}.$$

(n=0, 1, 2, ...)

$$a_0 = \frac{1}{\mu_e}\left\{A_1 \frac{\partial}{\partial r}\left[j_n(\mu_e r)\right] + A_2 \frac{\partial}{\partial r}\left[n_n(\mu_e r)\right]\right\} + $$
$$+ \frac{\tilde{\lambda}_n}{\mu_t r}\left[C_1 j_n(\mu_t r) + C_2 n_n(\mu_t r)\right],$$

$$a_1 = \frac{1}{\mu_e r}\left[A_1 j_n(\mu_e r) + A_2 n_n n_n(\mu_e r)\right] +$$
$$+ \frac{1}{\mu_t r}\left\{C_1 \frac{\partial}{\partial r}\left[r j_n(\mu_t r)\right] + C_2 \frac{\partial}{\partial r}\left[r n_n(\mu_t r)\right]\right\}.$$

(n=0, 1, 2, ...)

$$a_2 = B_1 j_n(\mu_t r) + B_2 n_n(\mu_t r),$$

$$Y^s = \sin k\psi P_n^k(\cos\varphi), \quad \tilde{\lambda} = m(m+1), \qquad (10)$$

$$\lambda_t = m(m+1), \quad (m = 0,1,2,\ldots).$$

$$Y_{kn}^c = P_n^k(\cos\varphi)\cos k\psi, \quad Y_{kn}^s = P_n^k(\cos\varphi)\sin k\psi,$$

where

$$P_n^k(x) = \left(1-x^2\right)^{k/2} \frac{d^k\left[P_n(x)\right]}{dx^k}(x = \cos\varphi),$$

$$P_n(x) = \begin{bmatrix} \frac{1}{2^n n!}\frac{d}{dx}\left(x^2-1\right)^n = \\ = \frac{1\cdot 3\cdot 3\cdot\ldots\cdot(2n-1)}{n!}\times \\ \times\left[x^n - \frac{n(n-1)}{2(2n-1)}x^{n-1} + \ldots\right] \end{bmatrix},$$

k is the half of the amount of nodal meridians or the amount of nodal median planes. The function $P_n(\cos\varphi)$ is said to be n-th order Legendre polynomial. Let's give some first values of the indicated functions:

$$P_0(x) = 1, \ P_1(x) = x, \ P_2(x) = \frac{1}{2}\left(3x^2 - 1\right),$$

$$P_n^0 = P_n(x),$$

$$P_1^1(x) = \left(1-x^2\right)^{1/2}, \ P_2^1(x) = 3x\left(1-x^2\right)^{1/2},$$

$$P_2^2(x) = 3\left(1-x^2\right).$$

The function $P_0(x)$ is a constant and characterizes vibrations only along the radius. The function $P_1(\cos\varphi)$ describes the movement of the oscillating sphere, where the plane $\beta = \frac{\pi}{2}$ is a nodal plane. In the general case, the n-th order Legendre polynomial has nodal planes (circumferences) where $j_n(z)$ and $n_n(z)$ are Bessel and Neimann spherical functions connected with cylindrical functions by the formulas [9, 10]:

$$j_n(z) = \sqrt{\frac{p}{2z}}B, \ n_n(z) = \sqrt{\frac{p}{2z}}B^*,$$

$$B = J_{n+\frac{1}{2}}(z), \quad B^* = N_{n+\frac{1}{2}}(z).$$

In the case of a solid filler, the expression (10) for a_i (i=0, 1.2) is simplified:

$$a_0 = \frac{1}{\mu_e}\left\{A_1 \frac{\partial}{\partial r}\left[j_n(\mu_e r)\right]\right\} + \frac{\tilde{\lambda}_n j_n(\mu_t r)}{\mu_t r}C_1,$$

$$a_1 = \frac{j_n(\mu_e r)}{\mu_e r}A_1 + \frac{1}{\mu_t r}\left\{C_1 \frac{\partial}{\partial r}\left[r j_n(\mu_t r)\right]\right\}, \qquad (11)$$

$$a_2 = B_1 j_n(\mu_t r).$$

The coefficients determined by expression (11) are included in the displacements of the common points on the contact surface $r=R$ of the coating with the liquid medium.

5. 3. Construction of the frequency equation of oscillation using the variation principle and solution by the Newton method

Applying the Hamilton-Ostrogradsky variation principle, the frequency equation is constructed.

As a result, for the total energy of an ortotropic, spherical shell inhomogeneous in thickness, stiffened with rings and dynamically contacting with soil let's obtain

$$J = V + \sum_{j=1}^{k_2}\left(\Pi_j + K_j\right) + A_0, \qquad (12)$$

here k_2 is the amount of rings.

The total energy of the system under investigation is supplemented with contact and boundary conditions. At the junction of the filler and the shell, conditions for the equality of displacement components:

$$s_\varphi = u, \ s_\psi = \vartheta, \ s_r = w, \ (r = R), \qquad (13)$$

and equality of pressures

$$q_r = -\sigma_{rr}, \ q_\varphi = -\sigma_{r\varphi}, \ q_\psi = -\sigma_{r\psi}, \ (r = R), \qquad (14)$$

are set, where

$$\sigma_{rr}=\lambda_s\, div\,\vec{s}+2\mu_s\frac{\partial s_r}{\partial r},$$

$$\sigma_{r\beta}=\mu_s\left[\frac{1}{r}\frac{\partial s_r}{\partial\phi}+r\frac{\partial}{\partial r}\left(\frac{s_\phi}{r}\right)\right], \quad (15)$$

$$\sigma_{r\beta}=\mu_s\left[\frac{1}{r\sin\beta}\frac{\partial s_r}{\partial\theta}+r\frac{\partial}{\partial r}\left(\frac{s_\phi}{r}\right)\right].$$

The conditions of rigid contact between the shell and the rods are considered to be satisfied:

$$u_j(y)=u(x_j,y),\;\; \upsilon_j(y)=\upsilon(x_j,y), \quad (16)$$
$$w_j(y)=w(x_j,y),\;\; \varphi_j(y)=\varphi_2(x_j,y),\;\; \varphi_{kpj}(y)=\varphi_1(x_j,y).$$

Using the stress formulas (15) and projections of displacements of the filler (9), let's obtain:

$$\sigma_{r\varphi}=b_1\frac{\partial}{\partial\varphi}Y^s-b_2\frac{1}{\sin\varphi}Y^s,$$

$$\sigma_{r\psi}=-b_2\frac{\partial}{\partial\varphi}Y^s-b_1\frac{1}{\sin\varphi}Y^s, \quad (17)$$

$$\sigma_{rr}=b_0Y^c,$$

where

$$b_1=\mu_s\left[r\frac{\partial}{\partial r}\frac{a_1}{r}+\frac{a_0}{r}\right],\;\; b_2=\mu_s r\frac{\partial}{\partial r}\left(\frac{a_2}{r}\right),$$

$$b_0=\lambda_s\left[\frac{1}{r^2}\frac{\partial}{\partial r}(a_0r^2)-\tilde{\lambda}_n\frac{a_1}{r}\right]+2\mu_s\frac{\partial a_0}{\partial r}.$$

Using the expressions for a_0, a_1, a_2, let's transform b_0, b_1 and b_2 As a result:

$$b_0=\left|\begin{matrix}\left\{\begin{matrix}-\frac{4\mu_s}{\mu_e r}\frac{\partial j_n(\mu_e r)}{\partial r}+\\ +\frac{2\mu_s}{\mu_e r^2}\left[\lambda_n-\left(\frac{\lambda_s}{2\mu_s}+1\right)\mu_e^2r^2j_n(\mu_e r)\right]\end{matrix}\right\}\\ A_1+\frac{2\mu_s}{\mu_t r}\lambda_n\left[\frac{\partial j_n(\mu_t r)}{\partial r}-\frac{j_n(\mu_t r)}{r}\right]C_1\end{matrix}\right|,$$

$$b_1=\left\{\begin{matrix}\frac{2\mu_s}{\mu_e}\left[-\frac{1}{r^2}j_n(\mu_e r)+\frac{1}{r}\frac{\partial j_n(\mu_e r)}{\partial r}\right]A_1+\\ +\frac{\mu_s}{\mu_t}\left[\begin{matrix}\left(\frac{2\lambda_n}{r^2}-\mu_t^2-\frac{2}{r^2}\right)j_n(\mu_e r)-\\ -\frac{2}{r}\frac{\partial j_n(\mu_t r)}{\partial r}\end{matrix}\right]C_1\end{matrix}\right\}, \quad (18)$$

$$b_2=\mu_s\left[\frac{\partial j_n(\mu_t r)}{\partial r}-\frac{j_n(\mu_t r)}{r}\right]B_1.$$

Substituting in (17) and (18) $r=R$ let's find the contact stresses q_r, q_φ, q_ψ:

$$q_{r\varphi}=-b_1\frac{\partial}{\partial\varphi}Y^c+b_2\frac{k}{\sin\varphi}Y^s,$$

$$q_{r\psi}=b_2\frac{\partial}{\partial\varphi}Y^c+b_1\frac{k}{\sin\varphi}Y^s, \quad (19)$$

$$q_{rr}=-b_0Y^c,$$

where

$$b_1=\left(\begin{matrix}\frac{2\mu_s}{\mu_e}\left[-\frac{1}{r^2}j_n(\mu_e R)+\frac{1}{R}\frac{\partial j_n(\mu_e R)}{\partial r}\right]A_1+\\ +\frac{\mu_s}{\mu_t}\left[\begin{matrix}\left(\frac{2\lambda_n}{r^2}-\mu_t^2-\frac{2}{R^2}\right)j_n(\mu_t R)-\\ -\frac{2}{r}\frac{\partial j_n(\mu_t R)}{\partial r}\end{matrix}\right]\end{matrix}\right),$$

$$b_2=\mu_s\left[\frac{\partial j_n(\mu_t R)}{\partial r}-\frac{j_n(\mu_t R)}{r}\right]B_1,$$

$$b_0=\left(\begin{matrix}\left\{\begin{matrix}-\frac{4\mu_s}{\mu_e r}\frac{\partial j_n(\mu_e R)}{\partial r}+\\ +\frac{2\mu_s}{\mu_e R^2}\left[\lambda_n-\left(\frac{\lambda_s}{2\mu_s}+1\right)\mu_e^2R^2j_n(\mu_e R)\right]\end{matrix}\right\}A+\\ +\frac{2\mu_s}{\mu_t R}\lambda_n\left[\frac{\partial j_n(\mu_t R)}{\partial r}-\frac{j_n(\mu_t R)}{R}\right]C_1\end{matrix}\right).$$

Let's present displacements of the spherical shell, that we will need for further investigation of the problem of free vibrations of a stiffened heterogeneous spherical shell with an elastic filler.

Finding natural frequency of free vibrations of a stiffened heterogeneous spherical shell with filler for all possible forms of vibrations is very difficult. Therefore, those displacements of the shell are chosen that are in good argument with the movement of the filler. These solutions are obtained from (9) if in (10) in the functions $a_i(r)$ i=0, 1, 2 let's set r=R:

$$u=a_i(r)\frac{\partial}{\partial\varphi}Y^C-a_j(r)\frac{k}{\sin\varphi}Y^S,$$

$$\vartheta=-a_j(r)\frac{\partial}{\partial\varphi}Y^C-a_i(r)\frac{k}{\sin\varphi}Y^S, \quad (20)$$

$$w=a_0(r)Y^C,$$

where

$$a_0(R)=\frac{1}{\mu_e}\left\{A_1\frac{\partial}{\partial r}\left[j_n(\mu_e R)\right]\right\}+\frac{\tilde{\lambda}_n j_n(\mu_t R)}{\mu_t r}C_1,$$

$$a_1=\frac{j_n(\mu_e R)}{\mu_e r}A_1+\frac{1}{\mu_t r}\left\{C_1\frac{\partial}{\partial r}\left[Rj_n(\mu_t R)\right]\right\},$$

$$a_2(R)=B_1j_n(\mu_t R).$$

Using q_r, q_φ, q_ψ stress vector components (19) and the expression (20) for displacements, it is possible to calculate the work A_0:

$$A_0=\int_0^{2\pi}\int_0^{\pi}\left\{\begin{aligned}&\left[\begin{aligned}&b_{01}a_{01}\left(Y^c\right)^2+b_{11}a_{11}\left(\frac{\partial}{\partial\varphi}Y^c\right)^2+\\&+b_{11}a_{11}\left(\frac{k}{\sin\varphi}Y^c\right)^2\end{aligned}\right]A_1^2+\\&+\left[\begin{aligned}&b_{02}a_{02}\left(Y^c\right)^2+b_{12}a_{12}\left(\frac{\partial}{\partial\varphi}Y^c\right)^2+\\&+b_{12}a_{12}\left(\frac{k}{\sin\varphi}Y^c\right)^2\end{aligned}\right]C_1^2+\\&+b_{21}j_n\left(\mu_tR\right)\left(\frac{k}{\sin\varphi}Y^c\right)^2B_1^2+\\&+\left[\begin{aligned}&\left(b_{01}a_{02}+b_{02}a_{01}\right)\left(Y^c\right)^2+\\&+\left(b_{11}a_{12}+b_{12}a_{11}\right)\left(\frac{\partial}{\partial\varphi}Y^c\right)^2+\\&+\left(b_{11}a_{12}+b_{12}a_{11}\right)\left(\frac{k}{\sin\varphi}Y^c\right)^2\end{aligned}\right]A_1C_1+\\&+\left[\begin{aligned}&-2b_{11}j_n\left(\mu_tR\right)\frac{k}{\sin\varphi}Y^c\frac{\partial}{\partial\varphi}Y^c+\\&+b_{21}a_{11}Y^c\frac{\partial}{\partial\varphi}\end{aligned}\right]A_1B_1+\\&+\left[\begin{aligned}&-2b_{21}j_n\left(\mu_tR\right)\frac{k}{\sin\varphi}Y^c\frac{\partial}{\partial\varphi}Y^c+\\&+b_{21}a_{12}Y^c\frac{\partial}{\partial\varphi}\end{aligned}\right]C_1B_1\end{aligned}\right\},\qquad(21)$$

where

$$b_{11}=\frac{2\mu_s}{\mu_e}\left[-\frac{1}{r^2}j_n\left(\mu_eR\right)+\frac{1}{R}\frac{\partial j_n\left(\mu_eR\right)}{\partial r}\right],$$

$$b_{12}=\frac{\mu_s}{\mu_t}\left[\left(\frac{2\lambda_n}{r^2}-\mu_t^2-\frac{2}{R^2}\right)j_n\left(\mu_tR\right)-\frac{2}{R}\frac{\partial j_n\left(\mu_tR\right)}{\partial r}\right],$$

$$b_{21}=\mu_s\left(\frac{\partial j_n\left(\mu_tR\right)}{\partial r}-\frac{j_n\left(\mu_tR\right)}{r}\right),$$

$$b_{01}=\left\{-\frac{4\mu_s}{\mu_er}\frac{\partial j_n\left(\mu_eR\right)}{\partial r}+\frac{2\mu_s}{\mu_eR^2}\left[\lambda_n-\left(\frac{\lambda_s}{2\mu_s}+1\right)\mu_e^2R^2j_n\left(\mu_eR\right)\right]\right\},$$

$$b_{02}=\frac{2\mu_s}{\mu_tR}\lambda_n\left[\frac{\partial j_n\left(\mu_tR\right)}{\partial r}-\frac{j_n\left(\mu_tR\right)}{R}\right],$$

$$a_{01}=\frac{1}{\mu_e}\left\{\frac{\partial}{\partial r}\left[j_n\left(\mu_eR\right)\right]\right\},\quad a_{02}=\frac{\tilde{\lambda}_nj_n\left(\mu_tR\right)}{\mu_tr},$$

$$a_{11}=\frac{j_n\left(\mu_eR\right)}{\mu_er},\quad a_{12}=\frac{1}{\mu_tr}\left\{\frac{\partial}{\partial r}\left[Rj_n\left(\mu_tR\right)\right]\right\}.$$

Using the movements of the shell (20) and the conditions of rigid contact between the shell and the rods (16) for the movement of the rod points, let's obtain:

$$\begin{aligned}&u_j=a_1(R)\frac{\partial}{\partial\varphi}Y^{cj}-a_2(R)\frac{k}{\sin\varphi}Y^{sj},\\&\vartheta_j=-a_2(R)\frac{\partial}{\partial\varphi}Y^{cj}-a_1(R)\frac{k}{\sin\varphi}Y^{sj},\\&w_j=a_o(R)Y^{cj},\end{aligned}\qquad(22)$$

where

$$Y^{sj}=\sin k\psi_jP_n^k(\cos\varphi),\quad Y^{cj}=\cos k\psi_jP_n^k(\cos\varphi).$$

Formulas for the movement of rod points (20) and formulas for kinetically and potential energy (5) and (6) can be computed for the energies of the supporting elements. Because of the unwieldiness of the expression obtained, it is not listed here.

Substituting (18) in (11), let's obtain a function with respect to the unknown constants A_1, B_1, C_1:

$$J=a_1A_1^2+a_2B_1^2+a_3C_1^2+a_4A_1B_1+a_5B_1C_1+a_6A_1C_1.\qquad(23)$$

From the stationery condition $\frac{\partial J}{\partial A_1}=0,\ \frac{\partial J}{\partial B_1}=0,\ \frac{\partial J}{\partial C_1}=0$ with using (21) a system of homogeneous algebraic equations to find constants A_1, B_1, C_1 using the stationary condition, i. e. by equating the specific derivatives obtained by unknown to zero.

$$\begin{cases}2a_1A_1+a_4B_1+a_6C_1=0,\\a_4A_1+2a_2B_1+a_5C_1=0,\\a_6A_1+a_5B_1+2a_3C_1=0.\end{cases}\qquad(24)$$

If the system is homogeneous, for the subsistence of its nontrivial solution let's equate the main determinants to zero. As a result, let's obtain a frequency equation:

$$\begin{vmatrix}2a_1&a_4&a_6\\a_4&2a_2&a_5\\a_6&a_5&2a_3\end{vmatrix}=0,\qquad(25)$$

The equation (20) was implemented for the following values of data [7]:

$$b_{11}=18.3\text{ GPa},\ \tilde{b}_{12}=2.77\text{ GPa},\ \tilde{b}_{22}=25.2\text{ GPa},$$

$$\tilde{b}_{66}=3.5\text{ GPa},\quad\tilde{\rho}=\rho_j=1850\,\frac{\text{kg}}{\text{m}^3},$$

$$\tilde{E}_j=6.67\cdot\frac{10^9\text{N}}{\text{m}^2}\cdot v_1=v_2=0.35,$$

$$\alpha=0.4,\quad h=0.45\text{ mm},\quad h_j=0.45\text{ mm},\quad F_j=5.2\text{ mm}^2,$$

$$R=160\text{ cm},\quad n=8,\quad I_{kp.j}=0.23\text{ mm}^4,$$

$$I_{xj}5.1\text{ mm}^4,\quad I_{zj}=1.3\text{ mm}^4,\quad\delta=\tilde{\delta}_{11}/\tilde{b}_{22}.$$

The results are calculations are in Fig. 2 in the depedence of the periodicity parameter

$$\omega_1=\omega/\omega_0,\quad\omega_0=\sqrt{\frac{\tilde{b}_{11}}{\left(1-v_1^2\right)r^2\tilde{\rho}}},$$

of the system on the amount of rings k_1, in Fig. 3 in the form of depedence on the heterogeneity parameter α for spherical shell with different property orthotropic material. As can be seen from Fig. 2 increasing the amount of rings, natural vibrations of the construction increases. The case $k_1=0$ corresponds to smooth spherical shell and from the graph for this case $\omega_1=0.87$.

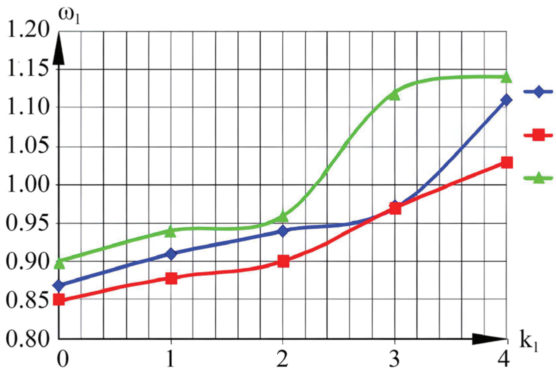

Fig. 2. Dependence of the number of frequency parameter on the number of rings k_1

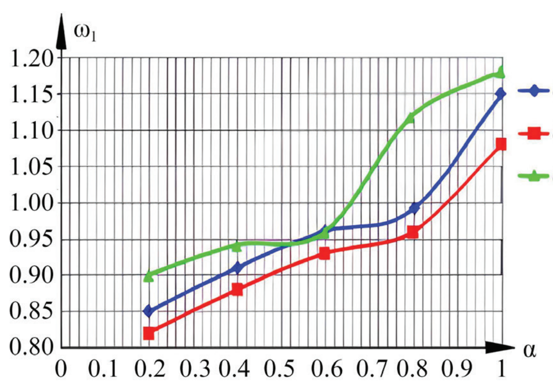

Fig. 3. Dependence of the frequency parameter on heterogeneity parameter α

The curves in the figure show that strengthening the orthotropic properties of the material of the spherical shell, compared to the isotropic case the natural vibrations frequency of the construction increases. Calculations show that increase in the value of heterogeneity parameter and strengthening the orthotropic property of the spherical shell causes increase in natural vibrations frequency of the construction. This is explained by the fact that increase in the value of heterogeneity parameter, increases rigidity parameters of the material of the spherical shell.

6. Discussion of the results of the study of free oscillations of an inhomogeneous spherical shell with a filler

The results obtained include the correct physical and mathematical factors.

The existing results refers to the reinforced shell without filler and the unsupported shell with filler. In contrast to these works [12], the proposed work considers a reinforced shell.

Research is carried out for a thin shell and a purely elastic filler.

In studies, it is assumed that the reinforced shell is homogeneous and the filler is purely elastic. These disadvantages can be overcome by assuming the filler is viscoelastic.

These studies can be developed for a thick shell with various fasteners and with a viscoelastic filler. However, taking into account the more real properties of the shell and filler materials, one may encounter mathematical difficulties.

For the first time in such a formulation of the problem, the frequency equation of the considered construction is constructed. The task is multiparametric. Among these parameters, the number of reinforcing elements and the heterogeneity parameter are selected.

Solving the frequency equation, the found eigenvalues, i. e. resonant frequencies, leading to the destruction of the structure as a whole. To avoid these frequencies, reinforcement of the spherical shell is assumed. As can be seen from Fig. 2, with an increase in the number of rings, the specific natural frequencies of the structure increase. The curves in the figure show that as the orthotropy property of the spherical coating material increases, the specific natural frequencies of the structure increase in comparison with the isotropic state. Fig. 2 allows to select the number of reinforcing elements. It can be seen from Fig. 3 that an increase in the value of the inhomogeneity parameter and an increase in the orthotropic property of the spherical coating material leads to an increase in the frequency of self-oscillation of the structure. This is explained by the fact that an increase in the value of the inhomogeneity parameter increases the stiffness parameters of the spherical coating material.

The problem for a smooth spherical shell with a filler and for a reinforced spherical shell without a filler has been solved. But it is not solved for a reinforced spherical shell with a filler.

Here it is assumed that the shell is thin, the reinforcing elements are thin and homogeneous, and the filler is elastic and homogeneous.

The disadvantage relates to the selection of solutions to the vector equation describing the behavior of the placeholder. To find another solution to this equation that makes it impossible to solve the considered problem.

This research can be developed for a three-dimensional shell, for inhomogeneous reinforcing elements and for a viscoelastic and inhomogeneous filler. It is mathematically difficult to solve these problems in an exact formulation. They can be solved only for approximate models.

7. Conclusions

1. A physical and mathematical model has been constructed to study free oscillations of a reinforced spherical shell with a filler that is inhomogeneous in thickness. Unlike the known models, the studied model additionally takes into account the heterogeneity of the shell material and reinforcement. There are models – a homogeneous spherical shell with a filler and a reinforced homogeneous spherical shell without a medium. The model under study takes into account both heterogeneity, environment, and reinforcements at the same time.

2. The found frequencies are simultaneously resonant. To avoid these frequencies, reinforcement of the spherical shell is assumed. As the number of rings increases, the special

dance frequencies of the structure increase. As the orthotropy properties of the spherical coating material increase, the specific vibration frequencies of the structure increase in comparison with the isotropic state.

3. Calculations for positive inhomogeneity parameters show that, the increase in the price of the inhomogeneity parameter and the strengthening of the orthotroph properties of the material of the spherical coating lead to an increase in the specific oscillation frequencies of the structure. This is due to the fact that the increase in the price of the non-uniformity parameter increases the hardness of the material of the spherical coating. At negative values of the uniformity parameter, the specific oscillation frequencies are less than the isotropic frequencies of the material of the spherical coating. In other words, it is possible to strengthen or weaken the structure by creating inhomogeneity.

Conflict of interest

The authors declare that they have no conflict of interest in relation to this research, whether financial, personal, authorship or otherwise, that could affect the research and its results presented in this paper.

Acknowledgments

We express our gratitude to the Professor of the Department of Mechanics of the Azerbaijan University of Architecture and Construction, Doctor of Mathematical Sciences, R. Iskandarov for his help in constructing a computational model of free oscillations of a reinforced inhomogeneous spherical shell with a filler.

References

1. Latifov, F. S. (1988). Asymptotic analysis of the problem of free vibrations of a spherical shell with viscous elastic filler. In: Mechanics of deformable solids. Baku: Elm, 159–167.
2. Latifov, F. S., Agayev, R. N. (2018). Oscillations of Longitudinally Reinforced Heterogeneous Orthotropic Cylindrical Shell with Flowing Liquid. International Journal on Technical and Physical Problems of Engineering (IJTPE), 10 (34), 41–45. Available at: https://www.iotpe.com/IJTPE/IJTPE-2018/IJTPE-Issue34-Vol10-No1-Mar2018/7-IJTPE-Issue34-Vol10-No1-Mar2018-pp41-45.pdf
3. Agayev, R. (2020). Vibrations of an inhomogeneous cylindrical shell dynamically interacting with motive fluid and stiffened with rings. International Journal on Technical and Physical Problems of Engineering (IJTPE), 12 (42), 31–34. Available at: https://www.iotpe.com/IJTPE/IJTPE-2020/IJTPE-Issue42-Vol12-No1-Mar2020/6-IJTPE-Issue42-Vol12-No1-Mar2020-pp31-34.pdf
4. Agayev, R. N. (2020). Vibrations of a heterogeneous cylindrical shell stiffened with rings and dynamically contacting with flowing fluid. International Journal on Technical and Physical Problems of Engineering (IJTPE), 12 (43), 83–87. Available at: https://www.iotpe.com/IJTPE/IJTPE-2020/IJTPE-Issue43-Vol12-No2-Jun2020/15-IJTPE-Issue43-Vol12-No2-Jun2020-pp83-87.pdf
5. Iskanderov, R. A., Tabatabaei, J. M. (2020). Vibrations of fluid-filled inhomogeneous cylindrical shells strengthened with lateral ribs. International Journal on Technical and Physical Problems of Engineering (IJTPE), 12 (42), 121–125. Available at: https://www.iotpe.com/IJTPE/IJTPE-2020/IJTPE-Issue42-Vol12-No1-Mar2020/23-IJTPE-Issue42-Vol12-No1-Mar2020-pp121-125.pdf
6. Tabatabaei, J. M. (2022). Vibrations of longitudinally stiffened elastic medium- contacting cylindrical sheels. Tran. Natl. Acad. Sci. Azerb. Ser. Phys.-Tech. Math. Sci. Mechanics, 42 (7), 23–42.
7. Suleimenov, U., Zhangabay, N., Utelbayeva, A., Azmi Murad, M. A., Dosmakanbetova, A., Abshenov, K. et al. (2022). Estimation of the strength of vertical cylindrical liquid storage tanks with dents in the wall. Eastern-European Journal of Enterprise Technologies, 1 (7 (115)), 6–20. doi: https://doi.org/10.15587/1729-4061.2022.252599
8. Fomin, O., Lovska, A., Khara, M., Nikolaienko, I., Lytvynenko, A., Sova, S. (2022). Adapting the load-bearing structure of a gondola car for transporting high-temperature cargoes. Eastern-European Journal of Enterprise Technologies, 2 (7 (116)), 6–13. doi: https://doi.org/10.15587/1729-4061.2022.253770
9. Kovalchuk, V., Sobolevska, Y., Onyshchenko, A., Bal, O., Kravets, I., Pentsak, A. et al. (2022). Investigating the influence of the diameter of a fiberglass pipe on the deformed state of railroad transportation structure "embankment-pipe." Eastern-European Journal of Enterprise Technologies, 2 (7 (116)), 35–43. doi: https://doi.org/10.15587/1729-4061.2022.254573
10. Gameliak, I., Dmytrychenko, A., Tsybulskyi, V., Kharchenko, A. (2022). Determining the effect of reinforcing a cement-concrete coating of bridges on the stressed-strained state of structures. Eastern-European Journal of Enterprise Technologies, 1 (7 (115)), 21–31. doi: https://doi.org/10.15587/1729-4061.2022.251189
11. Akhundov, V. M. (2020). Method for Calculating the Near-Surface Effect in Piecewise Homogeneous Bodies at Large Deformations Based on a Two-Level Approach. Mechanics of Composite Materials, 56 (2), 169–184. doi: https://doi.org/10.1007/s11029-020-09870-w
12. Amiro, I. Ya., Zarutskii, V. A. (1980). Theory of ribbed shells. Methods for Calculating Shells. Kyiv, 367.