ESTIMATING THE STABILITY OF STEADY MOTION OF VIBRATION MACHINES OPERATING ON THE SOMERFELD EFFECT USING AN EMPIRICAL METHOD

One-, two-, and three-mass vibration machines with translational motion of platforms and a vibration exciter of a ball, roller, or pendulum type with several loads were studied. The empirical criterion for the onset of auto-balancing was applied in the extended formulation.

It has been established that a single-mass vibration machine has one resonant speed, and:

– at the after-resonance speeds of rotation of loads synchronously with the rotor, the auto-balancing mode becomes stable;

– at the pre-resonance speeds of rotation of loads, loads tend to gather together.

In a dual-mass vibration machine, there are two resonant speeds and one additional speed located between two resonant ones. The auto-balancing mode is stable when the loads rotate synchronously with the rotor at the following speeds:

– between the first resonant speed and the additional speed;

– greater than the second resonant speed.

At other speeds of rotation of loads, loads tend to gather together.

The three-mass vibration machine has three resonant speeds and two additional speeds, located one by one between adjacent resonant speeds. The auto-balancing mode is stable when the loads rotate synchronously with the rotor at the following speeds:

– between the first resonant speed and the first additional speed;

– between the second resonant speed and the second additional speed;

– greater than the third resonant speed.

At other speeds of rotation of loads, loads tend to gather together.

In a single-mass vibration machine, the value of the resonant speed does not depend on the viscosity of supports. In dual-mass and three-mass vibration machines, all characteristic speeds depend on the viscosity of supports. With small forces of viscous resistance, the values of these speeds are close to the characteristic speeds found in the absence of resistance forces

Keywords: inertial vibration exciter, resonant vibration machine, steady state mode of motion, Sommerfeld effect, stability of motion

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1. Introduction

Rotor machines with auto-balancers have significantly nonlinear properties. Such machines can simultaneously have several stable steady state motion modes. Under an auto-balancing mode of motion of the machine, the loads rotate synchronously with the rotor and eliminate static imbalance in their correction plane. Under stuck modes, the loads come

together, cannot accelerate, lag behind the rotor, and get stuck in the vicinity of the resonant frequencies. Because of this, it is proposed to use these auto-balancers to excite resonant vibrations in vibration machines for various purposes.

There is a general problem in determining the conditions under which certain steady state modes of motion of a rotor machine with auto-balancers can be implemented.

One-, two-, and three-mass vibration machines with translational motion of platforms are widely used. It is relevant for such vibration machines to determine the conditions under which the auto-balancing mode of motion is possible, and under which the machine is inclined to a certain mode of jamming.

Information on the onset in practice of a certain mode of jam or auto-balancing is necessary for the design of both vibration machines and auto-balancers to balance the rotors on the go.

2. Literature review and problem statement

Auto-balancers of pendulum, ball, roller type are used in equipment for balancing fast-rotating rotors on the go [1]. The rotor with auto-balancers has an auto-balancing mode of motion. Under it, the loads rotate synchronously with the rotor while occupying one of the auto-balancing positions, which is why there are no vibrations. In the case of three or more loads, the auto-balancing mode is one or more parametric family of steady state motion modes.

Along with the auto-balancing mode of motion, the rotor machine has other steady state motions on which the rotor is not balanced.

Thus, a rotor machine may have resonant motion modes caused by the Sommerfeld effect [2]. Loads are assembled together, cannot accelerate to the speed of rotation of the rotor, and get stuck at one of the resonant speeds of the rotor. These stuck modes, in particular, were found for a rotor that executes:

– spatial motion and a two-ball auto-balancer [3];

– flat motion and a two-ball auto-balancer [4];

– flat motion, and the rotor is mounted on isotropic supports attached to a massive elastic-viscous foundation and a two-ball auto-balancer is installed on the rotor [5];

– flat motion, and the rotor is rigidly mounted on an elastic-viscous fixed platform that performs rectilinear translational motion and a two-ball auto-balancer is installed on the rotor [6].

Taking into account the results reported in [3, 4], paper [7] proposed to use ball, roller, or pendulum auto-balancers as exciters of resonant vibrations.

Ways to use the Sommerfeld effect to build resonant vibration machines in the case of a single unbalanced mass have been theoretically investigated:

– for a three-mass vibration machine on one of the platforms of which a wind wheel with unbalanced mass is installed [8];

– for a single-mass vibration machine, on the platform of which a pendulum vibration exciter is installed [9];

– for a single-mass vibration machine, on the platform of which an unbalanced inertial vibration exciter is installed [10].

The disadvantage of using one mass is that if the rotor with unbalanced mass accelerates and passes the resonant frequency, then vibrations occur with a greater frequency. In this case, the centrifugal forces acting on the rotor increase in proportion to the square of the angular velocity of rotation of the rotor. This can overload the rotor.

When using two or more loads, the vibration machine has auto-balancing modes of motion. This can be used to automatically turn off the vibration exciter in case of acceleration of loads to the speed of rotation of the rotor. But, according to the available materials, such a problem has not yet been solved.

Determining the conditions for the onset of auto-balancing is a complex mathematical problem. To solve it, the most commonly used are:

– method of synchronization of mechanical systems [11];

– method of separation of motions into fast and slow [12];

– theory of stability of stationary motions of nonlinear autonomous systems [13];

– empirical method for determining the conditions for the onset of auto-balancing [14].

Mathematical methods in [11–13] are based on the theory of stability of motions and methods of small parameter. Therefore, these methods are too cumbersome and require separate application for each type of an auto-balancer or vibration exciter.

The most effective method for determining the stability conditions of the auto-balancing mode of motion is an empirical criterion [14]. It makes it possible to obtain generalized conditions that are applicable to the vibration exciter of any type. The criterion is applicable in cases of both static [14] and dynamic rotor balancing [15]. Also, the results obtained allow for an extended interpretation. Thus, the criterion can answer the question of what mode of motion is possible with unfulfilled conditions for the onset of auto-balancing.

3. The aim and objectives of the study

The purpose of our work is to determine the conditions under which an auto-balancing mode of motion or a certain mode of jamming in vibration machines with translational motion of platforms and a resonant vibration exciter of ball, roller, pendulum type is possible. This will make it possible to enable the occurrence of a certain mode of load jam in the vibration exciter or to turn off the vibration exciter in the case of undesirable acceleration of loads to the speed of rotation of the rotor.

To accomplish the aim, the following tasks have been set:

– to find the specified conditions for a single-mass vibration machine;

– to find the specified conditions for a two-mass vibration machine;

– to find the specified conditions for a three-mass vibration machine.

4. The study materials and methods

The object of research is the conditions under which an auto-balancing mode of motion or a certain mode of getting stuck in vibration machines is possible. To search for them, an empirical criterion for the onset of auto-balancing was used [14] in a slightly expanded formulation. The criterion gives the conditions applicable to vibration exciters of any type with two or more loads. According to the criterion, we investigated the reaction of the center of the vibration ex-

citer to the total unbalanced mass of loads. Actually used is that the loads in the vibration exciter deviate in the direction of deviation of the center of the vibration exciter from the axis of rotation. From empirical considerations, it is assumed that if the center of the vibration exciter, on average, deviates in one revolution of loads:

– towards the total unbalanced mass of loads, the loads tend to gather together;

– opposite to the vector of the total unbalanced mass of loads, the loads tend to occupy an auto-balancing position.

The mathematical notation of the criterion is as follows. Let the loads in the vibration exciter perform a certain steady state motion, on which all together rotate at a constant angular velocity w and do not change the relative position. Let the total unbalanced mass $\vec{S}$. be formed. We consider the motion of the point K (the center of the vibration exciter), at which the longitudinal axis of the rotor crosses the perpendicular plane of the total unbalanced mass. The point K on the steady state motion will deviate from the equilibrium position by the vector $\vec{r}_K(t,\vec{S})$, where t is time.

Let us introduce the following functional

$$f(\vec{S},\omega)=\frac{\omega}{2\pi}\int_0^{2\pi/\omega}\vec{S}\,\vec{r}_K(t,\vec{S})\mathrm{d}t. \qquad (1)$$

Then:

– for the local stability of the auto-balancing mode, it is necessary and sufficient that the loads rotate synchronously with the rotor and point K under the action of any unbalanced mass $\vec{S}$ is deviated from the longitudinal axis of the rotor on average per one rotor revolution, opposite to the unbalanced mass vector

$$f(\vec{S},\omega)<0; \qquad (2)$$

– for the emergence of steady state motions, in which the loads are tightly pressed against each other, it is enough that the point K under the action of the total unbalanced mass deviates from the longitudinal axis of the rotor on average for one revolution of the rotor, towards the vector of total unbalanced mass

$$f(\vec{S},\omega)>0. \qquad (3)$$

The criterion is applied in the following sequence:

1) describe the physical and mechanical model of a vibration machine with an auto-balancer (vibration exciter) of a certain type;

2) derive differential equations of motion of the vibration machine under the assumption that the loads have created constant unbalanced mass, and all together rotate at a constant angular velocity;

3) determine the steady state motion of the rotor;

4) build functional (1);

5) from condition (2) of the negative functional, define the conditions for the onset of auto-balancing, and from condition (3) of the positive functional, define the conditions for the emergence of steady state modes under which the loads are tightly pressed against each other.

The extension of the criterion is that the unbalanced mass is not necessarily infinitesimal, and the loads may lag behind the rotor when rotating. This makes it possible to consider cases where loads tend to gather together.

5. Results of investigating the conditions of stability or the emergence of various steady state motions

5. 1. A single-mass vibration machine
5. 1. 1. Description of the model of a single-mass vibration machine, differential equations of motion in dimensionless form

The model of a single-mass vibration machine is shown in Fig. 1 [7]. The vibration machine has a platform of mass M. A vibration exciter is installed on the platform – ball, roller (Fig. 1, *b*), or pendulum (Fig. 1, *c*) type. The platform moves progressively in a vertical direction. The platform is supported by an elastic-viscous support with a coefficient of stiffness k and viscosity b. The position of the platform is determined by the coordinate y, and, in the position of static equilibrium of the platform, $y=0$.
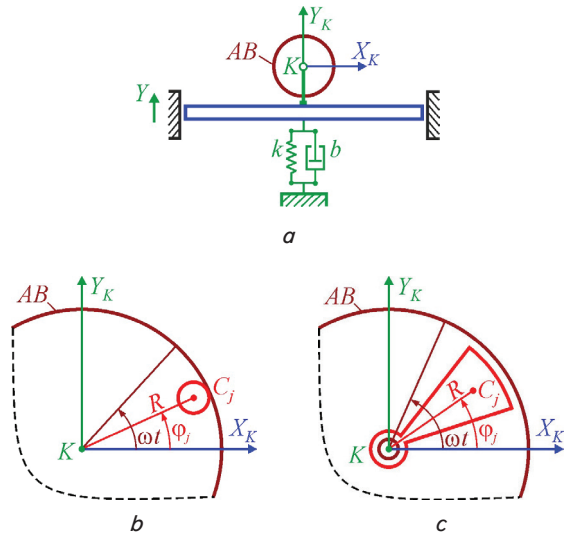

Fig. 1. Single-mass vibration machine, model, and kinematics of motion [7]: *a* — platform; *b* — ball or roller; *c* — pendulum

The vibration exciter (housing) has a mass M_c and rotates around the shaft, point K, with a constant angular velocity w. The center of mass of the housing is at point K. The position of the housing is determined relative to the X_K, Y_K axes by angle wt, where t is the time.

The vibration exciter has N identical loads. The mass of one load is m. The center of mass of the load moves in a circle of radius R with the center at point K (Fig. 1, *b*, *c*). The position of load number j relative to the housing is determined by angle j_j, $/j=\overline{1,N}/$. The viscous resistance force acts on the load when moving relative to the housing, having a module $b_W R|\varphi'_j-\omega|$, $/j=\overline{1,N}/$, where b_W is the coefficient of strength of viscous resistance and the stroke in magnitude denotes the time derivative t. The action of the forces of gravity is neglected.

Differential equations of motion of a single-mass vibration machine in dimensionless form take the following form [7]:

$$\ddot{y}+2h\dot{y}+y+\ddot{s}_y=0,$$

$$\ddot{\varphi}_j+\varepsilon\beta(\dot{\varphi}_j-n)+\varepsilon\ddot{y}\cos\varphi_j=0,\ /j=\overline{1,N}/. \qquad (4)$$

In (4), the dot over the quantity denotes a dimensionless time derivative, and:

– dimensionless variables and time

$$s_x = \frac{1}{N}\sum\nolimits_{j=1}^{N}\cos\varphi_j, \; s_y = \frac{1}{N}\sum\nolimits_{j=1}^{N}\sin\varphi_j,$$

$$\tau = \tilde{\omega}, \; y = YM_\Sigma / (NmR); \tag{5}$$

– dimensionless parameters

$$h = b / (2M_\Sigma\tilde{\omega}), \quad n = \omega / \tilde{\omega},$$

$$\varepsilon = Nm / (\kappa M_\Sigma), \quad \beta = b_W M_\Sigma / (Nm^2\tilde{\omega}). \tag{6}$$

In turn, in (5), (6):
– total mass of the platform and vibration exciter

$$M_\Sigma = M + M_c + Nm; \tag{7}$$

– characteristic time scale

$$\tilde{\omega} = \sqrt{k / M_\Sigma}; \tag{8}$$

– dimensionless coefficient equal to, respectively, for a ball, roller, and pendulum

$$\kappa = \left\{ 7/5, 3/2, 1 + J_C / (mR^2) \right\}. \tag{9}$$

Note that in real vibration machines, dimensionless parameters e, h are values of the first order of smallness.

5. 1. 2. Application of the empirical criterion

We introduce the total unbalanced mass of loads on the steady state motion

$$s_x = s\cos(q\tau), \quad s_y = -s\sin(q\tau), \tag{10}$$

where q is the dimensionless angular speed of rotation of loads.

Taking into account (10), the first equation in (4) takes the form

$$\ddot{y} + 2h\dot{y} + y = sq^2\sin(q\tau). \tag{11}$$

Note that in the absence of resistance forces, the vibration machine has one single resonant speed

$$q_1 = 1. \tag{12}$$

The partial solution to the differential equation (11) takes the following form

$$y(\tau,q,s) = X_1(\tau,q,s)\sin(q\tau)) + X_2(\tau,q,s)\cos(q\tau), \tag{13}$$

where

$$X_1(\tau,q,s) = \frac{q^2 s(1-q^2)}{(1-q^2)^2 + 4h^2 q^2},$$

$$X_2(\tau,q,s) = -\frac{2q^2 shq}{(1-q^2)^2 + 4h^2 q^2}. \tag{14}$$

The scalar product of the vector of the total unbalanced mass (10) by the vector of displacement of the platform:

$$y(\tau,q,s)\cdot s_y(\tau) = sX_1(\tau,q,s)\sin^2(q\tau) - $$
$$-sX_2(\tau,q,s)\sin(q\tau)\cos(q\tau).$$

Averaging in one rotor revolution gives

$$\frac{q}{2\pi}\int_0^{2\pi/q} y(\tau,q,s)\cdot s_y(\tau)d\tau = 0.5\cdot sX_1(\tau,q,s). \tag{15}$$

Taking into account (14), from (15) we obtain the following condition for the stability of the auto-balancing mode

$$q > q_1. \tag{16}$$

So, at the after-resonance speeds of rotation of loads synchronously with the rotor, the auto-balancing mode becomes stable. At the pre-resonance speeds of rotation of loads, loads tend to gather together. The resulting condition (16) does not depend on the strength of the viscous resistance in the support and on the magnitude of the unbalanced mass.

5. 2. A dual-mass vibration machine
5. 2. 1. Description of the generalized model of a two-mass vibration machine, differential equations of motion in dimensionless form

The model of a two-mass vibration machine is shown in Fig. 2 [7]. The vibration machine consists of two platforms with a mass of M_1 and M_2. Each platform is held by external elastic-viscous supports with the coefficients of stiffness k_i and viscosity b_i, /i=1,2/. The platform is connected by an internal elastic-viscous support with a stiffness coefficient k_{12} and a viscosity coefficient b_{12}.
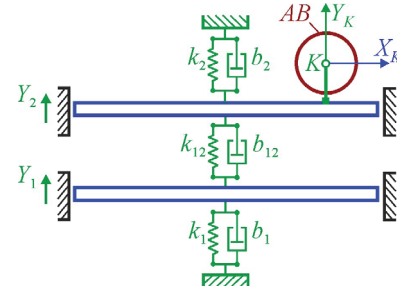

Fig. 2. Generalized model of a two-mass vibration machine [7] – kinematics of platform motion

Platforms can only move rectilinearly progressively owing to fixed guides. The coordinates Y_1, Y_2 of the platforms are counted from the positions of the static equilibrium of the platforms.

On the second platform, a vibration exciter is installed – ball, roller (Fig. 1, b), or pendulum (Fig. 1, c) type.

Differential equations of motion in dimensionless form [7]:

$$\ddot{y}_1 + 2h_1\dot{y}_1 + n_1^2 y_1 + 2h_{12}(\rho\dot{y}_1 - \dot{y}_2) + n_{12}^2(\rho y_1 - y_2) = 0,$$

$$\ddot{y}_2 + 2h_2\dot{y}_2 + n_2^2 y_2 - 2h_{12}(\rho\dot{y}_1 - \dot{y}_2) - $$
$$-n_{12}^2(\rho y_1 - y_2) + \ddot{s}_y = 0,$$

$$\ddot{\varphi}_j + \varepsilon\beta(\dot{\varphi}_j - n) + \varepsilon\ddot{y}_2\cos\varphi_j = 0, \; / j = \overline{1,N} / . \tag{17}$$

In (17):
– dimensionless variables and time

$$s_x = \frac{1}{N}\sum_{j=1}^{N}\cos\varphi_j, \quad s_y = \frac{1}{N}\sum_{j=1}^{N}\sin\varphi_j,$$

$$\tau = \tilde{\omega}t, \quad y_1 = Y_1 / \left(\rho \tilde{Y}\right), \quad y_2 = Y_2 / \tilde{Y}; \qquad (18)$$

– dimensionless parameters

$$n_1^2 = k_1 / \left(M_1 \tilde{\omega}^2\right), \quad n_{12}^2 = k_{12} / \left(M_{2\Sigma} \tilde{\omega}^2\right),$$

$$n_2^2 = k_2 / \left(M_{2\Sigma} \tilde{\omega}^2\right), \quad h_1 = b_1 / \left(2M_1 \tilde{\omega}\right),$$

$$h_{12} = b_{12} / \left(2M_{2\Sigma} \tilde{\omega}\right), \quad h_2 = b_2 / \left(2M_{2\Sigma} \tilde{\omega}\right),$$

$$n = \omega / \tilde{\omega}, \quad \varepsilon = Nm / \left(\kappa M_{2\Sigma}\right),$$

$$\beta = b_W M_{2\Sigma} / \left(Nm^2 \tilde{\omega}\right). \qquad (19)$$

In turn, in (18), (19):
– characteristic scales ($\tilde{\omega}$ not yet defined)

$$\rho = M_{2\Sigma} / M_1, \quad \tilde{Y} = NmR / M_{2\Sigma}; \qquad (20)$$

– the total mass of the second platform with a vibration exciter

$$M_{2\Sigma} = M_2 + M_c + Nm; \qquad (21)$$

– coefficient k from (9).

5. 2. 2. Application of the empirical criterion

Let the loads create total unbalanced mass with (10). Then the first two equations in system (17) will take the form

$$\ddot{y}_1 + 2h_1 \dot{y}_1 + n_1^2 y_1 + 2h_{12}\left(\rho \dot{y}_1 - \dot{y}_2\right) + n_{12}^2\left(\rho y_1 - y_2\right) = 0,$$

$$\ddot{y}_2 + 2h_2 \dot{y}_2 + n_2^2 y_2 - 2h_{12}\left(\rho \dot{y}_1 - \dot{y}_2\right) -$$
$$-n_{12}^2\left(\rho y_1 - y_2\right) + \ddot{s}_y = sq^2 \sin q\tau. \qquad (22)$$

Search for a partial solution to this system in the form

$$y_i\left(\tau,q,s\right) = X_{2i-1}\left(q,s\right)\sin\left(q\tau\right) +$$
$$+X_{2i}\left(q,s\right)\cos\left(q\tau\right), \ /i = 1,2/. \qquad (23)$$

We substitute (23) in (22) and collect the coefficients before sin(qt), cos(qt). We obtain the next system of equations to search for $X_i, \ /i = \overline{1,4}/$:

$$\mathbf{A}(q)\mathbf{X}(q,s) = \mathbf{B}(q,s). \qquad (24)$$

In (24)

$$A(q) = \begin{pmatrix} a_{11}(q) & a_{12}(q) & a_{13}(q) & a_{14}(q) \\ -a_{12}(q) & a_{11}(q) & -a_{14}(q) & a_{13}(q) \\ \rho a_{13}(q) & \rho a_{14}(q) & a_{33}(q) & a_{34}(q) \\ -\rho a_{14}(q) & \rho a_{13}(q) & -a_{34}(q) & a_{33}(q) \end{pmatrix},$$

$$\mathbf{X}(q,s) = \|X_i\|_1^4, \quad \mathbf{B}(q,s) = \begin{pmatrix} 0 & 0 & sq^2 & 0 \end{pmatrix}^T, \qquad (25)$$

where

$$a_{11}(q) = n_1^2 + \rho n_{12}^2 - q^2, \ a_{12}(q) = -2q\left(h_1 + \rho h_{12}\right),$$

$$a_{13}(q) = -n_{12}^2, \ a_{14}(q) = 2qh_{12},$$

$$a_{33}(q) = n_2^2 + n_{12}^2 - q^2, \ a_{34}(q) = -2q\left(h_2 + h_{12}\right). \qquad (26)$$

Scalar product of the total unbalanced mass vector (10) by the vector of displacement of the second platform

$$y_2\left(\tau,q,s\right) \cdot s_y\left(\tau\right) = sX_3\left(q,s\right)\sin^2\left(q\tau\right) +$$
$$+sX_4\left(q,s\right)\sin\left(\left(q\tau\right)\right)\cos\left(q\tau\right). \qquad (27)$$

Averaging in one rotor revolution gives

$$\frac{q}{2\pi}\int_0^{2\pi/q} y_2\left(\tau,q,s\right) \cdot s_y\left(\tau\right)\mathrm{d}\tau = 0.5 \cdot sX_3\left(\Omega,s\right). \qquad (28)$$

We introduce the determinants into consideration

$$\Delta(q) = \left|A(q)\right| = \begin{Bmatrix} a_{11}(q)a_{33}(q) - \\ -\rho\left[a_{13}^2(q) - a_{14}^2(q)\right] - \\ -a_{12}(q)a_{34}(q) \end{Bmatrix}^2 +$$
$$+ \begin{bmatrix} 2\rho a_{13}(q)a_{14}(q) - a_{12}(q)a_{33}(q) - \\ -a_{11}(q)a_{34}(q) \end{bmatrix}^2,$$

$$\Delta_3(q,s) = \begin{vmatrix} a_{11}(q) & a_{12}(q) & 0 & a_{14}(q) \\ -a_{12}(q) & a_{11}(q) & 0 & a_{13}(q) \\ \rho a_{13}(q) & \rho a_{14}(q) & sq^2 & a_{34}(q) \\ -\rho a_{14}(q) & \rho a_{13}(q) & 0 & a_{33}(q) \end{vmatrix} =$$
$$= sq^2 \left\langle \begin{aligned} \rho & \begin{Bmatrix} a_{11}(q)\left[a_{14}^2(q) - a_{13}^2(q)\right] - \\ -2a_{12}(q)a_{13}(q)a_{14}(q) \end{Bmatrix} + \\ & +a_{33}(q)\left[a_{11}^2(q) + a_{12}^2(q)\right] \end{aligned} \right\rangle. \qquad (29)$$

Note that in the presence of resistance forces, D(q)>0. Then

$$X_3\left(q,s\right) = \Delta_3\left(q,s\right) / \Delta(q). \qquad (30)$$

Taking into account (28) to (30), we obtain the following condition for the stability of the auto-balancing mode

$$f(q) = \rho \begin{Bmatrix} a_{11}(q)\left[a_{14}^2(q) - a_{13}^2(q)\right] - \\ -2a_{12}(q)a_{13}(q)a_{14}(q) \end{Bmatrix} +$$
$$+a_{33}(q)\left[a_{11}^2(q) + a_{12}^2(q)\right] < 0. \qquad (31)$$

Note that the condition does not depend on the total unbalanced mass but depends on the forces of viscous resistance in supports.

5. 2. 3. The case of the lack of resistance forces

In the absence of resistance forces

$$f(q) = a_{11}(q)\left[a_{33}(q)a_{11}(q) - \rho a_{13}^2(q)\right], \qquad (32)$$

$$\Delta(q) = \left[a_{11}(q)a_{33}(q) - \rho a_{13}^2(q)\right]^2. \qquad (33)$$

From equation (33), taking into account (26), we find

$$\Delta(q) = \begin{Bmatrix} q^4 - \left[n_1^2 + n_2^2 + (1+\rho)n_{12}^2\right]q^2 + \\ +n_1^2 n_2^2 + n_{12}^2\left(n_1^2 + \rho n_2^2\right) \end{Bmatrix}^2 = 0. \qquad (34)$$

From (34), we find two resonant rotor speeds:

$$q_{1/2} = \frac{1}{\sqrt{2}}\left[n_1^2 + n_2^2 + (1+\rho)n_{12}^2 \mp \sqrt{D}\right]^{\frac{1}{2}}, \tag{35}$$

where

$$D = \left[n_1^2 - n_2^2 - (1-\rho)n_{12}^2\right]^2 + 4\rho n_{12}^4 > 0. \tag{36}$$

The positive real root of the equation $a_{11}(q)=0$ is

$$\tilde{q} = \left(n_1^2 + \rho n_{12}^2\right)^{\frac{1}{2}}. \tag{37}$$

Since $q_1 < q_2$ and $\left(\tilde{q}^2 - q_1^2\right)\left(q_2^2 - \tilde{q}^2\right) = \rho n_{12}^4 > 0,$ then

$$q_1 < \tilde{q} < q_2. \tag{38}$$

The roots q_1 and q_2 are the resonant frequencies of the vibration machine, and the root $\tilde{q}$ is some additional rotor speed.
Since

$$f(0) = \left[n_1^2 n_2^2 + \left(n_1^2 + \rho n_2^2\right)n_{12}^2\right]\left(n_1^2 + \rho n_{12}^2\right) > 0,$$

the auto-balancing mode is stable when the loads rotate synchronously with the rotor at the following speeds:

– between the first resonant speed and the additional speed;

– greater than the second resonant speed.

At other speeds of rotation of loads, loads tend to gather together.

Note that the small forces of viscous resistance in the supports do not change the qualitative behavior of the system but somewhat change the characteristic speeds.

5. 3. A three-mass vibration machine
5. 3. 1. Description of the generalized model of a three-mass vibration machine, differential equations of motion in dimensionless form

A generalized model of a three-mass vibration machine is shown in Fig. 3 [7]. The vibration machine consists of three platforms with a mass of M_1, M_2 and M_3. Each platform is held by external elastic-viscous supports with a coefficient of stiffness k_i and a viscosity coefficient b_i, /i=1, 2, 3/. The platforms are connected to each other by internal elastic-viscous supports with stiffness coefficients k_{12}, k_{13}, k_{23} and viscosity coefficients b_{12}, b_{13}, b_{23}.

Platforms can only move rectilinearly progressively owing to fixed guides. The coordinates Y_1, Y_2, Y_3 of the platforms are counted from the positions of the static equilibrium of the platforms.
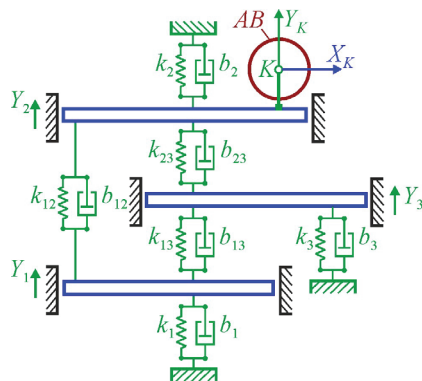

Fig. 3. Generalized model of a three-mass vibration machine [7] – kinematics of platform motion

A vibration exciter is installed on the second platform – ball, roller (Fig. 1, *b*), or pendulum (Fig. 1, *c*) type.

Differential equations of motion of the vibration machine in dimensionless form [7]:

$$\ddot{y}_1 + 2h_1\dot{y}_1 + n_1^2 y_1 + 2h_{12}\left(\rho_1\dot{y}_1 - \dot{y}_2\right) + \\ + n_{12}^2\left(\rho_1 y_1 - y_2\right) + 2h_{13}\left(\rho_1\dot{y}_1 - \rho_3\dot{y}_3\right) + \\ + n_{13}^2\left(\rho_1 y_1 - \rho_3 y_3\right) = 0,$$

$$\ddot{y}_2 + 2h_2\dot{y}_2 + n_2^2 y_2 - 2h_{12}\left(\rho_1\dot{y}_1 - \dot{y}_2\right) - \\ - n_{12}^2\left(\rho_1 y_1 - y_2\right) + 2h_{23}\left(\dot{y}_2 - \rho_3\dot{y}_3\right) + \\ + n_{23}^2\left(y_2 - \rho_3 y_3\right) + \ddot{s}_y = 0,$$

$$\ddot{y}_3 + 2h_3\dot{y}_3 + n_3^2 y_3 - 2h_{13}\left(\rho_1\dot{y}_1 - \rho_3\dot{y}_3\right) - \\ - n_{13}^2\left(\rho_1 y_1 - \rho_3 y_3\right) - 2h_{23}\left(\dot{y}_2 - \rho_3\dot{y}_3\right) - \\ - n_{23}^2\left(y_2 - \rho_1 y_1\right) = 0,$$

$$\ddot{\varphi}_j + \varepsilon\beta\left(\dot{\varphi}_j - n\right) + \varepsilon\ddot{y}_2\cos\varphi_j = 0, \quad / j = \overline{1, N} /. \tag{39}$$

In (39):
– dimensionless variables and time

$$y_1 = Y_1/\left(\rho_1\tilde{Y}\right), \quad y_2 = Y_2/\tilde{Y}, \quad y_3 = Y_3/\left(\rho_3\tilde{Y}\right),$$

$$s_x = \frac{1}{N}\sum_{j=1}^{N}\cos\varphi_j, \quad s_y = \frac{1}{N}\sum_{j=1}^{N}\sin\varphi_j, \quad \tau = \tilde{\omega}t; \tag{40}$$

– dimensionless parameters:

$$h_1 = \frac{b_1}{2M_1\tilde{\omega}}, \quad h_2 = \frac{b_2}{2M_{2\Sigma}\tilde{\omega}}, \quad h_3 = \frac{b_3}{2M_3\tilde{\omega}},$$

$$h_{12} = \frac{b_{12}}{2M_{2\Sigma}\tilde{\omega}}, \quad h_{13} = \frac{b_{13}}{2M_{2\Sigma}\tilde{\omega}}, \quad h_{23} = \frac{b_{23}}{2M_{2\Sigma}\tilde{\omega}},$$

$$n_1^2 = \frac{k_1}{M_1\tilde{\omega}^2}, \quad n_2^2 = \frac{k_2}{M_{2\Sigma}\tilde{\omega}^2}, \quad n_3^2 = \frac{k_3}{M_3\tilde{\omega}^2},$$

$$n_{12}^2 = \frac{k_{12}}{M_{2\Sigma}\tilde{\omega}^2}, \quad n_{13}^2 = \frac{k_{13}}{M_{2\Sigma}\tilde{\omega}^2}, \quad n_{23}^2 = \frac{k_{23}}{M_{2\Sigma}\tilde{\omega}^2},$$

$$n = \frac{\omega}{\tilde{\omega}}, \quad \varepsilon = \frac{Nm}{\kappa M_{2\Sigma}}, \quad \beta = \frac{b_w M_{2\Sigma}}{Nm^2\tilde{\omega}}. \tag{41}$$

In turn, in (40), (41), characteristic scales ($\tilde{\omega}$ not yet defined)

$$\tilde{Y} = NmR/M_{2\Sigma}, \quad \rho_1 = M_{2\Sigma}/M_1, \quad \rho_3 = M_{2\Sigma}/M_3. \tag{42}$$

In (41), (42) M_{2S} from (21), k from (9).

5. 3. 2. Application of the empirical criterion

Let the loads create total unbalanced mass (10). Then the first three equations in system (39) will take the form

$$\ddot{y}_1 + 2h_1\dot{y}_1 + n_1^2 y_1 + 2h_{12}\left(\rho_1\dot{y}_1 - \dot{y}_2\right) + n_{12}^2\left(\rho_1 y_1 - y_2\right) + \\ + 2h_{13}\left(\rho_1\dot{y}_1 - \rho_3\dot{y}_3\right) + n_{13}^2\left(\rho_1 y_1 - \rho_3 y_3\right) = 0,$$

$$\ddot{y}_2 + 2h_2\dot{y}_2 + n_2^2 y_2 - 2h_{12}\left(\rho_1\dot{y}_1 - \dot{y}_2\right) - n_{12}^2\left(\rho_1 y_1 - y_2\right) + \\ + 2h_{23}\left(\dot{y}_2 - \rho_3\dot{y}_3\right) + n_{23}^2\left(y_2 - \rho_3 y_3\right) + \ddot{s}_y = sq^2\sin(q\tau),$$

$$\ddot{y}_3 + 2h_3\dot{y}_3 + n_3^2 y_3 - 2h_{13}(\rho_1\dot{y}_1 - \rho_3\dot{y}_3) -$$
$$-n_{13}^2(\rho_1 y_1 - \rho_3 y_3) -$$
$$-2h_{23}(\dot{y}_2 - \rho_3\dot{y}_3) - n_{23}^2(y_2 - \rho_1 y_1) = 0. \tag{43}$$

We search for a partial solution to this system in the form

$$y_i(\tau,q,s) = X_{2i-1}(q,s)\sin(q\tau) +$$
$$+X_{2i}(q,s)\cos(q\tau), \ /i = \overline{1,3}/. \tag{44}$$

We substitute (44) in (43) and collect the coefficients before $\sin(qt)$, $\cos(qt)$. We obtain the next system of equations to search for X_i, $/i = \overline{1,6}/$:

$$\mathbf{A}(q)\mathbf{X}(q,s) = \mathbf{B}(q,s). \tag{45}$$

In (45)

$$\mathbf{A}(q) = \left\|a_{ij}(q)\right\|_1^6, \quad \mathbf{X}(q,s) = \left\|X_i\right\|_1^6,$$
$$\mathbf{B}(q,s) = \begin{pmatrix} 0 & 0 & sq^2 & 0 & 0 & 0 \end{pmatrix}^{\mathrm{T}}, \tag{46}$$

where, in turn

$$a_{11}(q) = n_1^2 + \rho_1(n_{12}^2 + n_{13}^2) - q^2,$$

$$a_{12}(q) = -2q\left[h_1 + \rho_1(h_{12} + h_{13})\right], \ a_{13}(q) = -n_{12}^2,$$

$$a_{14}(q) = 2qh_{12}, \ a_{15} = -n_{13}^2\rho_3, \ a_{16}(q) = 2qh_{13}\rho_3,$$

$$a_{21}(q) = -a_{12}(q), \ a_{22}(q) = a_{11}(q), \ a_{23}(q) = -a_{14}(q),$$

$$a_{24}(q) = a_{13}(q), \ a_{25}(q) = -a_{16}(q), \ a_{26} = a_{15}(q),$$

$$a_{31}(q) = -\rho_1 n_{12}^2, \ a_{32}(q) = 2q\rho_1 h_{12},$$

$$a_{33}(q) = n_2^2 + n_{12}^2 + n_{23}^2 - q^2, \ a_{34}(q) = -2q(h_2 + h_{12} + h_{23}),$$

$$a_{35} = -n_{23}^2\rho_3, \ a_{36}(q) = 2qh_{23}\rho_3, \ a_{41}(q) = -a_{32}(q),$$

$$a_{42}(q) = a_{31}(q), \ a_{43}(q) = -a_{34}(q), \ a_{44}(q) = a_{33}(q),$$

$$a_{45}(q) = -a_{36}(q), \ a_{46} = a_{35}(q), \ a_{51}(q) = -\rho_1 n_{13}^2,$$

$$a_{52}(q) = 2q\rho_1 h_{13}, \ a_{53}(q) = -n_{23}^2, \ a_{54}(q) = 2qh_{23},$$

$$a_{51}(q) = -\rho_1 n_{13}^2, \ a_{52}(q) = 2q\rho_1 h_{13}, \ a_{53}(q) = -n_{23}^2,$$

$$a_{54}(q) = 2qh_{23}, \ a_{55}(q) = n_3^2 + (n_{13}^2 + n_{23}^2)\rho_3 - q^2,$$

$$a_{56}(q) = -2q\left[h_3 + (h_{13} + h_{23})\rho_3\right], \ a_{61}(q) = -a_{52}(q),$$

$$a_{62}(q) = a_{51}(q), \ a_{63}(q) = -a_{54}(q), \ a_{64}(q) = a_{53}(q),$$

$$a_{65}(q) = -a_{56}(q), \ a_{66} = a_{55}(q). \tag{47}$$

Coefficients in the laws of motion (44) are determined from the following formulas

$$\mathbf{X}(q,s) = \mathbf{A}^{-1}(q)\mathbf{B}(q,s). \tag{48}$$

The scalar product of the vector of total unbalanced mass (10) by the vector of displacement of the second platform takes the following form (27). Averaging per revolution

of the rotor is (28). Then, the empirical condition for the stability of the auto-balancing mode

$$X_3(\Omega,s) < 0. \tag{49}$$

We introduce the determinant into the consideration

$$\Delta(q) = |A(q)| = 0. \tag{50}$$

Note that in the presence of resistance forces $\Delta(q) > 0$. We introduce the determinant

$$\Delta_3(q) = \begin{vmatrix} a_{11}(q) & a_{12}(q) & a_{14}(q) & a_{15}(q) & a_{16}(q) \\ a_{21}(q) & a_{22}(q) & a_{24}(q) & a_{25}(q) & a_{26}(q) \\ a_{41}(q) & a_{42}(q) & a_{44}(q) & a_{45}(q) & a_{46}(q) \\ a_{51}(q) & a_{52}(q) & a_{54}(q) & a_{55}(q) & a_{56}(q) \\ a_{61}(q) & a_{62}(q) & a_{64}(q) & a_{65}(q) & a_{66}(q) \end{vmatrix}. \tag{51}$$

Then

$$X_3(q,s) = sq^2 \Delta_3(q)/\Delta(q). \tag{52}$$

Empirical condition for the stability of the auto-balancing mode

$$\Delta_3(q) < 0. \tag{53}$$

This condition depends on the forces of viscous resistance in the supports and does not depend on the value of the total unbalanced mass.

5. 3. 3. The case of the lack of resistance forces
In the absence of resistance forces in the supports

$$\Delta_3(q) = \begin{pmatrix} a_{11}a_{33}a_{55} - a_{11}a_{35}a_{53} - \\ -a_{13}a_{31}a_{55} + a_{13}a_{51}a_{35} + \\ +a_{31}a_{15}a_{53} - a_{15}a_{33}a_{51} \end{pmatrix}(a_{11}a_{55} - a_{15}a_{51}), \tag{54}$$

$$\Delta_0(q) = \begin{pmatrix} a_{11}a_{33}a_{55} - a_{11}a_{35}a_{53} - a_{13}a_{31}a_{55} + \\ +a_{13}a_{51}a_{35} + a_{31}a_{15}a_{53} - a_{15}a_{33}a_{51} \end{pmatrix}^2. \tag{55}$$

The equation $D_0(q) = 0$ defines the resonant oscillation frequencies of the system. The three-mass vibration machine has three resonant oscillation frequencies q_1, q_2, q_3 ($q_1 < q_2 < q_3$) and three corresponding forms of platform oscillations. Because of this, the first multiplier in (54) gives three resonant frequencies of the vibration machine. The second multiplier in (54) takes the following form

$$a_{11}a_{55} - a_{15}a_{51} = q^4 - bq^2 + c, \tag{56}$$

where

$$b = n_1^2 + \rho_1(n_{12}^2 + n_{13}^2) +$$
$$+n_3^2 + \rho_3(n_{23}^2 + n_{13}^2) > 0,$$

$$c = (n_1^2 + \rho_1 n_{13}^2)\left[n_3^2 + \rho_3(n_{23}^2 + n_{13}^2)\right] +$$
$$+\rho_1 n_{13}^2(n_3^2 + \rho_3 n_{23}^2) > 0. \tag{57}$$

Equation (57) gives the following two additional characteristic speeds

$$\tilde{q}_{1/2} = \left(b \mp \sqrt{d}\right)^{\frac{1}{2}} \Big/ \sqrt{2}, \qquad (58)$$

where

$$d = b^2 - 4c = \left\{ \begin{array}{l} n_1^2 + \rho_1\left(n_{12}^2 + n_{13}^2\right) - \\ -\left[n_3^2 + \rho_3\left(n_{23}^2 + n_{13}^2\right)\right] \end{array} \right\}^2 + 4\rho_1\rho_2 n_{13}^4 > 0. \qquad (59)$$

Additional speeds are always there and positive. It can be shown that $q_1 < \tilde{q}_1 < q_2 < \tilde{q}_2 < q_3$.

Since $D_3(0) > 0$, the auto-balancing mode is stable when the loads rotate synchronously with the rotor at the following speeds:

– between the first resonant speed and the first additional speed;

– between the second resonant speed and the second additional speed;

– greater than the third resonant speed.

At other speeds of rotation of loads, loads tend to gather together.

It is worth noting that the small forces of viscous resistance almost do not change the magnitude of the five characteristic speeds found.

6. Discussion and interpretation of the obtained conditions of stability or the emergence of various steady state motions

The empirical criterion for the onset of auto-balancing in the extended formulation made it possible to obtain the conditions of stability or the emergence of various steady state motions of one-, two-, and three-mass vibration machines.

In the case of a single-mass vibration machine, there is one single resonant frequency (12), and its value does not depend on the forces of viscous resistance in the supports. At the pre-resonance speeds of rotation of loads, loads tend to gather together regardless of the speed of rotation of the rotor. At the after-resonance speeds of rotation of loads synchronously with the rotor, the auto-balancing mode of motion of the vibration machine becomes stable.

In a dual-mass vibration machine, there are two resonant speeds (35) and one additional speed (37), located between two resonant ones. The auto-balancing mode is stable when the loads rotate synchronously with the rotor with the speeds between the first resonant speed and the additional speed, and when the rotor rotates at speeds greater than the second resonant speed. At other speeds of rotation of loads, loads tend to gather together. All three speeds depend on the viscosity of the supports. But with small forces of viscous resistance, the values of these speeds almost do not change.

In a three-mass vibration machine, there are three resonant speeds and two additional speeds (58), located one by one between adjacent resonant speeds. The auto-balancing mode is stable when the loads rotate synchronously with the rotor with speeds:

– between the first resonant speed and the first additional speed;

– between the second resonant speed and the second additional speed;

– greater than the third resonant speed.

At other speeds of rotation of loads, loads tend to gather together. All five speeds depend on the viscosity of the sup-

ports. But with small forces of viscous resistance, the values of these speeds almost do not change.

In the case of a single-, two-, or three-mass vibration machine at the resonant speeds of rotation of the rotor, when accelerating loads to the speed of rotation of the rotor, the auto-balancing mode will occur. In this case, the vibration exciter will automatically turn off. To return a certain mode of jam, it is necessary to reduce the speed of rotation of the rotor (and loads) to the range at which loads tend to gather together. After the occurrence of the stuck mode, the speed of rotation of the rotor can be increased.

The resulting conditions of stability or the emergence of various steady state motions take place for a vibration exciter of any type with two or more loads. Such universal results allow us to obtain only an empirical criterion for the onset of auto-balancing in an extended formulation.

It should be noted that the empirical criterion is an approximate method and therefore makes it possible to obtain the limits of stability of steady state motions approximately. The results are the more accurate, the smaller the mass of loads relative to the mass of the system, the greater the viscous resistance forces acting on the load [14, 15].

On the other hand, as in other approximate methods, it is impossible to establish exact ratios of smallness between the parameters of the system. Therefore, the obtained limits of stability or the emergence of various steady state motions must be additionally checked.

In the future, it is planned to investigate the features of the origin and disappearance of various steady state motions for a particular vibration machine (antiphase, anti-resonance, etc.) with computational experiments.

7. Conclusions

1. In a single-mass vibration machine, there is one single resonant speed that does not depend on the forces of viscous resistance in the supports, and:

– at the after-resonance speeds of rotation of loads synchronously with the rotor, the auto-balancing mode of motion of the vibration machine becomes stable;

– at the pre-resonance speeds of rotation of loads, loads tend to gather together.

2. In a dual-mass vibration machine, there are two resonant speeds and one additional speed located between two resonant ones. The auto-balancing mode is stable when the loads rotate synchronously with the rotor with speeds:

– between the first resonant speed and the additional speed;

– greater than the second resonant speed.

At other speeds of rotation of loads, loads tend to gather together. All three characteristic speeds depend on the viscosity of the supports. But with small forces of viscous resistance, the values of these speeds are close to the characteristic speeds found in the absence of resistance forces.

3. A three-mass vibration machine has three resonant speeds and two additional speeds, located one by one between adjacent resonant speeds. The auto-balancing mode is stable when the loads rotate synchronously with the rotor with speeds:

– between the first resonant speed and the first additional speed;

– between the second resonant speed and the second additional speed;

– greater than the third resonant speed.

At other speeds of rotation of loads, loads tend to gather together. All five characteristic speeds depend on the viscosity of the supports. But with small forces of viscous resistance, the values of these speeds are close to the characteristic speeds found in the absence of resistance forces.

Conflicts of interest

The authors declare that they have no conflict of interest in relation to this research, whether financial, personal, authorship or otherwise, that could affect the research and its results presented in this paper.

Data availability

All data are available in the main text of the manuscript.

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