This paper considers the construction of a mathematical model of the movement of an autonomous mobile robot (AMR) in variable configuration, taking into account the relationship of the dynamic parameters of a mechanical system.

As an example, the design of AMR with a manipulator is considered.

The object of this study is the dynamics of AMR with a manipulator. The peculiarities of the dynamics of AMR with the manipulator are due to the change in the position of the center of mass of the system with the relative movement of the manipulator and the commensurate non-diagonal and diagonal elements of the inertia tensor calculated relative to the axes of the base coordinate system. The construction of the mathematical model was carried out according to the Nyton-Euler method. The resulting mathematical model contains:

– an equation of motion of the center of mass of the AMR system of variable configuration along the trajectory in the inertial coordinate system;

– an equation of angular motion of AMR in variable configuration in the inertial coordinate system;

– an equation of motion of the manipulator with respect to AMR. In a general case, the center of mass of the AMR platform moves in a horizontal plane. Establishing the relationship of dynamic parameters of the mechanical system will make it possible to maintain functionality and ensure the orientation of AMR in vertical planes despite the movement of the manipulator. As an object of control, AMR with a manipulator is a multi-connected system with a cross-internal connection of control channels, which is formed by the dynamic parameters of a mechanical system. Based on the results of mathematical modeling using the proposed model, it is possible to develop algorithms for adaptive control using cross-connection of channels. This will make it possible to identify reserves to reduce energy consumption, increase stability, improve the efficiency and survivability of AMR in variable configuration during autonomous work under extreme conditions

Keywords: autonomous mobile robot, manipulator, mathematical model, dynamics, dynamic parameters relationship

CONSTRUCTION OF A MATHEMATICAL MODEL OF THE DYNAMICS OF AN AUTONOMOUS MOBILE ROBOT OF VARIABLE CONFIGURATION

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1. Introduction

Robots and manipulators (M) were designed to perform tasks that a person cannot cope with in some circumstances. For example, AMR of a variable configuration performs operations under dangerous or harmful conditions for humans. Increasingly, M is used for military purposes [1, 2], in space [3, 4], in nuclear energy [5], to overcome the consequences of man-made and natural disasters [6]. For work under extreme conditions, remotely controlled AMR with M are typically used [1–6]. To ensure effective remote control of AMR with M, it is necessary to design:

– a kinematic scheme of AMR with M with actuators;
– an information system of AMR with M;
– a mathematical model taking into account the dynamic relationships of control channels;
– control algorithms and software;
– technology of transmission of commands from the operator.

To study the dynamics, such AMR can be considered as M on a mobile basis or a system of "carrier and transferred body" [1–6]. In [7], it is noted that in order to accurately model the dynamics of AMR with M, it is necessary to take into account the interaction between M and the platform, the interconnection of simultaneous rotation and sliding of

rigid arms, as well as non-holonomic restrictions to prevent slipping and skidding of the platform.

A feature of the dynamics of AMR with a manipulator as a system of bodies is [8]:

– changing the position of the center of mass of the system with the relative movement of the manipulator;

– commensuration of non-diagonal and diagonal elements of the inertia tensor calculated relative to the axes of the base coordinate system associated with the center of mass of the AMR platform. Changing the position of the center of mass of the system causes the occurrence of disturbing momenta and causes a loss of orientation of the AMR in vertical planes. The loss of orientation or fall of AMR in the working area under extreme conditions not only leads to the impossibility of performing technological operations and causes material losses. When transporting radiation or explosive objects, a fall of AMR can have catastrophic consequences.

The non-diagonality of the tensor of the inertia of the body system relative to the basic coordinate system causes [8]:

– the occurrence of inconsistency of the main central axes of inertia of the system of bodies with the axes connected to the center of mass of the AMR platform by the basic coordinate system;

– inconsistency of control actions with the directions of the main central axes of inertia of the system;

– interdependence of control channels.

When providing control commands to the drives of the AMR chassis, the non-diagonality of the inertia tensor causes the occurrence of angular velocities in the directions perpendicular to the direction of movement. The control system tries to work out perturbations and prevent the growth of angular velocities in the directions perpendicular to the direction of movement. Despite the operation of the control system, due to the non-diagonality and instability of the inertia tensor and cross-connection of the control channels, AMR with M loses orientation. Additional operation of the control system increases energy consumption, reduces battery life, and reduces the performance of AMR.

The movable base expands the service characteristics of M but imposes more stringent requirements on drives, devices of control, and surveillance systems.

The cycle of work of AMR with M can be divided into several stages [9]:

– movement of the AMR platform from the starting point S to the finish point F with a fixed manipulator in compliance with the requirements of optimality (speed, performance, or efficiency, etc.);

– performing technological operations with a manipulator, attached or information equipment with a fixed or movable platform;

– return of the AMR platform to the starting point S (or movement to a predetermined end point K) with a fixed manipulator.

At the first stage of operation, AMR is a trolley with an adaptive control system that moves under predetermined environmental conditions. To ensure the steady movement of the AMR from point S to point F, the control system must, based on the results of the exploration of the working area, draw up a dangerous route and generate control commands for each chassis drive. If in a mechanical system there is a relationship between dynamic characteristics and control channels, then the impact on the chassis drives causes disturbances in the drives of the manipulator. Such perturbations cause an increase in the generalized coordinates of M, as a result of which M begins to move relative to the platform and the system loses stability. Thus, in order to comply with the requirements of optimality, even for moving AMR with a fixed M, it is necessary to first establish the relationship of dynamic characteristics, to perform mathematical and simulation modeling of a mechanical system. Mathematical modeling of AMR dynamics of variable configuration without taking into account the relationship of the dynamic parameters of the mechanical system predetermines methodological and computational errors.

The relevance of this scientific topic is related to the need to build a mathematical model of AMR dynamics with a manipulator, taking into account the interrelationships of the dynamic parameters of a mechanical system. Such a mathematical model is the scientific basis for the development of adaptive control algorithms with a unit of current object self-identification. Synthesis of commands for actuators is carried out programmatically online after analyzing the dynamic parameters of a mechanical system, taking into account the interconnection of control channels. In practice, this will preserve the functionality and ensure the orientation of the AMR in vertical planes despite external disturbances and movements of the manipulator.

2. Literature review and problem statement

When working remotely under extreme a priori uncertain environmental conditions, various AMR designs of variable configuration are used. The use of AMR with manipulators is also appropriate in the implementation of technological operations with radiating [5] or explosive objects [1, 2]. In the operation of such AMRs, the manipulator executes technological operations with the movement of the object of manipulation (cargo, tools, instrumentation) according to a given law of motion.

In different parts of the AMR trajectory of the changing configuration, the load parameters may change: geometric dimensions, shape, mass distribution. These features should be taken into account when building a mathematical model since the coefficients of differential equations of dynamics are functions of the generalized coordinates of the manipulator and depend on the law of motion and load parameters.

The presence of a movable base increases the maneuverability of manipulation AMR, expands the working space, and increases the number of permissible configurations of the manipulator, but reduces the accuracy of positioning the grip (attachments or tools).

Dynamic processes for M on a moving platform can be divided into three driving modes:

– controlled movement of the platform along a given route with a "checked" M;

– performance of technological operations M with the platform stopped;

– M executes technological operations with controlled movement of the platform.

The mathematical model of the dynamics of the mechanical system "AMR with M" should be built and loaded into the microprocessor of the control system in a general case. The equations of the dynamics of a mechanical system for the listed modes of motion are special cases and are derived from a general mathematical model under certain conditions. In addition, the mathematical model should reflect the dynamic features of the studied design of AMR with M.

The controlled movement of the platform with the "checked" manipulator is well researched for various chassis design options. The "checked" manipulator changes the coordinates of the center of mass of the system but the elements of the inertia tensor are stationary [8]. When implementing this mode of movement, AMR is a controlled trolley that moves along the trajectory. The synthesis of the law of controlling the flat motion of a robot-trolley is a typical task: several methods of stabilizing movement along a given trajectory are known. In [10], the dynamics of a 4-wheelchair equipped with fixed devices were investigated. The mathematical model of the dynamics of the all-wheel drive and front-wheel drive trolley when cornering is given. In this case, the platform is considered a solid, the center of mass of the platform is the center of mass of the system. The linearized mathematical model presented in [10] does not reflect the relationship of dynamic parameters and control channels.

When moving the AMR with M along a given route, the control system must ensure minimal deviation of the center of mass of the platform from the specified trajectory. At the same time, the control moments are directed along the axes of the basic coordinate system associated with the center of mass of the AMR platform.

In [11], an unmanned vehicle is considered as a multistate hinge system. To describe the kinematics of the trolley,

a modified Denavit-Hartenberg method was used. To build a dynamic model, the Newton-Euler algorithm is used. The proposed model takes into account the interrelationships of all parts of the vehicle, which makes it more representative for the development of adaptive control algorithms. Control is implemented using the Lyapunov functions. Taking into account the interrelationships of dynamic parameters ensures reliable tracking of the reference trajectory when kept in lanes, lane changes, avoiding obstacles, and during critical driving situations. It is noted that multi-body models provide more information, which is usually neglected when using a closed-form model. The model presented in [11] cannot be used to describe the dynamics of AMR with M but the expediency of using the Nyton-Euler method has been proven.

Paper [12] considers dynamic modeling of mobile multi-body systems. This class of systems includes spatial wheeled vehicles subjected to constant rigid contacts. To build a nonlinear dynamic model, the Newton-Euler algorithm was used in combination with a projective approach based on an explicit contact model. The results of the implementation of the proposed method for modeling the dynamics of the bicycle are provided. Based on the mathematical model, an algorithm for controlling the movement along the trajectory has been developed. It has been proven that passive asymptotic stabilization of a bicycle can be ensured when it moves at sufficient initial speeds. The model presented in [12] does not take into account the peculiarities of the dynamics of AMR with M. At the same time, the effectiveness of the proposed control algorithms for cross-communication of channels is questionable.

Paper [13] describes dynamic models of tracked unmanned ground vehicles (UGV) in three-dimensional space instead of a simplified two-dimensional projection on a single plane in traditional vehicle theory. The determination of Euler angles is carried out relative to the coordinate system fixed on the main body. A dynamic model for rectilinear movement and steering modes is proposed. with the use of spatial forces and moments. Dynamic models are built according to the Newton-Euler method and the Lagrange method. With the help of the developed models, it is possible to establish rules for the distribution of vehicle load to ensure the stability of movement in three-dimensional space and implement the appropriate control algorithms. The model presented in [13] does not take into account the non-diagonality and nonstationarity of the tensor of inertia, especially the dynamics of AMR with M.

In [14], for the equations of controlled motion of the robot along the trajectory, a replacement of variables was found, which allows the linearization of the mathematical model on feedback. The synthesis of the law of control for an arbitrary "target path" was carried out taking into account the restrictions on phase coordinates and control actions. For AMR with M, linearization of the mathematical model is possible only for individual modes of motion. At the same time, the effectiveness of the proposed control algorithms for cross-linking channels is questionable.

Work [15] states that slipping the wheel can cause a significant deterioration in handling while driving a mobile robot. This paper proposes a method to prevent wheel slippage using a nonlinear predictive driving model. The limitations included in the optimization task change the force of interaction between each wheel and the ground. This approach has been tested under a dynamic modeling environment using the example of a wheeled mobile robot Pioneer 3-DX, which performs box pushing manipulations. For autonomous operation of AMR with M under extreme or predetermined conditions, the proposed method of preventing wheel slippage can be useful.

The trolley dynamics equations track properly the deviation of the trolley's center of mass from a given trajectory. But, in [12–15], issues related to the expediency and justification of the limits of application of the linearized mathematical model remained unresolved. In addition, the mathematical models proposed in [12–15] do not reflect the relationship of the dynamic parameters of the mechanical system. In [16], the design schemes for the formation of control actions on the chassis drives, taking into account the interconnection of the control channels, are given. The proposed schemes should be used in the development and design of driving chassis system for AMR with M; this will take into account and compensate for the non-diagonality and nonstationarity of the inertia tensor. But the mathematical models given in [16] cannot be used to describe the movement of M relative to the AMR platform.

M performs some technological operations with the platform stopped. Under this mode, the mathematical model of the system contains the equation of M dynamics relative to a non-moving base. Most often, the dynamics of manipulators are described using the Newton-Euler method and the Lagrange-Euler method [17].

When moving M relative to the AMR platform, the main central axes of inertia of the body system do not coincide with the axes of the base coordinate system [8]. As a result, the AMR inertia tensor in the coordinate system associated with the platform is non-diagonal and non-stationary. In [8], the results of the analysis of the elements of the inertia tensor of AMR are given. It is accepted that the mass M with a load of 10 % by weight of the AMR chassis. It is proved that the centrifugal and axial moments of inertia of AMR with a manipulator (relative to the axes of the base coordinate system) depend on the time and values of the generalized coordinates [8]. These features should be taken into account when building a mathematical model of movement M with a movable platform of AMR.

If M performs technological operations with controlled movement of the AMR platform, then the mathematical model should take into account the interrelationships of the dynamic parameters of the mechanical system. Under this mode of operation, it is advisable to consider the dynamics equation in relation to the coordinate system associated with the docking node of M and the platform. This approach is used in [18] where the mathematical model M of the spacecraft is presented, built according to the Newton-Euler method, taking into account the non-diagonality and nonstationarity of the inertia tensor. In [18], the mathematical model is constructed in the assumption that the mass of M is 1 % of the mass of the spacecraft's hull, and the center of mass of the spacecraft's hull is the center of mass of the mechanical system. Therefore, the mathematical model proposed in [18] does not take into account the change in the position of the center of mass of the system with the relative motion of the manipulator.

Paper [3] reports the results of the study of the dynamics and control of cosmic M with friction in the joints. To describe joint friction, the Coulomb, Stribek, and Lugre friction models are considered. The dynamic equation of the system is built by a recursive method based on the principle

of variation in the speed of Jourdain. An active controller has been developed to track the trajectory of the system. The reliability of the proposed dynamic model is verified by comparing the results of numerical modeling and the results obtained from the ADAMS software. A link has been established between joint friction and low-speed movement in the process of tracking the trajectory. It is proved that friction in the joints of M changes the dynamic characteristics of the mechanical system and reduces the accuracy of control. The materials provided indicate that for the synthesis of effective control, it is necessary to apply mathematical models taking into account the relationship of the dynamic parameters of the mechanical system.

Paper [4] considered the technology of telerobotics for remote control of space M. Implementation of telerobotics makes it possible for M to copy the movements of the operator. Commands on M drive are formed simultaneously by the encoders of the control device. It is indicated that the optimality of the generated trajectory of M links depends on the professional training of the operator. However, no criteria have been established for assessing the compliance of movements of M and operator movements. It is also not determined whether the M mathematical model takes into account the relationship of the dynamic parameters of a mechanical system.

Paper [5] reports the design of a multifunctional service platform (MFMP). The MFMP model was built in accordance with the maintenance requirements of the Thermonuclear Test Reactor (CFETR). The mathematical model MFMP is constructed using the Newton-Euler method. The equivalent driving force of each drive is determined using an equivalent inverse Jacobi matrix. The accuracy of the model is tested by modeling in the ADAMS software environment. Based on the Newton-Euler method and the improved virtual principle of operation, a control system with stability control has been developed. The presented mathematical model is not applicable for AMR with M since the dynamic features of the mechanical system are not taken into account. But the presented methods of compiling and checking the accuracy of the mathematical model should be used.

Paper [6] reports a remote control system of M with five degrees of mobility. M is mounted on a movable chassis and is used as a robot – a savior. The operator remotely controls M using the computer interface, which displays the movement of the robot and records the current values of the characteristics of M. The FPGA controller controls M servo drives and exchanges data with the computer through a wireless module. To provide remote control over M in real time an interactive mixed programming interface VC and MATLAB has been built. The mathematical model of the robot-savior is constructed using software, so it is difficult to assess how much the relationship between dynamic parameters and control channels is taken into account. In addition, the developed system does not provide feedback from M to the operator, that is, it cannot be used to manipulate fragile or explosive objects.

Paper [7] reports a symbolic algorithm for obtaining the equation of motion of N-link M installed on a mobile platform. The kinematic scheme of M contains rotary-prismatic connections. To build the kinematic and dynamic equations of motion of the system, the Gibbs-Appel recursive formula is used. To increase the computational efficiency of the proposed algorithm, all mathematical operations are performed using matrices 3×3 and 3×1 only. It should be noted that the

nonholonomic characteristics of a moving platform complicate the control equation. In addition, the proposed model does not make it possible to separate the effects of dynamic parameters on the interconnection of control channels.

In [19], a mobile wheeled platform with two manipulators is considered. The authors believe that the increase in the number of relationships between manipulators and the mobile platform, as well as the non-holonomic limitations of the mobile base, make manual derivation of motion equations almost impossible. To build a mathematical model of dynamics, an automatic system approach is proposed. To avoid calculating the "Lagrange multipliers" associated with nonholonomic constraints, motion equations are derived according to the Gibbs-Apelle recursive formula. To increase efficiency, all mathematical operations are implemented with matrices 3×3 and 3×1. The application of the proposed method for a wheeled platform with two manipulators demonstrates the possibility of building equations of motion but does not reflect the relationship of control channels.

In [20], pseudo symbolic dynamic modeling (PSDM) was presented to build simplified dynamic models of M, the design of which contains up to 7 degrees of mobility. The presented algorithm makes it possible to generate code in real time, simulate dynamics, and increase the efficiency of the model by eliminating minimally important elements. In addition, the authors of [20] developed an implementation of the algorithm in the MATLAB environment, which is publicly available. But in [20] the issues remained unresolved, related to the expediency and validity of the exclusion of minimally important elements.

In [21], a virtual model of kinematics and dynamics of M in MATLAB&SIMULINK was developed. The presented model uses a PID controller, and the dynamics equations are obtained by the Lagrange-Euler method. Work [22] considers the dynamic modeling of spatial three-link M using symbolic and numerical methods. To derive the equations of dynamics in the form of a space of states, an algorithm based on the Newton-Euler method is proposed. The algorithm is implemented in the Maple system, the simulation was carried out in MATLAB&SIMULINK. The mathematical models proposed in [20–22] do not reflect the relationship of the dynamic parameters of the mechanical system.

The above review of the literature proves that the application of the Newton-Euler method makes it possible to build a mathematical model to describe the three modes of motion of AMR with M. When overcoming the consequences of man-made and natural disasters, AMR with M performs technological operations in an a priori indefinite external environment. The working space of the AMR manipulator may be limited (for example, the width of the slot). The microprocessor of the control system should synthesize commands into executive drives in real time to take into account changes in workspace. This necessitates a change in the orientation of AMR in vertical planes, narrowing the set of values of the generalized coordinates of M, reducing the size of the working space of M, reducing the number of valid configurations of M. Mathematical modeling in real time will determine the optimal strategy for moving the manipulator with the controlled movement of the AMR platform. Taking into account the relationship of the dynamic parameters of the mechanical system could make it possible:

– to preserve the functionality and ensure the orientation of AMR in vertical planes despite the movement of the manipulator;

– to identify reserves to reduce energy consumption and increase the stability of AMR of a variable configuration when operating under a priori uncertain environmental conditions;

– to improve the efficiency and survivability of AMR of a variable configuration in overcoming the consequences of man-made and natural disasters.

Analysis of the literature [3–22] suggests that it is expedient to conduct a study to build a mathematical model of AMR of the changing configuration, taking into account the relationship of the dynamic parameters of the system.

3. The aim and objectives of the study

The aim of this study is to build a mathematical model of the dynamics of AMR of variable configuration, taking into account the relationship of dynamic parameters of a mechanical system. This will make it possible to identify reserves to reduce energy consumption, increase stability, improve the efficiency and survivability of AMR of a variable configuration.

To accomplish the aim, the following tasks have been set:

– to build the equation of motion of the center of mass of the AMR system of variable configuration along the trajectory, taking into account the relationship of the dynamic parameters of the mechanical system;

– to establish the influence of the relationship of the dynamic parameters of the mechanical system on the equation of angular motion of AMR of the variable configuration;

– to determine the relationship of the dynamic parameters of the mechanical system when the manipulator moves relative to the AMR platform.

4. Materials and research methods

4. 1. Object and hypothesis of research

The object of this study is the dynamics of AMR with the manipulator. The main hypothesis is the relationship of the dynamic parameters of the mechanical system. The peculiarities of the dynamics of AMR with the manipulator are due to the change in the position of the center of mass of the system with the relative movement of the manipulator and the commensurate of the non-diagonal and diagonal elements of the inertia tensor calculated relative to the axes of the base coordinate system.

While building a mathematical model, the following assumptions were accepted:

– the density of the AMR platform and the density of movable structural elements does not change over time;

– all links of the manipulator are absolutely rigid;

– the elements of kinematic pairs are absolutely rigid;

– the drive elements in the hinges are absolutely rigid;

– there is no elastic connection between the moving elements and the AMR platform.

4. 2. The design of an autonomous mobile robot of variable configuration

The design scheme of AMR with the manipulator is shown in Fig. 1. The structure consists of an all-wheel drive AMR 4-wheel platform, and an anthropomorphic manipula-

tor composed of a ring rotating around a vertical axis, and rod links – hands connected by rotary kinematic pairs of the fifth class.
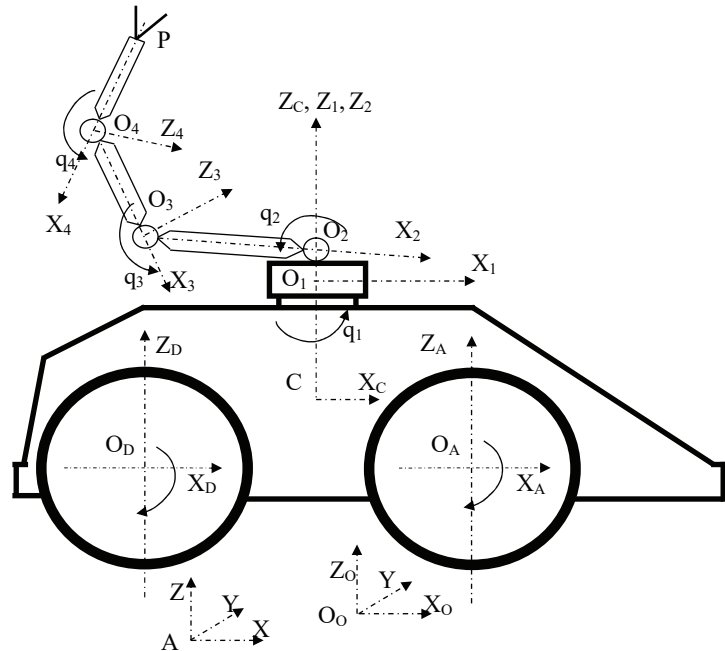

Fig. 1. Scheme of coordinate systems for constructing a mathematical model of an autonomous mobile robot with a manipulator

In the assumption that all links are absolutely solid bodies, AMR with a manipulator is a dynamic system of non-deformed bodies with non-holonomic stationary connections.

4. 3. Coordinate systems

To build a mathematical model, I shall introduce the right-hand Cartesian coordinate systems. In Fig. 1, the following notation is used:

– $AXYZ$ – inertial coordinate system;

– $CX_CY_CZ_C$ – movable base coordinate system. The reference point is connected to the point C – the center of mass of the platform. The axes are parallel to the main central axes of the inertia of the AMR platform. The CZ_C axis is perpendicular to the plane of the platform movement, coincides with the local vertical, and is directed upwards to the manipulator ring, CX_C located in the plane of movement of the platform and directed towards the movement, the CY_C axis is located in the plane of movement of the platform and complements the coordinate system to the right-hand one;

– $OX_0Y_0Z_0$ – movable coordinate system. The reference point is connected to the point O – the center of mass of the system of bodies. The axes are parallel to the axes of coordinate system $CX_CY_CZ_C$;

– $O_1X_1Y_1Z_1$ – connected movable coordinate system. The reference point is connected to the point O_1 – the center of mass of the ring (docking node). The axes coincide with the main central axes of the inertia of the ring. In the initial position of the manipulator ring the axes of the coordinate systems $O_1X_1Y_1Z_1$ are parallel to the axes of the coordinate system $CX_CY_CZ_C$;

– $O_iX_iY_iZ_i$ (for $i=2, 3, 4$) – connected moving coordinate systems. The reference point is connected to the point O_i – the center of the i-th kinematic pair. The O_iX_i axes coincide with the longitudinal axes of the rod links of the manipulator's hand, in the initial position of the links of the manipu-

lator the axes of the coordinate systems $O_iX_iY_iZ_i$ are parallel to the axes of the coordinate system $CX_CY_CZ_C$;

– $MX_MY_MZ_M$ – movable base coordinate system. The reference point is connected to the point M – the center of mass of the manipulator. In the initial position of the manipulator, the axes of the $MX_MY_MZ_M$ coordinate systems are parallel to the axes of the coordinate system $CX_CY_CZ_C$.

4. 4. Theoretical methods

The conclusions, scientific assumptions, and recommendations formulated in this paper are based on the fundamental positions of linear algebra, theoretical mechanics, and the theory of differential equations. The construction of a mathematical model was carried out according to the Nyton-Euler method.

The resulting mathematical model contains:

– an equation of motion of the center of mass of the AMR system of variable configuration along the trajectory in the inertial coordinate system;

– an equation of angular motion of AMR of variable configuration in the inertial coordinate system;

– an equation of motion of the manipulator with respect to AMR.

To construct the equation of motion of the center of mass of AMR of a variable configuration along the trajectory in the inertial coordinate system, the theorem on the center of mass of a solid was applied [23, 24].

To derive the equation of angular motion of AMR with a movable manipulator, a theorem on the change in the kinetic moment of a system of solids was employed [23, 24].

To compile the equation of motion of the manipulator relative to the AMR platform, the theorem on the moments of the amount of motion was applied [23, 24].

In the transformations of equations, the concept of a local derivative in the designated coordinate system is applied.

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5. The results of research of the mathematical model of the dynamics of an autonomous mobile robot of variable configuration

5. 1. Equation of motion of the center of mass of an autonomous mobile robot of variable configuration along the trajectory

Applying the theorem on the motion of the center of mass of a solid [23, 24], the following is recorded for the manipulator and platform in the inertial coordinate system

$$m_M \cdot \frac{d^2\overline{r}_M}{dt^2} = \sum_j \overline{F}_j, \tag{1}$$

$$m_C \cdot \frac{d^2\overline{r}_C}{dt^2} = \sum_i \overline{F}_i, \tag{2}$$

where $\sum_j \overline{F}_j$, $\sum_i \overline{F}_i$ are the sums of the external forces and forces O_1 in the hinge (Fig. 2), acting on the elementary

masses of the manipulator m_j and platform m_i and reduced to their centers of mass.
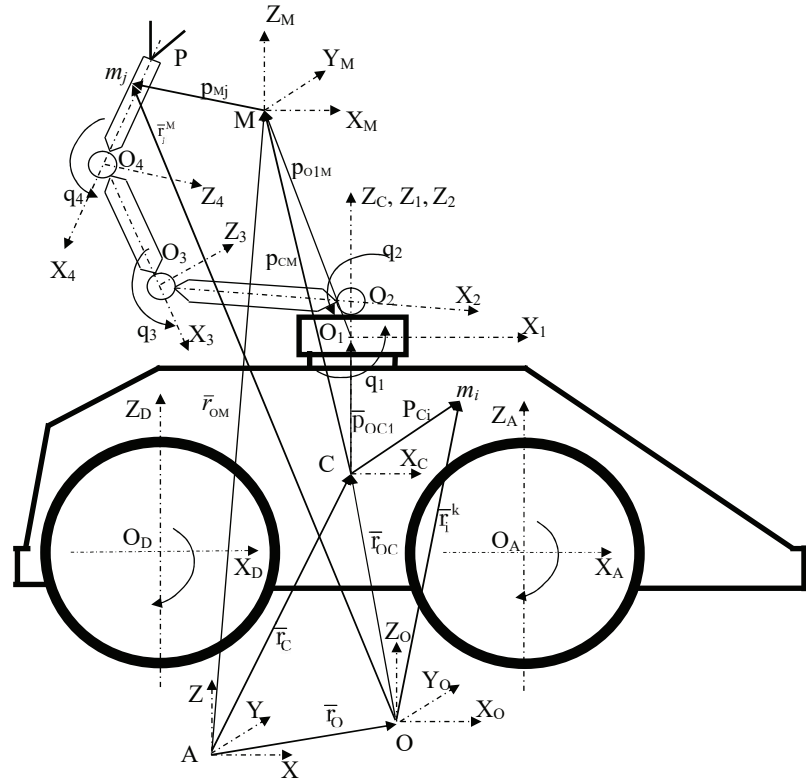

Fig. 2. Estimation scheme for an autonomous mobile robot with a manipulator

Given that $\overline{r}_M = \overline{r}_C + \overline{p}_{CM}$ and $v = i \cup j$, equations (1) and (2) were added:

$$m_M \cdot \frac{d^2\overline{r}_M}{dt^2} + m_C \cdot \frac{d^2\overline{r}_C}{dt^2} =$$

$$= m_M \cdot \frac{d^2\overline{r}_C}{dt^2} + m_M \cdot \frac{d^2\overline{p}_{CM}}{dt^2} + m_C \cdot \frac{d^2\overline{r}_C}{dt^2} =$$

$$= (m_M + m_C) \cdot \frac{d^2\overline{r}_Y}{dt^2} + m_M \cdot \frac{d^2\overline{p}_{CM}}{dt^2} = \sum_v \overline{F}_v,$$

or

$$m \cdot \frac{d^2\overline{r}_C}{dt^2} + m_M \cdot \frac{d^2\overline{p}_{CM}}{dt^2} = \sum_v \overline{F}_v. \tag{3}$$

Considering $\overline{p}_{CM} = \overline{p}_{CO1} + \overline{p}_{O1M}$, then

$$\frac{d\overline{p}_{CM}}{dt} = \frac{d\overline{p}_{CO1}}{dt} + \frac{d\overline{p}_{O1M}}{dt}, \tag{4}$$

where $\overline{p}_{CO1}$ – the radius-vector of point O_1 relative to the center of mass of the platform of point C; $\overline{p}_{O1M}$ – radius-vector of the center of mass of the manipulator of point M relative to O_1.

Suppose that the coordinate system $CX_CY_CZ_C$ moves relative to the inertial coordinate system $AXYZ$ at an angular velocity $\overline{\Omega}$, and the coordinate system $O_1X_1Y_1Z_1$ moves relative to the coordinate system $CX_CY_CZ_C$ with angular velocity $\overline{w}_1$. Applying local derivatives, the following is received:

$$\frac{d\overline{p}_{CO1}}{dt} = \frac{\tilde{d}_0\overline{p}_{CO1}}{dt} + \overline{\Omega} \times \overline{p}_{CO1} = \overline{\Omega} \times \overline{p}_{CO1}, \tag{5}$$

where $\dfrac{\tilde{d}_0}{dt}$ is the operator of the local derivative in the coordinate system $CX_C Y_C Z_C$; $\bar{\Omega}$, – angular velocity of the coordinate system $CX_C Y_C Z_C$ relative to the inertial coordinate system $AXYZ$.

Using the rule of adding vectors and the concept of a local derivative, the following is recorded

$$\frac{d\bar{p}_{O1M}}{dt} = \frac{\tilde{d}_0 \bar{p}_{O1M}}{dt} + \bar{\Omega} \times \bar{p}_{O1M} =$$
$$= \frac{\tilde{d}_1 \bar{p}_{O1M}}{dt} + \bar{w}_1 \times \bar{p}_{O1M} + \bar{\Omega} \times \bar{p}_{O1M} =$$
$$= \frac{\tilde{d}_1 \bar{p}_{O1M}}{dt} + (\bar{w}_1 + \bar{\Omega}) \times \bar{p}_{O1M}, \qquad (6)$$

where $\dfrac{\tilde{d}_0}{dt}$ is the operator of the local derivative in the coordinate system $CX_C Y_C Z_C$; $\dfrac{\tilde{d}_0}{dt}$ – operator of the local derivative in the coordinate system $O_1 X_1 Y_1 Z_1$; $\bar{\Omega}$ –– angular velocity of the coordinate system $CX_C Y_C Z_C$ relative to the inertial coordinate system $AXYZ$; $\bar{w}_1$ – angular velocity of the coordinate system $O_1 X_1 Y_1 Z_1$ relative to the coordinate system $CX_C Y_C Z_C$ (or relative to the coordinate system $OX_0 Y_0 Z_0$).

Substituting expressions (5) and (6) into formula (4):

$$\frac{d\bar{p}_{CM}}{dt} = \bar{\Omega} \times \bar{p}_{CO1} + \frac{\tilde{d}_1 \bar{p}_{O1M}}{dt} + (\bar{w}_1 + \bar{\Omega}) \times \bar{p}_{O1M} =$$
$$= \bar{\Omega} \times \bar{p}_{CM} + \frac{\tilde{d}_1 \bar{p}_{O1M}}{dt} + \bar{w}_1 \times \bar{p}_{O1M},$$

the following expression for the second derivative is recorded

$$\frac{d^2 \bar{p}_{CM}}{dt^2} = \frac{d}{dt}(\bar{\Omega} \times \bar{p}_{CM}) + \frac{d}{dt}\left(\frac{\tilde{d}_1 \bar{p}_{O1M}}{dt}\right) + \frac{d}{dt}(\bar{w}_1 \times \bar{p}_{O1M}) = \frac{d\bar{\Omega}}{dt} \times \bar{p}_{CM} + \bar{\Omega} \times \frac{d\bar{p}_{CM}}{dt} + \frac{d}{dt}\left(\frac{\tilde{d}_1 \bar{p}_{O1M}}{dt}\right) +$$

$$+ \bar{\Omega} \times \frac{\tilde{d}_1 \bar{p}_{O1M}}{dt} + \frac{d\bar{w}_1}{dt} \times \bar{p}_{O1M} + \bar{w}_1 \times \frac{d\bar{p}_{O1M}}{dt} = \left(\frac{\tilde{d}_0 \bar{\Omega}}{dt} + \bar{\Omega} \times \bar{\Omega}\right) \times \bar{p}_{CM} + \bar{\Omega} \times \left(\frac{\tilde{d}_0 \bar{p}_{CM}}{dt} + \bar{\Omega} \times \bar{p}_{CM}\right) +$$

$$+ \frac{\tilde{d}_1}{dt} \times \left(\frac{\tilde{d}_1 \bar{p}_{O1M}}{dt}\right) + \bar{w}_1 \times \frac{\tilde{d}_1 \bar{p}_{O1M}}{dt} + \bar{\Omega} \times \frac{\tilde{d}_1 \bar{p}_{O1M}}{dt} + \left(\frac{\tilde{d}_0 \bar{w}_1}{dt} + \bar{\Omega} \times \bar{w}_1\right)\bar{p}_{O1M} + \bar{w}_1 \times \left(\frac{\tilde{d}_0 \bar{p}_{O1M}}{dt} + \bar{\Omega} \times \bar{p}_{O1M}\right) =$$

$$= \frac{\tilde{d}_0 \bar{\Omega}}{dt} \times \bar{p}_{CM} + \bar{\Omega} \times \frac{\tilde{d}_0 \bar{p}_{CM}}{dt} + \bar{\Omega} \times (\bar{\Omega} \times \bar{p}_{CM}) + \frac{\tilde{d}_1^2 \bar{p}_{O1M}}{dt^2} + \bar{w}_1 \times \frac{\tilde{d}_1 \bar{p}_{O1M}}{dt} + \bar{\Omega} \times \frac{\tilde{d}_1 \bar{p}_{O1M}}{dt} +$$

$$+ \left(\frac{\tilde{d}_1 \bar{w}_1}{dt} + \bar{w}_1 \times \bar{w}_1 + \bar{\Omega} \times \bar{w}_1\right) \times \bar{p}_{O1M} + \bar{w}_1 \times \frac{\tilde{d}_0 \bar{p}_{O1M}}{dt} + \bar{w}_1 \times (\bar{\Omega} \times \bar{p}_{O1M}) = \frac{\tilde{d}_0 \bar{\Omega}}{dt} \times (\bar{p}_{CO1} + \bar{p}_{O1M}) +$$

$$+ \bar{\Omega} \times \frac{\tilde{d}_0}{dt}(\bar{p}_{CO1} + \bar{p}_{O1M}) + \bar{\Omega} \times (\bar{\Omega} \times \bar{p}_{CM}) + \frac{\tilde{d}_1^2 \bar{p}_{O1M}}{dt^2} + (\bar{w}_1 + \bar{\Omega}) \times \frac{\tilde{d}_1 \bar{p}_{O1M}}{dt} + \frac{\tilde{d}_1 \bar{w}_1}{dt} \times \bar{p}_{O1M} +$$

$$+ (\bar{\Omega} \times \bar{w}_1) \times \bar{p}_{O1M} + \bar{w}_1 \times \left(\frac{\tilde{d}1 \bar{p}_{O1M}}{dt} + \bar{w}_1 \times \bar{p}_{O1M}\right) + \bar{w}_1 \times (\bar{\Omega} \times \bar{p}_{O1M}) = \frac{\tilde{d}_0 \bar{\Omega}}{dt} \times (\bar{p}_{CO1} + \bar{p}_{O1M}) +$$

$$+ \bar{\Omega} \times \frac{\tilde{d}_0 \bar{p}_{CO1}}{dt} + \bar{\Omega} \times \frac{\tilde{d}_0 \bar{p}_{O1M}}{dt} + \bar{\Omega} \times (\bar{\Omega} \times \bar{p}_{CM}) + \frac{\tilde{d}_1^2 \bar{p}_{O1M}}{dt^2} + (\bar{w}_1 + \bar{\Omega}) \times \frac{\tilde{d}_1 \bar{p}_{O1M}}{dt} + \frac{\tilde{d}_1 \bar{w}_1}{dt} \times \bar{p}_{O1M} +$$

$$+ \bar{w}_1 \times \frac{\tilde{d}_1 \bar{p}_{O1M}}{dt} + (\bar{\Omega} \times \bar{w}_1) \times \bar{p}_{O1M} + \bar{w}_1 \times (\bar{w}_1 \times \bar{p}_{O1M}) + \bar{w}_1 \times (\bar{\Omega} \times \bar{p}_{O1M}) = \frac{\tilde{d}_0 \bar{\Omega}}{dt} \times \bar{p}_{CO1} +$$

$$+ \frac{\tilde{d}_0 \bar{\Omega}}{dt} \times \bar{p}_{O1M} + \bar{\Omega} \times \left(\frac{\tilde{d}_1 \bar{p}_{O1M}}{dt} + \bar{w}_1 \times \bar{p}_{O1M}\right) + \frac{\tilde{d}_1^2 \bar{p}_{O1M}}{dt^2} + (\bar{w}_1 + \bar{\Omega}) \times \frac{\tilde{d}_1 \bar{p}_{O1M}}{dt} + \frac{\tilde{d}_1 \bar{w}_1}{dt} \times \bar{p}_{O1M} +$$

$$+ \bar{w}_1 \times \frac{\tilde{d}_1 \bar{p}_{O1M}}{dt} + \bar{\Omega} \times (\bar{\Omega} \times \bar{p}_{CM}) + (\bar{\Omega} \times \bar{w}_1) \times \bar{p}_{O1M} + \bar{w}_1 \times (\bar{\Omega} \times \bar{p}_{O1M}) + \bar{w}_1 \times (\bar{w}_1 \times \bar{p}_{O1M}).$$

Thus, after the transformations

$$\frac{d^2 \bar{p}_{CM}}{dt^2} = \frac{\tilde{d}_0 \bar{\Omega}}{dt} \times \bar{p}_{CO1} + \frac{\tilde{d}_0 \bar{\Omega}}{dt} \times \bar{p}_{O1M} +$$
$$+ \bar{\Omega} \times \left(\frac{\tilde{d}_1 \bar{p}_{O1M}}{dt} + \bar{w}_1 \times \bar{p}_{O1M}\right) + \frac{\tilde{d}_1^2 \bar{p}_{O1M}}{dt^2} +$$
$$+ (\bar{w}_1 + \bar{\Omega}) \times \frac{\tilde{d}_1 \bar{p}_{O1M}}{dt} + \frac{\tilde{d}_1 \bar{w}_1}{dt} \times \bar{p}_{O1M} +$$
$$+ \bar{w}_1 \times \frac{\tilde{d}_1 \bar{p}_{O1M}}{dt} + \bar{\Omega} \times (\bar{\Omega} \times \bar{p}_{CM}) +$$
$$+ (\bar{\Omega} \times \bar{w}_1) \times \bar{p}_{O1M} + \bar{w}_1 \times (\bar{\Omega} \times \bar{p}_{O1M}) + \bar{w}_1 \times (\bar{w}_1 \times \bar{p}_{O1M}). \qquad (7)$$

If one considers the intervals of movement of AMR with a fixed manipulator, then the relative speed and acceleration of the center of mass of the manipulator is zero, that is,

$$\bar{w}_1 = 0; \quad \frac{\tilde{d}_1 \bar{w}_1}{dt} = 0; \quad \frac{\tilde{d}_1 \bar{p}_{O1M}}{dt} = 0; \quad \frac{\tilde{d}_1^2 \bar{p}_{O1M}}{dt^2} = 0.$$

Under these conditions, expression (7) takes the form:

$$\frac{d^2 \bar{p}_{CM}}{dt^2} = \frac{\tilde{d}_0 \bar{\Omega}}{dt} \times \bar{p}_{CO1} + \frac{\tilde{d}_0 \bar{\Omega}}{dt} \times \bar{p}_{O1M} + \bar{\Omega} \times (\bar{\Omega} \times \bar{p}_{CM}) =$$
$$= \frac{\tilde{d}_0 \bar{\Omega}}{dt} \times \bar{p}_{CM} + \bar{\Omega} \times (\bar{\Omega} \times \bar{p}_{CM}). \qquad (8)$$

If one considers the intervals of movement of AMR at which the angular velocities are much smaller than linear velocities, in expression (7) one can neglect terms that contain the product of angular velocities as second-order small ones. In this case, expression (7) takes the form:

$$\frac{d^2 \bar{p}_{CM}}{dt^2} = \frac{\tilde{d}_0 \bar{\Omega}}{dt} \times \bar{p}_{CO1} + \frac{\tilde{d}_0 \bar{\Omega}}{dt} \times$$
$$\times \bar{p}_{O1M} + \bar{\Omega} \times \frac{\tilde{d}_1 \bar{p}_{O1M}}{dt} +$$
$$+ \frac{\tilde{d}_1^2 \bar{p}_{O1M}}{dt^2} + (\bar{w}_1 + \bar{\Omega}) \times$$
$$\times \frac{\tilde{d}_1 \bar{p}_{O1M}}{dt} + \frac{\tilde{d}_1 \bar{w}_1}{dt} \times \bar{p}_{O1M} +$$
$$+ \bar{w}_1 \times \frac{\tilde{d}_1 \bar{p}_{O1M}}{dt} = \frac{\tilde{d}_0 \bar{\Omega}}{dt} \times \bar{p}_{CM} +$$
$$+ \frac{\tilde{d}_1^2 \bar{p}_{O1M}}{dt^2} + \bar{w}_1 \times \frac{\tilde{d}_1 \bar{p}_{O1M}}{dt} + \bar{\Omega} \times$$
$$\times \frac{\tilde{d}_1 \bar{p}_{O1M}}{dt} + (\bar{w}_1 + \bar{\Omega}) \times$$
$$\times \frac{\tilde{d}_1 \bar{p}_{O1M}}{dt} + \frac{\tilde{d}_1 \bar{w}_1}{dt} \times \bar{p}_{O1M} =$$
$$= \frac{\tilde{d}_0 \bar{\Omega}}{dt} \times \bar{p}_{CM} + \frac{\tilde{d}_1^2 \bar{p}_{O1M}}{dt^2} +$$
$$+ 2(\bar{w}_1 + \bar{\Omega}) \times \frac{\tilde{d}_1 \bar{p}_{O1M}}{dt} +$$
$$+ \frac{\tilde{d}_1 \bar{w}_1}{dt} \times \bar{p}_{O1M}. \qquad (9)$$

For the specified intervals of movement of AMR, equation (3) takes the form:

– when moving AMR with a fixed manipulator:

$$m \cdot \frac{d^2\bar{r}_{OC}}{dt^2} + m_M \cdot \left[\frac{\tilde{d}_0\bar{\Omega}}{dt} \times \bar{p}_{CM} + \bar{\Omega} \times (\bar{\Omega} \times \bar{p}_{CM}) \right] = \sum_v \bar{F}_v; \quad (10)$$

– when moving AMR with limited angular velocities:

$$m \cdot \frac{d^2\bar{r}_{OC}}{dt^2} + m_M \cdot \begin{bmatrix} \frac{\tilde{d}_0\bar{\Omega}}{dt} \times \bar{p}_{CM} + \frac{\tilde{d}_1^2\bar{p}_{O1M}}{dt^2} + \\ +2(\bar{w}_1 + \bar{\Omega}) \times \frac{\tilde{d}_1\bar{p}_{O1M}}{dt} + \\ +\frac{\tilde{d}_1\bar{w}_1}{dt} \times \bar{p}_{O1M} \end{bmatrix} = \sum_v \bar{F}_v; \quad (11)$$

– when moving AMR with the manipulator movable relative to the platform:

$$m \cdot \frac{d^2\bar{r}_{OC}}{dt^2} +$$

$$+m_M \cdot \begin{bmatrix} \frac{\tilde{d}_0\bar{\Omega}}{dt} \times \bar{p}_{CO1} + \frac{\tilde{d}_0\bar{\Omega}}{dt} \times \bar{p}_{O1M} + \\ +\bar{\Omega} \times \left(\frac{\tilde{d}_1\bar{p}_{O1M}}{dt} + \bar{w}_1 \times \bar{p}_{O1M} \right) + \\ +\frac{\tilde{d}_1^2\bar{p}_{O1M}}{dt^2} + (\bar{w}_1 + \bar{\Omega}) \times \frac{\tilde{d}_1\bar{p}_{O1M}}{dt} + \\ +\frac{\tilde{d}_1\bar{w}_1}{dt} \times \bar{p}_{O1M} + \bar{w}_1 \times \frac{\tilde{d}_1\bar{p}_{O1M}}{dt} + \\ +\bar{\Omega} \times (\bar{\Omega} \times \bar{p}_{CM}) + \\ +(\bar{\Omega} \times \bar{w}_1) \times \bar{p}_{O1M} + \\ +\bar{w}_1 \times (\bar{\Omega} \times \bar{p}_{O1M}) + \\ +\bar{w}_1 \times (\bar{w}_1 \times \bar{p}_{O1M}) \end{bmatrix} = \sum_v \bar{F}_v. \quad (12)$$

If one takes into account the ratio $m_M \cdot \bar{p}_{OM} = m_C \cdot \bar{p}_{CO}$ and $\bar{p}_{CM} = \bar{p}_{CO} + \bar{p}_{OM}$ (Fig. 3), then

$$m_M \frac{\tilde{d}_0\bar{\Omega}}{dt} \times \bar{p}_{CM} = m_M \frac{\tilde{d}_0\bar{\Omega}}{dt} \times (\bar{p}_{CO} + \bar{p}_{OM}) =$$

$$= \frac{\tilde{d}_0\bar{\Omega}}{dt} \times (m_M\bar{p}_{CO} + m_M\bar{p}_{OM}) =$$

$$= \frac{\tilde{d}_0\bar{\Omega}}{dt} \times (m_M\bar{p}_{CO} + m_C\bar{p}_{CO}) =$$

$$= (m_M + m_C)\frac{\tilde{d}_0\bar{\Omega}}{dt} \times \bar{p}_{CO} = m\frac{\tilde{d}_0\bar{\Omega}}{dt} \times \bar{p}_{CO};$$

$$m_M\bar{\Omega} \times (\bar{\Omega} \times \bar{p}_{CM}) = \bar{\Omega} \times (\bar{\Omega} \times m_M(\bar{p}_{CO} + \bar{p}_{OM})) =$$

$$= \bar{\Omega} \times (\bar{\Omega} \times (m_M\bar{p}_{OM} + m \; \bar{p} \quad)) =$$

$$= \bar{\Omega} \times (\bar{\Omega} \times (m_M\bar{p}_{CO} + m_C\bar{p}_{CO})) = m\bar{\Omega} \times (\bar{\Omega} \times \bar{p}_{CO}). \quad (13)$$

So, from the equation of moving the center of mass of AMR with a fixed manipulator (10), after substituting formulas (13), the following is obtained:

$$m \cdot \frac{d^2\bar{r}_{OC}}{dt^2} + m\frac{\tilde{d}_0\bar{\Omega}}{dt} \times \bar{p}_{CO} + m_M\bar{\Omega} \times (\bar{\Omega} \times \bar{p}_{CM}) =$$

$$= m \cdot \left(\frac{d^2\bar{r}_{OC}}{dt^2} + \frac{\tilde{d}_0\bar{\Omega}}{dt} \times \bar{p}_{CO} \right) + m_M\bar{\Omega} \times (\bar{\Omega} \times \bar{p}_{CM}) = \sum_v \bar{F}_v. \quad (14)$$
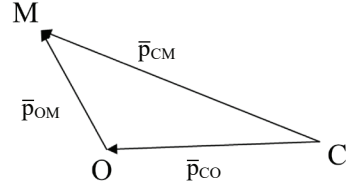

Fig. 3. Estimation scheme: O — center of mass of the system; M — the center of mass of the manipulator; C — the center of mass of the platform of an autonomous mobile robot

If AMR moves at limited angular velocities, then after substituting formulas (13), the following is obtained from equation (11):

$$m \cdot \left(\frac{d^2\bar{r}_{OC}}{dt^2} + \frac{\tilde{d}_0\bar{\Omega}}{dt} \times \bar{p}_{CO} \right) +$$

$$+m_M \cdot \begin{pmatrix} \frac{\tilde{d}_1^2\bar{p}_{O1M}}{dt^2} + 2(\bar{w}_1 + \bar{\Omega}) \times \\ \times \frac{\tilde{d}_1\bar{p}_{O1M}}{dt} + \frac{\tilde{d}_1\bar{w}_1}{dt} \times \bar{p}_{O1M} \end{pmatrix} = \sum_v \bar{F}_v. \quad (15)$$

For the sections of the trajectory on which AMR moves with a movable manipulator, taking into account formulas (13), (12) is recorded in the form:

$$m \cdot \left(\frac{d^2\bar{r}_{OC}}{dt^2} + \frac{\tilde{d}_0\bar{\Omega}}{dt} \times \bar{p}_{CO} + \bar{\Omega} \times (\bar{\Omega} \times \bar{p}_{CO}) \right) +$$

$$+m_M \cdot \begin{bmatrix} \frac{\tilde{d}_1^2\bar{p}_{O1M}}{dt^2} + \\ +2(\bar{w}_1 + \bar{\Omega}) \times \frac{\tilde{d}_1\bar{p}_{O1M}}{dt} + \\ +\frac{\tilde{d}_1\bar{w}_1}{dt} \times \bar{p}_{O1M} + \\ +\bar{\Omega} \times (\bar{w}_1 \times \bar{p}_{O1M}) + \\ +(\bar{\Omega} \times \bar{w}_1) \times \bar{p}_{O1M} + \bar{w}_1 \times (\bar{\Omega} \times \bar{p}_{O1M}) + \\ +\bar{w}_1 \times (\bar{w}_1 \times \bar{p}_{O1M}) \end{bmatrix} = \sum_v \bar{F}_v. \quad (16)$$

Equation (16) is the equation of motion of the center of mass of AMR of a variable configuration along a trajectory in an inertial coordinate system in a general case. There are some cases of the equation of motion of the center of mass of AMR; (14) – if the manipulator does not move, (15) – if the system moves at a limited angular velocity.

5. 2. The equation of angular motion of an autonomous mobile robot with a movable manipulator

Using the theorem on the change in the kinetic moment of the system of solids, the following is recorded for the manipulator:

$$\frac{d}{dt} \sum_j \bar{r}_j \times m_j\bar{v}_j = \bar{M}^M,$$

where $\bar{v}_j = \frac{d\bar{r}_j}{dt};$

$$\sum_j \frac{d}{dt}\bar{r}_j \times m_j\bar{v}_j + \sum_j \bar{r}_j \times m_j\frac{d\bar{v}_j}{dt} = \bar{M}^M,$$

where m_j is the mass of the j-th elementary part of the manipulator; $\overline{r}_j$ – radius-vector of the j-th elementary part of the manipulator relative to the beginning of the inertial coordinate system (Fig. 2); $\overline{v}_j$ – linear velocity of the j-th point of the manipulator relative to the beginning of the inertial coordinate system; $\overline{M}^M$ – the main vector of the moment of external forces and forces in the hinges acting on the manipulator, relative to the beginning of the inertial coordinate system.

Since $\dfrac{d}{dt}\overline{r}_j \times m_j \overline{v}_j = 0$, as a vector product of collinear vectors, then

$$\sum_j \overline{r}_j \times m_j \frac{d\overline{v}_j}{dt} = \overline{M}^M.$$

Fig. 2 shows that $\overline{r}_j = \overline{r}_0 + \overline{p}_{Oj}$ hence

$$\sum_j \overline{r}_0 \times m_j \frac{d\overline{v}_j}{dt} + \sum_j \overline{p}_{Oj} \times m_j \frac{d\overline{v}_j}{dt} = \overline{M}^M,$$

where $\overline{r}_0$ is the radius-vector of point O relative to the beginning of the inertial coordinate system; $\overline{p}_{Oj}$ – radius-vector of the j-th elementary part of the manipulator relative to point O.

Based on the theorem on the change in the amount of motion of the manipulator, the following is recorded:

$$\frac{d}{dt}\sum_j m_j \overline{v}_j = \overline{F}^M,$$

where $\overline{F}^M$ is the main vector of external forces and forces in hinges acting on the manipulator.

Then:

$$\overline{r}_0 \times \overline{F}^M + \sum_j \overline{p}_{Oj} \times m_j \frac{d\overline{v}_j}{dt} = \overline{M}^M.$$

Taking into account that , we obsess:

$$\overline{M}^M - \overline{r}_0 \times \overline{F}^M = \overline{M}_0^M$$

$$\sum_j \overline{p}_{oj} \times m_j \frac{d\overline{v}_j}{dt} = \overline{M}_0^M,$$

where $\overline{M}_0^M$ is the main vector of the moment of external forces and forces in the hinges acting on the manipulator, relative to point O.

Fig. 2 shows that $\overline{v}_j = \overline{v}_0 + \dfrac{d\overline{p}_{oj}}{dt}$, where $\overline{v}_0$ is the linear velocity of point O relative to the beginning of the inertial coordinate system, then

$$\sum_j \overline{p}_{Oj} \times m_j \frac{d}{dt}\left(\overline{v}_0 + \frac{d\overline{p}_{Oj}}{dt}\right) =$$

$$= \sum_j \overline{p}_{Oj} \times m_j \frac{d\overline{v}_0}{dt} + \sum_j \overline{p}_{Oj} \times m_j \frac{d^2\overline{p}_{Oj}}{dt^2} =$$

$$= \sum_j \overline{p}_{Oj} m_j \times \frac{d\overline{v}_0}{dt} +$$

$$+ \sum_j \overline{p}_{Oj} \times m_j \frac{d^2\overline{p}_{oj}}{dt^2} =$$

$$= \overline{p}_{OM} m^M \times \frac{d\overline{v}_0}{dt} + \sum_j \overline{p}_{Oj} \times m_j \frac{d^2\overline{p}_{Oj}}{dt^2} = \overline{M}_0^M. \tag{17}$$

Formula (18) takes into account that $\sum_j \overline{p}_{Oj} m_j = \overline{p}_{OM} m^M$, where $\overline{p}_{OM}$ is the radius-vector of the center of mass of the manipulator of point M relative to the center of mass of the system of point O; m^M – mass of the manipulator.

Applying the theorem on the change of the kinetic moment, for the casing (platform) of the AMR, the following is recorded

$$\frac{d}{dt}\sum_i \overline{r}_i \times m_i \overline{v}_i = \overline{M}^C,$$

where m_i is the mass of the i-th elementary part of AMR body; $\overline{r}_i$ – radius- vector of the i-th elementary part of AMR body relative to the beginning of the inertial system; $\overline{v}_i$ – linear velocity of the i-th point of AMR relative to the beginning of the inertial coordinate system; $\overline{M}^C$ – the main vector of the moment of external forces and forces in the hinges acting on AMR hull relative to the beginning of the inertial coordinate system.

Fig. 2 shows that $\overline{r}_i = \overline{r}_0 + \overline{p}_{Oi}$ hence

$$\sum_i \overline{r}_0 \times m_i \frac{d\overline{v}_i}{dt} + \sum_i \overline{p}_{Oi} \times m_i \frac{d\overline{v}_i}{dt} = \overline{M}^C,$$

where $\overline{r}_0$ is the radius-vector of point O relative to the beginning of the inertial coordinate system; $\overline{p}_{Oi}$ – radius-vector of the i-th elementary part of the platform relative to the center of mass of the system of point O.

Take into account that $\sum_i \overline{p}_{Oi} m_i = \overline{p}_{OC} m^C$, where $\overline{p}_{Oi}$ is the radius-vector of the i-th elementary part of AMR housing relative to point O; $\overline{p}_{OC}$ – radius-vector of the center of mass of the hull of AMR relative to the point O; m^C – mass of AMR body. After transformations, the following is obtained:

$$\overline{p}_{OC} m^C \times \frac{d\overline{v}_0}{dt} + \sum_i \overline{p}_{Oi} \times m_i \frac{d^2\overline{p}_{Oi}}{dt^2} = \overline{M}_0^C, \tag{18}$$

where $\overline{M}_0^C$ is the main vector of the moment of external forces and forces in the hinges acting on AMR hull relative to point O.

Adding (17) and (18), the following is obtained:

$$\overline{p}_{OM} m^M \times \frac{d\overline{v}_0}{dt} + \sum_j \overline{p}_{Oj} \times m_j \frac{d^2\overline{p}_{Oj}}{dt^2} +$$

$$+ \overline{p}_{OC} m^C \times \frac{d\overline{v}_0}{dt} + \sum_i \overline{p}_{Oi} \times m_i \frac{d^2\overline{p} Oi}{dt^2} = \overline{M}_0^C + \overline{M}_0^M,$$

$$\left(\overline{p}_{OM} m^M + \overline{p}_{OC} m^C\right) \times \frac{d\overline{v}_0}{dt} +$$

$$+ \sum_j \overline{p}_{Oj} \times m_j \frac{d^2\overline{p}_{Oj}}{dt^2} + \sum_i \overline{p}_{Oi} \times m_i \frac{d^2\overline{p}_{Oi}}{dt^2} = \overline{M}_0, \tag{19}$$

where $\overline{M}_0$ is the main vector of the moment of external forces relative to the center of mass of the system of point O. Since $\overline{p}_{OM} m^M + \overline{p}_{OC} m^C = 0$, as the static moment of mass m^M and m^C relative to the center of mass of the system, equation (19) can be written as:

$$\sum_j \overline{p}_{Oj} \times m_j \frac{d^2\overline{p}_{Oj}}{dt^2} + \sum_i \overline{p}_{Oi} \times m_i \frac{d^2\overline{p}_{Oi}}{dt^2} = \overline{M}_0. \tag{20}$$

The derivatives $\dfrac{d^2\bar{p}_{Oj}}{dt^2}$ and $\dfrac{d^2\bar{p}_{Oi}}{dt^2}$ were derived using the concept of a local derivative:

$$\frac{d\bar{p}_{Oj}}{dt}=\frac{\tilde{d}_0\bar{p}_{Oj}}{dt}+\bar{\Omega}\times\bar{p}_{Oj},$$

where $\dfrac{\tilde{d}_0}{dt}$ is the operator of the local derivative in the co-ordinate system $CX_cY_cZ_c$; $\bar{\Omega}$ – angular velocity of the co-ordinate system $CX_cY_cZ_c$ relative to the inertial coordinate system $AXYZ$.

Fig. 2 shows that $\bar{p}_{Oj}=\bar{p}_{OO1}+\bar{p}_{O1j}$, where $\bar{p}_{OO1}$ is the radius-vector of the center of the kinematic pair (docking node) O_1 relative to the point O; $\bar{p}_{O1j}$ is the radius-vector of the j-th elementary part of the manipulator relative to the hinge O_1, then

$$\frac{d\bar{p}_{Oj}}{dt}=\frac{d}{dt}\left(\bar{p}_{OO1}+\bar{p}_{O1j}\right)=\frac{\tilde{d}_0\bar{p}_{OO1}}{dt}+\frac{\tilde{d}_0\bar{p}_{O1j}}{dt}+\bar{\Omega}\times\bar{p}_{j}.$$

Using the rule of adding vectors and the concept of a local derivative, the following is recorded

$$\frac{d\bar{p}_{Oj}}{dt}=\frac{\tilde{d}_0\bar{p}_{OO1}}{dt}+\frac{\tilde{d}_0\bar{p}_{O1j}}{dt}+\bar{\Omega}\times\bar{p}_{Oj},$$

$$\frac{\tilde{d}_0\bar{p}_{O1j}}{dt}=\frac{\tilde{d}_1\bar{p}_{O1j}}{dt}+\bar{w}_1\times\bar{p}_{O1j},$$

where $\dfrac{\tilde{d}_1}{dt}$ is the operator of the local derivative in the coordinate system $O_1X_1Y_1Z_1$, $O_1X_1Y_1Z_1$, $\bar{w}_1$ is the angular velocity of the coordinate system $O_1X_1Y_1Z_1$ relative to the coordinate system $OXYZ$.

By analogy, the following is obtained

$$\frac{\tilde{d}_1\bar{p}_{O1j}}{dt}=\frac{\tilde{d}_2\bar{p}_{O1O2}}{dt}+\frac{\tilde{d}_2\bar{p}_{O2j}}{dt}+\bar{w}_2\times\bar{p}_{O2j}=$$
$$=\frac{\tilde{d}_2\bar{p}_{O2j}}{dt}+\bar{w}_2\times\bar{p}_{O2j},$$

$$\frac{\tilde{d}_2\bar{p}_{O2j}}{dt}=\frac{\tilde{d}_3\bar{p}_{O2O3}}{dt}+\frac{\tilde{d}_3\bar{p}_{O3j}}{dt}+\bar{w}_3\times\bar{p}_{O3j}=$$
$$=\frac{\tilde{d}_3\bar{p}_{O3j}}{dt}+\bar{w}_3\times\bar{p}_{O3j},$$

$$\frac{\tilde{d}_3\bar{p}_{O3j}}{dt}=\frac{\tilde{d}_4\bar{p}_{O3O4}}{dt}+\frac{\tilde{d}_4\bar{p}_{O4j}}{dt}+\bar{w}4\times\bar{p}_{O4j}=$$
$$=\frac{\tilde{d}_4\bar{p}_{O4j}}{dt}+\bar{w}_4\times\bar{p}_{O4j},$$

where $\dfrac{\tilde{d}_2}{dt}$ is the operator of the local derivative in the coordinate system $O_2X_2Y_2Z_2$; $\bar{w}_2$ – angular velocity of the coordinate system $O_2X_2Y_2Z_2$ relative to the coordinate system $O_1X_1Y_1Z_1$; $\dfrac{\tilde{d}_3}{dt}$ – operator of the local derivative in the coordinate system $O_3X_3Y_3Z_3$; $\bar{w}_3$ – angular velocity of the coordinate system $O_3X_3Y_3Z_3$ relative to the coordinate system $O_2X_2Y_2Z_2$;

$\dfrac{\tilde{d}_4}{dt}$ – operator of the local derivative in the coordinate system $O_4X_4Y_4Z_4$; $\bar{w}_4$ – angular velocity of the coordinate system $O_4X_4Y_4Z_4$ relative to the coordinate system $O_3X_3Y_3Z_3$.

Taking into account

$$\frac{\tilde{d}_2\bar{p}_{O1O2}}{dt}=\frac{\tilde{d}_3\bar{p}_{O2O3}}{dt}=\frac{\tilde{d}_4\bar{p}_{O3O4}}{dt}=\frac{\tilde{d}_4\bar{p}_{O4j}}{dt}=0,$$

after the substitution, the following is obtained

$$\frac{d\bar{p}_{Oj}}{dt}=\frac{\tilde{d}_0\bar{p}_{OO1}}{dt}+\bar{w}_4\times\bar{p}_{O4j}+\bar{w}_3\times\bar{p}_{O3j}+$$
$$+\bar{w}_2\times\bar{p}_{O2j}+\bar{w}_1\times\bar{p}_{O1j}+\bar{\Omega}\times\bar{p}_{Oj},$$

$$\frac{d^2\bar{p}_{Oj}}{dt}=\frac{\tilde{d}_0^2\bar{p}_{OO1}}{dt}+\bar{\Omega}\times\frac{\tilde{d}_0\bar{p}_{OO1}}{dt}+$$
$$+\frac{\tilde{d}_0\left(\bar{w}_4\times\bar{p}_{O4j}\right)}{dt}+\bar{\Omega}\times\left(\bar{w}_4\times\bar{p}_{O4j}\right)+$$
$$+\frac{\tilde{d}_0\left(\bar{w}_3\times\bar{p}_{O3j}\right)}{dt}+\bar{\Omega}\times\left(\bar{w}_3\times\bar{p}_{O3j}\right)+$$
$$+\frac{\tilde{d}_0\left(\bar{w}_2\times\bar{p}_{O2j}\right)}{dt}+\bar{\Omega}\times\left(\bar{w}_2\times\bar{p}_{O2j}\right)+$$
$$+\frac{\tilde{d}_0\left(\bar{w}_1\times\bar{p}_{O1j}\right)}{dt}+\bar{\Omega}\times\left(\bar{w}_1\times\bar{p}_{O1j}\right)+$$
$$+\frac{\tilde{d}_0\left(\bar{\Omega}\times\bar{p}_{Oj}\right)}{dt}+\bar{\Omega}\times\left(\bar{\Omega}\times\bar{p}_{Oj}\right). \tag{21}$$

By analogy, record for $\bar{p}_{oi}$

$$\frac{d\bar{p}_{Oi}}{dt}=\frac{\tilde{d}_0\bar{p}_{OO1}}{dt}+\frac{\tilde{d}_0\bar{p}_{O1i}}{dt}+\bar{\Omega}\times\bar{p}_{Oi};$$

$$\frac{d^2\bar{p}_{Oi}}{dt}=\frac{\tilde{d}_0^2\bar{p}_{OO1}}{dt}+\frac{\tilde{d}_0^2\bar{p}_{O1i}}{dt}+\frac{\tilde{d}0\left(\bar{\Omega}\times\bar{p}_{Oi}\right)}{dt}+$$
$$+\bar{\Omega}\times\frac{\tilde{d}_0\bar{p}_{OO1}}{dt}+\bar{\Omega}\times\frac{\tilde{d}_0\bar{p}_{Oi}}{dt}+\bar{\Omega}\times\left(\bar{\Omega}\times\bar{p}_{Oi}\right), \tag{22}$$

where $\dfrac{\tilde{d}_0}{dt}$ is the operator of the local derivative in the coordinate system $CX_CY_CZ_C$; $\bar{\Omega}$ – angular velocity of the coordinate system $CX_CY_CZ_C$ relative to the inertial coordinate system $AXYZ$.

Since the design of AMR platform can provide for the linear movement of points relative to the coordinate system $CX_CY_CZ_C$ (for example, for a tracked chassis), then, in a general case,

$$\frac{\tilde{d}_0\bar{p}_{Oi}}{dt}\neq 0$$

and

$$\frac{\tilde{d}_0\bar{p}_{O1i}}{dt}\neq 0.$$

Substituting (21) and (22) in (20), the following is obtained:

$$\sum_j \bar{p}_{Oj} \times m_j \left\{ \frac{\tilde{d}_0^2 \bar{p}_{OO1}}{dt} + \bar{\Omega} \times \frac{\tilde{d}_0 \bar{p}_{OO1}}{dt} + \frac{\tilde{d}_0 \left(\bar{w}_4 \times \bar{p}_{O4j} \right)}{dt} + \right. \\ + \bar{\Omega} \times \left(\bar{w}_4 \times \bar{p}_{O4j} \right) + \\ + \frac{\tilde{d}_0 \left(\bar{w}_3 \times \bar{p}_{O3j} \right)}{dt} + \bar{\Omega} \times \left(\bar{w}_3 \times \bar{p}_{O3j} \right) + \\ + \frac{\tilde{d}_0 \left(\bar{w}_2 \times \bar{p}_{O2j} \right)}{dt} + \bar{\Omega} \times \left(\bar{w}_2 \times \bar{p}_{O2j} \right) + \\ + \frac{\tilde{d}_0 \left(\bar{w}_1 \times \bar{p}_{O1j} \right)}{dt} + \bar{\Omega} \times \left(\bar{w}_1 \times \bar{p}_{O1j} \right) + \\ \left. + \frac{\tilde{d}_0 \left(\bar{\Omega} \times \bar{p}_{Oj} \right)}{dt} + \bar{\Omega} \times \left(\bar{\Omega} \times \bar{p}_{Oj} \right) \right\} +$$

$$+ \sum_i \bar{p}_{Oi} \times m_i \left\{ \frac{\tilde{d}_0^2 \bar{p}_{OO1}}{dt} + \bar{\Omega} \times \frac{\tilde{d}_0 \bar{p}_{OO1}}{dt} + \frac{\tilde{d}_0^2 \bar{p}_{O1i}}{dt} \right. \\ + \bar{\Omega} \times \frac{\tilde{d}_0 \bar{p}_{O1i}}{dt} + \\ \left. + \frac{\tilde{d}_0 \left(\bar{\Omega} \times \bar{p}_{Oi} \right)}{dt} + \bar{\Omega} \times \left(\bar{\Omega} \times \bar{p}_{Oi} \right) \right\} = \bar{M}_0. \quad (23)$$

Let's simplify equation (23):
1) In a general case,

$$\left(\sum_j \bar{p}_{Oj} \times m_j + \sum_i \bar{p}_{Oi} \times m_i \right) \times \\ \times \left(\frac{\tilde{d}_0^2 \bar{p}_{OO1}}{dt} + \bar{\Omega} \times \frac{\tilde{d}_0 \bar{p}_{OO1}}{dt} \right) = 0,$$

because

$$\sum_j \bar{p}_{Oj} \times m_j + \sum_i \bar{p}_{Oi} \times m_i = 0,$$

as a static moment of mass relative to the center of mass of the system; then equation (23) will take the following form:

$$\sum_j \bar{p}_{Oj} \times m_j \left\{ \frac{\tilde{d}_0 \left(\bar{w}_4 \times \bar{p}_{O4j} \right)}{dt} + \bar{\Omega} \times \left(\bar{w}_4 \times \bar{p}_{O4j} \right) + \right. \\ + \frac{\tilde{d}_0 \left(\bar{w}_3 \times \bar{p}_{O3j} \right)}{dt} + \bar{\Omega} \times \left(\bar{w}_3 \times \bar{p}_{O3j} \right) + \\ + \frac{\tilde{d}_0 \left(\bar{w}_2 \times \bar{p}_{O2j} \right)}{dt} + \bar{\Omega} \times \left(\bar{w}_2 \times \bar{p}_{O2j} \right) + \\ + \frac{\tilde{d}_0 \left(\bar{w}_1 \times \bar{p}_{O1j} \right)}{dt} + \bar{\Omega} \times \left(\bar{w}_1 \times \bar{p}_{O1j} \right) + \\ \left. + \frac{\tilde{d}_0 \left(\bar{\Omega} \times \bar{p}_{Oj} \right)}{dt} + \bar{\Omega} \times \left(\bar{\Omega} \times \bar{p}_{Oj} \right) \right\} +$$

$$+ \sum_i \bar{p}_{Oi} \times m_i \left\{ \frac{\tilde{d}_0^2 \bar{p}_{O1i}}{dt} + \bar{\Omega} \times \frac{\tilde{d}_0 \bar{p}_{O1i}}{dt} + \right. \\ \left. \frac{\tilde{d}_0 \left(\bar{\Omega} \times \bar{p}_{Oi} \right)}{dt} + \bar{\Omega} \times \left(\bar{\Omega} \times \bar{p}_{Oi} \right) \right\} = \bar{M}_0. \quad (24)$$

2) If one considers the intervals of movement of AMR with a fixed manipulator, then the angular velocities in the kinematic pairs of the manipulator are zero $\bar{w}_1 = \bar{w}_2 = \bar{w}_3 = \bar{w}_4 = 0$. Under these conditions, expression (24) takes the form:

$$\sum_j \bar{p}_{Oj} \times m_j \left\{ \frac{\tilde{d}_0 \left(\bar{\Omega} \times \bar{p}_{Oj} \right)}{dt} + \bar{\Omega} \times \left(\bar{\Omega} \times \bar{p}_{Oj} \right) \right\} +$$

$$+ \sum_i \bar{p}_{Oi} \times m_i \left\{ \frac{\tilde{d}_0^2 \bar{p}_{O1i}}{dt} + \bar{\Omega} \times \frac{\tilde{d}_0 \bar{p}_{O1i}}{dt} + \right. \\ \left. \frac{\tilde{d}_0 \left(\bar{\Omega} \times \bar{p}_{Oi} \right)}{dt} + \bar{\Omega} \times \left(\bar{\Omega} \times \bar{p}_{Oi} \right) \right\} = \bar{M}_0. \quad (25)$$

3) If one considers the intervals of movement of AMR at which the angular velocities are much smaller than linear velocities, in expression (24) one can neglect terms that contain the product of angular velocities as small second-order. After the mathematical transformations in equation (24), the following is obtained:

$$\sum_j \bar{p}_{Oj} \times m_j \left\{ \frac{\tilde{d}_0 \left(\bar{w}_4 \times \bar{p}_{O4j} \right)}{dt} + \frac{\tilde{d}_0 \left(\bar{w}_3 \times \bar{p}_{O3j} \right)}{dt} + \right. \\ + \frac{\tilde{d}_0 \left(\bar{w}_2 \times \bar{p}_{O2j} \right)}{dt} + \\ \left. + \frac{\tilde{d}_0 \left(\bar{w}_1 \times \bar{p}_{O1j} \right)}{dt} + \frac{\tilde{d}_0 \left(\bar{\Omega} \times \bar{p}_{Oj} \right)}{dt} \right\} +$$

$$+ \sum_i \bar{p}_{Oi} \times m_i \left\{ \frac{\tilde{d}_0^2 \bar{p}_{O1i}}{dt} + \bar{\Omega} \times \frac{\tilde{d}_0 \bar{p}_{O1i}}{dt} + \frac{\tilde{d}_0 \left(\bar{\Omega} \times \bar{p}_{Oi} \right)}{dt} \right\} = \bar{M}_0. \quad (26)$$

Equations (24) to (26) is the equation of angular motion of AMR with the manipulator as a system of bodies relative to the inertial coordinate system.

5. 3. The equation of motion of the manipulator relative to the body of an autonomous mobile robot

Consider the manipulator as a body transferred in space by the AMR body. Based on the theorem on the moments of the amount of motion, the following is recorded for the manipulator:

$$\frac{d}{dt} \sum_j \bar{r}_j \times m_j \bar{v}_j = \bar{M}^M,$$

$$\sum_j \frac{d}{dt} \bar{r}_j \times m_j \bar{v}_j + \sum_j \bar{r}_j \times m_j \frac{d\bar{v}_j}{dt} = \bar{M}^M.$$

Since $\frac{d}{dt} \bar{r}_j \times m_j \bar{v}_j = 0$, as a vector product of collinear vectors, then

$$\sum_j \bar{r}_j \times m_j \frac{d\bar{v}_j}{dt} = \bar{M}^M.$$

Fig. 2 shows that $\bar{r}_j = \bar{r}_0 + \bar{p}_{OO1} + \bar{p}_{O1j}$ hence

$$\sum_j \left(\bar{r}_0 + \bar{p}_{OO1} + \bar{p}_{O1j} \right) \times m_j \frac{d\bar{v}_j}{dt} = \bar{M}^M, \quad (27)$$

where $\bar{r}_0$ is the radius-vector of point O relative to the reference of the inertial coordinate system; $\bar{p}_{Oj}$ – radius-vector of the j-th elementary part of the manipulator relative to the point O.

From Newton's theorem on the amount of motion, the following is obtained:

$$\sum_j m_j \frac{d\overline{v}_j}{dt} = \overline{F}^M,$$

then

$$(\overline{r}_0 + \overline{p}_{OO1}) \times \overline{F}^M + \sum_j \overline{p}_{O1j} \times m_j \frac{d\overline{v}_j}{dt} = \overline{M}^M,$$

or

$$\sum_j \overline{p}_{O1j} \times m_j \frac{d\overline{v}_j}{dt} = \overline{M}_{O1}^M. \qquad (28)$$

Formula (28) takes into account that

$$\overline{M}_{O1}^M = \overline{M}^M - (\overline{r}_0 + \overline{p}_{OO1}) \times \overline{F}^M$$

is the main moment applied to the manipulator relative to the docking node (point O_1).

The speed of the elementary mass of the manipulator m_j is equal to:

$$\overline{v}_j = \frac{d\overline{r}_j}{dt} = \frac{d(\overline{r}_0 + \overline{p}_{Oj})}{dt} = \frac{d\overline{r}_0}{dt} + \frac{d\overline{p}_{Oj}}{dt} = \overline{v}_0 + \frac{d\overline{p}_{Oj}}{dt}.$$

Fig. 2 shows that $\overline{p}_{Oj} = \overline{p}_{OC} + \overline{p}_{CO1} + \overline{p}_{O1j}$, then

$$\frac{d\overline{p}_{Oj}}{dt} = \frac{d\overline{p}_{OC}}{dt} + \frac{d\overline{p}_{CO1}}{dt} + \frac{d\overline{p}_{O1j}}{dt} = \frac{d\overline{p}_{OC}}{dt} + \frac{d\overline{p}_{O1j}}{dt}. \qquad (29)$$

Equation (29) takes into account that $\frac{d\overline{p}_{CO1}}{dt} = 0$, since $\overline{p}_{CO1} = \text{const}$.

Calculate the individual terms in equation (29) using the concept of a local derivative:

$$\frac{d\overline{p}_{OC}}{dt} = \frac{\tilde{d}_0 \overline{p}_{OC}}{dt} + \overline{\Omega} \times \overline{p}_{OC};$$

$$\frac{d\overline{p}_{O1j}}{dt} = \frac{\tilde{d}_0 \overline{p}_{O1j}}{dt} + \overline{\Omega} \times \overline{p}_{O1j} =$$

$$= \frac{\tilde{d}_1 \overline{p}_{O1j}}{dt} + \overline{w}_1 \times \overline{p}_{O1j} + \overline{\Omega} \times \overline{p}_{O1j}.$$

The following is obtained after the transformation:

$$\overline{v}_j = \overline{v}_0 + \frac{\tilde{d}_0 \overline{p}_{OC}}{dt} + \overline{\Omega} \times \overline{p}_{OC} +$$

$$+ \frac{\tilde{d}_1 \overline{p}_{O1j}}{dt} + \overline{w}_1 \times \overline{p}_{O1j} + \overline{\Omega} \times \overline{p}_{O1j},$$

then

$$\frac{d\overline{v}_j}{dt} = \frac{d\overline{v}_0}{dt} + \frac{d}{dt}\left(\frac{\tilde{d}_0 \overline{p}_{OC}}{dt}\right) + \frac{d\overline{\Omega}}{dt} \times \overline{p}_{OC} +$$

$$+ \overline{\Omega} \times \frac{\tilde{d}\overline{p}_{OC}}{dt} + \frac{d}{dt}\left(\frac{\tilde{d}_1 \overline{p}_{O1j}}{dt}\right) +$$

$$+ \frac{d\overline{w}_1}{dt} \times \overline{p}_{O1j} + \overline{w}_1 \times \frac{d\overline{p}_{O1j}}{dt} +$$

$$+ \frac{d\overline{\Omega}}{dt} \times \overline{p}_{O1j} + \overline{\Omega} \times \frac{d\overline{p}_{O1j}}{dt}. \qquad (30)$$

Considering

$$\frac{d}{dt}\left(\frac{\tilde{d}_0 \overline{p}_{OC}}{dt}\right) = \frac{\tilde{d}_0^2 \overline{p}_{OC}}{dt} + \overline{\Omega} \times \frac{\tilde{d}_0 \overline{p}_{OC}}{dt},$$

and

$$\frac{d}{dt}\left(\frac{\tilde{d}_1 \overline{p}_{O1j}}{dt}\right) = \frac{\tilde{d}_0}{dt}\left(\frac{\tilde{d}_1 \overline{p}_{O1j}}{dt}\right) + \overline{\Omega} \times \frac{\tilde{d}_1 \overline{p}_{O1j}}{dt} =$$

$$= \frac{\tilde{d}_1^2 \overline{p}_{O1j}}{dt} + \overline{w}_1 \times \frac{\tilde{d}_1 \overline{p}_{O1j}}{dt} + \overline{\Omega} \times \frac{\tilde{d}_1 \overline{p}_{O1j}}{dt},$$

then equation (30) takes the form

$$\frac{d\overline{v}_j}{dt} = \frac{d\overline{v}_0}{dt} + \frac{\tilde{d}_0^2 \overline{p}_{OC}}{dt} + \overline{\Omega} \times \frac{\tilde{d}_0 \overline{p}_{OC}}{dt} + \frac{d\overline{\Omega}}{dt} \times \overline{p}_{OC} +$$

$$+ \overline{\Omega} \times \frac{\tilde{d}\overline{p}_{OC}}{dt} + \frac{\tilde{d}_1^2 \overline{p}_{O1j}}{dt} + \overline{w}_1 \times \frac{\tilde{d}_1 \overline{p}_{O1j}}{dt} +$$

$$+ \overline{\Omega} \times \frac{\tilde{d}_1 \overline{p}_{O1j}}{dt} + \frac{d\overline{w}_1}{dt} \times \overline{p}_{O1j} + \overline{w}_1 \times \frac{d\overline{p}_{O1j}}{dt} +$$

$$+ \frac{d\overline{\Omega}}{dt} \times \overline{p}_{O1j} + \overline{\Omega} \times \frac{d\overline{p}_{O1j}}{dt}. \qquad (31)$$

Substitute (31) in (28):

$$\sum_j \overline{p}_{O1j} \times m_j \begin{pmatrix} \dfrac{d\overline{v}_0}{dt} + \dfrac{\tilde{d}_0^2 \overline{p}_{OC}}{dt} + \overline{\Omega} \times \dfrac{\tilde{d}_0 \overline{p}_{OC}}{dt} + \\[2mm] + \dfrac{d\overline{\Omega}}{dt} \times \overline{p}_{OC} + \overline{\Omega} \times \dfrac{\tilde{d}\overline{p}_{OC}}{dt} + \\[2mm] + \dfrac{\tilde{d}_1^2 \overline{p}_{O1j}}{dt} + \overline{w}_1 \times \dfrac{\tilde{d}_1 \overline{p}_{O1j}}{dt} + \\[2mm] + \overline{\Omega} \times \dfrac{\tilde{d}_1 \overline{p}_{O1j}}{dt} + \dfrac{d\overline{w}_1}{dt} \times \overline{p}_{O1j} + \\[2mm] + \overline{w}_1 \times \dfrac{d\overline{p}_{O1j}}{dt} + \\[2mm] + \dfrac{d\overline{\Omega}}{dt} \times \overline{p}_{O1j} + \overline{\Omega} \times \dfrac{d\overline{p}_{O1j}}{dt} \end{pmatrix} = \overline{M}_{O1}^M.$$

Since

$$\sum_j \overline{p}_{O1j} m_j = m_M \overline{p}_{O1M},$$

the following is obtained from equation (31):

$$m_M \overline{p}_{O1M} \begin{pmatrix} \dfrac{d\overline{v}_0}{dt} + \dfrac{\tilde{d}_0^2 \overline{p}_{OC}}{dt} + \overline{\Omega} \times \dfrac{\tilde{d}_0 \overline{p}_{OC}}{dt} + \\[2mm] + \dfrac{d\overline{\Omega}}{dt} \times \overline{p}_{OC} + \overline{\Omega} \times \dfrac{\tilde{d}\overline{p}_{OC}}{dt} \end{pmatrix} +$$

$$+ \sum_j m_j \overline{p}_{O1j} \times \begin{pmatrix} \dfrac{\tilde{d}_1^2 \overline{p}_{O1j}}{dt} + \overline{w}_1 \times \dfrac{\tilde{d}_1 \overline{p}_{O1j}}{dt} + \\[2mm] + \overline{\Omega} \times \dfrac{\tilde{d}_1 \overline{p}_{O1j}}{dt} + \\[2mm] + \dfrac{d\overline{w}_1}{dt} \times \overline{p}_{O1j} + \overline{w}_1 \times \dfrac{d\overline{p}_{O1j}}{dt} + \\[2mm] + \dfrac{d\overline{\Omega}}{dt} \times \overline{p}_{O1j} + \overline{\Omega} \times \dfrac{d\overline{p}_{O1j}}{dt} \end{pmatrix} = \overline{M}_{O1}^M. \qquad (32)$$

Formula (32) is the equation of motion of the manipulator relative to AMR housing in the general case.

6. Discussion of research results

Using the Newton-Euler method, a mathematical model of AMR dynamics of a variable configuration was built. A feature of the results obtained is to take into account the relationship of the dynamic parameters of the mechanical system.

As an example, the structure of AMR with a manipulator, which is designed to overcome the consequences of man-made and natural disasters, is considered. The features of the dynamics of AMR with the manipulator are due to the change in the position of the center of mass of the system with the relative motion of the manipulator and the commensuration of the non-diagonal and diagonal elements of the inertia tensor, calculated relative to the axes of the base coordinate system.

The proposed mathematical model describes three modes:
– controlled movement of the platform along a given route with fixed M;
– movement of AMR, provided that the angular velocities are significantly less than linear velocities;
– M performs technological operations with controlled movement of the platform.

The equations of the mathematical model are derived for the specified modes of motion of AMR with M. If one considers the intervals of movement of AMR at which the angular velocities are much smaller than linear velocities, then in the equations of dynamics one can neglect terms that contain the product of angular velocities, as small second-order. If one considers the intervals of movement of AMR with a fixed manipulator, then the angular velocities in the kinematic pairs of the manipulator are zero.

The mathematical model of the dynamics of AMR of changing configuration, taking into account the relationship of the dynamic parameters of the mechanical system, contains:
– equation (16) – the equation of motion of the center of mass of AMR of a variable configuration along a trajectory in an inertial coordinate system in general. In addition, the equation of motion of the center of mass of AMR in some cases is derived: (14) – if the manipulator does not move, and (15) – if the system moves at a limited angular velocity;
– equation (24) – the equation of angular motion of AMR of a variable configuration in an inertial coordinate system. The equations of angular motion of AMR in some cases are given: (25) – if the manipulator does not move, and (26) – if the system moves at a limited angular velocity;
– (32) – the equation of motion of the manipulator relative to the AMR housing in the general case.

The AMR chassis control system provides movement in a limited space (work area) along a given trajectory using GPS navigation or orientation on a map or beacons. Within the working area, static and (or) moving obstacles are possible, which must be overcome with the slightest deviation from the specified route. The mathematical models of dynamics presented in [11–15] provide reliable tracking by the trolley of the reference trajectory but the considered design does not imply the presence of moving elements (M or attachments). In equations (14) to (16), the relationship of the dynamic parameters of the system from the direction and magnitude of the radius-vector $\bar{p}_{OC}$. is established. The shift of the center of mass of the platform relative to the center of mass of the sys-

tem of bodies by the $\bar{p}_{OC}$ radius-vector is due to the presence of moving elements (M or attachments). The application of equations (14) to (16) provides tracking by the AMR platform of the reference trajectory in the presence of moving elements (M or attachments). This will prevent the occurrence of angular velocities in the directions perpendicular to the direction of movement of the platform, avoid slipping and skidding of the platform, and maintain the orientation of AMR with M.

Remote control of the manipulator provides high quality indicators of technological operations both with a non-moving and mobile chassis. The mathematical models proposed in [3–7, 17–19] do not take into account the change in the position of the center of mass of the system with the relative motion of the manipulator. Models [19–22] do not make it possible to separate the influence of dynamic parameters on the interconnection of control channels.

The design of the grip and the kinematic scheme of the manipulator provides for the possibility of working with fragile or explosive objects. Equations (24) to (26) make it possible to establish the influence of dynamic parameters of the system on generalized speeds $\bar{w}_j$ and accelerations $\dfrac{d\bar{w}_j}{dt}$ in the kinematic pairs of the manipulator. The movable base of M causes a change in the working area and an increase in the position error of the grip pole. The application of equations (24) to (26) provides software adjustment of the dynamic parameters of the manipulator when moving the AMR platform along the trajectory.

Equation (32) enables control and software adjustment of the dynamic parameters of the manipulator and contact forces in the grip. The application of equation (32) makes it possible to fix the object and ensure its reliable retention when moving the AMR platform along the trajectory.

Adjusting the operation of the control system reduces energy consumption, prolong the time of autonomous work, and increases the productivity of AMR.

The limitations of this study are due to the consideration of only three modes of movement of AMR with M and the assumption that the elements of the kinematic scheme of M and the AMR platform are absolutely rigid bodies. For other modes of motion of AMR with M, it is necessary to refine the mathematical model based on reasonable assumptions and simplifications. The limitations of this study must be taken into account when trying to apply the developed mathematical model in practice, as well as in further theoretical studies. The disadvantage of the study is that the dynamics of kinematic pairs are not taken into account.

Advancing this study is the development of an adaptive control system with elements of artificial intelligence. As an object of control, AMR with a manipulator is a multi-connected system with a cross-internal connection of control channels, which is formed by the dynamic parameters of a mechanical system. According to the results of mathematical modeling using the proposed model, it is possible to develop algorithms for adaptive control using cross-connection of channels. This will reduce the number of responses of the control system, save energy, and prolong the time of autonomous work of AMR with M.

7. Conclusions

1. The equation of motion of the center of mass of the AMR system with a manipulator along the trajectory along the trajectory, taking into account the relationship of the

dynamic parameters of the mechanical system, is built. At the same time, the influence of the dynamic parameters of the system on the direction and magnitude of the radius-vector $\bar{p}_{OC}$ – the shift of the center of mass of the platform relative to the center of mass of the system of bodies – was established.

2. The influence of the relationship of dynamic parameters of a mechanical system on the equation of angular motion of AMR of changing configuration has been established. This takes into account the commensurate of the non-diagonal and diagonal elements of the inertia tensor, calculated relative to the axes of the base coordinate system.

3. The interrelation of dynamic parameters of the mechanical system during the movement of the manipulator relative to the AMR platform is determined. The influence of system parameters on generalized speeds $\bar{w}_j$ and accelerations $\dfrac{d\bar{w}_j}{dt}$ in the kinematic pairs of the manipulator has been established. Thus, it is possible to control the contact forces in the grip, and if necessary, fix the object and ensure that it is held reliably.

Conflicts of interest

The author declares that she has no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

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Data availability

All data are available in the main text of the manuscript.

References

1. Lopota, A., Spassky, B. (2020). Mobile ground-based robot systems for professional use. Robotics and Technical Cybernetics, 8 (1), 5–17. doi: https://doi.org/10.31776/rtcj.8101

2. Tsarichenko, S., Antokhin, E., Chernova, P., Dementey, V. (2020). The state and problems of standardization and unification of military ground robot systems. Robotics and Technical Cybernetics, 8 (1), 18–23. doi: https://doi.org/10.31776/rtcj.8102

3. Liu, X.-F., Li, H.-Q., Chen, Y.-J., Cai, G.-P. (2015). Dynamics and control of space robot considering joint friction. Acta Astronautica, 111, 1–18. doi: https://doi.org/10.1016/j.actaastro.2015.02.010

4. Liu, G., Geng, X., Liu, L., Wang, Y. (2019). Haptic based teleoperation with master-slave motion mapping and haptic rendering for space exploration. Chinese Journal of Aeronautics, 32 (3), 723–736. doi: https://doi.org/10.1016/j.cja.2018.07.009

5. Li, D., Lu, K., Cheng, Y., Zhao, W., Yang, S., Zhang, Y., Li, J., Shi, S. (2020). Dynamic analysis of multi-functional maintenance platform based on Newton-Euler method and improved virtual work principle. Nuclear Engineering and Technology, 52 (11), 2630–2637. doi: https://doi.org/10.1016/j.net.2020.04.017

6. Sun, H., Zhang, Y., Xue, J., Wu, Z. (2014). The remote control system of the manipulator. Proceedings of the 33rd Chinese Control Conference. doi: https://doi.org/10.1109/chicc.2014.6896388

7. Korayem, M. H., Shafei, A. M. (2015). Motion equation of nonholonomic wheeled mobile robotic manipulator with revolute-prismatic joints using recursive Gibbs–Appell formulation. Applied Mathematical Modelling, 39 (5-6), 1701–1716. doi: https://doi.org/10.1016/j.apm.2014.09.030

8. Ashhepkova, N. (2022). Analysis of the inertia tensor of autonomous mobile robot. Technology Audit and Production Reserves, 1 (2 (63)), 36–40. doi: https://doi.org/10.15587/2706-5448.2022.252712

9. Ashchepkova, N. S. (2020). Algorithm for adaptive control of autonomous mobile robot. Science and Education a New Dimension, VIII (30 (244)), 41–44. doi: https://doi.org/10.31174/SEND-NT2020-244VIII30-10

10. Ashchepkova, N. S., Ashchepkov, S. A., Kapera, S. S. (2018). Dynamics of transport robot model during the turns. Science and Education a New Dimension, VI (19 (171)), 26–29. doi: https://doi.org/10.31174/send-nt2018-171vi19-05

11. Chebly, A., Talj, R., Charara, A. (2017). Coupled Longitudinal and Lateral Control for an Autonomous Vehicle Dynamics Modeled Using a Robotics Formalism. IFAC-PapersOnLine, 50 (1), 12526–12532. doi: https://doi.org/10.1016/j.ifacol.2017.08.2190

12. Mauny, J., Porez, M., Boyer, F. (2017). Symbolic Dynamic Modelling of Locomotion Systems with Persistent Contacts - Application to the 3D Bicycle. IFAC-PapersOnLine, 50 (1), 7598–7605. doi: https://doi.org/10.1016/j.ifacol.2017.08.1007

13. Ma, Y. (2020). Dynamics of tracked UGVs in three-dimensional space. Dynamics and Advanced Motion Control of Off-Road UGVs, 77–94. doi: https://doi.org/10.1016/b978-0-12-818799-9.00003-7

14. Gilimyanov, R. F., Pesterev, A. V., Rapoport, L. B. (2008). Motion control for a wheeled robot following a curvilinear path. Journal of Computer and Systems Sciences International, 47 (6), 987–994. doi: https://doi.org/10.1134/s1064230708060129

15. Bertoncelli, F., Ruggiero, F., Sabattini, L. (2019). Wheel Slip Avoidance through a Nonlinear Model Predictive Control for Object Pushing with a Mobile Robot. IFAC-PapersOnLine, 52 (8), 25–30. doi: https://doi.org/10.1016/j.ifacol.2019.08.043

16. Ashchepkova, N. S. (2021). Control of a dynamic object with a non-diagonal andnon-stationary inertia tensor moving along a trajectory. Modern engineering and innovative technologies, 18 (2), 44–52. Available at: https://www.moderntechno.de/index.php/meit/issue/view/meit18-02/meit18-02

17.   Bai, S., Zhou, L., Wu, G. (2014). Manipulator Dynamics. Handbook of Manufacturing Engineering and Technology, 1855–1872. doi: https://doi.org/10.1007/978-1-4471-4670-4_91

18.   Ashchepkova, N. S., Sheptun, Yu. D. (1997). Mathematical model of the motion of a space vehicle with a manipulator. Space Science and Technology, 3 (5-6), 34–42. doi: https://doi.org/10.15407/knit1997.05.034

19.   Korayem, M. H., Shafei, A. M., Seidi, E. (2014). Symbolic derivation of governing equations for dual-arm mobile manipulators used in fruit-picking and the pruning of tall trees. Computers and Electronics in Agriculture, 105, 95–102. doi: https://doi.org/10.1016/j.compag.2014.04.013

20.   Lloyd, S., Irani, R., Ahmadi, M. (2021). A numeric derivation for fast regressive modeling of manipulator dynamics. Mechanism and Machine Theory, 156, 104149. doi: https://doi.org/10.1016/j.mechmachtheory.2020.104149

21.   Khurpade, J., Dhami, S. S., Banwait, S. S. (2018). A Virtual Model of 2D Planar Manipulator Dynamics. 2018 International Conference on Smart Systems and Inventive Technology (ICSSIT). doi: https://doi.org/10.1109/icssit.2018.8748674

22.   Tian, S. X., Wang, S. Z. (2011). Dynamic Modeling and Simulation of a Manipulator with Joint Inertia. Information and Automation, 10–16. doi: https://doi.org/10.1007/978-3-642-19853-3_2

23.   Bulhakov, V. M., Yaremenko, V. V., Chernysh, O. M., Berezovyi, M. H. (2019). Teoretychna mekhanika. Kyiv: TsUL, 640.

24.   Kuzo, I. V., Zinko, Ya. A., Vankovych, T.-N. M. et al. (2017). Teoretychna mekhanika. Kharkiv: Folio, 576.