

*This paper reports a method that numerically models the deformation and accumulation of hidden damage in structural elements that are in an inhomogeneous thermal field and are exposed to radiation. Materials considered are those exhibiting orthotropy (transversal isotropy) of long-term properties. The problem is stated as an boundary – initial value one. To solve it, the finite element method and the initial-difference method of time integration are used. To simulate the anisotropy of the process of accumulation of hidden damage, a damage tensor is applied. The development of irradiation swelling strains is described using the equation for a limited temperature range and a specific fluence value. The results of numerical modeling of creep and damage in plates in tension with circular notches, which are in an inhomogeneous temperature field, are considered. The material of the plates is a titanium alloy VT1-0. It was found that the effect of irradiation significantly, up to 6–7%, increases the level of deformation in the plate. Radiation significantly, by almost 4 times, reduces the time until the completion of the hidden fracture of the plates. It was found that orthotropy of radiation swelling properties leads to redistribution of areas with significant strains and damage values. It has been established that the effect of irradiation swelling also qualitatively changes the nature of the distribution of maximum damage in the plate, which extends to a fairly large area. Such results are due to the additional effect of irradiation swelling strains on the rate of general irreversible deformation and redistribution of stresses*

**Keywords:** creep, irradiation swelling, modeling, orthotropic material, damage tensor, titanium alloy

UDC 539.3  
DOI: 10.15587/1729-4061.2023.272317

# DETERMINING THE INFLUENCE OF IRRADIATION SWELLING ON CREEP AND DAMAGE IN ELEMENTS WITH ORTHOTROPIC MATERIAL PROPERTIES

**Dmytro Breslavsky**

Doctor of Technical Sciences, Professor, Head of Department\*  
Institute of Mechanics - Otto von Guericke University  
Universitätsplatz, 2, Magdeburg, G10-048, Germany, 39106

**Volodymyr Mietielov**

Corresponding author  
PhD\*

E-mail: vometel@gmail.com

**Oksana Tatarinova**

PhD, Associate Professor\*

\*Department of Computer Modeling of Processes and Systems  
National Technical University «Kharkiv Polytechnic Institute»  
Kyrpychova str., 2, Kharkiv, Ukraine, 61002

Received date 07.11.2022

Accepted date 13.01.2023

Published date 28.02.2023

**How to Cite:** Breslavsky, D., Mietielov, V., Tatarinova, O. (2023). Determining the influence of irradiation swelling on creep and damage in elements with orthotropic material properties. *Eastern-European Journal of Enterprise Technologies*, 1 (7 (121)), 6–13. doi: <https://doi.org/10.15587/1729-4061.2023.272317>

## 1. Introduction

Determining the residual life of the structural elements of the of nuclear reactors vessel internals (RVI) is an important task. However, conducting full-fledged experimental studies of materials and structural elements in an uniaxial, and even more complex stress state, is a very expensive task, which also requires a long time. In this regard, making a decision on extending the resource of the equipment involves data obtained using the methods of mathematical modeling and computer simulation. The processes occurring in materials under the joint action of thermal forces and radiation fields are quite complex. In this regard, the use for the analysis of typical engineering software systems in most cases does not make it possible to assess the degree of deformation and fracture of structural elements. In this regard, an important task arises for the development or improvement of calculation methods that allow saving resources and time in determining the resource of already operating RVI and in the design of new ones.

## 2. Literature review and problem statement

Statements of numerical modeling problems are boundary-initial value due to the need to assess the long-term

behavior of elements. Significant nonlinearity of processes requires the development of special modeling methods, one of the important components of which is the construction and application of adequate equations of state [1, 2].

For the manufacture of structural elements of RVI, various metals and alloys are used [3]. Much attention was paid to the description of the deformation properties and strength of many of them, primarily specialized steels, zirconium alloys, uranium due to the fact that the main elements of the core and fuel were made of them. A number of models have been built to describe the main effects manifested in materials during long-term operation under irradiation conditions, primarily irradiation creep and swelling [3]. The description of irradiation creep can often be performed in a linear formulation, so this task is not very challenging. Irradiation swelling is a complex nonlinear process. For its prediction, complex dependences on neutron fluence or accumulated dose, temperature, time, and material parameters are used.

The behavior of light alloys under the conditions of action of thermal forces and radiation fields has been studied much less. This is largely due to the anisotropy of the properties of these materials, including long-term ones [3]. One of the most common classes of materials used in nuclear energy are titanium alloys, due to their low density, relatively

high strength properties, and low thermal stress [4]. Data from studies of their behavior under irradiation conditions, obtained in the first decades of their use, are analyzed in [5], for the next twenty-five years – in [6]. Further use of titanium alloys determines the continuation of research into their physical and mechanical properties, including at high temperatures and radiation exposure [7]. The main conclusions from the results obtained during this time are as follows: under reactor conditions, structural elements made of titanium alloys can be operated in the temperature range up to 550–570 °C [5, 6]. As in most materials, at elevated temperatures, the physical and mechanical properties change with a decrease in their values [6, 7]. When irradiated with fluence values of the order of  $5 \cdot 10^{22}$  neutron/cm<sup>2</sup> (more than 0.1 MeV), strains of radiation swelling occur in the material of the alloys while their value significantly depends on the temperature. However, for most titanium alloys, the irradiation swelling strains in the temperature range of 450–550 °C are moderate and reach no more than 2 % [5, 6]. It is also noted that along with the anisotropy of mechanical properties, both instantaneous and long-term, anisotropy of the properties of irradiation swelling may occur, including depending on previous changes in the structure of the material [8, 9].

Analysis of the long-term behavior of structural elements operated at elevated temperatures requires an adequate description of time-dependent processes – creep and accumulation of hidden damage [1, 2]. In the case of the anisotropic nature of the latter, complex state equations are used to describe the evolution of the state of the material, using tensor parameters of damage [1, 10]. A number of similar equations have been built, among which we note the classical models given in [1]. Despite the closed form of evolution equations, all of them in the general three-dimensional case require long-term experimental research [11]. That is, the power, efficiency, and physical reliability of tensor models to describe damage is often offset by the need for very costly long-term experiments to determine the values of the constants that are included in them. If such experiments are performed and the constants are defined, the advantages of using tensor models are indisputable. It should also be noted that in most cases at present it is possible to build state equations to describe the two-dimensional stress state that occurs in thin-walled structural elements. In this case, the material is described using models of orthotropy (transversal isotropy) [11].

After constructing and verifying the equations of state, it is possible to fully formulate the basics of the calculation method. The vast majority of boundary problems are solved by the finite element method (FEM) [12]. This is due to the complexity of the geometry and boundary conditions of the structural elements. In the case of boundary-initial value problems, difference methods of time integration are attached to FEM [12]. Features of solving such problems for the case of taking into account the effect of irradiation can be found in [2].

It should be noted that there are practically no methods of numerical modeling, which would provide an opportunity to assess the stress-strain state and lifetime values of structural elements under complex laws of accumulation of damage and irradiation swelling. There are also no results of such calculation studies, which are necessary for engineers. Thus, it is necessary to devise an estimation method for analyzing the deformation and strength of the RVI elements with orthotropic properties of materials. Due to its universality and prevalence, the basis of the method should include FEM

while tensor models should be used to construct evolution equations to describe damage.

---

### 3. The aim and objectives of the study

---

The aim of this work is to determine the effect of irradiation swelling on the course of creep processes, accompanied by the accumulation of hidden damage, in materials with orthotropic long-term properties. Achieving the goal will determine the practical possibility of performing cycles of calculated studies of the resource of structural elements of RVI.

To accomplish the aim, the following tasks have been set:

- to devise an estimation numerical method for solving the boundary- initial value problems of orthotropic creep using evolution equations to describe the accumulation of damage that adequately reflects its non-isotropic nature, and irradiation swelling of the material;
- to formulate the equation of state of irradiation swelling, effective for a given temperature range and the value of fluence, and to determine the parameters included in it;
- to check the implemented method of numerical modeling and conduct a cycle of calculations for its verification.

---

### 4. The study materials and methods

---

The object of our study is the nature and level of deformation and hidden damage in notched plates made of materials with orthotropic properties of irradiation swelling and damage.

The main hypothesis of the study assumes the possibility of constructing equations of state with the orthotropic nature of the spread of swelling strains for limited temperature ranges and taking into account the tensor nature of damage.

The main research method is numerical modeling of creep and damage processes in structural elements, in the material of which irradiation swelling strains develop. The construction of the method involves the main approaches and components of the finite element method (FEM) [12], which were previously used to analyze the deformation and damage accumulation of structural elements with isotropic properties of the material [2]. The solution of the initial problems is performed using difference methods, the schemes of the predictor-corrector method are used [2]. The equation of state, which is implemented to the calculated functions, is constructed for a material with orthotropic properties of creep and accumulation of damage. To reflect the anisotropy of the properties of hidden damage, the kinetic (evolution) equation is formulated for the tensor of damage.

To construct equations of creep state accompanied by damage, experimental results obtained for samples from titanium alloy VT1-0 were used [13, 14]. The equation for the rate of irradiation swelling strains is formulated after the analysis of experimental data obtained for titanium alloys at high temperatures [5, 6].

In this study, it is believed that the direction of the axes of the coordinate system of the plate coincides with the directions of the main axes of orthotropy. The methodology of research with a different coordinate system is reported, for example, in [1, 11], and can be applied if necessary. Also, the development of irradiation creep strains possible during irradiation is not taken into account. The range of stresses corresponds to the area of elastic deformation. These assumptions

and simplifications are accepted to identify the basic laws of the processes under consideration.

**5. Analysis of the effect of irradiation swelling on creep and damage in notched titanium plates**

**5. 1. Numerical modeling method**

Most problems related to the creep theory for materials with anisotropic (orthotropic) properties, given the limited experimental data, can be solved mainly in a two-dimensional statement. Therefore, the region  $\Omega$ , which is a bounded curve  $\Gamma$ , is considered. The Cartesian coordinate system  $x_i$  ( $i=1,2$ ) is used. It is suggested that on part of the contour of region  $\Gamma_1$ , the displacements are set  $u_i|_{\Gamma_1} = \tilde{u}_i$ . Part of the contour  $\Gamma_2$  is loaded with traction  $p_i(x,t)$ , and the volumetric forces are considered absent. The case of a range of stresses that does not exceed the yield limit is considered. Elastic properties of the material are accepted to be isotropic, as well as creep properties - transversal-isotropic. The case of deformation at a variable temperature in the range of 500–560 °C, when they can be considered the same for a given interval, is analyzed.

According to the Lagrange approach, we consider the case of small strains. The following notations are adopted:  $\underline{u} = \underline{u}(x_i, t)$  – displacement vector;  $\underline{\varepsilon} = \underline{\varepsilon}(x_i, t)$  – strain tensor;  $\underline{\sigma} = \underline{\sigma}(x_i, t)$  – stress tensor,  $t$  – time. The hypothesis of strain additivity is involved:

$$\underline{\varepsilon} = \underline{e} + \underline{\varepsilon}^T + \underline{c} + \underline{\varepsilon}^{sw}. \tag{1}$$

It is believed that the total strain consists of elastic  $\underline{e}$ , thermoelastic  $\underline{\varepsilon}^T$ , creep  $\underline{c}$  and irradiation swelling  $\underline{\varepsilon}^{sw}$  strains. In this case, the main system of equations from the creep theory for the two-dimensional statement of the problem will take the following form [2]:

$$\begin{aligned} \sigma_{ij,j} &= 0, \quad \sigma_{ij}n_j = p_i(x), \quad x \in \Gamma_2; \\ \varepsilon_{ij} &= \frac{1}{2}(u_{i,j} + u_{j,i}), \quad x \in \Omega, \quad u_i|_{\Gamma_1} = \tilde{u}_i, \quad x \in \Gamma_1, \quad i = 1, 2; \\ \sigma_{ij} &= C_{ijkl}(\varepsilon_{kl} - \varepsilon_{kl}^T - c_{kl} - \varepsilon_{kl}^{sw}), \quad \varepsilon_{ij}^T = \alpha_{ij}(T - T_0)\delta_{ij}; \\ u(x_i, 0) &= u_0(x), \quad c_{ij}(x_i, 0) = 0, \quad \varepsilon_{ij}^{sw}(x_i, 0) = 0. \end{aligned} \tag{2}$$

Here,  $n(n_1, n_2)$  denotes the unit vector of the normal to the contour of the body,  $C$  – the tensor of the elastic constants of the material,  $T = T(x_i)$  – the function of the distribution of temperature by region  $\Omega$ ,  $T_0$  – the initial value of temperature,  $\alpha_{ij}$  – coefficients of linear expansion of the material,  $\delta_{ij}$  – Kronecker delta. The initial conditions – vector  $u_0$  – are determined by the solution of system (2) at the initial thermoelastic stress-strain state.

To system (2), it is necessary to add the equation of state. The equation of creep, accompanied by damage, obtained in [13, 14], is applied:

$$\dot{\underline{c}} = \tilde{B} \frac{\bar{\sigma}_v^{n-1}}{(1-\eta)^n} [\bar{B}] \underline{\sigma}; \tag{3}$$

$$\begin{aligned} \dot{\underline{\omega}} &= d_{1111}^{p/2} \frac{\sigma_D^{p-2}}{(1-\eta)^{p+s-1}} [\bar{D}] \underline{\sigma}, \quad \dot{\eta} = d_{1111}^{p/2} \frac{\sigma_D^p}{(1-\eta)^{p+s}}, \\ \eta(0) &= 0, \quad \eta(t_*) = 1, \end{aligned} \tag{4}$$

where  $\underline{\omega}$  is the damage tensor.  $\sigma_v = \underline{\sigma}^T [\bar{B}] \underline{\sigma}$ ,  $\sigma_D = \underline{\sigma}^T [\bar{D}] \underline{\sigma}$  – equivalent stresses, which are common invariants of stress and material constants tensors;  $\eta = \eta(t)$  is a scalar measure for the fracture criterion. At the time of completion of the hidden damage accumulation process,  $\eta_*(t_*) = 1$ . The matrix  $[\bar{B}]$  contains the components of the tensor of the creep properties of the material  $b_{ijkl}$ :

$$\begin{aligned} [\bar{B}] &= \begin{bmatrix} 1 & \beta_{12} & 0 \\ \beta_{12} & \beta_{22} & 0 \\ 0 & 0 & 4\beta \end{bmatrix}, \quad \beta_{12} = -\frac{1}{2}b_{1111}, \\ \beta_{22} &= \frac{b_{2222}}{b_{1111}}, \quad 4\beta = \frac{b_{1212}}{b_{1111}}; \end{aligned} \tag{5}$$

The matrix  $[\bar{D}]$  contains the components of the tensor of material damage properties  $d_{ijkl}$ :

$$\begin{aligned} [\bar{D}] &= \begin{bmatrix} 1 & \delta_{12} & 0 \\ \delta_{21} & \delta_{22} & 0 \\ 0 & 0 & 4\delta \end{bmatrix}, \quad \delta_{12} = -\frac{1}{2}d_{1111}, \\ \delta_{22} &= \frac{d_{2222}}{d_{1111}}, \quad 4\delta = \frac{d_{1212}}{d_{1111}}. \end{aligned} \tag{6}$$

The components of the tensors  $b_{ijkl}$  and  $d_{ijkl}$  are determined by the results of experiments on creep and fracture of samples cut from the material in three directions. For the case of sheet materials made by rolling, these are directions along, across the rolling, and at an angle of 45° to them.  $\tilde{B}$ ,  $n$ ,  $p$ ,  $s$  are constants determined experimentally. Details of the numerical procedures applied to the determination of constants and components of property tensors under consideration can be found in [13–15].

The equations for determining the values of the tensor components of irradiation swelling strains during neutron irradiation, as is known from [3], are of the same volumetric nature as thermoelastic strains. The state equation is built for the rates of the components of tensor  $\underline{\varepsilon}^{sw}$ :

$$\dot{\varepsilon}_{ij}^{sw} = \frac{1}{3} \dot{S}_\Phi (\dot{\Phi}, t, T, \dots) \delta_{ij}. \tag{7}$$

Here,  $S_\Phi$  is a function of neutron fluence  $\Phi$ , time, temperature, and parameters that determine the swelling process [16].

To build a method for solving the formulated boundary-initial value problem, a combination of FEM and the difference method of integration by time was chosen. The case of a plane stress state is considered. To use FEM algorithms, a transition to the vector-matrix statement of the problem was carried out. A triangular finite element is used. After the transformations, the main system of ordinary differential equations with respect to the nodes of the finite-element model was built [2, 14]:

$$\begin{aligned} [K] \{\dot{u}\} &= \{\dot{F}\} + \{\dot{F}^C\} + \{\dot{F}^{Sw}\}; \\ \{\dot{F}\} &= \sum_{N_p} \int_{S_2^p} [N^p]^T \{\dot{p}\} dV + \sum_{N_p} \int_{V_p} [\bar{B}]^T [C] \{\dot{\varepsilon}^T\} dV; \\ \{\dot{F}^C\} &= \sum_{N_p} \int_{V_p} [\bar{B}]^T [C] \{\dot{c}\} dV; \end{aligned}$$

$$\begin{aligned} \{ \dot{F}^{Sw} \} &= \sum_{N_\beta} \int_{V_\beta} [ \bar{B} ]^T [ C ] \{ \epsilon^{sw} \} dV; \\ \{ \dot{\epsilon} \} &= \frac{3}{2} \bar{B} \frac{\sigma_V^{n-1}}{(1-\eta)^k} [ \bar{B} ] \{ \sigma \}; \\ \{ \dot{\omega} \} &= d_{1111}^{p/2} \frac{\sigma_D^{p-2}}{(1-\eta)^{p+s-1}} [ \bar{D} ] \{ \sigma \}, \\ \dot{\eta} &= d_{1111}^{p/2} \frac{\sigma_D^p}{(1-\eta)^{p+s}}, \quad \eta(0) = 0, \quad \eta(t_*) = 1, \end{aligned} \quad (8)$$

where  $K$  is the stiffness matrix of the system;  $B$  – deformation matrix;  $u$  – global vector of nodal displacements;  $F$  is a vector of instantaneous nodal loads caused by traction and temperature strains;  $F^C$  – nodal loads caused by creep strains;  $F^{Sw}$  – nodal loads caused by irradiation swelling strains;  $C$  – matrix of elastic constants;  $N$  – matrix of forms;  $\beta$  – number of the finite element;  $V_\beta$  – the volume of a finite element;  $\sum_{N_\beta}$  – summation for all finite elements;  $S_2^\beta$  – the surface area of a finite element that is under the action of a traction. The notations of the vectors of the stress-strain state components and the damage vector in the finite element correspond to the notation of the tensors introduced above.

After solving system (8), at each step in time, the components of the stress-strain vectors, the vector, and the damage measure values in each finite element are determined. The calculation continues until the completion of the hidden damage accumulation time  $t_*$ , which is determined by the time of obtaining by a damage measure of its critical value (close to 1 [10]).

The calculations used the software package *FEM Creep* [2], supplemented with program units for solving problems of thermal conductivity. Features of the construction of finite-element algorithms and the method of calculation as a whole can be found in [2, 17].

### 5. 2. Irradiation swelling state equation

To determine the values of irradiation swelling strains, the data given in [5, 6] were used. The values of irradiation swelling strains given there show an increase from 0.5 % at 450 °C to 1.5 % at 550 °C. Due to incomplete representation of the data (only for two temperature limit values), their linear approximation was used for the case of isotropy of defect growth properties for the temperature range of 450–550 °C and a constant value of fluence of 5 10<sup>22</sup> neutron/cm<sup>2</sup>:

$$\epsilon_{ij}^{sw} = \frac{1}{3} a_{ij} \exp\left(-\frac{Q}{T}\right) t \delta_{ij}. \quad (9)$$

Here,  $a_{ij} = a = 1.27 \cdot 10^{-5} \text{ h}^{-1}$ ,  $Q = 4124 \text{ K}$  are constants determined by the processing of experimental data. There are no quantitative data on the anisotropy of irradiation swelling in the papers under consideration. In this regard, two variants of orthotropy (transversal isotropy) properties were considered, in which the value of the constants  $a_{ij}$  varies by 2 times compared to the value for the case of isotropy of swelling properties:

Variant 0 –  $a_{11} = a$ ;  $a_{22} = a$ . Variant 1 –  $a_{11} = 2a$ ;  $a_{22} = 0.5a$ . Variant 2 –  $a_{11} = 0.5a$ ;  $a_{22} = 2a$ .

### 5. 3. Results of numerical modeling of creep and damage accumulation in a notched titanium plate

For an example of using the method of numerical modeling, a model of a structural element was chosen, which, on

the one hand, has non-canonical geometry, and on the other hand, is typical for cycles of experimental and computational research. In this regard, a model of a plate with side circular notches, loaded with traction of 1 MPa, was adopted for modeling (Fig. 1).

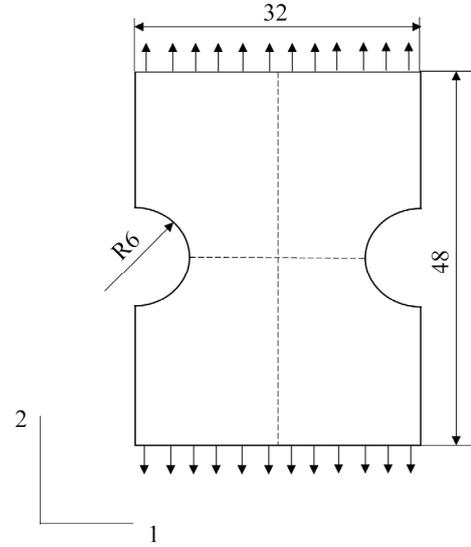


Fig. 1. Scheme of the plate with two circular notches

The plate is located in an inhomogeneous temperature field with a temperature range of 500–560 °C and is under the influence of radiation exposure. The material of the plate is titanium alloy VT1-0 (analog of IMI125 or T40). It is characterized by a significant anisotropy of creep properties and the damage accumulation. The constants to the equations of state of this alloy (5), (6) in this temperature range are determined by experimental studies [13] and are given in [14]. They are equal to  $n=p=5$ ,  $s=1$ :

$$\begin{aligned} b_{1111} &= 2,303 \cdot 10^{-4}, \quad b_{1122} = -1,151 \cdot 10^{-4}, \quad b_{2222} = 1,924 \cdot 10^{-4}, \\ b_{1212} &= 2,058 \cdot 10^{-4}, \quad (\text{MPa})^{2N/N+1} / (\text{hour})^{2/N+1}, \\ d_{1111} &= 3,542 \cdot 10^{-5}, \quad d_{1122} = -1,771 \cdot 10^{-5}, \\ d_{2222} &= 3,324 \cdot 10^{-5}, \quad d_{1212} = 3,127 \cdot 10^{-5}, \quad (\text{MPa})^{-2} / (\text{hour})^{2/k}. \end{aligned} \quad (10)$$

The direction of the axes of the Cartesian coordinate system of the estimation model (Fig. 1) coincides with the axes of the material symmetry of the plate.

Note that such structural elements of titanium alloys are used as RVI elements of equipment fastening [4].

Due to the symmetry of the plate, numerical modeling was carried out for its one-fourth part. Finite-element models with condensed number of nodes to the boundary of the circular notch were used. After conducting estimation studies of the solution convergence, a cycle of calculations was carried out for various cases of mutual influence of the presence and absence of irradiation swelling using isotropy and anisotropy of its properties.

First, estimation data on creep and damage accumulation in the plate were considered, without taking into account the effects of radiation. Fig. 2,  $a$  shows the results of solving the problem of stationary thermal conductivity. As you can see, the temperature field is significantly heterogeneous, with a maximum at the plate boundary.

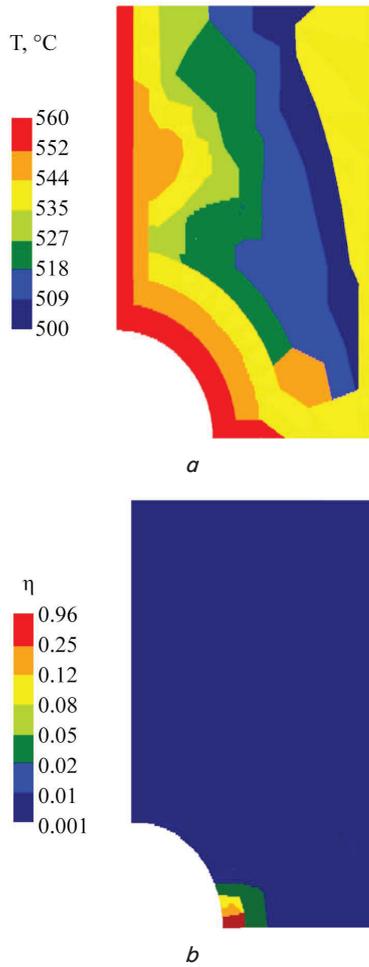


Fig. 2. Temperature field (a) and damage measures  $\eta$  distributions (b) in a notched titanium plate

Calculations of the thermoelastic stress state showed that the maximum stresses, as is known from theory [10], is at the top of the notch; its value is 38 MPa. According to the calculation of creep, accompanied by the accumulation of damage, it was established that the time until the completion of the hidden fracture of this plate is 4077 hours (approximately six months). The distribution of the damage measure for the moment of time preceding the fracture ( $t=4076.9$  h) is shown in Fig. 2, b, which demonstrates that the finishing of the hidden damage accumulation occurs at the top of the notch where there are maximum stresses. The accumulated irreversible strains, determined only by the creep of the material, are insignificant and do not exceed 0.1 %.

The following are the results of modeling the creep of these plates, taking into account the development of irradiation swelling strains. The results of the calculation of total strains, represented by the distributions of the von Mises strains  $\epsilon_{vM}$ , %, and measures of damage  $\eta$ , are given in Fig. 3, 4, respectively. Distributions according to Variant 0 (isotropic swelling) are presented in figures a, according to anisotropy variants for variants 1 and 2 in figures b and c, respectively, Fig. 3, 4.

Fig. 5, 6 show plots of time varying in normal components of damage tensor at points A and B (Fig. 5). Point A, in which the maximum stresses are present, is located near the top of the notch. Point B, also with a sufficiently high level of thermal stresses, is located inside the plate. In Fig. 5, the numbers of curves 1 and 2 correspond to the time dis-

tributions of component  $\omega_{22}$  at points A and B, respectively, the numbers of curves 2 and 4 – component  $\omega_{11}$  at the same points. Fig. 6 illustrates the variation of component  $\omega_{22}$  at point A for the case of isotropy of the properties of irradiation swelling (Variant 0, curve 1) and anisotropy (Variant 2, curve 2 and Variant 1, curve 3).

It was established by calculations that in the case of isotropy of the properties of irradiation swelling (Variant 0), the time before the completion of the hidden damage accumulation was  $t_{*0}=1197$  hours, in cases of anisotropy – for Variant 1,  $t_{*1}=757$  h, and, for Variant 2,  $t_{*2}=810$  h.

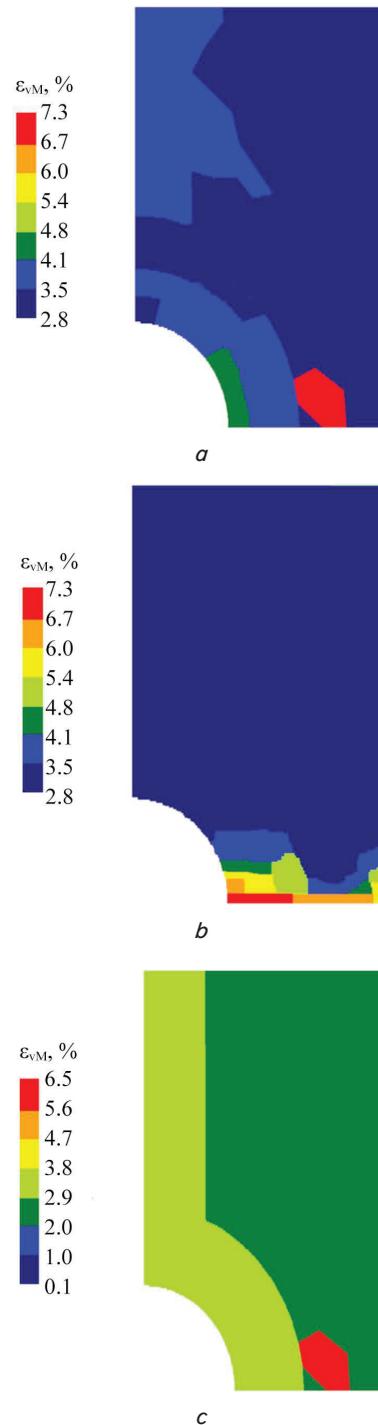


Fig. 3. Distribution of the von Mises strains in a notched titanium plate: a – according to Variant 0; b – according to Variant 1; c – according to Variant 2

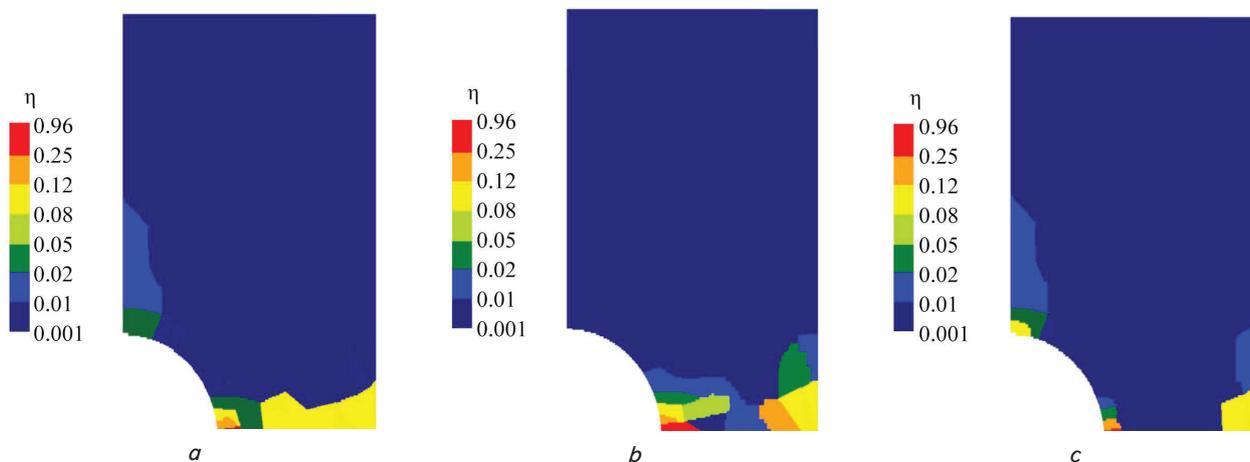


Fig. 4. Distribution of the damage measure in a notched titanium plate: *a* – according to Variant 0; *b* – according to Variant 1; *c* – according to Variant 2

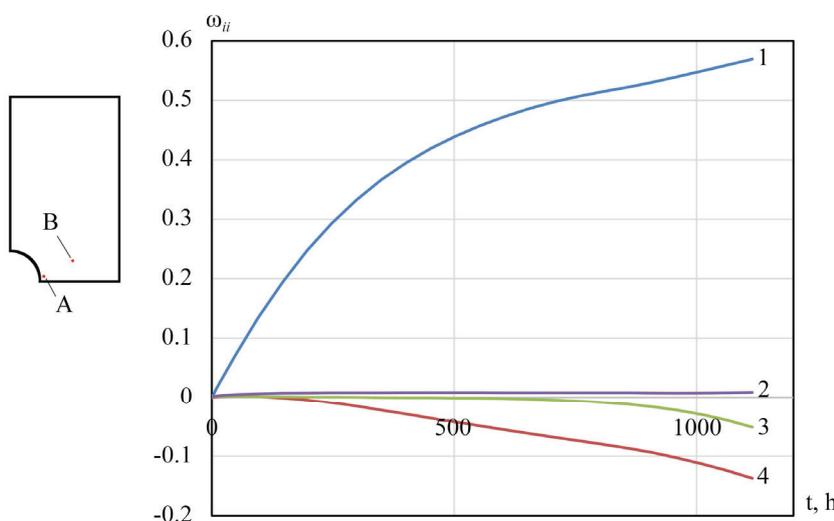


Fig. 5. Dependence of the components of the damage tensor on time: point A – curves 1, 4; point B – curves 2, 3;  $\omega_{22}$  – curves 1, 2;  $\omega_{11}$  – curves 3, 4

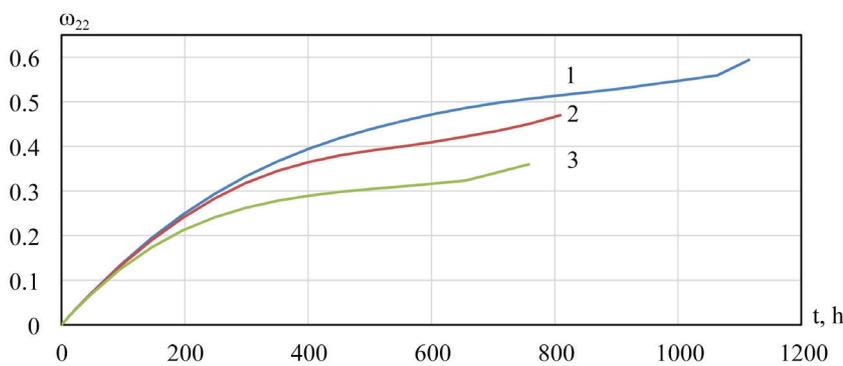


Fig. 6. Dependence of the component  $\omega_{22}$  of damage tensor at point A on the time for cases of isotropy (curve 1) and anisotropy of irradiation swelling properties (curves 2 and 3)

The obtained distributions of strains (Fig. 3), measures (Fig. 4), and components of the damage tensor (Fig. 5, 6) indicate that the developed calculation method provides an opportunity to analyze the influence of the development of irradiation swelling strains on the rate and nature of the processes of deformation and accumulation of damage.

### 6. Discussion of results of the numerical modeling of deformation and hidden damage

As can be seen from the results shown in Fig. 3, 4, the development of irradiation swelling strains significantly affects the level of deformation, several times higher than that characteristic of a pure creep process, and the time and nature of damage to the plates.

When comparing the time values until the completion of the hidden fracture  $t^*$  in the presence and absence of exposure to irradiation swelling, it is clear that radiation significantly, by almost 4 times, reduces the life of the plates. The presence of anisotropy of swelling properties also reduces the time value  $t^*$ , by about a quarter. In the simulation, the case of coincidence of the directions of the coordinate axes and the axes of symmetry of the material is considered. In this case, the difference between the values of the time before fracture when changing the direction of the maximum swelling intensity (that is, between Variants 1 and 2) is 53 hours. This is not very significant.

However, this anisotropy of swelling properties significantly affects the nature of the spread of damage zones (Fig. 2, *b*, 4). In the absence of radiation, the damage zone is localized at the top of the notch (Fig. 2, *b*), which corresponds to known theoretical and experimental data [10, 17]. In the case of taking into account the development of irradiation swelling strains, the zones

with significant damage are much larger, they are located over the entire area of the region, having the smallest width (Variants 0 and 1, Fig. 4, *a*, *b*). With preferential asymmetry in the axial direction (Variant 2, Fig. 4, *c*) and isotropy of swelling properties (Fig. 4, *a*, *c*), there is a limited area on the face of the plate where maximum temperature

values are present. Also, in all cases, there is a more pronounced tendency to increase damage in the central region, characterized by higher temperature values.

In all cases, the hidden damage accumulation is completed in the area of the notch top. The maximum value of the axial stress component  $\sigma_{22}$ , which takes place at point A (Fig. 5), mainly determines the maximum value of the damage tensor component  $\omega_{22}$  and, accordingly, the maximum rate of its growth (Fig. 5, 6). The presence of relatively large values of temperature stresses inside the plate (for example, point B, Fig. 5, is considered) does not lead to large values of damage due to relaxation of mechanical stresses and exposure to stresses caused by of irradiation swelling strains.

The effect of irradiation swelling anisotropy on the growth rate of the maximum component of the damage tensor  $\omega_{22}$  is illustrated in Fig. 6. The presence of a greater or lesser effect of the rate of development of swelling strains in direction 1, perpendicular to the axis of the plate, leads to a change in the growth rate  $\omega_{22}$ . But the effect on the total level of stresses of swelling anisotropy is greater than in the isotropic case, which causes a longer time of hidden fracture with it (curve 1, Fig. 6) and less in the anisotropic case (curves 2 and 3, Fig. 6).

The reason for the qualitative redistribution of areas with significant damage is the presence of a heterogeneous temperature field, which generates a heterogeneous distribution of strains, and therefore stresses caused by irradiation swelling. The values of these stresses make a significant qualitative contribution to the process of redistribution of stresses during creep (relaxation and growth at different points). The change in the nature of the stress state causes a different nature of the damage process. When using a physically based model using a damage tensor, it is more sensitive to changes in the components of stress tensor than when using a scalar parameter.

Due to the joint effect of changes in stresses caused by deformation of irradiation swelling, the total strains in the plate also increase significantly, reaching values of 6–7% (Fig. 3). That is, the level of deformity in the area between the notches increases by an order of magnitude. In the isotropic case, in addition to the region of maximum temperatures and stresses at the boundary in the notch, due to the strain compatibility inside the plate, a highly deformed zone arises (Fig. 3, *a*). The influence of anisotropy also qualitatively and quantitatively changes the nature of the deformed state. In anisotropy, described by Variant 1, that is, the maximum development of swelling strains in direction 1, the strains are localized in a narrow part of the plate. In Variant 2 (maximum rate in direction 2), the entire area of the plate is significantly deformed (Fig. 3, *c*).

Note that the region of maximum strains inside the plate occurs during the isotropy of swelling properties (Variant 0) and in Variant 2 with a maximum in the axial direction (Fig. 3, *a, c*). In Variant 1, due to the action of stresses in the direction perpendicular to the axial, their redistribution occurs, which leads to a decrease in strains in time.

Thus, on the example of this numerical study, the possibility of analyzing the effect of irradiation swelling on deformation and long-term strength of structural elements with anisotropic (orthotropic) properties of the material is demonstrated. Such studies were carried out only for the case of isotropy of damage properties, for example in [2]. For light alloys, this approach could lead to an incorrect

assessment of both quantitative (time before fracture, level of strains) and qualitative indicators (areas of maximum values and the nature of the spread of deformation and damage).

It should be noted that the proposed approach and method of calculation can be used in the analysis of long-term strength and resource of existing RVI elements. Modeling can be carried out both for elements made of light alloys with orthotropy properties, and from other metallic materials when formulating for them the relevant laws of accumulation of hidden damage.

The calculations did not take into account the irradiation creep strains. Their consideration in the isotropic case leads to the acceleration of the overall processes of growth of strains and damage [2, 16]. In the case of the availability of experimental data that will indicate the orthotropy of irradiation creep, numerical modeling due to the linearity of the equations will not present difficulties and can be carried out together with the calculation of thermal creep strains [2]. If it is known about the manufacture of structural elements from sheets at an angle to the direction of rolling, then the equations of this method should be supplemented by an algorithm for the transition to a new coordinate system [11].

In our work, numerical studies were performed for one material with a rather limited temperature range and neutron fluence, for one, albeit common, type of stress concentrators. To confirm the findings, it is necessary to continue research for other materials and characteristics of radiation irradiation and other structural elements.

---

## 7. Conclusions

---

1. An estimation method of the processes of deformation and accumulation of hidden damage in structural elements, which are made of materials that exhibit orthotropy of long-term properties, has been devised. The case of joint action of the load, inhomogeneous thermal field, and the influence of radiation irradiation is considered. The method is implemented in a two-dimensional statement, due to the best current possibility of obtaining experimental data on long-term deformation and fracture of materials under a plane stress state. In the presence of a complete set of experimental data, given the universality of FEM algorithms, the method can be expanded to the general case of a three-dimensional stress-strain state.

2. Experimental data on irradiation swelling of titanium alloys have been analyzed and the equation of state for a limited temperature range of 450–550 °C and a specific fluence value of  $5 \cdot 10^{22}$  neutron/cm<sup>2</sup> has been formulated. For these values of parameters, an equation was built to describe the rate of irradiation swelling strains. The value of the constants included in it has been determined:  $a=1.27 \cdot 10^{-5} \text{ h}^{-1}$ ,  $Q=4124 \text{ K}$ . This equation was built for the effective analysis of patterns of deformation in the considered temperature ranges and fluence. The procedure used in its elaboration can also be used for more complex operation conditions.

3. The analysis of the effect of irradiation swelling on deformation and accumulation of damage in notched plates made of titanium alloy VT1-0, which simulate parts of the fastening RVI elements has been performed. The value of time was determined before the fracture of the plates without taking into account the irradiation swelling strains ( $t=4077 \text{ h}$ ), and taking into account, and in the latter case the resource of the plates is four times less (1197 hours for swelling isotropy and from 757 h to 810 h for the case of

orthotropy of its properties). The derived distributions of the measure and components of the damage tensor showed a significant heterogeneity of the damage fields in the plate in case of anisotropy of swelling properties. It has been shown that the anisotropy of swelling quantitatively and qualitatively changes the deformed state of the plate – from the concentration in its narrow part between the notches to the deformability of the entire volume with mostly 2–3 times greater values of the intensity of von Mises strains.

---

#### Conflicts of interest

---

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

---

#### Funding

---

The work was partially funded by the Volkswagen Foundation “Visiting research program for refugee Ukrainian scientists” (Az. 9C184), (D. Breslavsky).

---

#### Data availability

---

The data will be provided upon reasonable request.

---

#### Acknowledgments

---

We thank the Volkswagen Foundation for supporting D. Breslavsky within the framework of “Visiting research program for refugee Ukrainian scientists” (Az. 9C184).

---

#### References

1. Lemaitre, J. (Ed.) (2001). Handbook of materials behavior models. Academic Press. Available at: <https://www.sciencedirect.com/book/9780124433410/handbook-of-materials-behavior-models>
2. Breslavskiy, D. V. (2020). Deformuvannya ta dohotryvala mitsnist konstruktyvnykh elementiv yadernykh reaktoriv. Kharkiv: Drukarnia Madryd, 249.
3. Nordlund, K. (2019). Historical review of computer simulation of radiation effects in materials. *Journal of Nuclear Materials*, 520, 273–295. doi: <https://doi.org/10.1016/j.jnucmat.2019.04.028>
4. Olander, D. R. (1976). Fundamental aspects of nuclear reactor fuel elements. United States. doi: <https://doi.org/10.2172/7343826>
5. Peterson, D. (1982). Swelling in Neutron Irradiated Titanium Alloys. *Effects of Radiation on Materials*, 260. doi: <https://doi.org/10.1520/stp34350s>
6. Mansur, L. K. (2008). Survey of Radiation Effects in Titanium Alloys. USA: Materials Science and Technology Division Oak Ridge National Laboratory. Available at: <https://info.ornl.gov/sites/publications/files/Pub12339.pdf>
7. Tabie, V. M., Li, C., Saifu, W., Li, J., Xu, X. (2020). Mechanical properties of near alpha titanium alloys for high-temperature applications - a review. *Aircraft Engineering and Aerospace Technology*, 92 (4), 521–540. doi: <https://doi.org/10.1108/aeat-04-2019-0086>
8. Leguey, T., Baluc, N., Schäublin, R., Victoria, M. (2005). Temperature dependence of irradiation effects in pure titanium. *Philosophical Magazine*, 85 (4-7), 689–695. doi: <https://doi.org/10.1080/14786430412331319992>
9. Jin, P., Shen, T.-L., Li, J., Yang, Y.-S., Liu, C., Cui, M.-H. (2023). Changes in the microstructure and mechanical properties of Ti–6Al–4V alloys induced by Fe ion irradiation at a high He generation rate. *Vacuum*, 207, 111639. doi: <https://doi.org/10.1016/j.vacuum.2022.111639>
10. Lemaitre, J., Chaboche, J.-L. (1990). *Mechanics of Solid Materials*. Cambridge University Press. doi: <https://doi.org/10.1017/cbo9781139167970>
11. Altenbach, H. (2022). Creep and Damage of Materials at Elevated Temperatures. *CISM International Centre for Mechanical Sciences*, 1–62. doi: [https://doi.org/10.1007/978-3-031-04354-3\\_1](https://doi.org/10.1007/978-3-031-04354-3_1)
12. Zienkiewicz, O. C., Taylor, R. L., Fox, D. (2014). *The Finite Element Method for Solid and Structural Mechanics*. Butterworth-Heinemann. doi: <https://doi.org/10.1016/c2009-0-26332-x>
13. Konkin, V. N., Morachkovskii, O. K. (1987). Creep and long-term strength of light alloys with anisotropic properties. *Strength of Materials*, 19 (5), 626–631. doi: <https://doi.org/10.1007/bf01524293>
14. Breslavskii, D. V., Metelev, V. A., Morachkovskii, O. K. (2015). Anisotropic Creep and Damage in Structural Elements Under Cyclic Loading. *Strength of Materials*, 47 (2), 235–241. doi: <https://doi.org/10.1007/s11223-015-9653-z>
15. Breslavskiy, D. V., Metel'ov, V. O., Morachkovskiy, O. K., Tatarinova, O. A. (2019). Short-Term Creep of St3 Steel Under Low-Frequency Cyclic Loading. *Strength of Materials*, 51 (5), 753–760. doi: <https://doi.org/10.1007/s11223-019-00124-2>
16. Breslavsky, D., Chuprynin, A., Morachkovsky, O., Tatarinova, O., Pro, W. (2019). Deformation and damage of nuclear power station fuel elements under cyclic loading. *The Journal of Strain Analysis for Engineering Design*, 54 (5-6), 348–359. doi: <https://doi.org/10.1177/0309324719874923>
17. Altenbach, H., Breslavsky, D., Mitielov, V., Tatarinova, O. (2019). Short Term Transversally Isotropic Creep of Plates Under Static and Periodic Loading. *Advanced Structured Materials*, 181–211. doi: [https://doi.org/10.1007/978-3-030-23869-8\\_9](https://doi.org/10.1007/978-3-030-23869-8_9)