UDC 514.18
DOI: 10.15587/1729-4061.2023.272400

CONSTRUCTION OF SPHERICAL NON-CIRCULAR WHEELS FORMED BY SYMMETRICAL ARCS OF LOXODROME

Tetiana Kresan
PhD, Associate Professor, Head of Department
Department of Natural, Mathematical and General Engineering Disciplines
Separated Subdivision of the National University of Life and Environmental Sciences of Ukraine «Nizhyn Agrotechnical Institute»
Shevchenko str., 10, Nizhyn, Ukraine, 16600

Ali Kadhim Ahmed
PhD, Lecturer
Department of Soil Science and Water Resources
College of Agriculture University of Diyala
Bagdad str., 1, Baqubah, Iraq, 32001

Serhii Pylypaka
Doctor of Technical Sciences, Professor, Head of Department*

Tatiana Volina
Corresponding author
PhD, Associate Professor*
E-mail: t.n.zaharova@ukr.net

Svitlana Semirnenko
PhD, Associate Professor
Department of Engineering Systems Design
Sumy National Agrarian University
Herasyma Kondratieva str., 160, Sumy, Ukraine, 40000

Viktor Trokhaniak
PhD, Associate Professor
Department of Heat and Power Engineering**

Irina Zakharova
PhD, Associate Professor
Department of Management of Education and Pedagogy of the Higher School
Sumy State Pedagogical University named after A. S. Makarenko
Romenska str., 87, Sumy, Ukraine, 40002

*Department of Descriptive Geometry, Computer Graphics and Design**
**National University of Life and Environmental Sciences of Ukraine
Heroyiv Oborony str., 15, Kyiv, Ukraine, 03041

Bevel gears are used to transmit torque between intersecting axes. They demonstrate high reliability and durability of work, as well as a constant gear ratio. The disadvantage of such a transmission is the mutual sliding of the surfaces of the teeth of the gears, which leads to the emergence of friction forces and wear of their working surfaces. In this regard, there is a task to design such bevel gears that would have no slip.

Non-circular wheels are understood as a pair of closed curves that rotate around fixed centers and at the same time roll over each other without sliding. They can serve as centroids for the design of cylindrical gears between parallel axes. If the axes of rotation of the wheels intersect, then the gears are called conical. An analog of gears between parallel axes, in which centroids are flat closed curves, for gears with intersecting axes are spherical closed curves. For a bevel gear with a constant gear ratio, such spherical curves are circles on the surface of the sphere, and with a variable gear ratio, spatial spherical curves.

This paper considers the construction of closed spherical curves that roll around each other without sliding when they rotate around the axes intersecting in the center of the spheres. These curves are formed from symmetrical arcs of the loxodrome, a curve that crosses all the meridians of the ball at a constant angle. This angle should be 45°, which ensures the intersection of the loxodrome at right angles. Analytical dependences have been derived underlying the calculations of profiles of spherical non-circular wheels and their visualization by means of computer graphics. The results could be used to design non-circular wheels for textile machines, hydraulic machine pumps, pump dispensers, etc.

Keywords: noncircular wheels, rolling, bevel gear, arc length, spherical curve, loxodrome

Received date 07.11.2022
Accepted date 09.01.2023
Published date 28.02.2023

How to Cite: *Kresan, T., Kadhim Ahmed, A., Pylypaka, S., Volina, T., Semirnenko, S., Trokhaniak, V., Zakharova, I. (2023). Construction of spherical non-circular wheels formed by symmetrical arcs of loxodrome. Eastern-European Journal of Enterprise Technologies, 1 (1 (121)), 44–50. doi: https://doi.org/10.15587/1729-4061.2023.272400*

1. Introduction

To transmit rotational motion between parallel axes, both round and non-circular wheels can be used. Their construction is reduced to finding flat closed curves. If the transmission of rotational motion occurs between intersecting axes, then the curves become spherical and are located on a sphere with a center at the point of intersection of the axes. Such gears are called bevel. Loxodrome arcs can be accepted as spherical curves. With proper selection of parameters, non-circular wheels can be formed from symmetrical loxodrome arcs, which can transmit rotational motion between the axes intersecting at a given angle.

Bevel gears are widely used in assemblies to transmit torque between intersecting axles. They have high reliability and durability of work, as well as a constant gear ratio. The disadvantage of such a transmission is the mutual sliding of the surfaces of the teeth of the gears, which leads to the emergence of friction forces and wear of their working surfaces. In this regard, the question arises of designing such bevel gears that would have no slipping, that is, the teeth could roll over each other without sliding.

Rolling without sliding of bevel surfaces one over one can be replaced by the rolling of spherical curves, which are lines of intersection of bevel surfaces with a sphere. Thus, the problem about the rolling of bevel surfaces can be reduced to the problem of rolling spherical curves. Such curves are closed and can be both continuous and consist of separate arcs. For the design of gears, these curves are the basis and are called centroids. The centroids of non-circular wheels for cylindrical gears are well studied. A classic example is the rotation of congruent ellipses around their focuses. Another example is the use of a logarithmic helix as a centroid. It is known that the analog of the logarithmic helix for cylindrical transmission is a spherical curve – a loxodrome – for a bevel gear. This curve can be used to construct a bevel gear centroid with a given angle between the axes of rotation. Design and manufacture of bevel gears are constantly being improved. Given this, any suggestions for improving their work deserve attention and are relevant.

2. Literature review and problem statement

The construction of geometric objects is one of the key tasks of engineering, which is solved with the help of various approaches. Thus, article [1] reports the results of research on the possibility of constructing multidimensional geometric objects by approximating the solution of differential equations; in [2], it is shown that the same goal can be achieved by multidimensional parabolic interpolation, and, in [3], by using geometric interpolants of space as a basis. This indicates a sufficient variety of methods for the geometric design of objects. But the geometric aspects of such modeling remain undisclosed.

In addition, it should be noted that the issue of increasing the durability of parts does not lose its relevance. To achieve this goal, paper [4] developed a fundamentally new method of sulfidation of surfaces by EDM doping; [5] – a technique for strengthening surfaces with cementation and nitriding; [6] – a procedure of coating with EDM doping. During the analysis of literary sources, it was found that it is proposed to increase the wear resistance of parts by strengthening their surface layer, which is a rather costly process. However, the task can be accomplished by fundamentally different methods. A variant to eliminate the cause of wear can be to eliminate friction between the working surfaces.

The non-circular wheel of a cylinder gear is formed from segments of straight lines, that is, it is a polygon. Features of the design of such wheels and the limitations that arise in this case are considered in [7]. The authors of [8] described the geometric design of the profile of a non-circular gear wheel with a centroid from connected arcs. Paper [9] proposes a technique of automated design of teeth for a non-circular wheel with an elliptical centroid. Article [10] reports a new type of gear pair – an eccentric screw, with the help of which the rotation of intersecting axes is realized with their simultaneous movement. Such a pair consists of a helical noncircular gear and eccentric helical curved face. In [11], a new composite profile of the tooth was proposed with an analysis of the dependence of design parameters in accordance with various elliptical eccentricities. The practical side of the issue of manufacturing non-circular wheels is revealed in [12], which presents a device for the manufacture of non-circular gears. In addition, paper [13] developed a technique for processing the working surface of the wheels, which can be used for grinding the ends of non-circular parts. In [14], it is proposed to improve the procedure for the manufacture of non-circular wheels by generating geometric features of non-circular contours. However, only a flat contour is considered. It should be noted that the problem of rolling bevel surfaces can be reduced to the problem of rolling spherical curves. The centroid of a bevel gear can be a spherical curve – a loxodrome. It can be used to construct a bevel gear centroid with a given angle between the axes of rotation. All this suggests that it is expedient to conduct a study on the design of pairs of spherical non-circular wheels that roll over each other without sliding, which minimizes friction and, accordingly, surface wear.

3. The aim and objectives of the study

The aim of this study is to devise a procedure for constructing pairs of spherical noncircular wheels from symmetrical loxodrome arcs, which rotate around the axes intersecting in the center of the sphere, and at the same time roll over each other without sliding. The absence of slipping will provide an increase in wear resistance and durability of a pair of wheels.

To accomplish the aim, the following tasks have been set:

– to derive the parametric equations of the loxodrome, that is, the curve that will intersect all the meridians at a given angle, with the visualization of the results obtained;

– to build spherical centroids of non-circular wheels from symmetrical arcs of the loxodrome;

– to consider the proposed approach on the example of designing spherical non-circular wheels under predetermined conditions.

4. The study materials and methods

The logarithmic helix intersects all radius-vectors that come out of the pole at a constant angle. From the two symmetrical arcs of the logarithmic helix, a closed line is formed, which can serve as a non-circular wheel. Two such closed congruent lines can rotate around the poles and at the same time roll over each other without sliding. An analog of the logarithmic helix on the plane is the loxodrome on the sphere – a curve that crosses all the meridians at a constant angle and twists around the pole of the sphere. This curve is the object of our study. Its use as a bevel gear centroid will ensure that the working surfaces of spherical wheels do not slip. Parametric equations of a sphere are:

$$X = R\sin\varepsilon\cos\gamma;$$

$$Y = R\sin\varepsilon\sin\gamma;$$

$$Z = R\cos\varepsilon, \tag{1}$$

where R is the radius of the ball; γ and ε are independent surface variables, and γ is numerically equal to the angular rotation of the surface point around the Oz axis, and ε is the second angle, the countdown of which starts from the upper pole of the sphere along the meridian towards the equator. The first quadratic form (linear element) of the sphere (1) is:

$$dS^2 = R^2 d\varepsilon^2 + R^2 \sin^2 \varepsilon \, d\gamma^2. \tag{2}$$

In the geometric interpretation, the linear element (2) can be represented as the hypotenuse of an infinitely small right triangle, the legs of which are infinitesimal increments of the parallel and the meridian (Fig. 1, *a*).

The angle β is the angle between the curve on the sphere and the meridian. For loxodrome, it must be constant. From a right triangle (Fig. 1, *a*), we write:

$$\mathrm{ctg}\beta = \frac{R d\varepsilon}{R \sin \varepsilon \, d\gamma}. \tag{3}$$

The above provisions formed the basis for the design of spherical non-circular wheels from symmetrical loxodrome arcs.

5. Results of the design of spherical non-circular wheels

5. 1. Constructing parametric equations of the loxodrome, which will intersect all the meridians at a given angle

In order for a line to be described on the surface of a sphere, it is necessary to establish a certain relationship between the variables ε and γ. Let this dependence take the form of $\varepsilon=\varepsilon(\gamma)$. In this case, expression (3) is converted to a differential equation:

$$\frac{d\varepsilon}{d\gamma} = \mathrm{ctg}\beta \sin \varepsilon. \tag{4}$$

Equation (4) has the following solution:

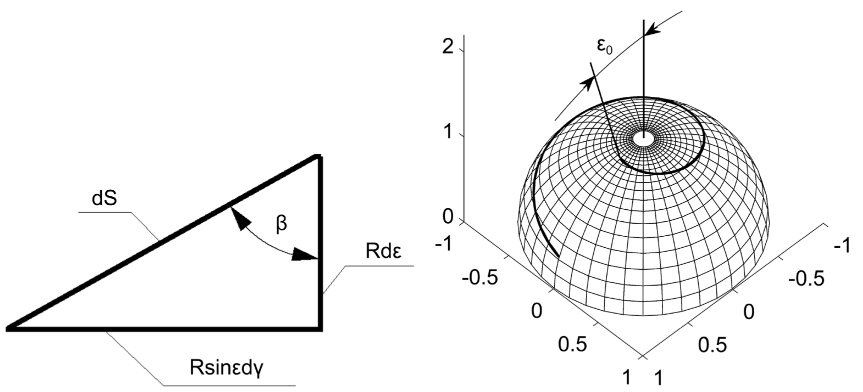
$$\varepsilon = 2\mathrm{arcctg}\left(e^{-\gamma \mathrm{ctg}\beta - c}\right), \tag{5}$$

where c is the integration constant.

We set the condition that at $\gamma=0$ the angle ε has an initial value of ε_0. Based on this, dependence (5) takes the final form:

$$\varepsilon = 2\mathrm{arctg}\left(e^{\gamma \mathrm{ctg}\beta} \mathrm{tg}\frac{\varepsilon_0}{2}\right). \tag{6}$$

The substitution of dependence (6) into the equation of the sphere (1) makes it possible to obtain parametric equations of the loxodrome, which will intersect all the meridians at a given angle β.



a *b*

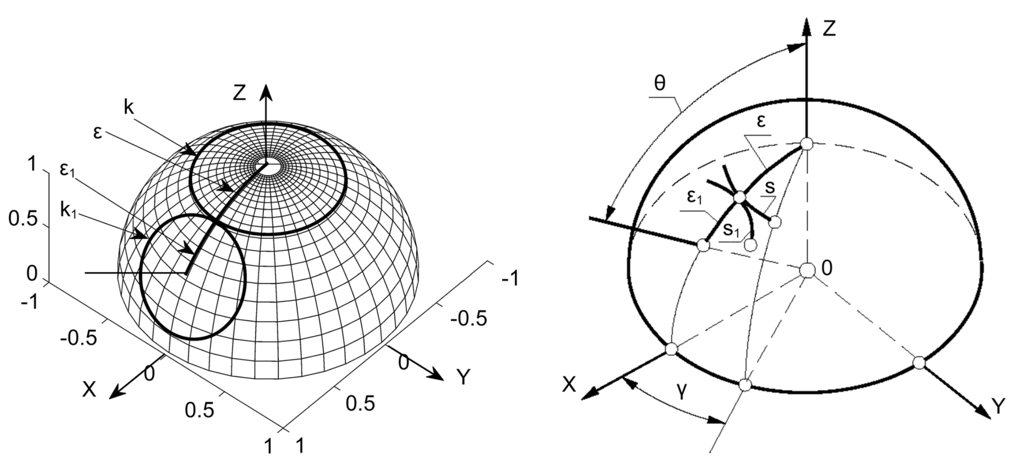
Fig. 1. Graphic illustrations for the construction of a loxodrome on the surface of a sphere: *a* — an elementary right triangle for determining the angle of β; *b* — loxodrome on the surface of the sphere

5. 2. Construction of spherical centroids of noncircular wheels from symmetrical loxodrome arcs

The devised approach is enough to build spherical wheels that will ensure the absence of slipping.

In Fig. 1, *b*, the loxodrome is constructed at $\varepsilon_0=15°$, $\beta=75°$, and the change in angle γ within $\gamma=0...2\pi$.

In the case when two cones rotate around the axes intersecting in the center of the sphere, the spherical wheels will be circles (Fig. 2, *a*).



a *b*

Fig. 2. Graphic illustrations for the construction of the second spherical wheel: *a* — spherical wheels *k* and k_1 are circles; *b* — arcs *s* and s_1 of spherical noncircular wheels with a common point of contact

For both round and non-round spherical wheels, the condition $\theta=\varepsilon+\varepsilon_1$ must be met, where θ is the angle between the axes of rotation, ε and ε_1 – angles from the axes of rotation to the point of contact of the wheels (Fig. 2, *a*, *b*). Hence, we find $\varepsilon_1=\theta-\varepsilon$, where the expression $\varepsilon=\varepsilon(\gamma)$ for the loxodrome is found in (6). The curve *k* of the first non-circular wheel we have is a loxodrome, which we obtain by substituting dependence (6) into the equation of the sphere (1).

Similarly, we shall look for the curve k_1 of the second non-circular wheel. To do this, we substitute $\varepsilon_1=\theta-\varepsilon$ in the equation of the sphere (1) with ε, and, instead of the angle γ – a similar angle of φ, between which we establish a relationship based on the equality of arcs *s* and s_1 (Fig. 2, *a*):

$$x_1 = R\sin\left[\theta - 2\operatorname{arctg}\left(e^{y\operatorname{ctg}\beta}\operatorname{tg}\frac{\varepsilon_0}{2}\right)\right]\cos\varphi;$$

$$y_1 = R\sin\left[\theta - 2\operatorname{arctg}\left(e^{y\operatorname{ctg}\beta}\operatorname{tg}\frac{\varepsilon_0}{2}\right)\right]\sin\varphi;$$

$$z_1 = R\cos\left[\theta - 2\operatorname{arctg}\left(e^{y\operatorname{ctg}\beta}\operatorname{tg}\frac{\varepsilon_0}{2}\right)\right]. \qquad (7)$$

The curve k_1, according to equations (7), is derived with the axis of rotation *OZ*. Then we transfer it to the desired position by turning to the angle θ around the *OY* axis.

With an increase in the angle γ from zero to a certain value on the sphere, the arc of the loxodrome of length *s* will be described (Fig. 2, *b*). The corresponding arc of the curve k_1 will have a length s_1. From the condition of rolling without sliding, these arcs should be equal: $s=s_1$. Find the expressions of their lengths and equate with each other. To do this, we shall use the expression of the linear element (2). We have already established a relationship between the variables $\varepsilon=\varepsilon(\gamma)$ in the form (6). In this case, the linear element (2) takes the form:

$$\frac{ds}{d\gamma} = R\sqrt{\left(\frac{d\varepsilon}{d\gamma}\right)^2 + \sin^2\varepsilon}\,. \qquad (8)$$

We substitute in (8) the expression $\varepsilon=\varepsilon(\gamma)$ from (6), as well as its derivative, and, after the simplifications, we obtain:

$$\frac{ds}{d\gamma} = \frac{R}{\sin\beta}\sin\left[2\operatorname{arctg}\left(e^{\gamma\operatorname{ctg}\beta}\operatorname{tg}\frac{\varepsilon_0}{2}\right)\right]. \qquad (9)$$

In the curve k_1 (7), the dependence of the angle φ is unknown. We shall consider it dependent on the angle γ: $\varphi=\varphi(\gamma)$. We find the linear element of the arc of the curve k_1 from known formula $s_1' = \sqrt{x_1'^2 + y_1'^2 + z_1'^2}$. Take the derivatives of equations (7), implying that the dependence $\varphi=\varphi(\gamma)$ is an unknown function and substitute into the above formula. After simplifications, we obtain:

$$\frac{ds_1}{d\gamma} = R\sqrt{\frac{4e^{2\gamma\operatorname{ctg}\beta}\operatorname{ctg}^2\beta\operatorname{tg}^2\frac{\varepsilon_0}{2}}{\left(1+e^{2\gamma\operatorname{ctg}\beta}\operatorname{tg}^2\frac{\varepsilon_0}{2}\right)^2} + \left(\frac{d\varphi}{d\gamma}\right)^2\sin^2\left[\theta - 2\operatorname{arctg}\left(e^{\gamma\operatorname{ctg}\beta}\operatorname{tg}\frac{\varepsilon_0}{2}\right)\right]}. \qquad (10)$$

If the arcs *s* and s_1 are equal, then their derivatives are equal. We equate expressions (9) and (10) with each other and solve with respect to $d\varphi/d\gamma$. Despite the cumbersomeness of the resulting expression, it is possible to integrate it. Of course, the operations described, including the differentiation of equations (7), would be unrealistic to perform manually. For this purpose, the software product of symbolic mathematics «Mathematica» was used. After integration, we obtain the dependence $\varphi=\varphi(\gamma)$ taking into account the initial conditions:

$$\varphi = \pi + \operatorname{tg}\beta\ln\left(\frac{\operatorname{ctg}\frac{\theta}{2} + e^{\gamma\operatorname{ctg}\beta}\operatorname{tg}\frac{\varepsilon_0}{2}}{\operatorname{ctg}\frac{\varepsilon_0}{2}\operatorname{tg}\frac{\theta}{2} - e^{\gamma\operatorname{ctg}\beta}}\operatorname{tg}\frac{\theta-\varepsilon_0}{2}\right). \qquad (11)$$

If the expression of the angle φ (11) is substituted into parametric equations (7), then they will describe the curve k_1, which must roll along the curve *k* (loxodrome), while rotating around the axes intersecting in the center of the sphere. To build a loxodrome, you need to set the values of constants R, β, ε_0. In Fig. 3, *a*, a solid line shows a loxodrome *k* constructed at $R=1$, $\beta=45°$, $\varepsilon_0=2.7°$ and a change in the angle γ within $\gamma=0...\pi$. To construct a curve k_1 using equations (7), you additionally need to set the angle θ. Two angles (ε_0 and θ) must be coordinated with each other. For example, we want that when rotating the curve by 180°, the curve k_1 must also rotate at the same angle, that is, at $\gamma=\pi$, expression (11) must also give a value $\varphi=\pi$. The *R* and β constants must be common to both curves, which will ensure their symmetry. So, there are two ways to ensure that $\varphi=\pi$ when substituting in (11) $\gamma=\pi$: set ε_0 and choose the desired value of θ, or vice versa. For example, in order for $\Theta=60°$, for ε_0, you need to give a value $\varepsilon_0=2.7°$. The curve k_1 constructed at these constant values coincides with the curve *k*, which suggests that it is also a loxodrome.

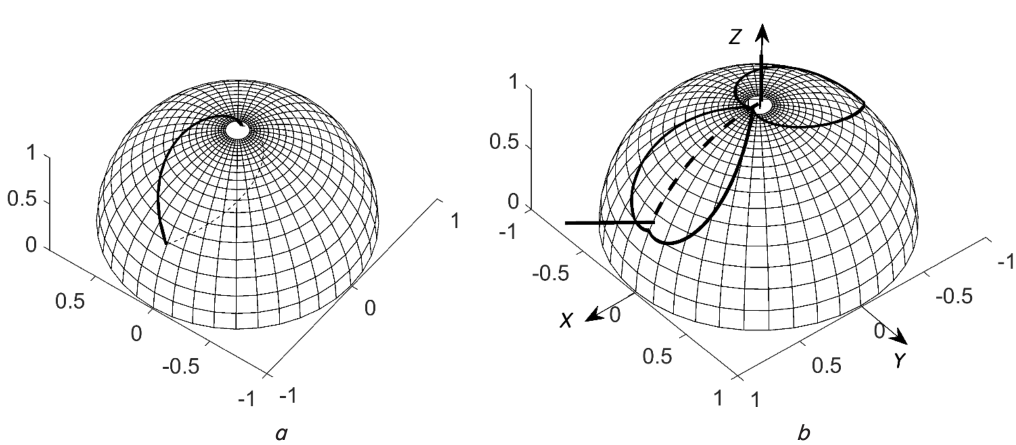


Fig. 3. Closed curves formed by symmetrical arcs of the loxodrome: *a* — curves *k* and k_1 coincide; *b* — curves *k* and k_1 are separated by turning the curve k_1 at angle θ

If we build a symmetrical arc on the surface of the sphere (Fig. 3, *a*, dashed line), then a closed non-circular wheel will be formed, having two right angles at the intersection points of the arcs. Turning one wheel at an angle of Θ=60° around the *OY* axis, we obtain two non-circular wheels with a common point of contact (Fig. 3, *b*). Meridian arc corresponding to the angular Θ=60° in Fig. 3, *b* is shown by a dashed line. In order to avoid jamming when rolling the resulting spherical wheels, the angle at the points of intersection of the arcs must be equal to a straight line or greater, that is, β≥45°.

5. 3. Application of the proposed approach to the design of spherical non-circular wheels under predetermined conditions

The above results allow us to expand the design of spherical centroids of non-circular wheels from symmetrical loxodrome arcs. For example, set the condition that at an angle Θ=60° the angles of rotation of the curves k nd k_1 are half the smaller: γ=φ=π/2. Then we obtain non-congruent wheels from symmetrical loxodrome arcs constructed in Fig. 4.

If we set the condition that the axes of rotation of spherical non-circular wheels (Fig. 4) intersect at right angles, then, according to the developed procedure, we find ε_0=17.37°. Non-circular spherical wheels for this case on the sphere of a single radius are built in Fig. 5, *a*.

Confirmation that the calculations are performed correctly is a demonstration of the rotation of the wheels around the axes, indicating the corresponding angles of rotation (Fig. 5). For example, in Fig. 5, *c*, one spherical wheel rotates around the *OZ* axis at an angle of γ=60°. The magnitude of the angle of rotation of the second wheel around the *OX* axis is found from formula (11) when substituting into it γ=60°. From it, we find φ=–35.8°. When rotating the second wheel around the *OX* axis at this angle, both wheels touch each other and the point of contact is located on the arc of the meridian connecting the centers of the wheels (the arc is depicted by a dashed line). On the sphere of a single radius, it is equal to the angular θ, that is, 90°.

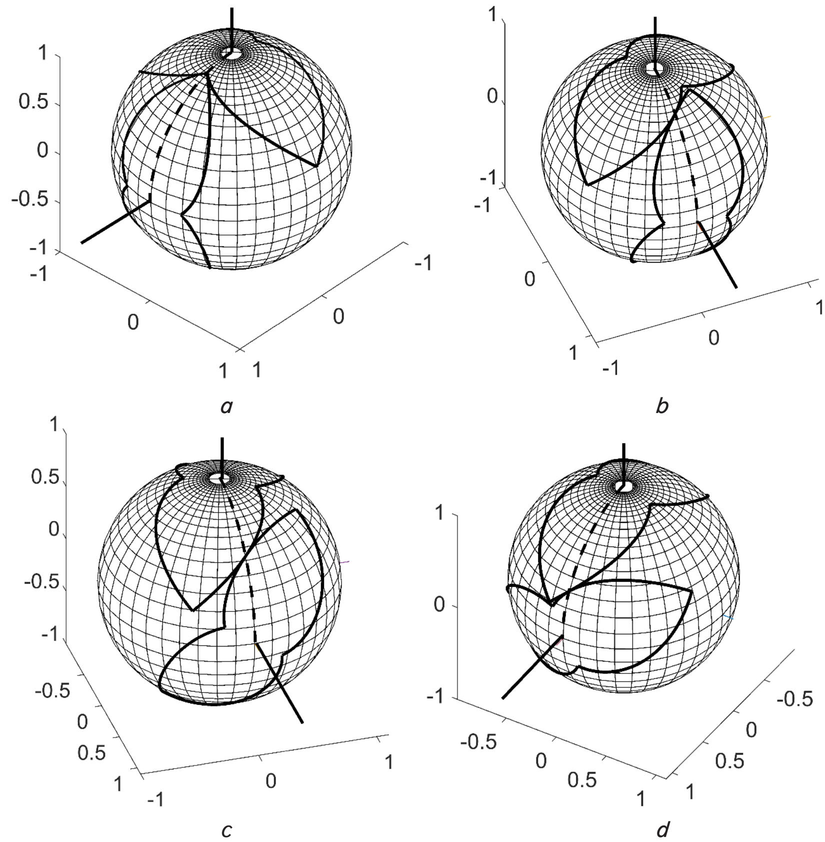


Fig. 5. Sequential rotation of spherical congruent noncircular wheels around the *OZ* and *OX* axes, respectively, at angles γ and φ: *a* – γ=0, φ=0; *b* – γ=30°, φ= –12.6°; *c* – γ=60°, φ= –35.8°; *d* – γ=80°, φ= –64.9°

6. Discussion of results of investigating the methodology for constructing spherical non-circular wheels

By analogy with cylindrical gears, in which the profile of a noncircular wheel is formed by a pair of symmetrical arcs of a logarithmic helix, an analytical expression (6) was obtained for a bevel gear. This expression makes it possible to build a spherical analog of the logarithmic helix – the loxodrome. To do this, you need to substitute expression (6) into the equation of the sphere. An example of the construction of a loxodrome arc is shown in Fig. 1, *b*.

When turning the arc of the loxodrome, which is the profile of the first non-circular wheel, at a certain angle γ, the curve of the profile of the second non-circular wheel will rotate at angle φ.

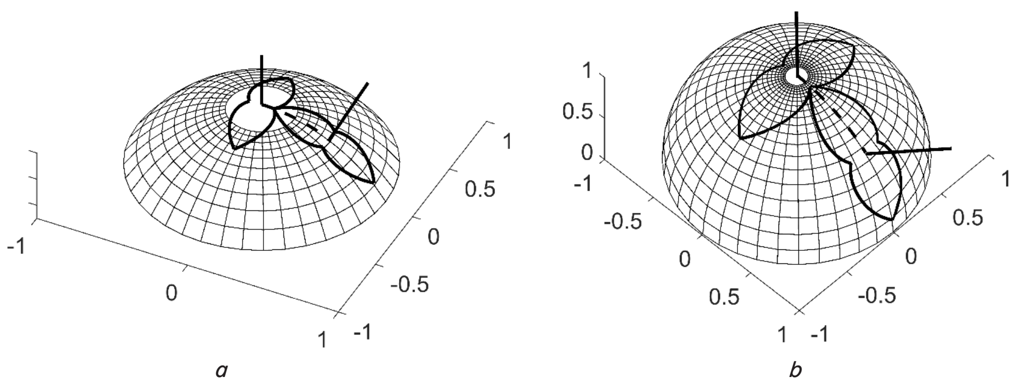


Fig. 4. Spherical congruent noncircular wheels formed by loxodrome arcs: *a* – angle Θ=25°; *b* – angle Θ=45°

Based on the condition that the curves roll one by one without sliding, that is, they pass the same lengths of their arcs, the relationship between the angles of γ and φ in the form (11) has been established. The resulting dependence (11) makes it possible to construct the profile of the second non-circular wheel when substituting it into parametric equations (7). It was found out during the construction and mathematically proved that the profile curve will be a congruent loxodrome. This allows us to build a closed non-circular wheel from two symmetrical congruent arcs of the loxodrome, as shown in Fig. 3, *a*. The beginning and end of the arc must intersect the meridian at an angle of 45°, which ensures the angle of intersection of the loxodrome at right angles. There can be no angle smaller than the right one since, in this case, rolling the wheels upon contact at these points will become impossible due to jamming. If two non-circular wheels are positioned as shown in Fig. 3, *b*, then they will be able to rotate around two mutually perpendicular axes and the point of their contact will move along the dashed line.

The developed procedure allows us to increase the number of elements of non-circular wheels. If, in Fig. 3, *b*, both wheels consist of one element, then in Fig. 4 – of two. In this case, you can set the angle between the axes around which the wheels rotate. This can be seen from Fig. 4 when comparing images in Fig. 4, *a*, *b*. As the angle decreases, the size of the wheels themselves also decreases. In Fig. 5, two non-circular wheels are built, the axes of which intersect at right angles. To confirm the reliability of the results obtained, four of their positions are shown when rotating around their axes. In this case, the angle of rotation γ for one wheel was set, and for the second – calculated from formula (11). When turning the wheels at the appropriate angles, the point of contact is always on the arc of the meridian depicted by the dashed line. The lengths of the arcs that the wheels pass during mutual rolling are equal, as seen visually, and are also confirmed mathematically by integrating expression (10). Consequently, the rolling of such non-circular bevel wheels occurs without slipping. The mathematical construction of non-circular wheels is demonstrated for one and two elements, which can be conditionally called teeth. Fig. 5 convincingly shows the possibility of transmitting rotational motion around intersecting axes using wheels with two teeth. In traditional bevel gears, two-pronged wheels do not exist. This makes it possible to form a profile of a bevel wheel, similar to a gear, and with a larger number of teeth.

In addition to the advantage, which is the absence of sliding between the surfaces of the teeth, which does not lead to wear of their surface, the disadvantages are also inherent in a transmission. The main one is the variable gear ratio within one tooth. However, there are mechanisms for which the gear ratio does not matter, or even useful. For example, in counting devices, it is important to have the number of full revolutions of the wheel, which is not affected by the variable gear ratio. The reliability of the proposed approach is based on the mathematical derivation of the obtained dependences and graphic images of non-circular wheels with two teeth built on the basis of these dependences.

The loxodrome can cross the meridians at any given angle. Accordingly, the intersection of two symmetrical arcs of the loxodrome will form twice the angle, which can be sharp. Theoretically constructed elements (teeth) with an acute angle at the top can also be components of non-circular wheels. However, in practice they will not work. The magnitude of the angle at the top of the tooth is subject to a limit of 90° since jamming will occur at a smaller angle.

The current study considered a pair of non-circular wheels with the same number of elements (teeth), which forms its limitations. The prospect of further research is to design pairs of non-circular wheels with a different number of elements, which will make it possible to obtain a different numerical ratio of the full revolutions of the wheels.

7. Conclusions

1. To design bevel non-circular wheels that can roll over each other, while rotating around the intersecting axes, it was necessary to find a spherical curve that could describe the profile of the wheels. By analogy, the transmission between parallel axes was taken, into which the non-circular wheels were outlined by the arcs of the logarithmic helix. A feature of this helix is that it intersects all radius-vectors in the polar coordinate system at a constant angle. An analog of such a curve on a sphere is a curve that crosses all the meridians at a constant angle and is called a loxodrome. Based on this, we have derived the internal equation of the loxodrome in curvilinear coordinates and its parametric equations in a form suitable for further use, in which the value of the specified angle appears.

2. After the loxodrome equation was built, the task arose to, with the help of its arcs, outline the prototype of a non-circular wheel on a sphere. At the same time, it was assumed that when rolling two non-circular spherical wheels while simultaneously rotating around their axes, which intersect in the center of the sphere, the angle θ between the axes must be constant. The sum of the angles ε and ε_1 from the axes to the point of contact must also be constant and equal to angle θ. In addition, when turning non-circular wheels at angles γ and φ around their axes, the arcs s and s_1 must be equal, which ensures that the wheels roll without sliding. Based on these requirements, an important equation was built that establishes an analytical relationship between the angles γ and φ.

3. Based on our analytical results, the profiles of spherical non-circular wheels were calculated according to predetermined conditions and their visualization by means of computer graphics. To verify the reliability of the results, both spherical non-circular wheels were rotated around their axes at the specified and found angles γ and φ, respectively. All four rotation options give correct images.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

Funding

The study was conducted without financial support.

Data availability

All data are available in the main text of the manuscript.

References

1. Konopatskiy, E., Voronova, O., Bezditnyi, A., Shevchuk, O. (2020). About one method of numeral decision of differential equalizations in partials using geometric interpolants. CPT2020 The 8th International Scientific Conference on Computing in Physics and Technology Proceedings. doi: https://doi.org/10.30987/conferencearticle_5fce27708eb353.92843700
2. Konopatskiy, E. V., Bezditnyi, A. A. (2020). Geometric modeling of multifactor processes and phenomena by the multidimensional parabolic interpolation method. Journal of Physics: Conference Series, 1441 (1), 012063. doi: https://doi.org/10.1088/1742-6596/1441/1/012063
3. Konopatskiy, E., Bezditnyi, A., Shevchuk, O. (2020). Modeling Geometric Varieties with Given Differential Characteristics and Its Application. Proceedings of the 30th International Conference on Computer Graphics and Machine Vision (GraphiCon 2020). Part 2, short31-1-short31-8. doi: https://doi.org/10.51130/graphicon-2020-2-4-31
4. Tarelnyk, V., Martsynkovskyy, V., Gaponova, O., Konoplianchenko, I., Dovzyk, M., Tarelnyk, N., Gorovoy, S. (2017). New sulphiding method for steel and cast iron parts. IOP Conference Series: Materials Science and Engineering, 233, 012049. doi: https://doi.org/10.1088/1757-899x/233/1/012049
5. Tarelnyk, V., Martsynkovskyy, V., Gaponova, O., Konoplianchenko, I., Belous, A., Gerasimenko, V., Zakharov, M. (2017). New method for strengthening surfaces of heat treated steel parts. IOP Conference Series: Materials Science and Engineering, 233, 012048. doi: https://doi.org/10.1088/1757-899x/233/1/012048
6. Tarel'nik, V. B., Martsinkovskii, V. S., Zhukov, A. N. (2017). Increase in the Reliability and Durability of Metal Impulse End Seals. Part 1. Chemical and Petroleum Engineering, 53 (1-2), 114–120. doi: https://doi.org/10.1007/s10556-017-0305-y
7. Pylypaka, S. F., Klendii, M. B., Trokhaniak, V. I., Kresan, T. A., Hryshchenko, I. Y., Pastushenko, A. S. (2021). External rolling of a polygon on closed curvilinear profile. Acta Polytechnica, 61 (1), 270–278. doi: https://doi.org/10.14311/ap.2021.61.0270
8. Lyashkov, A. A., Panchuk, K. L., Khasanova, I. A. (2018). Automated Geometric and Computer-aided Non-Circular Gear Formation Modeling. Journal of Physics: Conference Series, 1050, 012049. doi: https://doi.org/10.1088/1742-6596/1050/1/012049
9. Lyashkov, A. A., Reinhard, A. J., Murashev, G. E. (2019). Geometric And Computer Modeling of Forming the Gear Wheels with Elliptical Centroid. Journal of Physics: Conference Series, 1260 (11), 112018. doi: https://doi.org/10.1088/1742-6596/1260/11/112018
10. Lin, C., Wu, X. (2018). Calculation and Characteristic Analysis of Tooth Width of Eccentric Helical Curve-Face Gear. Iranian Journal of Science and Technology, Transactions of Mechanical Engineering, 43 (4), 781–797. doi: https://doi.org/10.1007/s40997-018-0239-9
11. Liu, J.-G., Tong, Z.-P., Yu, G.-H., Zhao, X., Zhou, H.-L. (2022). Design and Application of Non-Circular Gear with Cusp Pitch Curve. Machines, 10 (11), 985. doi: https://doi.org/10.3390/machines10110985
12. Bogdan-Roth, M., Romanet, M., Ripeanu, R. G. (2019). Device for machining non-circular gears. Proceedings on Engineering Sciences, 1 (1), 449–453. doi: https://doi.org/10.24874/pes01.01.059
13. Kalchenko, V., Kalchenko, V., Kolohoida, A., Yeroshenko, A., Kalchenko, D. (2022). Building a model of dressing the working surfaces of wheels during the two-side grinding of round end faces at CNC machines. Eastern-European Journal of Enterprise Technologies, 1 (1 (115)), 86–93. doi: https://doi.org/10.15587/1729-4061.2022.252642
14. Krawiec, P., Czarnecka-Komorowska, D., Warguła, Ł., Wojciechowski, S. (2021). Geometric Specification of Non-Circular Pulleys Made with Various Additive Manufacturing Techniques. Materials, 14 (7), 1682. doi: https://doi.org/10.3390/ma14071682