This paper considers the influence of hydrodynamic processes in the movement of the free surface of liquid in partially filled tractor tanks. Splashing liquid in partially filled containers is a significant problem in the study of functional stability of movement in the marine, aerospace, rail, and automotive industries. After all, it affects productivity and traffic safety. The same effect was observed when performing transportation work while delivering liquid cargoes in the agricultural sector. That was due to increasing the transportation speeds of wheeled tractors. In the procedure, using the Rayleigh theory of surface waves, a linearized problem of motion of the free surface of a liquid is obtained. Based on Helmholtz's theorem, the components of scalar and Laplace field vector potentials of fluid velocity vector are separated. The potential problem for translational motion of fluid, in which vortex component of the field is absent, is considered. Instead of the fluid velocity potential, a scalar fluid displacement potential in Rayleigh surface waves was introduced. Comparing the results of calculating fluid splashing with the work of other scientists, a high convergence of natural frequencies of partial oscillators in 3D space was found. This is noticeable in the last quarter of the filling of the tank, in which significant displacements of the deep liquid occur. A feature of the results is the introduction, instead of the real shape of the container, an equivalent form of a parallelepiped, the final shape of which depends on the level of fullness. The frequency properties of movement of the free surface of liquid based on the standard size of tanks used in agriculture are separated. The proposed improved methodology could be used to increase stability, controllability, and smoothness when operating tanks with a wheeled tractor

Keywords: cylindrical tank, free surface, equivalent shape, eigenfrequency, partial oscillator

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IMPROVING THE PROCEDURE FOR MODELING LOW-FREQUENCY OSCILLATIONS OF THE FREE SURFACE LIQUID IN A TRACTOR TANK

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1. Introduction

Transport is the driving sector of the economy of any state, which is designed to ensure the delivery of commodities in order to meet the needs of society. Thereby developing the economy of the state as a whole. An integral part of the successful functioning of sectors of the economy (agro-industrial, civil, construction, etc.) is the execution of transport operations when liquid cargoes are delivered by vehicles. For each of the cargo group, there are regulatory documents that standardize the rules of transportation because depending on the physicochemical composition of the liquid, compliance with volume, temperature, and other standards is required.

In view of the global food crises, agro-industrial complexes of any state are required to increase the production of food. One of such solutions is the systematic application of organic fertilizers, which requires the fulfillment of transport tasks for delivering liquid cargoes by tanks. Until recently, such operations were carried out by automotive vehicles but the constant increase in the production of energy-saturated tractors made it possible to change this trend. For high-volume tanks, it is the norm to introduce structural changes designed to reduce the flow of fluid (longitudinal and transverse partitions, installation of elastic elements between the tank and chassis of tanks, etc.). As regards tractor tanks, the introduction of internal partitions was impractical, at least for tanks with a carrying capacity of less than 20 tons, but with an increase in transport speeds, this norm becomes unacceptable. This nature of operation of partially filled tanks stimulates the formation of resonant oscillatory effects [1], which can lead to functional instability of the system, which may cause a deterioration in the technical and operational performance of vehicles. In addition, increased splashing of the free surface of the transported liquid is one of the factors in road accidents.

2. Literature review and problem statement

In work [2], the results of the study of the influence of changes in the acceleration of the center of mass of the system "tractor – transport and technological machine of variable mass" on the load of the tractor transmission are shown. It is shown that the problem of dynamics of the transported unit, which has a variable mass, is not solved in view of their energy efficiency and productivity. The outlined work is based on the determination of variable mass by determining the dynamic properties of the material point of variable mass, which is appropriate in the concept of studying the movement of a tractor tank during splashing. But issues related to the study of movement during transportation of a tractor tank, where a variable mass of the unit is observed only when the free surface of the liquid moves in a partially filled tank, remained unresolved.

Work [3] is aimed at assessing the oscillatory effects that act on the driver's condition during transportation of an agricultural tank. It is shown that when transporting a fully filled tank, the level of driver discomfort according to EU directive 2002/44 increases. The question of the influence of fluid vibrations on the smoothness and ergonomic performance of the wheeled tractor remains unresolved. An option to overcome the corresponding difficulties may be to avoid the resonant zones of action of fluctuations of the free surface of the liquid at the design stage of sprung systems for wheeled tractors and agricultural tanks. To this end, it is necessary to conduct comprehensive studies into the dynamics of movement, which cannot be avoided without taking into account the splashing of fluid.

Work [4] shows the results of the study of selection of the best size of the tank and partition in the OpenFOAM environment using the finite element method in order to minimize the horizontal displacement of the free surface of the liquid. It is shown that the presence of a vertical partition reduces the splashing of liquid in tanks. However, the introduction of partitions into an agricultural tank is inappropriate in view of its maintenance and inevitable getting into resonant zones when driving on complex soil coating. Works [5, 6] are aimed at studying the transitional three-dimensional motion of a liquid in a horizontal cylindrical tank of finite length. The authors of work [6] improved the approach from [5] by presenting a model based on the linear theory of free surface motion taking into account the theorem of translational addition of Graph for cylindrical Bessel functions. The stated improvement led to the formation of a large amount of numerical data (thirty-six longitudinal/transverse antisymmetric/symmetric dimensionless splash frequencies), which greatly complicates the process of interpretation of the obtained data and further implementation of the proposed model. In [7], the finite element method is used to study the vortex motion of a free surface based on an integral equation in the form of Galerkin. It is shown that to study the dynamics of movement of a horizontal tank it is enough to use only the mass of the first oscillator. The adequacy of this result remains unknown in view of the action of low-frequency and high-frequency oscillations of the liquid. Thus, the modern development of mathematical models that describe with exhaustive accuracy the oscillations of the free surface of a liquid (including the trajectories of individual droplets in air) leads to the complication of computational algorithms. Therefore, a variant of overcoming the corresponding difficulties may be the use of a combined model of the classical type, which is based on the concept of separation of vortex (pulse) and potential (convective) motion of the free surface and subsequent solution through the model of concentrated masses [8, 9]. In work [10] it is emphasized that this approach is the simplest and most popular method for analyzing the behavior of the free surface of a liquid in rectangular and cylindrical containers. However, studies have shown [11] that using this approach of an approximate splashing model has a worse convergence when compared with experimental data [12] when filled with liquid in the last half of the tank. In addition, the use of equivalent replacement of the real shape of the tank with a rectangular one [8, 9] has a worse convergence with experimental ones for higher harmonics. All this suggests that it is expedient to conduct research on improving the method for determining the movement of the free surface of a liquid in a tank, which increases the adequacy of the model without its mathematical complexity.

3. The aim and objectives of the study

The aim of this work is to refine the methodology for modeling low-frequency oscillations of liquid in a closed container by introducing a complex shape of tank when replacing the real one. This will make it possible to calculate dynamic indicators of the movement of a vehicle with tanks under difficult agricultural operating conditions. In addition, it will make it possible, at the design stage of agricultural tanks, to take into account the oscillatory effects that arise due to the splashing of liquid, and to avoid resonant phenomena.

To accomplish the aim, the following tasks have been set:

– to determine the low-frequency oscillations of a viscous liquid in a horizontal container with a free surface;

– to formalize the oscillatory movements of a liquid having a free surface in a closed horizontal tank, depending on the size of agricultural tanks;

– to establish the convergence of theoretical and experimental results at different levels of fullness of a cylindrical container.

4. The study materials and methods

4. 1. The object and hypothesis of the study

The object of our research is the hydrodynamic processes that occur when the free surface of a liquid moves in a partially filled cylindrical tank.

Such sources of low-frequency oscillations as the pulse of force and moment of force are applied to the shell of the tank from the side of the running system. Such an action can occur sporadically (when overcoming road obstacles) or be formed in a certain sequence (operation of a vacuum pump), forming attenuating free or non-attenuating forced oscillations of the liquid. The source of free low-frequency oscillations (and the transformer of forced oscillations) is the free surface of the liquid. Its fronts and kinematic perturbation waves propagate from the side surfaces of the tank, which leads to splashing of the liquid surface. This leads to a change in the magnitude of normal pressure in the opposite phase to a vertical displacement of the surface caused by gravitational and surface tension forces, and to free oscillations. The amplitude of free oscillations decreases with increasing depth according to the exponential law. Friction of the liquid causes the appearance of attenuated oscillations and limits the amplitudes of forced oscillations in the resonance zones.

The assumption that all types of linear oscillations of a liquid can be analyzed within the framework of a single approach based on the search for frequencies and shapes of natural (partial) oscillations is introduced. The complete group of rigid shell movements includes 3 displacements and 3 rotations. Deformations and elastic connections of shell oscillations are not taken into account since their frequency is much more than 100 Hz. With vertical oscillations, as indicated above, fluid overflow does not occur, and the oscillation frequency is determined by compression of the liquid. The eigenfrequency is approximately 500 Hz, so we do not take it into account (after all, it is necessary to take into account together with the deformation of the shell). The remaining 5 oscillations are divided into groups: oscillations in the *zOy* plane (displacement and rotation); oscillations in the *xOy* plane (displacement and rotation); rotation around the *Oy* axis. For the first two groups, the velocity vector field is flat, and for the third group – spatial.

4. 2. Physical model of fluid movement in a closed container

The calculation of the dynamic characteristics of an object with a mobile mass of viscous incompressible liquid in a tank today is carried out by using modern mathematical models that describe with exhaustive accuracy the oscillations of the free surface of the liquid. This situation leads to complication of design algorithms, in particular when taking into account the movement of the tank as part of the vehicle. Therefore, in the problems of studying the dynamics of the movement of tanks as part of a vehicle, it is advisable to use a combined model of the classical type, which combines the Navier-Stokes equations with the Rayleigh surface wave equations.

The formation of a physical model of fluid motion in a closed tank (Fig. 1) is based on the known Navier-Stokes equation, which takes into account internal friction in proportion to viscosity, supplementing it with taking into account the conditional force, which is associated with external friction of the fluid on the side walls of the tank. In turn, this conditional force will be transmitted to the inner parts of the tank through the partial layers of the liquid moving along with the walls:

$$\rho\frac{\partial\vec{V}}{\partial t}=-\text{grad}\,p-\rho\cdot\upsilon\cdot\text{rot}\,\text{rot}\,\vec{V}-f\cdot\left(\vec{V}-\vec{V}_{st}\right), \quad (1)$$

where $\vec{V}=\{Vx,Vy,Vz\}$, p, ρ – velocity, pressure, and density of the liquid; υ – kinematic viscosity of the liquid; t – time; $\vec{V}_{st}$ – tangential velocity of the side wall (it lies in the plane of the flat core of the flow); f – coefficient of friction, which depends on the shape and roughness of the flowing surface, as well as on the average level of tangential velocities (which makes the problem of modeling forced nonstationary oscillations of a liquid nonlinear).

Using Helmholtz's theorem, the velocity vector of a fluid decomposes into the scalar and vector potentials of the Laplace field. With the assumption that the Laplace component of the field is zero, the potential and vortex components of the fluid oscillation field are obtained:

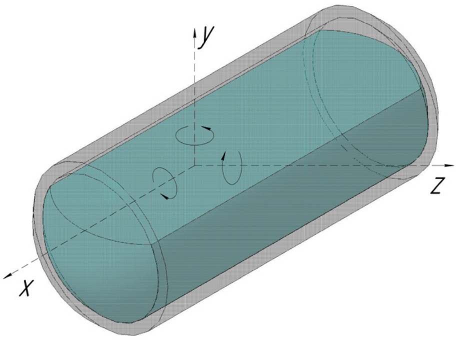

Fig. 1. Equivalent scheme of a liquid transport tank

$$\vec{V}=\text{grad}\,\varphi+\text{rot}\,\vec{b}=$$
$$=\text{grad}\left[\rho\frac{\partial\varphi}{\partial t}+p+f\cdot\left(\varphi-\varphi_{st}\right)\right]+$$
$$+\text{rot}\left[\rho\frac{\partial\vec{b}}{\partial t}+\rho\cdot\upsilon\,\text{rot}\,\text{rot}\,\vec{b}+f\cdot\left(\vec{b}-\vec{b}_{st}\right)\right], \quad (2)$$

where φ is the scalar potential, which, as a consequence of fluid incompressibility, is a harmonic function, i.e., corresponds to the Laplace equation div grad φ=0; $\vec{b}$ is a vector potential that, for a plane field perpendicular to vector $\vec{n}\in\{\vec{i},\vec{j},\vec{k}\}$, is determined by the corresponding scalar function.

Considering the potential problem for translational motion of a fluid in which there is no vortex component of the field, the potential of the fluid velocity is replaced by the scalar potential F of fluid displacement in surface Rayleigh waves, and also taking into account the average curvature of the surface (Gaussian formula):

$$p=\rho\cdot g\cdot\Delta y_p-\sigma\cdot\left[\frac{\partial^2\Delta y_p}{\partial x^2}+\frac{\partial^2\Delta y_p}{\partial z^2}\right]; \quad (3)$$

$$\Delta y_p=\frac{\partial F}{\partial y};\;\; F=\int\left(\varphi-\varphi_{st}\right)\text{d}t, \quad (4)$$

where Δy_n – local rise of the free surface from its initial position; σ – surface tension coefficient.

As a result, for the function F, which is the scalar potential of fluid displacement, the following boundary value problem was obtained:

$$\begin{cases}\vec{V}=\left\{\dfrac{\partial\dot{F}}{\partial x};\dfrac{\partial\dot{F}}{\partial y};\dfrac{\partial\dot{F}}{\partial z}\right\};\;\nabla^2F=0;\;\left.\dfrac{\partial F}{\partial\vec{n}}\right|_{G_0}=0;\\ \ddot{F}+f\cdot\dot{F}+g\cdot\dfrac{\partial F}{\partial y}-\dfrac{\sigma}{\rho}\cdot\left.\left[\dfrac{\partial^3F}{\partial y\partial x^2}+\dfrac{\partial^3F}{\partial y\partial z^2}\right]\right|_{G}=\ddot{F}_{st},\end{cases} \quad (5)$$

where $\left.\frac{\partial F}{\partial\vec{n}}\right|_{G_0}$ is the derivative in the direction of the normal to the wetted surface, and the field $\vec{V}$ corresponds to the velocities of the liquid relative to the shell.

The vortex component of the field of oscillations of the liquid is observed only in the near-wall layer of the liquid; it is necessary to take into account the vortex rise of the liquid, which causes a change in the vortex velocities of the wall in the direction of the longitudinal axis. Thus, for the vortex velocity field $\vec{V}_{ex}$ of the wall around the Ox axis:

$$\vec{V}_{st.ex}=\{\Omega_{st.x}\cdot y;-\Omega_{st.x}\cdot z\}=$$
$$=\Omega_{st.x}\cdot\{y;\,z\}+\Omega_{st.x}\cdot\{0;-2z\}. \quad (6)$$

In the right-hand side (6), the first component has a scalar harmonic potential $\Omega_{st.x}\cdot\{y;z\}$=grad($\Omega_{st.x}\cdot z\cdot y$), and the second component activates an uneven oscillatory displacement of the fluid along the Oz axis; this displacement does not lead to a vortex rise of the free surface.

If the shell of the tank rotates around the Ox axis with an angular velocity of $\Omega_{st.x}(t)$ and shifts along the Oz axis with a linear velocity $Vz_{st}(t)$, then this motion of the shell is described by the potential $\varphi_{st}(t)=Vz_{st.}(t)\cdot z+\Omega_{st.x}(t)\cdot z\cdot y$.

In view of this, the scalar potential of fluid displacement (5) changes:

– plane zOy:

$$\begin{cases} \vec{V} = \left\{ \dfrac{\partial \dot{F}}{\partial z}; \dfrac{\partial \dot{F}}{\partial y} \right\}; \ \nabla^2 F = 0; \ \left. \dfrac{\partial F}{\partial \vec{n}} \right|_{G_0} = 0; \\ \ddot{F} + f \cdot \dot{F} + g \cdot \dfrac{\partial F}{\partial y} + \dfrac{\sigma}{\rho} \cdot \left[\dfrac{\partial^3 F}{\partial y^3} \right]_G = \\ = -\left[a_{st.z}(t) + g \cdot \theta_{st.x}(t) \right] \cdot z, \end{cases} \qquad (7)$$

where $\theta_{st.x}(t) = \int \Omega_{st.x} \, dt$ is is the angle of torsion of the shell.

– plane xOy:

$$\begin{cases} \vec{V} = \left\{ \dfrac{\partial \dot{F}}{\partial z}; \dfrac{\partial \dot{F}}{\partial y} \right\}; \ \nabla^2 F = 0; \ \left. \dfrac{\partial F}{\partial \vec{n}} \right|_{G_0} = 0; \\ \ddot{F} + f \cdot \dot{F} + g \cdot \dfrac{\partial F}{\partial y} + \dfrac{\sigma}{\rho} \cdot \left[\dfrac{\partial^3 F}{\partial y^3} \right]_G = \\ = -\left[a_{st.x}(t) + g \cdot \theta_{st.z}(t) \right] \cdot x. \end{cases} \qquad (8)$$

– plane xOz:

$$\begin{cases} \vec{V} = \left\{ \dfrac{\partial \dot{F}}{\partial x}; \dfrac{\partial \dot{F}}{\partial y}; \dfrac{\partial \dot{F}}{\partial z} \right\}; \ \nabla^2 F = 0; \ \left. \dfrac{\partial F}{\partial \vec{n}} \right|_{G_0} = 0; \\ \ddot{F} + f \cdot \dot{F} + g \cdot \dfrac{\partial F}{\partial y} + \dfrac{\sigma}{\rho} \cdot \left[\dfrac{\partial^3 F}{\partial y^3} \right]_G = \dot{\Omega}_{st.y} \cdot z \cdot y. \end{cases} \qquad (9)$$

The excitatory factor in equation (9), which accompanies the oscillations of the fluid, is angular acceleration.

5. Results of investigating fluid movement in a partially filled container

5. 1. Procedure for determining low-frequency oscillations of a viscous liquid in a horizontal container with a free surface

A procedure for determining fluid oscillations primarily depends on the shape of the container. Since most tanks have a cylindrical shape, the use of mathematical analysis methods leads to the complication of design algorithms. It is proposed to perceive the cylindrical shape of the tank according to the following conditions: at $H \leq R$, the shape of a rectangle (Fig. 2, *a*), and, at $H > R$, a trapezoid (Fig. 2, *b*).

In view of this simplification of the shape and depending on the H level of the liquid in the tank, new attributes of parallelepipeds are calculated:

$$l = \sqrt{H \cdot (2R - H)};$$

$$S = \begin{cases} R^2 \arcsin\left(\dfrac{l}{R}\right) - l \cdot (R - H), \ H \leq R; \\ R^2 \left[\pi - \arcsin\left(\dfrac{l}{R}\right)\right] - l \cdot (R - H); \end{cases}$$

$$h = \begin{cases} \dfrac{S}{2l}, \ \ H \leq R; \\ \dfrac{S}{l + R}. \end{cases}$$

It should be noted that fluid oscillations in the tank are described by partial differential equations, that is, the Rayleigh surface wave model at the initial level is continuous. This is not entirely appropriate because the further construction of mathematical models of the "tank-vehicle" system is performed in a discrete way. But, as the study showed, the solutions to the continuum model can be interpreted as a solution to the system of ordinary differential equations for the so-called partial oscillators. Each partial oscillator is a conditional layer of liquid that shifts along the free surface and interacts with the shell by pressing on the side wall and bottom of the tank. To simplify the modeling process, it is permissible to consider a conditional layer of liquid as present, giving it a specific size, weight, and place in the tank.

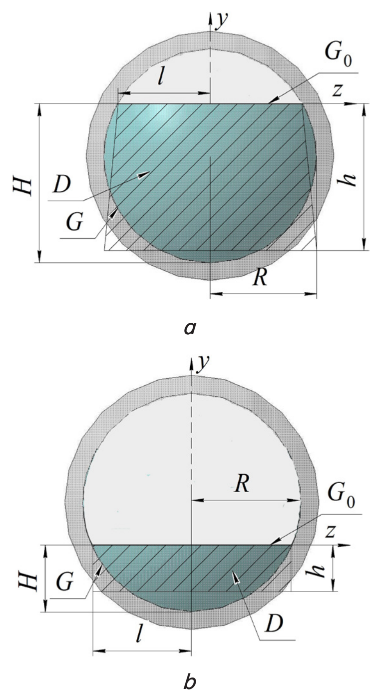

Fig. 2. Simplifying the shape of the tank: *a* – at $H \leq R$; *b* – $H > R$

The subsequent calculation of scalar potentials (7) to (9) is carried out using the classical method of mathematical analysis, namely the Fourier method of distribution of variables. The resulting potential in the form of a similar Fourier series is compared with the potential (7) to (9). As a result, an infinite system of ordinary differential equations of the second order with constant coefficients is derived:

– plane zOy:

$$F(t, z) = \sum_{k=0}^{\infty} T_m(t) \cdot (-1)^m \sin(\beta_m \cdot z) \cdot \mathrm{ch}(\beta_m \cdot y); \qquad (10)$$

$$\ddot{T}_m + f \cdot \dot{T}_m + \beta_m \cdot \left(g + \left(\dfrac{\sigma}{\rho}\right) \cdot \beta_m^{\ 2} \right) \cdot \mathrm{th}(\beta_m h) \cdot T_m = $$
$$= \dfrac{2}{l} \cdot \dfrac{\left(-a_{st.z}(t) - g \cdot \theta_{st.x}(t)\right)}{\beta_m^{\ 2} \cdot \mathrm{ch}(\beta_m h)}, \ m = 1, 2, \ldots ; \qquad (11)$$

$$\beta_m = \dfrac{\pi \cdot (2m - 1)}{2l}, \quad m = 1, 2, \ldots;$$

– plane xOy:

$$F(t, x, y) = \sum_{k=0}^{\infty} T_k(t) \cdot (-1)^k \sin(\lambda_k \cdot x) \cdot \mathrm{ch}(\lambda_k \cdot y); \qquad (12)$$

$$\ddot{T}_k + f\cdot\dot{T}_k + \lambda_k\cdot\left(g+\left(\frac{\sigma}{\rho}\right)\cdot\lambda_k^2\right)\cdot \mathrm{th}(\lambda_k h)\cdot T_k = \\ =\frac{2}{L}\cdot\frac{\left(-a_{st.x}(t)-g\cdot\theta_{st.z}(t)\right)}{\lambda_k^{2}\cdot \mathrm{ch}(\lambda_k h)},\ k=1,2,...; \qquad (13)$$

$$\lambda_k=\frac{\pi\cdot(2k-1)}{2L},\quad k=1,2,...;$$

– plane xOz:

$$F(t,x,z,y)=\sum_{m,k=0}^{\infty}\begin{matrix} T_{m,k}(t)\cdot(-1)^m\sin(\beta_m\cdot z)\times \\ \times(-1)^k\sin(\lambda_k\cdot x)\cdot \mathrm{ch}(\gamma_{m,k}\cdot y)\end{matrix}; \qquad (14)$$

$$\ddot{T}_{m,k} + f\cdot\dot{T}_{m,k} + \gamma_{m,k}\cdot\left(g+\frac{\sigma}{\rho}\cdot\gamma_{m,k}^2\right)\cdot \mathrm{th}(\gamma_{m,k} h)\cdot T_{m,k} = \\ =\frac{4}{l\cdot L}\cdot\frac{\left(-g\cdot\theta_{st.y}(t)\right)}{\beta_m^{2}\lambda_k^{2}\cdot \mathrm{ch}(\gamma_{m,k} h)}; \qquad (15)$$

$$\gamma_{m,k}=\sqrt{\beta_m^{2}+\lambda_k^{2}}=\pi\cdot\sqrt{\frac{(2m-1)^2}{4l^2}+\frac{(2k-1)^2}{4L^2}},\quad k,m=1,2,...$$

where λ_k, β_k and γ_k are wavenumbers; l and L – half width and half length of the tank; $T(t)$ – variable amplitude coefficient function.

Based on equations (11), (13), (15), it is possible to distinguish the natural frequencies of oscillation of layers of moving fluid. Each term from the sum (10), (12), (14) corresponds to a separate basis (eigen) shape of low-frequency oscillations of the liquid relative to the shell of the tank, that is, to some partial oscillator. For each oscillator, its mass fractions δM can be determined of the total mass of the liquid. To this end, it is enough to take into account the natural shape of the oscillator and find the average value of the derivative $\partial F/\partial\xi$ over the cross-sectional area D:

$$\frac{1}{S}\iint_D\frac{\partial F}{\partial \xi}\mathrm{d}S,\quad \xi=\{x,y,z\}.$$

Further, this result is multiplied by the Fourier coefficient (the relative component of external influence from equations (11), (13), (15)):

$$\begin{cases} zOy:\dfrac{2}{l\cdot\beta_m^{2}\mathrm{ch}(\beta_m h)}\Rightarrow\delta M_m=\dfrac{2}{\pi^2}\displaystyle\sum_{j=1}^{m}\dfrac{\mathrm{th}(\beta_j h)}{\beta_j^{3}h}; \\ xOy:\dfrac{2}{L\cdot\lambda_k^{2}\mathrm{ch}(\lambda_k h)}\Rightarrow\delta M_k=\dfrac{2}{\pi^2}\displaystyle\sum_{j=1}^{k}\dfrac{\mathrm{th}(\lambda_j h)}{\lambda_j^{3}h}; \\ xOz:\dfrac{4}{l\cdot L\cdot\beta_m^{2}\lambda_k^{2}\mathrm{ch}(\gamma_{m,k} h)}\Rightarrow\delta M_{m,k}= \\ =\dfrac{4}{\pi^4}\displaystyle\sum_{j=1}^{m}\sum_{i=1}^{k}\dfrac{\mathrm{th}(\gamma_{j,i}h)}{\beta_j^{2}\lambda_i^{2}(\gamma_{j,i}h)}. \end{cases}$$

Fig. 3 shows the eigenforms of potentials of low-frequency oscillations of liquid in three planes. It is noticeable that the natural shapes of oscillations change rapidly in all planes of motion. When deepened into a tank, the intensity of oscillations decreases, and for the second oscillator at the bottom it becomes barely noticeable. The direction of movement of the fluid is perpendicular to the level line, and the intensity of movement is inversely proportional to the distance between adjacent level lines. The main motion of the fluid is carried out by the first oscillator, its movement is the main one, other oscillators perform the corrective movement of the liquid – this is confirmed by establishing the total relative mass of the oscillators (Fig. 4).

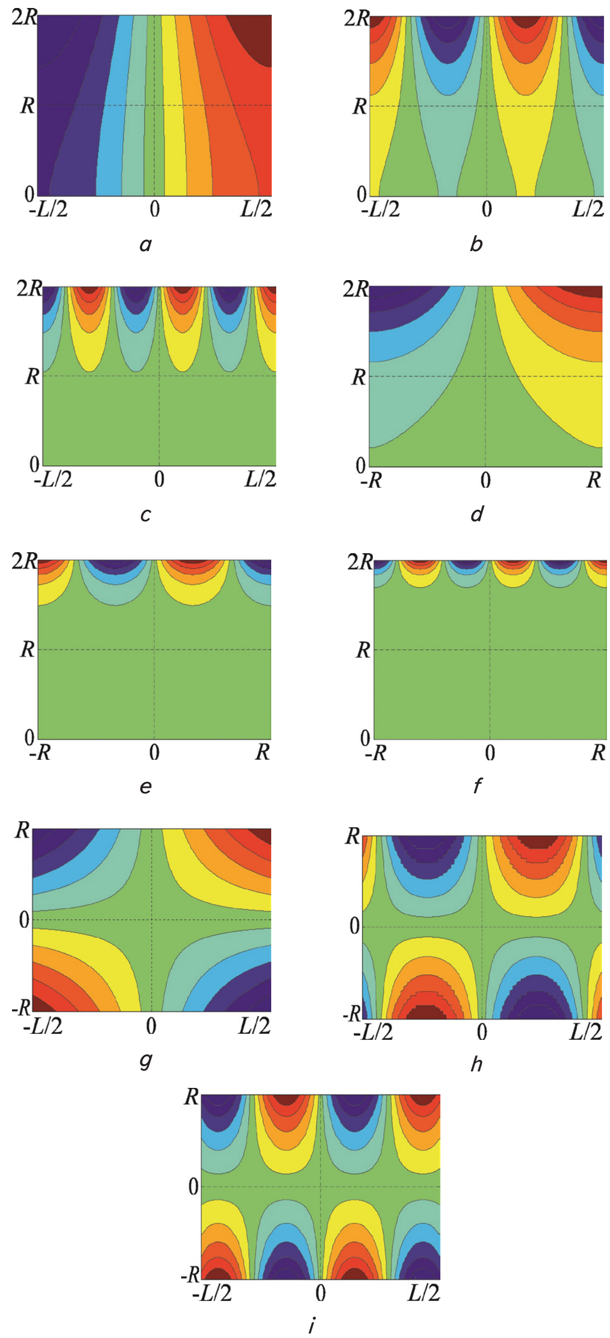

Fig. 3. Characteristics of eigenforms levels of low-frequency oscillation potentials: *a*–*c* – xOy plane; *d*–*f* – zOy plane; *g*–*i* – plane xOz; *a, d, g* – for oscillator 1; *b, e, h* – for oscillator 2; *c, f, i* – for oscillator 3

Based on the above procedure for modeling low-frequency oscillations of the free surface of a liquid, it becomes possible to determine the elastic-mass characteristics of fluid movement during transportation of a tractor tank.

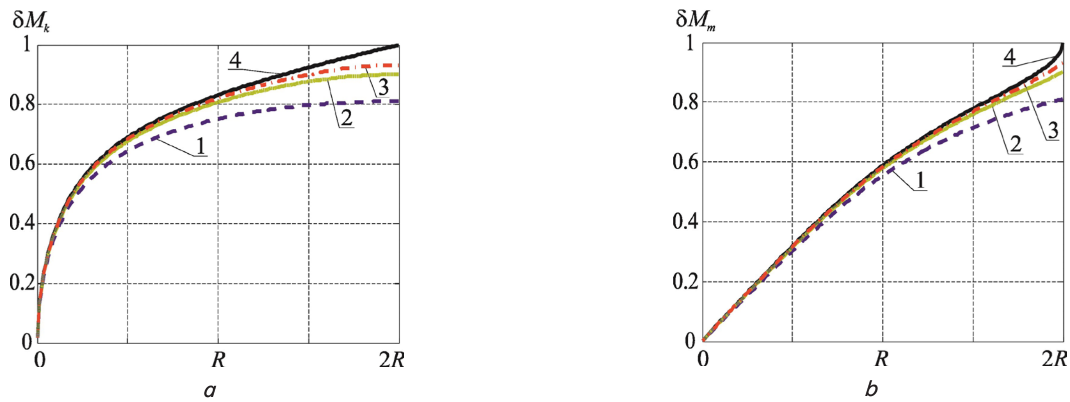

Fig. 4. The total relative mass of oscillators for the corresponding plane of oscillations depending on the liquid level: *a* – plane *xOy*; *b* – plane *zOy*; 1 – for 1 oscillator; 2 – for 2 oscillators; 3 – for 3 oscillators; 4 – for 100 oscillators

5. 2. Determination of splashing characteristics in partially filled agricultural tanks

Due to the above simplified methodology, it is possible to establish the range of change of longitudinal and transverse modes of dimensionless natural frequencies $\left(\upsilon_{k,m}=\omega_{k,m}^2 R/g\right)$ depending on the depth of filling the tractor tank (Fig. 5). From work [12] it is known that for tractor tanks the ratio of the length of the tank to its radius is within L/R=5÷8 and corresponds to a carrying capacity of 8÷20 tons.

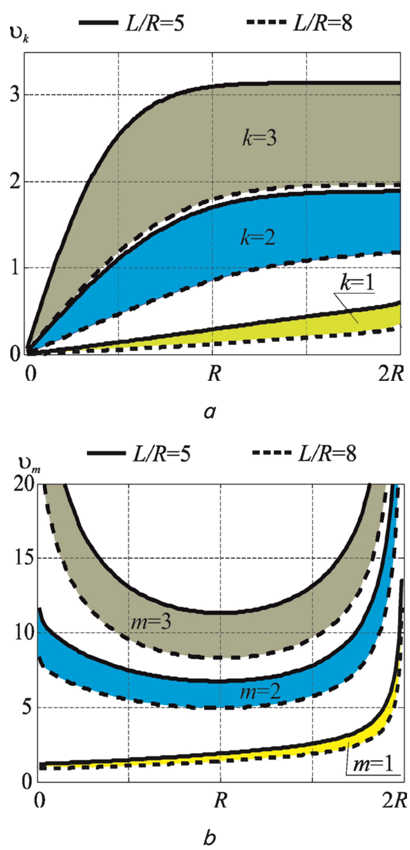

Fig. 5. Dependence of dimensionless natural frequencies on the filling depth of the tank: *a* – plane *xOy*; *b* – *zOy* plane

Fig. 5, *a* demonstrates that the increase in the natural frequencies of the second and third fluid layers exaggerate the frequency of the first approximately by $\sqrt{3}$ and $\sqrt{5}$, respectively. Analyzing the transverse natural frequencies (Fig. 5, *b*), we note the progressively increasing rate of increase in values under conditions of minimum and maximum filling of the tank with liquid.

5. 3. Comparison of the results obtained at different levels of fullness of a cylindrical container

In order to test the improved procedure for determining the characteristics of fluid movement in a horizontal tank, a comparison is made with the achievements of other scientists [5–8, 12]. In those works, a thorough study of the influence of the ratio of the length of the tank to its radius (L/R) and filling depth is carried out; the results (Fig. 6, 7) are represented in the form of dimensionless frequency dependences.

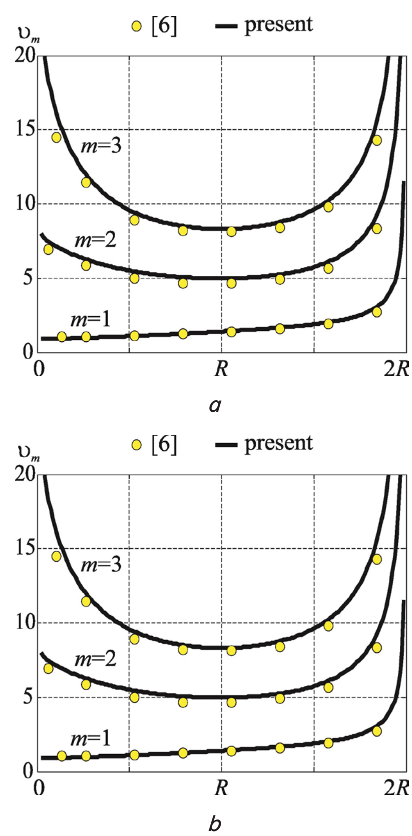

Fig. 6. Dependence of dimensionless natural frequencies on the filling depth of the tank (L/R=6): *a* – plane *zOy*; *b* – plane *xOz*

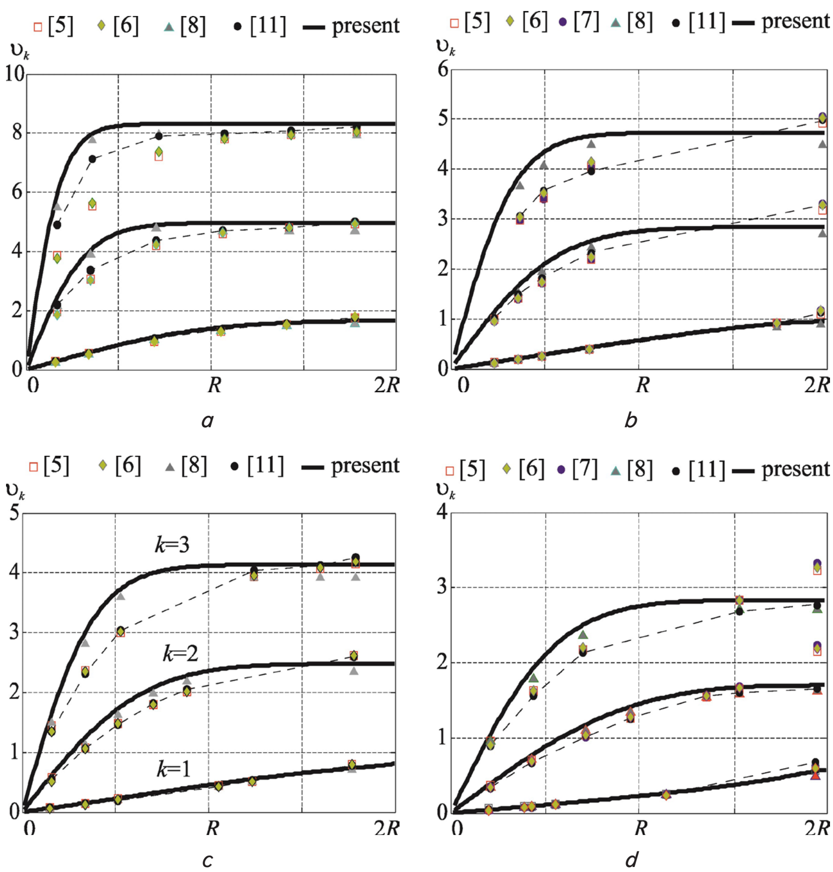

Fig. 7. Dependence of dimensionless natural frequencies in the longitudinal plane xOy on the filling depth of the tank:
a – L/R=2; b – L/R=3.5; c – L/R=4; d – L/R=5.8

The result in Fig. 7 proves that with decreasing value of the L/R ratio, the magnitude of the natural frequencies of oscillation of liquid layers increases.

6. Discussion of results of investigating the introduction of a complex shape when replacing the real shape of the container during the splashing of liquid

The formulated mathematical apparatus is constructed in a combined way from classical numerical methods. The results of elastic-mass characteristics of the movement of liquid layers, which are components of the free surface of the liquid with partial filling of a closed tank, are given.

From the dependences in Fig. 4, a sufficient condition has been established for determining adequate results of general calculations of the influence of low-frequency oscillations of a liquid that implies taking into account three partial oscillators. The first oscillator takes into account ~80 % of the mass of the oscillating liquid, the second – ~10 %, the third – ~3 %. In studies of high-frequency oscillations of fluid, it is necessary to increase the number of oscillators under consideration.

Fig. 6 shows the results of modeling the change in natural frequencies of layers (oscillators) from the depth of filling the tank in the planes zOy and xOz, which have sufficient consistency with the results of work [6]. Note that the use of the procedure of equivalent replacement of the real shape of the container with prismatic involves a drawback because it does not take into account the curvature of the walls of the tank (the effect of "fine water").

Performing a visual comparison of the results obtained by other scientists, it should be noted that the convergence with works [5–8, 12] for the first fluid oscillator is quite high. When considering the second and third layers of liquid, which are not determinative, their results differ in the filling range R/2÷3R/4. The current results are close to the results reported in [8] in which the model of motion of the free surface of a liquid is based on a similar procedure of equivalent replacement of the real shape of the container with a rectangular one. Comparing the results, we note that for the tank L/R=5.8, the current procedure is close to the results in [8, 12] when filling the last quarter of the tank for all oscillators studied. It should be noted that the results of work [12] are based on experimental studies.

The improved procedure has practical value since it make it possibles to comprehensively calculate the characteristics of movement under difficult operating conditions of a vehicle with tanks without using complex design approaches.

The use of the above procedure has limitations and cannot be used in the study of the movement of a vehicle with tanks in the course of transport and technological operations. In the study of such operations, it is necessary to take into account the rate of release of fluid from the tank because it is the main factor in the attenuation of fluid oscillations. The disadvantage of the study is that the approbation of theoretical and experimental studies does not cover the full range of sizes of agricultural tanks.

The prospect of further development of this study is to take into account the operation of the hydraulic mixer. Since this study is more aimed at solving the problems of transporting partially filled agricultural tanks, a mixer is usually introduced in their design. Taking into account this option during operation causes the formation of an additional excitatory effect on the movement of the free surface.

7. Conclusions

1. An approach to improving the methodology for determining low-frequency oscillations of a viscous liquid in a horizontal container with a free surface is proposed. The above approach is based on the use of boundary integral equations associated with the Helmholtz equations. This concept made it possible to consider the oscillatory motion of a liquid as non-vortex (except for the wall layer) and to use the method of partial oscillators, which made it possible to replace the continuum model of the liquid with a discrete one (with concentrated masses). The difference between the outlined method and the existing ones is the equivalent replacement of the real shape of the tank with the shape of a parallelepiped and, depending on the level of filling with liquid, this shape is a rectangle at H R and a trapezoid at $H>R$. The use of surface waves with vertical polarization under conditions of formation of a displacement of a liquid medium

is due to the displacement of particles of the medium parallel to the velocity vector. It was this characteristic that made it possible to get rid of integro-differential equations in the formation of a mathematical apparatus in describing low-frequency oscillations of a liquid.

2. The range of natural frequencies of the main layers of liquid involved in splashing of the free surface of the liquid in agricultural tanks with size L/R=5÷8 has been established. In longitudinal motion, the range of change in dimensionless natural frequency is:

$$\upsilon_{k=1}^{H=R} = 0.117 \div 0.287 \text{ and } \upsilon_{k=1}^{H=2R} = 0.312 \div 0.605;$$

$$\upsilon_{k=2}^{H=R} = 0.858 \div 1.700 \text{ and } \upsilon_{k=2}^{H=2R} = 1.175 \div 1.885;$$

$$\upsilon_{k=3}^{H=R} = 1.792 \div 3.099 \text{ and } \upsilon_{k=1}^{H=2R} = 1.964 \div 3.144.$$

With transverse motion, the range of change:

$$\upsilon_{m=1}^{H=R} = 1.395 \div 1.894; \ \upsilon_{m=2}^{H=R} = 4.964 \div 6.746;$$

$$\upsilon_{k=3}^{H=R} = 1.792 \div 3.099.$$

3. Based on a comparison of the results obtained by other scientists, a sufficiently high convergence of natural frequency calculations of partial oscillators in 3D space was found. Thus, with longitudinal motion, the maximum divergence for the first oscillator is up to 6.7 %; for the second – 9.4 %; for the third – 16.2 %. As regards the transverse and rotational planes, the minimum error is observed at $H=R$: for the first oscillator – up to 4.0 %, for the second – 6.5 %; for the third – 10.1 %. The use of the above procedure under conditions of minimum ($H\approx0$) and maximum ($H\approx2R$) filling in the transverse and rotating planes is not acceptable because the curvature of the tank walls is not taken into account, which leads to obtaining the maximum possible discrepancy.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

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Data availability

All data are available in the main text of the manuscript.

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