

*With technology innovation and market complexity, this paper examines a dual-channel supply chain with one manufacturer and one retailer. A manufacturer performs the encroachment by selling products through offline retail and direct online channels. It also provides an extended warranty service with the product sold in both channels. This study identifies the best pricing and extended warranty strategies as decision variables to maximize profits for the dual-channel supply chain under the demand substitution effect. Despite the centralized scenario, a decision-making model in the decentralized scenario is also developed using Stackelberg's game to determine optimal decision variables. The results demonstrate that the centralized scenario consistently outperforms the decentralized scenario. Thus, coordination contracts are proposed considering the profit structure, revenue-sharing and cost-sharing contracts to coordinate and promote a win-win situation for supply chain members. Furthermore, demand substitution effects are investigated for channel parameters represented in the initial market size, price sensitivity coefficient, and warranty length sensitivity coefficient on optimal decision variables. From the sensitivity analysis, these parameters have a different substitution effect on the optimal decision in each channel. The price and extended warranty length offered in a dual channel positively correlate with the market size and warranty length sensitivity coefficient of the corresponding channel. However, they negatively correlate with the price sensitivity coefficient. Accordingly, a similar situation occurs for the profit of each channel. Last, the discussion supports the theoretical results and generates managerial insights for practical decision-making*

**Keywords:** coordination mechanism, game theory, pricing decision, supply chain design

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# COORDINATING A DUAL-CHANNEL SUPPLY CHAIN WITH PRICING AND EXTENDED WARRANTY STRATEGIES UNDER DEMAND SUBSTITUTION EFFECTS

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## 1. Introduction

Rapid technological advancement and innovation have resulted in a substantial transformation of market and business interactions in current operations and management. Moreover, the Covid-19 pandemic has altered the business environment, where the focus has shifted from «physical first» to «digital first» and from «selling» to «serving» customers [1]. The manufacturer captures this as an opportunity to enlarge the encroachment practice, which is omnipresent with various innovations, such as a dual-channel supply chain. In a dual channel, a manufacturer typically separates its marketing efforts from retail offline channels and engages customers directly through an online channel [2]. The online channel presence is significant because it entices numerous customers from offline toward online channels and presents sales channels to serve more markets. In addition, this condition also affects the operational decisions of the manufacturer, particularly in the pricing strategy [3, 4]. Thus, manufacturers attempt to redesign their traditional channel systems by constructing a dual channel.

Despite the above situation, the responsibility of business players in the online transaction process and product quality

and reliability has provoked concern in online shopping. New products introduced at a high level of technology also become more complex and personalized, so customers increasingly consider product warranties and after-sales services [5]. Generally speaking, there are two kinds of warranties, base and extended warranties [6]. Although a base warranty is typically bundled with the product sold, more customers are considering purchasing extended warranty services recently [7]. Nevertheless, it mostly becomes the manufacturer's responsibility to repair or maintain when product failure occurs. With manufacturing encroachment through online channels, the manufacturer might offer extended warranties directly to customers, and customers also can reach the manufacturer to solve their issues [8]. As an illustration, Apple Inc. offers an extended program, «Apple Care+», allowing customers to repair their products beyond the base warranty. Performing the proposed trend of a warranty system by introducing an extended warranty service will encourage the manufacturer's online channel to become successful, highly rated, and trusted by customers.

While the dual-channel innovation and direct warranty provision empower the manufacturer, they create fierce new competition in supply chain sales channels. Furthermore,

demand substitution might occur when customers shift from one channel to another due to price, quality, or other factors [4]. As a result, it significantly affects the emergence of channel conflicts, changes in demand, and pricing strategies in a dual-channel system. In order to overcome these potentials and remain competitive in the market, all channel members should strategically coordinate their pricing and management decisions [9].

Based on these observations, the emergence of a dual channel is noteworthy since customers can eventually select between traditional retail and online direct channels. Some customers will efficiently switch to other channels when one of the channels runs out of stock. However, several gaps still need further analysis in coordinating the dual-channel supply chain with pricing and extended warranty strategies under the demand substitution effect. Therefore, a study on developing an appropriate design for pricing and extended warranty strategies is relevant to present technological advances for business resilience and competitive success in the market.

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## 2. Literature review and problem statement

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As market conditions and technology transform rapidly, a dual-channel supply chain has become a concern in operations and industrial management today. In such a situation, despite selling through a retailer, several manufacturers have initiated selling their products through direct sales channels. Thus, pricing strategy as one of the operational decisions is a major research issue still being developed. Paper [3] constructed a dual-channel supply chain model where the manufacturer sells the product thru a direct channel and multiple retail channels. They identified that the market share strongly impacts the optimal pricing strategy and effort level. Furthermore, the article [10] analyzed two platform innovation strategies adopted by the manufacturer and e-retailer. They designed scenarios where the manufacturer can utilize a platform to market products directly by paying fees. The results summarized that the manufacturer has a higher profit for platform innovation than the e-retailer.

Developing a dual-channel system has generated further competition in the current supply chain. Regarding channel management, scholars have conducted several analyses from the perspective of the manufacturer and retailer on channel decision-making and coordination to minimize the competition effect. They typically adopt mathematical models and game theory to propose a feasible coordination contract, achieve a win-win situation, such as wholesale price, buyback contract, and revenue, and cost sharing contracts [11–14]. Work [12] studied the decision-making and coordination under customer preferences for a dual-channel supply chain consisting of a manufacturer and an e-commerce platform. By using a game theory approach, models were presented in the decentralized and centralized decision scenarios to determine maximum profits. In a more complicated setting, the study [15] developed the pricing strategy in the decentralized and centralized models by considering customer value and value-added service in a dual-channel supply chain with a manufacturer and an online retail platform. The analysis demonstrated that profits and customer value under the centralized scenario are higher than in the decentralized scenario.

In light of the aforementioned research, channel management and decision-making methods could significantly improve the profit and business processes of the whole supply chain. Nevertheless, the intense competition and increasingly complex consumer demands have directed the elaboration of

other supply chain studies related to after-sale services to be studied further. Paper [5] considered manufacturer warranty services in a dual-channel supply chain for demand-enhancing services. The warranty service distributed by the retailer supports a win-win situation for the manufacturer and retailer with large initial market size.

In a highlight development related to warranty, the extended warranty program has achieved economic system recognition and has significantly progressed. Hence, several scholars working on operations and management research have broadened the study of extended warranties under the dual-channel model. Work [7] designed extended warranty strategies in a dual supply channel with a single manufacturer and a retailer under customer preferences. The findings indicated that the price reduces as the warranty is extended and no matter who offers the extended warranty, the supply chain will make significant profits. Afterwards, the article [8] determined the optimal strategy of the extended warranty services provider and the impact on customers in the dual-channel supply chain. They suggested that if the probability of product failure and extended warranty coverage are high, the manufacturer should offer the extended warranty service to gain more profit. In another interesting work, the study [9] examined the coordination decision-making model of e-commerce supply chains where the e-commerce platform offers an extended warranty. The results revealed that the centralized model offers the best pricing and low selling price.

In fact, most scholars have already discussed operating an online channel as a direct operations strategy by a manufacturer in the supply chain for marketing and sales. However, only a few scholars have considered utilizing the manufacturer's direct channel using an online platform as an after-sales service channel. Paper [16] presented the manufacturer's strategy by offering extended warranties for the online shopping supply chain. They stated that the manufacturer's pricing decision is critical in determining whether it is profitable to offer an extended warranty.

To the best of our knowledge, current studies in this area focus either on pricing strategies or extended warranty marketing in a dual-channel supply chain without concerning the potential demand substitution effect, particularly on the performance of each channel. Moreover, the coordination issue and the joint effect of pricing and extended warranty strategies on dual-channel performance have never been explored. Therefore, exploring the pricing and extended warranty strategies under the demand substitution effect is necessary to fill this gap. All this allows to argue that a study on designing a dual-channel supply chain coordination contract is appropriate and relevant. Considering the price and warranty length as decision variables, this study enriches and contributes to the study of operations management and dual channels.

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## 3. The aim and objectives of the study

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The aim of the study is to develop a game theory model of a dual-channel supply chain for determining optimal price and extended warranty length. These mechanisms allow each supply chain member in the whole system to maximize total profit and achieve a win-win situation.

To accomplish the aim, the following objectives have been set:

- to develop decision-making scenarios, centralized  $c$  and decentralized  $d$ , for determining the optimal pricing and extended warranty decisions under the demand substitution effect;

– to derive analytical propositions of decision variables for investigating the joint impact of pricing and extended warranty decisions on the profit of the supply chain system and each channel;

– to propose coordination contracts in revenue and cost-sharing contracts for coordinating supply chain members in each channel under the demand substitution effect and achieving a win-win situation;

– to perform numerical and sensitivity analyses in exploring the effect of demand substitution parameters on performance, pricing, and extended warranty decisions for each channel in a dual-channel supply chain.

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#### 4. Material and methods

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This paper examines a dual-channel supply chain that includes a manufacturer's retail and direct channels, which will be referred to as offline and online channels, respectively. In offline channel  $r$ , manufacturer  $M$  markets products to retailer  $R$ , then retailer  $R$  sells to end customers. Meanwhile, in online channel  $m$ , manufacturer  $M$  directly sells products to end customers via its online platform. Following the Stackelberg game, manufacturer  $M$  becomes the game's leader, whereas retailer  $R$  becomes the follower [11, 17]. The manufacturer  $M$  provides a wholesale price  $w_r$  to retailer  $R$  and an extended warranty length  $t_r$  to customers in offline channel  $r$ . Afterward, retailer  $R$  offers a retail price  $p_r$  to customers. By adopting technology innovation in an online platform, manufacturer  $M$  provides customers a selling price  $p_m$  and an extended warranty length  $t_m$  in online channel  $m$ . Assuming that manufacturer  $M$  produces a product at a unit cost and sells via retailer  $c_r$  and its online channel  $c_m$ , the manufacturer's wholesale price  $w_r$  is specified [18]. Therefore, the retailer's price  $p_r$  in offline channel  $r$  is its profit margin [16].

When customers purchase an extended warranty, period  $t$  is the aggregate of base and extended warranty periods [6]. As this study focuses on extended warranty length effects, the base warranty provided is constant and normalized to zero, which was also applied by papers [7] and [19]. Manufacturer  $M$  provides an extended warranty bundle for products sold on both channels, where the manufacturer incurs warranty cost  $c_w$  for defective products. Accordingly, the extended warranty sold is supposed to be equivalent to the product sold. The number of product failures through warranty length  $t$  is defined as  $N(t)$  [20]. It is supposed to be distributed with a cumulative distribution function  $N(t)=\lambda t^\beta$ , where  $\lambda$  is the failure rate,  $t$  is the extended warranty length, and  $\beta$  is the shape parameter in Weibull distribution. Then the extended warranty cost increases exponentially over time and can be formulated as  $c_w[N(t)]=c_w(\lambda t^\beta)$  [8]. To show channel differences, online channel  $m$ , as a newly developed market, has a lower initial market size than offline channel  $r$ ,  $\alpha_m < \alpha_r$ . All notations for this study are summarized as follows:

$i = r, m$  – index of channels, offline  $r$  and online  $m$ ;

$j = M, R$  – index of supply chain members, manufacturer  $M$  and retailer  $R$ ;

$k = c, d$  – index of decision-making scenarios, centralized  $c$  and decentralized  $d$ ;

$q_i$  – customer demand for  $i$  channel;

$c_i$  – production cost for  $i$  channel;

$c_w$  – warranty cost per defective unit,  $c_i > c_w$ ;

$\lambda_i$  – failure rate for  $i$  channel;

$\beta$  – product life cycle (shape parameter);

$\alpha$  – the initial market size;

$\alpha'$  – the initial market size for the other channel;

$\gamma$  – price sensitivity coefficient;

$\gamma'$  – price sensitivity coefficient for the other channel;

$\theta$  – warranty length sensitivity coefficient;

$\theta'$  – warranty length sensitivity coefficient for the other channel;

$w_r$  – wholesale price,  $p_i > w_r > c_i$ ;

$p_i$  – selling price for  $i$  channel;

$t_i$  – warranty length for  $i$  channel;

$\pi_{rR}$  – retailer profit in offline channel  $r$ ;

$\pi_{rM}$  – manufacturer profit in offline channel  $r$ ;

$\pi_{mM}$  – manufacturer profit in online channel  $m$ .

This study develops centralized and decentralized decision-making scenarios using game theory to maximize supply chain profit. In the centralized scenario, these decisions are determined to maximize the supply chain profit [4, 20]. Meanwhile, decisions are severed to maximize profit for each member under a decentralized scenario. Given the setting parameters, the demand in each channel is estimated. Afterwards, the optimal price and extended warranty length to maximize each channel's profit are determined under different decision-making scenarios. Coordination contracts are also proposed regarding the profit structure in revenue-sharing and cost-sharing contracts. In supply chain coordination, revenue-sharing and cost-sharing contracts are two common mechanisms that align the interests or incentives of different supply chain members and improve overall performance [14]. Following the study in [12], the supply chain members accept a contract where one member will compensate for incentivizing other members to join the coordination mechanism. At last, the demand substitution effects represented in initial market size, price, and extended warranty length sensitivity coefficients are investigated for different channels. In the following sections, the results of each approach are discussed in more detail.

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#### 5. Results of decision-making model development and coordination analysis

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##### 5.1. Developing demand and profit function for a dual-channel supply chain

In offline channel  $r$ , customer demand  $q_r$  decreases in proportion to its price  $p_r$  and other channel's warranty length  $t_m$ , while increasing in proportion to its extended warranty length  $t_r$  and other channel's price  $p_m$ , and vice versa. Similar to previous studies [6–9], this demand function is supposed to be linear, focusing on the key parameters investigated in this study. Hence, the demand functions for offline  $q_r$  and online  $q_m$  channels are defined as follows:

$$q_r = \alpha_r - \gamma_r p_r + \gamma'_r p_m + \theta_r t_r - \theta'_r t_m,$$

$$q_m = \alpha_m - \gamma_m p_m + \gamma'_m p_r + \theta_m t_m - \theta'_m t_r.$$

Let's note that  $\alpha$  denotes the initial market size,  $\gamma$  is the customer sensitivity to warranty length that describes the highest price that the customer's willingness to pay for a product, and  $\theta$  denotes the longest warranty duration provided by manufacturer  $M$ . At the same time, the pricing and extended warranty decisions influence the demand substitution effect. Thus,  $\gamma'$  and  $\theta'$  are the substitution degree between channels, where  $\gamma_r > \gamma'_r$  and  $\theta_r > \theta'_r$ . Two channels are presumed to have positive demands, so  $\alpha > 0$ ,  $\gamma > 0$ , and  $\theta > 0$  make the customer demand feasible and practical. Hence, the

manufacturer's profit  $\pi_{rM}$  and retailer profit  $\pi_{rR}$  in the offline channel and the manufacturer's profit  $\pi_{mM}$  in the online channel are defined as follows:

$$\pi_{rM}(t_r) = q_r(w_r - c_r) - c_w(\lambda_r t_r)^\beta. \tag{1}$$

$$\pi_{rR}(p_r) = q_r(p_r - w_r), \tag{2}$$

$$\pi_{mM}(p_m, t_m) = q_m(p_m - c_m) - c_w(\lambda_m t_m)^\beta. \tag{3}$$

In offline channel  $r$ , the manufacturer's profit  $\pi_{rM}$  encloses the revenue from selling the product to the retailer and the extended warranty cost. Meanwhile, the retailer's profit  $\pi_{rR}$  denotes the retailer's revenue from selling the product to the customers. For online channel  $m$ , the manufacturer's profit  $\pi_{mM}$  is estimated as the manufacturer's revenue from selling the product directly to customers and extended warranty cost.

### 5. 2. Developing decision-making scenarios for a dual-channel supply chain

#### 5. 2. 1. Centralized decision-making scenario

The supply chain members' pricing and extended warranty decisions across all channels are examined in this scenario. Accordingly, the optimal prices ( $p_r^*, p_m^*$ ) and extended warranty lengths ( $t_r^*, t_m^*$ ) are determined to maximize the profit for the supply chain system. From (1)–(3), the profit of the centralized scenario  $\pi_c$  is defined as follows:

$$\pi_c(p_r, t_r, p_m, t_m) = \pi_{rM} + \pi_{rR} + \pi_{mM}. \tag{4}$$

The optimal decision should satisfy a concave profit function. Hence, the concavity of the model is verified in Theorem 1. Afterward, the optimal prices and warranty lengths are determined from the necessary conditions in Theorem 2.

*Theorem 1.* The profit function  $\pi_c(p_r, t_r, p_m, t_m)$  is jointly concave in  $p_r, t_r, p_m$  and  $t_m$ .

*Proof of Theorem 1.* The eigenvalues are determined to support the Hessian matrix by calculating the following principle minors:

$$H(p_r, t_r, p_m, t_m) = \begin{bmatrix} \frac{\partial^2 \pi_c}{\partial p_r^2} & \frac{\partial^2 \pi_c}{\partial p_r \partial t_r} & \frac{\partial^2 \pi_c}{\partial p_r \partial p_m} & \frac{\partial^2 \pi_c}{\partial p_r \partial t_m} \\ \frac{\partial^2 \pi_c}{\partial t_r \partial p_r} & \frac{\partial^2 \pi_c}{\partial t_r^2} & \frac{\partial^2 \pi_c}{\partial t_r \partial p_m} & \frac{\partial^2 \pi_c}{\partial t_r \partial t_m} \\ \frac{\partial^2 \pi_c}{\partial p_m \partial p_r} & \frac{\partial^2 \pi_c}{\partial p_m \partial t_r} & \frac{\partial^2 \pi_c}{\partial p_m^2} & \frac{\partial^2 \pi_c}{\partial p_m \partial t_m} \\ \frac{\partial^2 \pi_c}{\partial t_m \partial p_r} & \frac{\partial^2 \pi_c}{\partial t_m \partial t_r} & \frac{\partial^2 \pi_c}{\partial t_m \partial p_m} & \frac{\partial^2 \pi_c}{\partial t_m^2} \end{bmatrix}.$$

Thus, there are:

$$|H_1| = -2\gamma_r < 0,$$

$$|H_2| = \frac{2c_w \beta \gamma_r (\beta - 1) (t_r \lambda_r)^\beta}{t_r^2} > 0,$$

$$|H_3| = -\frac{4c_w \beta \gamma_r \gamma_m (\beta - 1) (t_r \lambda_r)^\beta}{t_r^2} < 0,$$

$$|H_4| = \frac{4c_w^2 \beta^2 \gamma_r \gamma_m (\beta - 1)^2 (t_r \lambda_r)^\beta (t_m \lambda_m)^\beta}{t_r^2 t_m^2} > 0.$$

Since more than one determinant is negative, at least one eigenvalue is negative. It could be a maximum or a saddle function. As a result, necessary conditions prove in Theorem 1 that the profit function  $\pi_c(p_r, t_r, p_m, t_m)$  is jointly concave in  $p_r, t_r, p_m$ , and  $t_m$ .

By solving  $\partial \pi_c / \partial p_r = 0$ ,  $\partial \pi_c / \partial t_r = 0$ ,  $\partial \pi_c / \partial p_m = 0$ , and  $\partial \pi_c / \partial t_m = 0$ , the optimal prices and extended warranty lengths are given in the following theorem.

*Theorem 2.* The optimal price  $p_r^*, p_m^*$  and extended warranty lengths  $t_r^*, t_m^*$  are unique optimal decisions in offline  $r$  and online  $m$  channels under the centralized scenario that are presented as follows:

$$p_r^* = \frac{\alpha_r + \gamma_r c_r - \gamma'_m c_m + \theta_r t_r - \theta'_r t_m + p_m (\gamma'_r + \gamma'_m)}{2\gamma_r}. \tag{5}$$

$$t_r^* = \left( \frac{(\theta_r (p_r - c_r) + \theta'_m (c_m - p_m) \lambda_r^{-\beta})}{\beta c_w} \right)^{\frac{1}{\beta-1}}, \tag{6}$$

$$p_m^* = \frac{\alpha_m + \gamma_m c_m - \gamma'_r c_r + \theta_m t_m - \theta'_m t_r + p_r (\gamma'_r + \gamma'_m)}{2\gamma_m}, \tag{7}$$

$$t_m^* = \left( \frac{(\theta_m (p_m - c_m) + \theta'_r (c_r - p_r) \lambda_m^{-\beta})}{\beta c_w} \right)^{\frac{1}{\beta-1}}, \tag{8}$$

where  $\beta > 1$  and  $\gamma_r > \theta_r, \gamma_m > \theta_m$ . Substituting (5)–(8) into (4), the optimal profit for the supply chain system under a centralized scenario is given by:  $\pi_c^*(p_r^*, t_r^*, p_m^*, t_m^*)$ .

*Proof of Theorem 2.* To verify that the prices and extended warranty length given in (5)–(8) are unique optimal decisions under the centralized scenario, the first partial derivative of (4) is calculated for  $p_r, t_r, p_m$ , and  $t_m$ , then put the result equal to 0, there are:

$$\begin{aligned} \frac{\partial \pi_c}{\partial p_r} &= \alpha_r + \gamma_r c_r - \gamma'_m c_m - 2\gamma_r p_r + \\ &+ p_m (\gamma'_r + \gamma'_m) + \theta_r t_r - \theta'_r t_m = 0, \end{aligned}$$

$$\frac{\partial \pi_c}{\partial t_r} = \theta_r (p_r - c_r) + \theta'_m (c_m - p_m) - \frac{\beta c_w (t_r \lambda_r)^\beta}{t_r} = 0,$$

$$\begin{aligned} \frac{\partial \pi_c}{\partial p_m} &= \alpha_m + \gamma_m (c_m - 2p_m) - \gamma'_r c_r + \\ &+ p_r (\gamma'_r + \gamma'_m) + \theta_m t_m - \theta'_m t_r = 0, \end{aligned}$$

$$\frac{\partial \pi_c}{\partial t_m} = \theta_m (p_m - c_m) + \theta'_r (c_r - p_r) - \frac{\beta c_w (t_m \lambda_m)^\beta}{t_m} = 0.$$

By solving them individually, the results are presented as (5)–(8).

In subsequence, the impact of key parameters on the decision variables is explored under the centralized scenario as presented in Proposition 1. These parameters are initial market size  $\alpha$ , price sensitivity coefficient  $\gamma$ , and warranty length sensitivity coefficient  $\theta$ .

*Proposition 1.* Under the centralized decision-making scenario, there are:

$$1) \frac{\partial p_r^*}{\partial \alpha_r} > 0, \frac{\partial p_m^*}{\partial \alpha_m} > 0, \frac{\partial t_r^*}{\partial \alpha_r} > 0, \frac{\partial t_m^*}{\partial \alpha_m} > 0,$$

$$\frac{\partial \pi_c^*}{\partial \alpha_r} = \frac{\partial \pi_{r_c}^*}{\partial \alpha_r} > 0, \frac{\partial \pi_c^*}{\partial \alpha_m} = \frac{\partial \pi_{m_c}^*}{\partial \alpha_m} > 0;$$



$$2) \frac{\partial p_r^*}{\partial \gamma_r} < 0, \frac{\partial p_m^*}{\partial \gamma_m} < 0, \frac{\partial t_r^*}{\partial \gamma_r} < 0, \frac{\partial t_m^*}{\partial \gamma_m} < 0,$$

$$\frac{\partial \pi_c^*}{\partial \gamma_r} = \frac{\partial \pi_c^*}{\partial \gamma_r} < 0, \frac{\partial \pi_c^*}{\partial \gamma_m} = \frac{\partial \pi_c^*}{\partial \gamma_m} < 0;$$

$$3) \frac{\partial p_r^*}{\partial \theta_r} > 0, \frac{\partial p_m^*}{\partial \theta_m} > 0, \frac{\partial t_r^*}{\partial \theta_r} > 0, \frac{\partial t_m^*}{\partial \theta_m} > 0,$$

$$\frac{\partial \pi_c^*}{\partial \theta_r} = \frac{\partial \pi_c^*}{\partial \theta_r} > 0, \frac{\partial \pi_c^*}{\partial \theta_m} = \frac{\partial \pi_c^*}{\partial \theta_m} > 0.$$

In the centralized scenario, Proposition 1 indicates an increase in the initial market size  $\alpha$  and extended warranty length sensitivity coefficient  $\theta$  for each channel, increasing the optimal price  $p^*$  and extended warranty length  $t^*$  for the corresponding channel. Conversely, increasing the price sensitivity coefficient  $\gamma$  decreases both decision variables. In addition, it also presents the effect of parameter changes on optimal profit  $\pi_c^*$ , which tends to have the same effect as  $p^*$  and  $t^*$ . Under the centralized scenario, all parameters impact the whole profit of the dual channel. Furthermore, the profit for each channel is impacted by changes in each corresponding parameter. This situation indicates that there is a substitution effect between both channels.

### 5. 2. 2. Decentralized decision-making scenario

In a competitive market, a decentralized system is more prevalent and applicable in practice than a centralized. Despite locally maximizing the profit of supply chain members, each channel is separated in making decisions [20, 21]. By implementing backward induction for the Stackelberg game, offline channel  $r$  is discussed first and then online channel  $m$ .

In offline channel  $r$ , the retailer determines the optimal retail price  $p_r^*$  to maximize profit. Accordingly, given wholesale price  $w_r$ , the manufacturer determines the optimal warranty length  $t_r^*$  to maximize profit. The process converges, thereby following theorem is given by solving  $\partial \pi_{rR}/\partial p_r = 0$  and  $\partial \pi_{rM}/\partial t_r = 0$ .

*Theorem 3.* The optimal price  $p_r^*$  and extended warranty length  $t_r^*$  are unique optimal decisions in offline channel  $r$  under the decentralized scenario presented as follows:

$$p_r^* = \frac{\alpha_r + \gamma_r w_r + \gamma'_r p_m + \theta_r t_r - \theta'_r t_m}{2\gamma_r}, \quad (9)$$

$$t_r^* = \left( \frac{\theta_r \lambda_r^{-\beta} (w_r - c_r)}{\beta c_w} \right)^{1/\beta-1}. \quad (10)$$

Meanwhile, in online channel  $m$ , the manufacturer specifies the optimal price  $p_m^*$  and warranty length  $t_m^*$  to maximize the profit. Considering the concave nature of the profit function, the following theorem is generated by solving  $\partial \pi_{mM}/\partial p_m = 0$  and  $\partial \pi_{mM}/\partial t_m = 0$ .

*Theorem 4.* The optimal price  $p_m^*$  and extended warranty length  $t_m^*$  are unique optimal decisions in online channel  $m$  under the decentralized scenario presented as follows:

$$p_m^* = \frac{\alpha_m + \gamma_m c_m + \gamma'_m p_r + \theta_m t_m - \theta'_m t_r}{2\gamma_m}, \quad (11)$$

$$t_m^* = \left( \frac{\theta_m \lambda_m^{-\beta} (p_m - c_m)}{\beta c_w} \right)^{1/\beta-1}. \quad (12)$$

*Proof of Theorems 3 and 4.* To confirm that the prices and extended warranty length given in (9)–(12) are unique optimal decisions under the decentralized scenario, the first partial derivative of (1) and (2) are calculated for  $p_r$  and  $t_r$ , then put the result equal to 0. In addition, the first partial derivative of (3) is calculated for  $p_m$ , and  $t_m$ , then put the result equal to 0, there are:

$$\frac{\partial \pi_{rR}}{\partial p_r} = \alpha_r + \gamma_r w_r + \gamma'_r p_m - 2\gamma_r p_r + \theta_r t_r - \theta'_r t_m = 0,$$

$$\frac{\partial \pi_{rM}}{\partial t_r} = \theta_r (w_r - c_r) - \frac{\beta c_w (t_r \lambda_r)^\beta}{t_r} = 0,$$

$$\begin{aligned} \frac{\partial \pi_{mM}}{\partial p_m} &= \alpha_m + \gamma_m (c_m - 2p_m) + \\ &+ \gamma'_m p_r + \theta_m t_m - \theta'_m t_r = 0, \end{aligned}$$

$$\frac{\partial \pi_{mM}}{\partial t_m} = \theta_m (p_m - c_m) - \frac{\beta c_w (t_m \lambda_m)^\beta}{t_m} = 0.$$

Solving them one by one, respectively, the results are formulated as (9)–(12).

Substituting (9)–(12) into (4), the optimal profit under the decentralized scenario is represented as:  $\pi_d^*(p_r^*, t_r^*, p_m^*, t_m^*)$ . Furthermore, the impacts of other parameters on the decision variables and profit are characterized in Proposition 2.

*Proposition 2.* Under the decentralized decision-making scenario, there are:

$$1) \frac{\partial p_{rd}^*}{\partial \alpha_r} > 0, \frac{\partial p_{md}^*}{\partial \alpha_m} > 0, \frac{\partial t_{rd}^*}{\partial \alpha_r} > 0, \frac{\partial t_{md}^*}{\partial \alpha_m} > 0,$$

$$\frac{\partial \pi_d^*}{\partial \alpha_r} = \frac{\partial \pi_d^*}{\partial \alpha_r} > 0, \frac{\partial \pi_d^*}{\partial \alpha_m} = \frac{\partial \pi_d^*}{\partial \alpha_m} > 0,$$

$$2) \frac{\partial p_{rd}^*}{\partial \gamma_r} < 0, \frac{\partial p_{md}^*}{\partial \gamma_m} < 0, \frac{\partial t_{rd}^*}{\partial \gamma_r} < 0, \frac{\partial t_{md}^*}{\partial \gamma_m} < 0,$$

$$\frac{\partial \pi_d^*}{\partial \gamma_r} = \frac{\partial \pi_d^*}{\partial \gamma_r} < 0, \frac{\partial \pi_d^*}{\partial \gamma_m} = \frac{\partial \pi_d^*}{\partial \gamma_m} < 0,$$

$$3) \frac{\partial p_{rd}^*}{\partial \theta_r} > 0, \frac{\partial p_{md}^*}{\partial \theta_m} > 0, \frac{\partial t_{rd}^*}{\partial \theta_r} > 0, \frac{\partial t_{md}^*}{\partial \theta_m} > 0,$$

$$\frac{\partial \pi_d^*}{\partial \theta_r} = \frac{\partial \pi_d^*}{\partial \theta_r} > 0, \frac{\partial \pi_d^*}{\partial \theta_m} = \frac{\partial \pi_d^*}{\partial \theta_m} > 0.$$

As in the centralized scenario, Proposition 2 demonstrates that increasing the initial market size  $\alpha$  and extended warranty length coefficient  $\theta$  increases the optimal price  $p^*$ , extended warranty length  $t^*$ , and profit  $\pi_d$ . Meanwhile, an increase in the price sensitivity coefficient  $\gamma$  yields a decrease for all three. Furthermore, differences in the decision and profit variables for each channel are significantly affected by each change of key parameters in the corresponding channel.

**5. 2. 3. Comparative analysis for the development of decision-making scenarios**

In the following proposition, the analysis begins by comparing optimal decision variables,  $p^*$  and  $t^*$ , for a supply chain system under different channels and decision-making strategies.

*Proposition 3.* Comparing the optimal price, there are  $p_{rc}^* > p_{mc}^* > p_{rd}^* > p_{md}^*$ .

Proposition 3 explains the differences in the optimal prices for each channel under different decision-making scenarios. Similar to previous studies [18, 20], prices in the centralized scenario are higher than in the decentralized,  $p_{rc}^* > p_{rd}^*$  and  $p_{mc}^* > p_{md}^*$ . Moreover, offline channels  $r$  have higher prices than online channels  $m$  in both decision-making scenarios,  $p_{rc}^* > p_{mc}^*$  and  $p_{rd}^* > p_{md}^*$ .

*Proposition 4.* Comparing the optimal extended warranty length, there are  $t_{md}^* > t_{rc}^* > t_{rd}^* > t_{mc}^*$ .

In Proposition 4, offline channel  $r$  has a longer extended warranty length than online channel  $m$  under the centralized scenario,  $t_{rc}^* > t_{mc}^*$ . Conversely, in the decentralized scenario, the extended warranty length on online channel  $m$  is longer than offline channel  $r$ ,  $t_{md}^* > t_{rd}^*$ . It is easy to understand because the manufacturer, as the exclusive provider of the extended warranty, can offer lower prices to induce demand in a decentralized scenario. Meanwhile, in the centralized scenario, the manufacturer coordinates with the retailer so that they do not significantly differ in the extended warranty offered. In such circumstances, the article [22] also discovered a similar result in another development work of manufacturer encroachment.

*Proposition 5.* Comparing the profit in different decision-making coordination scenarios, there are  $\pi_c^* > \pi_d^*$ ,  $\pi_{rc}^* > \pi_{rd}^*$  and  $\pi_{mc}^* > \pi_{md}^*$ . Similarly, the member's profit in offline and online channels are  $\pi_{rRc} > \pi_{rRd} > \pi_{rMd} > \pi_{rMc}$  and  $\pi_{mMc} > \pi_{mMd}$ .

Proposition 5 shows that the centralized scenario always yields better profits than the decentralized scenario. However, in the manufacturer's problem, the decentralized scenario provides higher profits than the centralized scenario under offline channel  $r$  since the manufacturer wants to maximize its profit,  $\pi_{rMd} > \pi_{rMc}$ . Thus, dual channel development can increase the supply chain profitability.

*Proposition 6.* Comparing the profit in different channels, there are  $\pi_{rc}^* = \pi_{mc}^*$  and  $\pi_{md}^* > \pi_{rd}^*$ . Similarly, the member's profit in each channel are  $\pi_{mMc} > \pi_{rMc}$  and  $\pi_{mMd} > \pi_{rMd}$ .

Proposition 6 demonstrates that the online channel  $m$  has the exact profit as the offline channel  $r$  in a centralized scenario,  $\pi_{rc}^* = \pi_{mc}^*$ . Meanwhile, the profit of online channel  $m$  is higher than offline channel  $r$  in a decentralized scenario,  $\pi_{md}^* > \pi_{rd}^*$ . Furthermore, the profit comparison of supply chain members in Proposition 6 only involves the manufacturer with two channels, offline and online. Thus, the manufacturer earns more profits online than offline channels,  $\pi_{mMc} > \pi_{rMc}$  and  $\pi_{mMd} > \pi_{rMd}$ . In summary, Propositions 5 and 6 reveal that the centralized scenario consistently outperforms the decentralized scenario. Therefore, a coordination contract is proposed to obtain a win-win situation for all members on each channel.

**5. 3. Developing coordination contracts for a dual-channel supply chain**

**5. 3. 1. Revenue sharing contract**

This contract is an agreement between supply chain members in which they share a portion of the revenue generated by the sale of a product or service [12, 13]. Under

this type of contract, the profit is split between the parties according to a predetermined percentage, which encourages them to work together to maximize sales and revenue.

In this proposed contract, the proportion of channels' revenue for offline channel  $r$  is defined as  $X$ , whereas online channel  $m$  is  $1-X$  ( $0 < X < 1$ ). In offline channel  $r$ , the retailer determines price  $p_{rX}$  to maximize profit. Accordingly, the manufacturer has optimal warranty length  $t_{rX}$  to maximize profit. Meanwhile, in online channel  $m$ , the manufacturer defines price  $p_{mX}$  and warranty length  $t_{mX}$  to maximize profits. However, the profit in the coordination contract should coincide with the centralized model. Thus, these necessary conditions should be satisfied  $p_{rc} = p_{rX}$ ,  $t_{rc} = t_{rX}$ ,  $p_{mc} = p_{mX}$ , and  $t_{mc} = t_{mX}$ . By substituting (1)–(3), there are  $\pi_{rc} = q_r(p_{rc} - c_r) - c_w(\lambda_r t_{rc})^\beta$  and  $\pi_{mc} = q_m(p_{mc} - c_m) - c_w(\lambda_m t_{mc})^\beta$ . As a result, profits of offline and online channels for this coordination contract are defined as follows:

$$\pi_{rX} = Xq_r(p_{rc} - c_r) - c_w(\lambda_r t_{rc})^\beta, \tag{13}$$

$$\pi_{mX} = q_m(1 - X)(p_{mc} - c_m) - c_w(\lambda_m t_{mc})^\beta. \tag{14}$$

Note that  $\pi_{rX} + \pi_{mX} = \pi_c$ . To ensure the feasibility of this contract, the profit inequations,  $\pi_{rX} > \pi_{rd}$  and  $\pi_{mX} > \pi_{md}$ , should be satisfied. After simplifying (13) and (14), the upper and lower bounds of revenue sharing proportions  $X$  are formulated as follows:

$$X \leq \frac{\pi_{rd} + c_w(\lambda_r t_{rc})^\beta}{q_r(p_{rc} - c_r)} = \bar{X}, \tag{15}$$

$$X \geq 1 - \frac{\pi_{md} + c_w(\lambda_m t_{mc})^\beta}{q_m(p_{mc} - c_m)} = \underline{X}. \tag{16}$$

From the analysis above, Corollary 1 is formulated as follows.

*Corollary 1.* When a revenue sharing contract is used to coordinate a dual-channel decentralized model, both channels estimate the proportion as  $X \in (\underline{X}, \bar{X})$ .

The manufacturer and retailer might use a revenue sharing proportion  $X$  within the range in Corollary 1 to coordinate the decentralized model in a dual-channel. Consequently, each channel resolves the channel conflict and achieves a win-win situation to supply chain members.

**5. 3. 2. Cost sharing contract**

A cost-sharing contract is an agreement in which the costs of producing and delivering a product or service are shared between supply chain members [14]. This contract encourages collaboration and efficiency in the supply chain, as each member is incentivized to minimize costs to maximize their profits.

In this contract setting, the proportion of channels' cost for offline channel  $r$  is  $Y$ , whereas online channel  $m$  is  $1-Y$  ( $0 < Y < 1$ ). In offline channel  $r$ , the retailer determines retail price  $p_{rY}$  to maximize profit. Afterwards, the manufacturer sets optimal warranty length  $t_{rY}$  to maximize profit. Meanwhile, in online channel  $m$ , the manufacturer specifies price  $p_{mY}$  and warranty length  $t_{mY}$  to maximize the profit. As in a revenue sharing contract, these necessary conditions should be satisfied  $p_{rc} = p_{rY}$ ,  $t_{rc} = t_{rY}$ ,  $p_{mc} = p_{mY}$ , and  $t_{mc} = t_{mY}$ . Hence, the profit of the offline and online channels for this contract is defined as follows:

$$\pi_{rY} = q_r(p_{rc} - c_r) - c_w Y (\lambda_r t_{rc})^\beta, \quad (17)$$

$$\pi_{mY} = q_m(p_{mc} - c_m) - c_w (1-Y) (\lambda_m t_{mc})^\beta. \quad (18)$$

Despite  $\pi_{rY} + \pi_{mY} = \pi_c$ , the profit inequations,  $\pi_{rY} > \pi_{rd}$  and  $\pi_{mY} > \pi_{md}$ , should be satisfied. In subsequence, the upper and lower bounds of cost sharing proportions  $Y$  are formulated as follows:

$$Y \leq \frac{q_r(p_{rc} - c_r) - \pi_{rd}}{c_w (\lambda_r t_{rc})^\beta} = \bar{Y}, \quad (19)$$

$$Y \geq 1 + \frac{\pi_{md} - q_m(p_{mc} - c_m)}{c_w (\lambda_m t_{mc})^\beta} = \underline{Y}. \quad (20)$$

From the analysis above, Corollary 2 is formulated as follows.

**Corollary 2.** When a cost-sharing contract is used to coordinate a dual-channel decentralized model, both channels estimate the proportion as  $Y \in (\underline{Y}, \bar{Y})$ .

Here, all members in the dual channel use cost-sharing proportions  $Y$  in the range in Corollary 2 to coordinate the decentralized model.

As a result, the profit of each channel increases and a win-win situation is established.

#### 5.4. Computational analysis for a dual-channel supply chain

In order to confirm the model developed earlier and show the impact of key parameters on optimal decisions, this section presents numerical and sensitivity analysis. A real case is examined as the motivation for developing the model. A manufacturer engaged in electronics and home appliances has been selling its products through offline retailers. With technology innovation and market competition, a manufacturer performs manufacturer encroachment by constructing its online channel via a website and application platform. So far, the manufacturer has also offered an extended warranty to differentiate it from competitors. Due to increasingly complex and dynamic demands, a manufacturer has difficulty deciding the optimal strategy for pricing and extended warranty on its dual channels.

##### 5.4.1. Numerical analysis

The numerical example has been employed to illustrate this study's pricing and extended warranty decision cases. With referring the previous literatures [7, 9], other relevant parameters were selected to manage results effectively while demonstrating impacts under different scenarios:  $\alpha_r = 2000$ ,  $\alpha_m = 1500$ ,  $\gamma_r = \gamma_m = 6$ ,  $\gamma'_r = \gamma'_m = 4$ ,  $\theta_r = \theta_m = 0.6$ ,  $\theta'_r = \theta'_m = 0.4$ ,  $w_r = 300$ ,  $c_r = 200$ ,  $c_m = 150$ ,  $c_w = 100$ ,  $\beta = 2$ , and  $\lambda_r = \lambda_m = 0.4$ .

Let's initially analyze the impact of optimal prices  $p$  and extended warranty lengths  $t$  of retailer  $R$  and manufacturer  $M$  on the profit under different decision-making scenarios. Table 1 shows that the optimal price in a centralized scenario is higher than in a decentralized scenario. However, the optimal extended warranty length on the online channel is less when in the centralized decision. As a result, the supply chain profit in the centralized scenario is more significant than in a decentralized scenario. Hence, let's propose coordination contacts to coordinate the decentralized scenario.

Table 1

Optimal Results under Different Decision-Making Scenarios

Scenario	$p_r$	$p_m$	$t_r$	$t_m$	$\pi_{rR}$	$\pi_{rM}$	$\pi_{mM}$	$\pi$
Centralized $c$	550	500	2	2	175131	69945	245077	490153
Decentralized $d$	431	344	2	4	103381	78702	225311	407394

In addition, every profit function for the retailer and manufacturer in different channels is jointly concave related to the pairs  $(p_r, t_r)$  and  $(p_m, t_m)$ . Fig. 1 describes how price and extended warranty length simultaneously impact the corresponding profit under the centralized and decentralized scenarios.

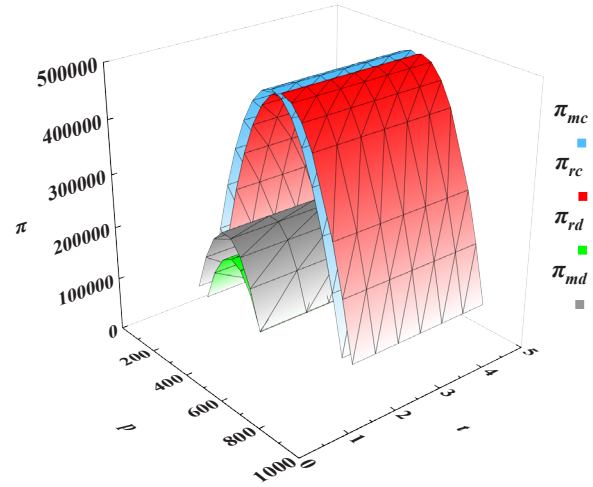


Fig. 1. Joint effect of price  $p$  and extended warranty length  $t$  on profit  $\pi$

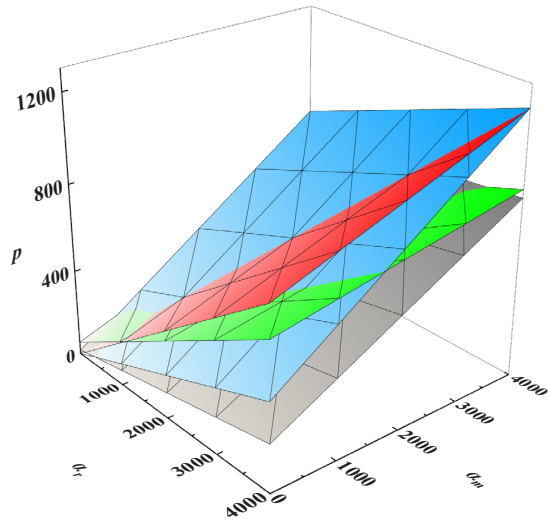
Therefore, these results ensure Theorem 1–4.

##### 5.4.2. Sensitivity analysis

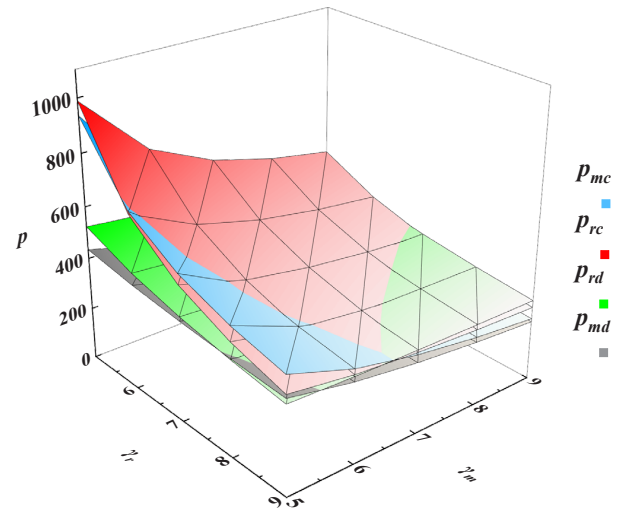
This section discusses the substitution effects of several key parameters on optimal decision variables and supply chain performance for each channel under different models. These key parameters are initial market size  $\alpha$ , price sensitivity coefficient  $\gamma$ , and warranty length coefficient  $\theta$ . Firstly, the impact of changing the initial market size is examined for offline market  $\alpha_r$  and online market  $\alpha_m$ , which varies between 0 and 4000. In Fig. 2, increasing the market size of  $\alpha$  on a particular channel will increase the price  $p$ , extended warranty length  $t$ , and profit on the corresponding channel but decrease on another channel.

In subsequence, the price sensitivity coefficient  $\gamma$  in both channels is changed to investigate the substitution effect of this parameter from 4 to 10. Fig. 3 demonstrates that increasing price sensitivity coefficient  $\gamma$  in either channel decreases price  $p$ , extended warranty length  $t$ , and profit in that channel.

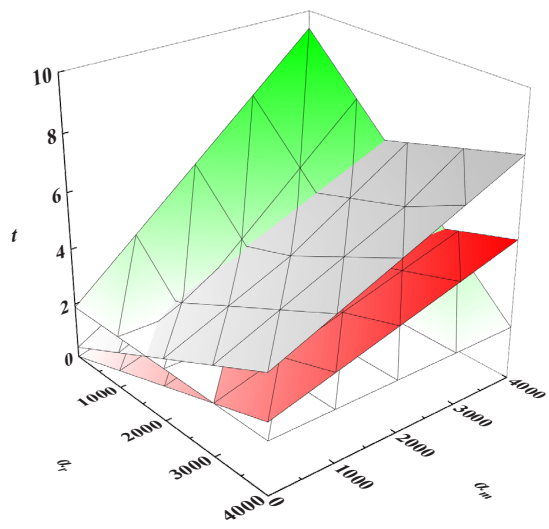
To this end, the effect of extended warranty length sensitivity  $\theta$  for different channels is analyzed by taking values within  $[0, 1]$ . Fig. 4 explains that as extended warranty length sensitivity  $\theta$  in a particular channel increases, there will only be an increase in extended warranty lengths  $t$  with the corresponding channel. However, no matter how warranty length sensitivity  $\theta$  changes, price  $p$  and profit in the offline channel consistently outperform the online channel. From this section, the results have confirmed Propositions 1 and 2.



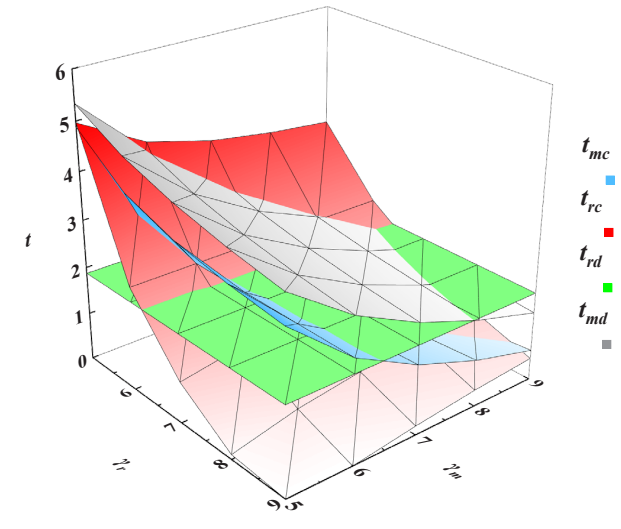
a



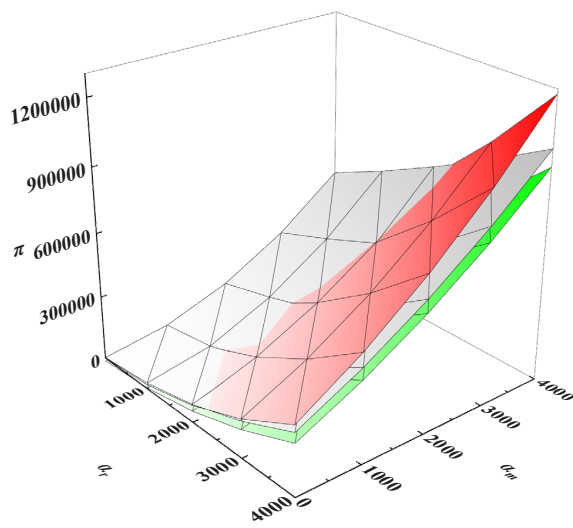
a



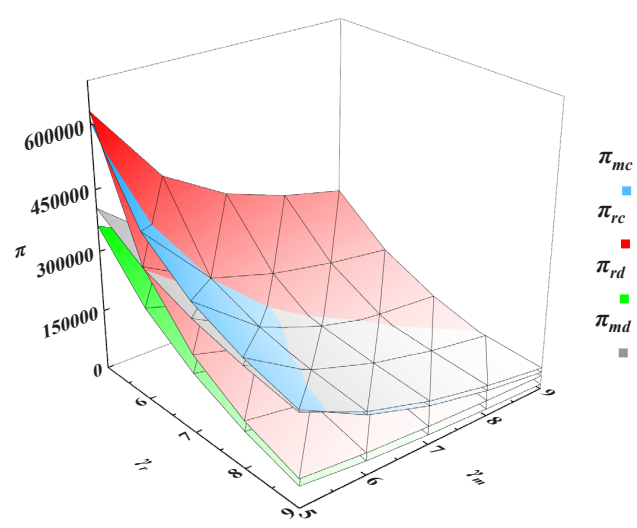
b



b



c



c

Fig. 2. Substitution effects of market size for offline channel  $\alpha_r$  and online channel  $\alpha_m$ : a – price  $p$ ; b – warranty length  $t$ ; c – profit  $\pi$

Fig. 3. Substitution effects of price sensitivity coefficient for offline channel  $\gamma_r$  and online channel  $\gamma_m$ : a – price  $p$ ; b – warranty length  $t$ ; c – profit  $\pi$



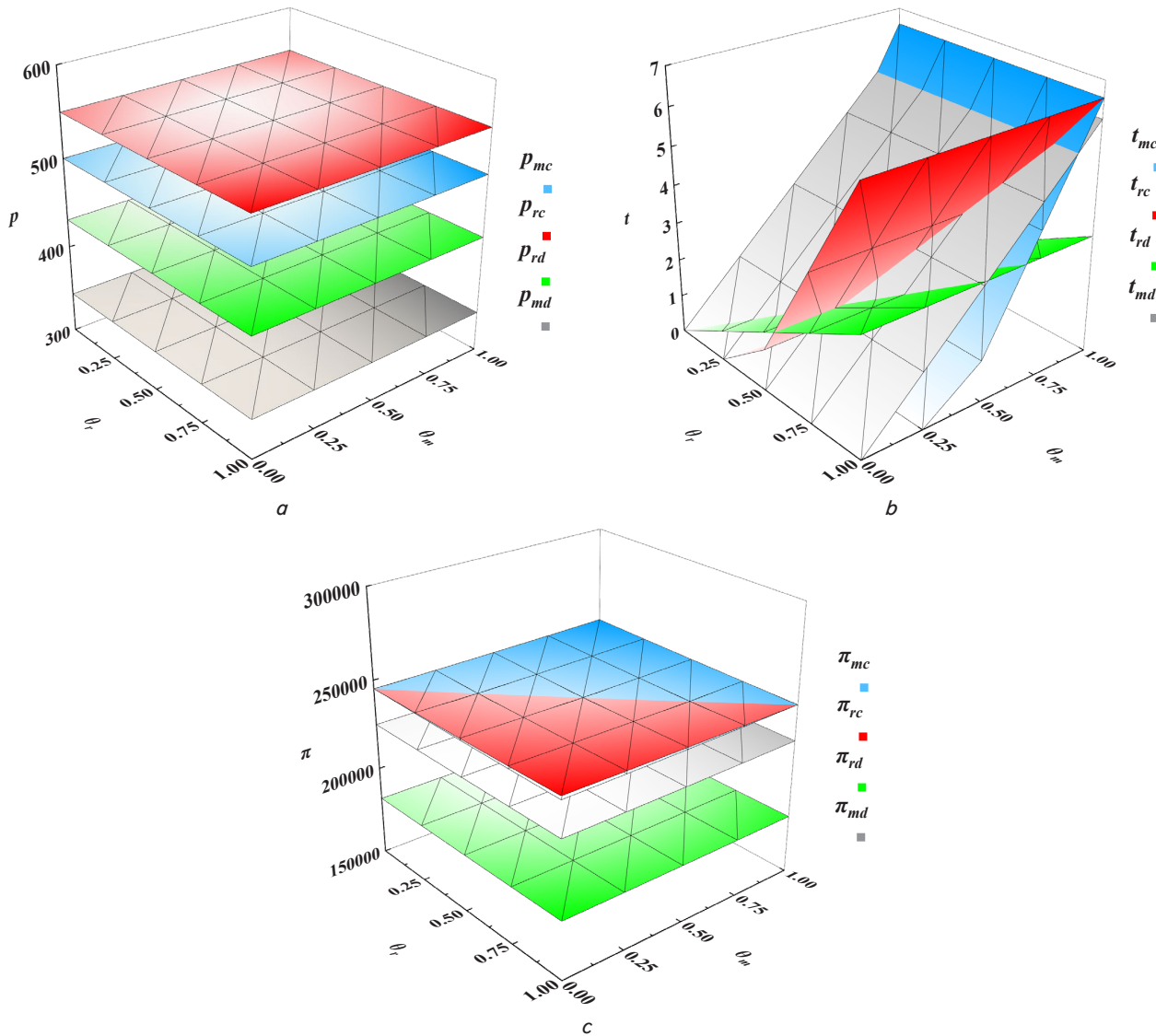


Fig. 4. Substitution effects of extended warranty length sensitivity for offline channel  $\theta_r$  and online channel  $\theta_m$ :  
 a – price  $p$ ; b – warranty length  $t$ ; c – profit  $\pi$

**5. 4. 3. Coordination contract analysis**

In order to coordinate and improve the performance of a dual-channel supply chain, two coordination contracts are proposed under profit structures, revenue-sharing and cost-sharing contracts. A revenue-sharing contract involves sharing a portion of the revenue generated by selling a product or service. In contrast, the cost sharing involves sharing the costs associated with production and distribution [12, 23].

In Table 1, the supply chain is uncoordinated because  $\pi_c \neq \pi_d$ . First, the proposed revenue-sharing contract in Corollary 1 is performed to obtain revenue-sharing proportion  $X: 0.08 < X < 0.74$ . As a revenue sharing proportion  $X$  changes in range, decentralized scenario profits shift to the coordination contracts profits equal to the centralized scenario. Using a similar procedure, share proportions for the cost-sharing contract  $Y$  in Corollary 2 is generated and presented in Table 2.

Table 2

Coordination Results under Profit structures

Coordination Model	$X/Y$	$\pi_{md}$	$\pi_{mX}/\pi_{mY}$	$\pi_{rd}$	$\pi_{rX}/\pi_{rY}$	$\pi_c$	$\pi_X/\pi_Y$
Revenue sharing contract ( $X$ )	0.08	225311	225311	182083	264842	490153	490153
	0.24		245077		245077		490153
	0.54		282344		207809		490153
	0.74		308070		182083		490153
Cost sharing contract ( $Y$ )	-258	225311	225311	182083	264842	490153	490153
	0.24		245077		245077		490153
	540.63		286438		203715		490153
	823.26		308070		182083		490153

Within the range of  $X$  and  $Y$ , each profit for the coordinated model has been achieved the same as the centralized model,  $\pi_X = \pi_Y = \pi_C = 490153$ . Moreover, the profit of each channel increases more than the profit in the decentralized scenario. For example, in the revenue-sharing contract,  $\pi_{mX} \geq \pi_{md} = 225311$  and  $\pi_{rX} \geq \pi_{rd} = 182083$  when sharing proportion  $X$  shifts within  $0.08 < X < 0.74$ . Accordingly, the supply chain system has been coordinated and achieved a win-win situation.

## 6. Discussion of decision-making and coordination results for a dual-channel supply chain with pricing and extended warranty strategies under demand substitution effects

A dual-channel supply chain allows companies to capture a larger market share by reaching customers through multiple channels. However, it requires a careful balance between the preferences of the manufacturer and the retailer. A manufacturer should determine prices and attractive warranty strategies for online and offline channels. Meanwhile, a retailer needs to offer their customers a unique value proposition. Using game theory models can help a decision-maker understand how different pricing and warranty strategies will impact the performance of a channel and its competitors. This paper determines optimal pricing and extended warranty strategies to maximize profits in a dual-channel supply chain by developing centralized and decentralized decision-making scenarios summarized in Theorems 2 to 4. Afterwards, numerical and sensitivity analyses are performed to support them.

As Table 1, although the optimal extended warranty length is identical  $t_r = 2$ , the optimal price on offline channel  $r$  with a centralized decision is higher than a decentralized decision,  $p_{rc} = 550 > p_{rd} = 431$ . Similarly, on online channel  $m$ , the optimal price is higher when the decision is centralized,  $p_{mc} = 500 > p_{md} = 344$ . However, the optimal extended warranty length on the online channel is less when in the centralized decision,  $t_{m_c} = 2 < t_{m_d} = 4$ . This situation occurs because the member of the decentralized scenario aims to maximize its profit individually. Hence, these results confirm Propositions 3 and 4. In addition, the cooperation of supply chain members could substantially enhance the supply chain's profit,  $\pi_c = 490153 > \pi_d = 407394$  (Fig. 1). In offline channel  $r$ , the retailer gains more profits in the centralized scenario,  $\pi_{rc} = 175131 > \pi_{rd} = 103381$ . However, manufacturers' profits are higher with a decentralized scenario,  $\pi_{rmd} = 78702 > \pi_{rmc} = 69945$ . Thus, the retailer enjoys cooperating with the manufacturer to earn more profit. In contrast, the manufacturer prefers to perform independently instead of cooperating with the retailer to maximize profit. This analysis supports the research question regarding the manufacturer's rationale for developing its sales channels. In online channel  $m$ , which the manufacturer performs and manages its channel independently, manufacturers' profits are increased with a centralized scenario,  $\pi_{m_c} = 245077 > \pi_{m_d} = 225311$ . As a result, both channels have the same profit in the centralized scenario,  $\pi_m = \pi_c = 245077$ . However, in the decentralized scenario, the online channel's profit is more significant than that of the offline channel,  $\pi_{m_d} = 225311 > \pi_{rd} = 182083$ . These analyzes verify Propositions 5 and 6. Hence, coordination contracts are proposed to coordinate the decentralized scenario in achieving a win-win situation for supply chain members in each channel.

Furthermore, the demand substitution effect of several key parameters is explored on optimal decision variables and supply chain profit for each channel under different models. First, increasing in market size for a particular channel  $\alpha_i$  increases the warranty length in the corresponding channel  $t_i$  but decreases the warranty length in another channel, as shown in Fig. 2. Accordingly, demand for these channels grows, providing incentives to increase selling prices  $p_i$  and earn higher profits  $\pi_i$ . Meanwhile, another channel shifts its pricing strategy by lowering prices to sustain its competitiveness. Thus, like studies in [7] and [20], large market sizes allow supply chain members to increase their sales and achieve more performance. Second, from Fig. 3, an increase in price sensitivity coefficient  $\gamma$  in a particular channel tends to lower the price of the corresponding channel dramatically more than another channel. This situation occurs because the channel should provide lower prices to maintain customer demand and reduce profit losses. Thus, channel members reduce the warranty length  $t$  offered to cover costs and remain profitable. In turn, if the price sensitivity coefficient  $\gamma$  increases for a particular channel, the profit  $\pi$  decreases rapidly compared to the other. Third, a channel sensitive to the shift of warranty length parameter will be more sensitive to demand, so it will increase the warranty length (Fig. 4). At the same time, since there is a price margin with the retailer in offline channel  $r$ , price  $p$  in the offline channel consistently outperforms the online channel. As a result, the increased sensitivity of the extended warranty length on offline  $\theta_r$  and online  $\theta_m$  channels imperceptibly increases profit  $\pi$ . However, channel members should continue providing extended warranties to stay competitive and increase profitability.

The coordination contracts proposed in the revenue-sharing and cost-sharing contracts under Corollaries 1 and 2, respectively, resulted in a range change for the sharing proportions. From the range of  $X$  and  $Y$  in Table 2, both coordination contracts establish equilibrium at the centralized scenario,  $\pi_{mc} = \pi_{rc} = 245077$ . Increasing sharing proportions for  $X$  and  $Y$  tend to increase online channel profit  $\pi_m$  and decrease offline channel profit  $\pi_r$ , and vice versa. Thus, as the decision maker, the manufacturer would determine sharing proportions  $X$  and  $Y$  regarding the market bargaining power [12, 14]. Despite considering the nature of structured incentives, revenue-sharing contracts focus on maximizing revenue and profits, while cost-sharing contracts focus on minimizing costs. Therefore, revenue-sharing contracts might be more suitable for situations focusing on driving sales and revenue growth. On the other hand, cost-sharing contracts might be more appropriate for reducing costs and improving efficiency.

In this study, the analysis addresses the gaps in existing studies. Thus, some managerial insights are generated for decision-makers or managers to apply these results in real industrial systems. These aim to align incentives and increase overall profitability for supply chain members. The optimal warranty length and price depend on the level of demand substitution effects between channels. For example, if the offline channel raises its price too high, customers may substitute for purchasing products from the online channel, resulting in lost sales for the offline channel. Nevertheless, the optimal pricing and warranty strategy in a dual-channel supply chain depends on various factors of the profit structure, including market size, price, and extended warranty length that represent the competition level. Accordingly, a manager should carefully analyze these factors to develop a strategy

that maximizes profits while minimizing the substitution effect. Moreover, this study emphasizes the importance of coordination between different supply chain members. By collaborating to develop pricing and warranty strategies that optimize profits for both channels, the manufacturer and retailer can create a win-win situation that improves overall supply chain performance.

However, there are some limitations of the study. First, supply chain members are assumed to be rational and have perfect information. Hence, exploring the effect of the bounded rationality of supply chain members, asymmetric and incomplete information is worth further research. Second, this study focuses on developing coordination contracts regarding the profit structure. Future research could further investigate other coordination contracts or how bargaining power among supply chain members can affect pricing decisions and supply chain performance. Involving those concerns will be an interesting direction to enhance the developed model.

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## 7. Conclusions

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1. This study examines a decision-making scenario in a dual-channel supply chain under the demand substitution effect, including initial market size, price sensitivity coefficient, and warranty length sensitivity coefficient. These scenarios generate optimal pricing and extended warranty strategies to maximize profits in a dual-channel supply chain. This study develops centralized and decentralized decision-making scenarios using game theory and mathematical models. Centralized scenarios result in optimal decisions to maximize the profit of the whole supply chain. Meanwhile, the decentralized scenario results in a decision to maximize profits individually for each supply chain member. The optimal decision for each scenario is briefly and precisely presented in Theorem 1–4.

2. Based on the comparative analysis, several analytical propositions are derived mathematically for investigating the joint impact of pricing and extended warranty decisions on the profit of the supply chain system and each channel under different decision-making scenarios. Propositions 3 to 6 show that the centralized decision-making scenario consistently outperforms the decentralized scenario in developing optimal decisions for price and extended warranties across supply chain systems and retailers. As a result, the total profit of the centralized scenario is higher than the decentralized scenario. However, without the manufacturer's encroachment on dual channels, the decentralized scenario is more profitable for the manufacturer.

3. Two coordinating contracts under the profit structure, revenue and cost, are proposed to coordinate supply chain

members in each channel and achieve a win-win situation. With the share proportions of revenue-sharing and cost-sharing contracts formulated analytically in Corollaries 1 and 2, the analysis demonstrates that the total profit of the earlier decentralized scenario has achieved the centralized scenario. In addition, the supply chain members' profits increase, thus resulting in a win-win situation.

4. Numerical and sensitivity analyzes are performed to explore the effect of demand substitution parameters on performance, pricing, and extended warranty decisions for each channel in a two-channel supply chain. Based on numerical analysis, the optimal decision for price and warranty length in the centralized scenario is 1.5 to 2 times higher than in the decentralized scenario. As a result, cooperation between members in a centralized scenario can increase the total profit by approximately 20 % higher than in a decentralized scenario in a dual-channel supply chain. Moreover, the demand sensitivity to a particular channel changes the optimal decision of the potential business and operating situations. Based on the sensitivity analysis results, an increase in market size and warranty length sensitivity coefficient increases optimal decision variables. However, an increase in the price sensitivity coefficient decreases optimal decision variables. Both of these analyzes confirm the resulting Propositions 1–6. Thus, some managerial insights are generated to assist managers in such circumstances.

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## Conflict of Interest

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The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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The study was performed without financial support.

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## Data Availability

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Data will be made available on reasonable request.

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