This paper investigates the influence of hydrodynamic conditions for entering the initial section of the channel located after local obstacles of various types. It is shown that the head losses in the valves and bends of pipelines and in various control elements can be several times higher than those in straight sections of the pipeline. It was established that the assumption about the rectangular shape of the velocity diagram at the entrance to the hydrodynamic initial section does not correspond to the flow pattern in real channels of technological equipment. It is proved that with the manifestation of inertia forces in the flow at the initial section of the channel, hydrodynamic energy losses usually increase, velocity and stress fields are significantly deformed. Given this, it seemed expedient to conduct a study into the processes of flow of viscous liquids in the initial section, located after local obstacles of various types. Experimental and analytical studies have confirmed that there is a significant influence of boundary conditions at the entrance to the initial section on the formation of velocity diagrams and energy loss along its length. The analytical-numerical solution to the system of differential equations describing such flow is given. While solving, the system of equations, by appropriate transformations, takes the form of a nonlinear integral-differential equation. This makes it possible to obtain correct dependences for determining the length of the velocity distribution and energy loss in the investigated section of the channel. The results of calculations of velocity fields in the region of local obstacles agree well with known ideas of the flow pattern, which is observed in physical experiments and the results of analytical solutions. The quantitative difference in results ranges within 12–20 % depending on the Reynolds number. Thus, there is reason to assert that the results of studies reported here could be the basis for devising a procedure of hydrodynamic calculation aimed at structural and operational improvement of existing and designed technological equipment

Keywords: local obstacles of various types, conditions for entering the initial section, numerical solution

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DETERMINING A FLOW STRUCTURE IN THE REGION OF LOCAL OBSTACLES OF DIFFERENT TYPES TAKING INTO ACCOUNT THE HYDRODYNAMIC CONDITIONS FOR ENTERING THE INITIAL SECTION

Serhii Nosko
Corresponding author
PhD, Associate Professor*
E-mail: noskosv@ukr.net
Dmytro Kostiuk
PhD, Senior Lecturer*
Oleksandr Haletskyi
PhD, Senior Lecturer*
Ihor Nochnichenko
PhD, Associate Professor*
*Department of Applied Hydro-Aeromechanics and Mechatronics
National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute"
Peremohy ave., 37, Kyiv, Ukraine, 03056

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1. Introduction

In many industries, there is now a need to transport working fluids through channels of technological equipment that have complex geometric shapes. The magnitudes of local obstacles can reach large values. For example, head losses in valves and bends of pipelines can be several times higher than those in straight sections of the pipeline. Studies have shown that the properties of polymer solutions in many cases are determined not only by the processing modes but also by the conditions for the passage of the latter through the channels of structural elements of equipment, which significantly affects the quality of articles. When the polymer melt flows along hydrotracts, local hydraulic losses are created, which are caused mainly by the deformation of the flow and changes in its speed and direction of flow, accompanied by the formation of vortices and stagnant zones. An important role belongs to such factors as additional pressure losses caused by destabilization of the flow in the equipment channels, which determine the power of the hydraulic drive and its inertia during automatic control.

The relevance of studies into the flow of working fluids in channels with local obstacles is due to the need to improve the structural shapes of channels and is dictated by the increase in the complexity of technological processes, as well as the need to simultaneously ensure the predefined product quality.

Thus, the analysis and evaluation of this issue introduce significant adjustments to the methodology for performing the hydrodynamic calculation of technological equipment.

2. Literature review and problem statement

Numerical integration of equations using methods of finite differences, volumes, and finite elements makes it possible to derive approximate solutions to complete equations of fluid motion with sufficiently high accuracy.

In [1], analytical and numerical studies into the flow of a viscoelastic fluid obeying the Walters B-model in the input region of a flat channel formed between two parallel plates are reported. Velocity profiles were built by using a boundary layer theory. The results show that fluid elasticity has a stabilizing effect on the development of velocity profiles. The distance of the unstable flow shifts further downstream (that is, to a fully developed section) of the channel as the Deborah number increases. The authors do not take into account the development of inertial forces in the flow, which lead to an increase in the length of the flow stabilization section and additional energy losses.

In [2], numerical simulation of laminar flows in pipes and channels was carried out. The influence of entry conditions with a rectangular shape of velocity diagrams is considered, the development of flow in radial and transverse arrangements is investigated. The results indicate that the hydrodynamic input lengths from the flow tube input simulation are significantly lower compared to the input length at flat input simulation for low Reynolds numbers. This means that it is not determined how the vortex processes take place in the input area of the channel subject to changes in entry conditions. From a practical point of view, this may lead to difficulties in calculating the hydrodynamic parameters of the forming equipment.

In [3], complete nonstationary Navier-Stokes equations in velocity-pressure variables were used, the numerical method was applied to solve the problem of fluid motion in a plane channel with a sudden narrowing. For the solution, the method of finite differences was employed. The features of the flow structure were investigated in the area of the site of sudden narrowing of the cross-section of the channel. Studies have established that the velocity, pressure fields and the length of the support zone before narrowing depend on the Reynolds number and the contraction parameter. With the help of calculations, the main circulation zone and vortex structure were identified, which were observed at Reynolds numbers greater than 400. However, it should be noted that in the cited paper, in the calculations, the assumption was made that in the input section of the channel the condition of a fully developed flow is satisfied, and the horizontal velocity has a parabolic Poiseil profile. This means that the assumption did not take into account the conditions for entering the hydrodynamic initial site. From a practical point of view, this can cause errors in determining energy losses in geometrically complex areas of the channels of technological equipment.

The authors of [4], to describe the laminar motion of an incompressible viscous fluid in a flat rectilinear channel, used nonstationary two-dimensional Navier-Stokes equations as the base. The universal discrete analog of the Navier-Stokes equations was obtained in the form of a system of linear algebraic equations, which are solved by the iterative method. The efficiency of the difference scheme and the solution algorithm are tested on the example of calculating the flow in the initial section of a flat rectilinear channel. It was shown that the velocity diagram at the entrance to the initial section of the channel has the simplest geometry in the form of a rectangle.

However, it should be noted that work [4] does not take into account the actions of inertial forces in the flow at the initial section of the channel. It is very difficult to practically obtain a rectangular velocity field profile. Since, in the initial section following any local obstacle, for example, a controlling element (throttle, flap, valve, etc.), the diagram will differ significantly from the rectangular one.

These phenomena certainly lead to additional energy losses of the main stream.

In [5], simulation of the stationary state of the fluid at low Reynolds numbers is performed. The flow through the channel with successive smooth contractions and expansions is considered. The hydrodynamic characteristics of the fluid of vortex models that develop downstream are analyzed. It is shown that elastic vortices are formed with a greater narrowing of the channel and that there are critical Weissenberg numbers for each contraction coefficient where the flow passes from non-vortex to vortex. However, it should be noted that the cited paper does not present the influence of the velocity profile on the processes of development of circulation flows. From a practical point of view, this can lead to errors in modeling unstabilized flows of viscoelastic fluid in the channels of technological equipment.

The study of the processes of flow of Newtonian fluids through channels of circular geometry is reported in work [6]. The purpose of this work was to experimentally investigate the hydrodynamic and thermal inlet lengths for laminar flows in a smooth horizontal pipe. The development of hydrodynamic nonisothermal flow under conditions of stimulated and mixed convection is shown. Two smooth round test areas with internal diameters of 4 mm and 11.5 mm and a maximum length-to-diameter ratio of 1373 and 872, respectively, were used. Differential pressure measurements were carried out at Reynolds numbers from 460 to 3200 at different heat flows. But the authors of the cited work, when calculating the hydrodynamic length of the stabilization section of a nonisothermal flow in smooth horizontal pipes, neglected the shape of the velocity profile at the inlet. This does not make it possible to estimate the change in the hydrodynamic conditions of entry into the channel on the flow structure, the redistribution of pressure along the length and, as a result, to obtain correct analytical dependences.

In [7], the results of mathematical modeling of polymer flow in the inlet channel of the extrusion head are presented. The model of viscoelastic fluid flow in the channel with a sharp narrowing, characteristic of the extrusion head, is considered. In the mathematical statement of the problem, a scheme is adopted that in the upper region of the flow the liquid has a fully developed profile, then it benters the extrusion head, characterized by a sharp narrowing of the flow. To describe the flow of an incompressible fluid, the Navier-Stokes equations and the flow continuity equation were used. Calculations were carried out using the control volume method using the modified SIMPLER algorithm. The calculations show that with a smooth transition from a wide part of the channel to a narrow part, which is an element of the extrusion head, there is a slight decrease in the circulation flow region with small stress relaxation times. The theoretical results obtained in the study are compared with the experimental results in work [8].

In work [9], the problem of input flows of viscoelastic liquids in a flat channel with narrowing coefficients of 8:1 is investigated [9]. Studies of axisymmetric nonisothermal and non-stationary flow have been carried out.

In the mathematical formulation of the problem, it is assumed that the isothermal flow of non-Newtonian fluids in narrowing channels is described by the equations of motion and continuity. The calculations were carried out using the method of control volumes in the Open Foam software environment, and a constant speed and pressure were set at the input to the channel. Studies have established the dynamics of the behavior of the circulation zone and its interaction with the angular flow at different Weissenberg numbers. It was established that during the absorption of angular circulation flow near the sharp edge of the channel, stress and pressure peaks occur, but the issue of eliminating vortex structures is not considered. Obviously, this is due to the complexity of numerical modeling of these processes using the method of control volumes. From a practical point of view, this can cause difficulties in calculating the forming channels of extrusion equipment since the most important feature of the fluid flow in such channels is the deformation of the velocity profile along its length. The movement of fluid occurs in the presence of inertial forces and additional energy losses arising from this. In this regard, these assumptions make it difficult to obtain a certain performance with given energy losses.

In all the works [1–9], finite element methods were used that allow modeling hydrodynamic processes in geometrically complex regions. However, none of the works that consider the problems of viscous and abnormally viscous fluid motion in local obstacles took into account the conditions of entry into the hydrodynamic initial section. That is, as boundary conditions in the calculations, assumptions were made that the velocity diagram at the entramce to the initial section of the channel has the simplest geometry in the form of a rectangle. Or, in the input section of the channel, the condition of a fully developed flow is satisfied, and the horizontal velocity has a parabolic Poiseil profile.

These assumptions do not sufficiently correspond to the real flow processes in the channels of technological equipment since when inertia forces manifest themselves in the flow, hydrodynamic energy losses usually increase, velocity and stress fields are significantly deformed.

In this regard, it is extremely important to know the laws of change in the hydrodynamic characteristics of the viscous rarity flow under the action of inertial forces in the initial section and the size of such a section. Given that the channel lengths used in technological equipment are relatively insignificant, determining the length of the initial section is particularly relevant.

Therefore, there is reason to believe that insufficient certainty of the influence of hydrodynamic conditions of entry into the initial section, which is located after local obstacles, necessitates research into this area.

3. The aim and objectives of the study

The aim of this study is to determine the flow structure in the area of local obstacles of various types, taking into account the hydrodynamic conditions for entering the initial section, which makes it possible to determine the size of such a section and energy loss along its length.

To accomplish the aim, the following tasks have been set:

to obtain approximate solutions to complete equations of fluid motion in the region of local obstacles using the analytical-numerical method;

– based on the results of experimental studies, establish the boundary and initial conditions for the flow of a viscous fluid and solve a system of algebraic equations for finding the function of local velocities;

– to compare the results of solving the problem with existing theoretical and experimental data and make adjustments to the estimation dependences to determine pressure losses at the initial section of the channel.

4. The study materials and methods

4. 1. The object and hypothesis of the study

The object of our study is an unstabilized flow of viscous liquids in the region of local obstacles of various types.

Simulation of inertial laminar flow of viscous incompressible fluid in channels with local obstacles was carried out using numerical integration of equations. Using finite difference methods, approximate solutions to complete fluid motion equations are obtained.

Since any model is a simplified, incomplete reflection of the original, it is advisable to check the adopted model for a reliable description of the object. Therefore, the next stage of the study is to check the adequacy of the model and the compliance of its modeled object. This was achieved by comparing the results of the study with experimental data; a good correlation was obtained.

4. 2. Investigated materials and equipment used in the experiment

The studies were conducted using the following working media: water, aqueous solutions of sodium salt of carboxymethyl-cellulose (CMC) and polyvinyl alcohol (PVA). Determination of rheological properties of model liquids was carried out on samples of CMC and PVA at concentrations of solutions from 2 % to 10 % (dry matter), at temperatures of 20, 40, 60, 80, 100 °C, and gradients of flow shear velocity 1.5–310 s^{-1}.

The determination of the systematic error of the rotary viscometer "Reotest-2" (GDR) was carried out according to the following procedure: viscosymmetric measurements of the reference liquid (technical glycerin) were carried out, the viscosity of which is known and does not depend on the shear rate.

The scheme of the experimental installation and the results of rheological tests are reported in [10].

The study was carried out on an experimental installation in the laboratory "KPI named after Igor Sikorsky». The design of the working section of the installation made it possible to investigate local obstacles of several types: sudden narrowing (sudden expansion) with narrowing coefficients of 2:1, 3:1, 4:1, and a damper type regulatory body. In addition, the design of the experimental site provided for the installation of special inserts that allow changing the conditions for entering the channel with a smaller cross section. In the experiments, we used the following angles of entry into the channel of a smaller cross section: 10°; 15°; 30°; 45°; 60°, and transitions in the form of radii (concave, convex) with respect to the flow and lemniskates. This provided different input conditions for the flow.

4. 3. Procedure for measuring the kinematic parameters of the flow

In determining the kinematic characteristics of the flow in the region of local obstacles, a visualization method was applied, which involves photographing particles-tags introduced into the stream and illuminated by the light source.

The applied method of studying secondary currents and circulation zones provides the required accuracy of measurements (about 8–10 %), has a wide information content and universality [11].

To determine the local velocities in the initial section of the channel, a laser speed meter (LSM) based on the Doppler effect was used. In this case, the procedure given in works [12, 13] was used.

The joint use of physical and computational experiments in the study of the object makes it possible, on the one hand, to reduce the number of full-scale expensive measurements, and on the other – to improve mathematical models.

5. Results of investigating the influence of hydrodynamic conditions for entering the initial section of the channel

5. 1. Obtaining approximate solutions to complete fluid motion equations in the region of local obstacles using the analytical-numerical method

It should be noted that the results of studies [14–16] showed the dependence of the length of the hydrodynamic initial section on the geometric features of the channel, the type of fluid flowing (its rheological properties), flow regime, etc.

It should be noted that the length of the initial section can be determined from the following formula:

$$L_I = \mathrm{const}\,\mathrm{Re}\,A, \tag{1}$$

or

$$L_I = \mathrm{const}\,ZA, \quad Z = A\rho u_a \frac{\dot{\gamma}}{\tau_w},$$

where Re is the Reynolds number; ρ – liquid density; γ – shear velocity; A – characteristic channel size; τ_w – shear stress at the wall; const – a constant depending on the geometric features of the channel, the rheological properties of the fluid, and the conditions of its flow in the area of entry to the channel.

Of particular importance is the fact that the use of the second formula from (1) is advisable in the study of the flow of abnormally viscous liquids. To determine *const* in equation (1) and clarify the latter for a particular channel, it is necessary to solve the corresponding equations of motion of a viscous fluid and compare the results obtained with the flow characteristics for a stabilized flow regime.

It should be noted that most researchers believe that the forces of inertia in the flow can be neglected in cases in which the difference between the velocity diagrams for the initial section and the section of the stabilized flow is less than 1 %.

Based on these statements, the flow of a viscous fluid in a flat-slit channel in the region of the hydrodynamic initial section was considered. The coordinate system shown in Fig. 1 was chosen.

The equations describing such a flow for a viscous Newtonian fluid when mass forces are neglected were written as follows:

$$\begin{cases} \upsilon_x \dfrac{\partial \upsilon_x}{\partial x} + \upsilon_y \dfrac{\partial \upsilon_x}{\partial y} = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial x} + \nu\left(\dfrac{\partial^2 \upsilon_x}{\partial x^2} + \dfrac{\partial^2 \upsilon_x}{\partial y^2}\right); \\ \upsilon_x \dfrac{\partial \upsilon_y}{\partial x} + \upsilon_y \dfrac{\partial \upsilon_y}{\partial y} = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial y} + \nu\left(\dfrac{\partial^2 \upsilon_y}{\partial x^2} + \dfrac{\partial^2 \upsilon_y}{\partial y^2}\right), \end{cases} \tag{2}$$

where υ_x, υ_y are the projections of the velocity vector onto the coordinate axis; p – hydrostatic pressure.

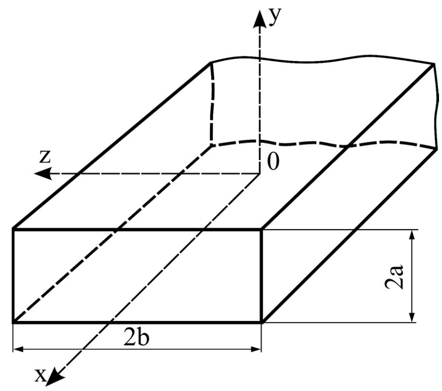

Fig. 1. Scheme for calculating a flat-slit channel in the area of the hydrodynamic initial section

It should be noted that solving the system of equations (2) analytically for the initial section is very difficult. Usually, such equations are solved using numerical methods [17, 18].

In this case, system (2), by appropriate transformations (differentiation of the first equation by y and the second by x and subtraction from one another), was written as:

$$\begin{cases} \upsilon_x\left(\dfrac{\partial^2 \upsilon_x}{\partial x \partial y} - \dfrac{\partial^2 \upsilon_y}{\partial x^2}\right) + \upsilon_y\left(\dfrac{\partial^2 \upsilon_x}{\partial y^2} - \dfrac{\partial^2 \upsilon_y}{\partial y \partial x}\right) = \\ = \dfrac{\partial}{\partial y}\nu\nabla^2\upsilon_x - \dfrac{\partial}{\partial x}\nu\nabla^2\upsilon_y; \\ \upsilon_y = -\displaystyle\int \dfrac{\partial \upsilon_x}{\partial x}\partial y. \end{cases} \tag{3}$$

It should be noted that if the value of υ_y, determined from the second equation of system (3), is substituted into the first equation, then the equations in question were reduced to one differential equation with respect to $_x$:

$$\upsilon_x\left(\frac{\partial^2 \upsilon_x}{\partial x \partial y} + \int_0^a \frac{\partial^3 \upsilon_x}{\partial x}\mathrm{d}y\right) - \left(\frac{\partial^2 \upsilon_x}{\partial y^2} + \frac{\partial^2 \upsilon_x}{\partial x^2}\right)\int_0^n \frac{\partial \upsilon_x}{\partial x}\mathrm{d}y =$$
$$= \nu\left(2\frac{\partial^2 \upsilon_x}{\partial x^2 \partial y} + \frac{\partial^3 \upsilon_x}{\partial y^3} + \int_0^a \frac{\partial^4 \upsilon_x}{\partial x^4}\mathrm{d}y\right) + \left(\frac{\partial^2 \upsilon_x}{\partial x^2} + \frac{\partial^2 \upsilon_x}{\partial y^2}\right)\frac{\partial \upsilon_x}{\partial y}, \tag{4}$$

in which υ_x is a function of x and y coordinates.

For further transformations of equation (4), we introduce the following dimensionless variables:

$$\xi = x/a; \;\; \eta = y/a; \;\; u = \upsilon_x/\upsilon_{\max}, \tag{5}$$

where a – half the height of the channel (Fig. 1); $\upsilon_{\max}$ – velocity on the channel axis at the end of the initial section.

Taking into account notation (5), equation (4) took the following form:

$$u\left(\frac{\partial^2 u}{\partial \xi \partial \eta} + \int_0^1 \frac{\partial^3 u}{\partial \xi^3}\mathrm{d}\eta\right) - \left(\frac{\partial^2 u}{\partial \eta^2} + \frac{\partial^2 u}{\partial \xi^2}\right)\int_0^1 \frac{\partial u}{\partial \xi}\mathrm{d}\eta =$$
$$= \frac{1}{\mathrm{Re}}\left(2\frac{\partial^3 u}{\partial \xi^2 \partial \eta} + \frac{\partial^3 u}{\partial \eta^3} + \int_0^1 \frac{\partial^4 u}{\partial \xi^4}\mathrm{d}\eta\right). \tag{6}$$

Based on these results, considering the above transformations, the system of equations (2) took the form of a nonlinear integral-differential equation with respect to the function $u(\xi, \eta)$.

5. 2. Determining the boundary and initial conditions and solving a system of algebraic equations to find the function of local velocities

The boundary conditions for the considered case of flow in the initial section should take into account the conditions of fluid movement when entering the initial section and at the end of it, as well as the conditions for adhesion of the liquid to the walls of the channel.

It should be noted that in most cases, the velocity diagram at the entrance to the initial section is taken rectangular [19–22]. This assumption negatively affects the results of calculations. At the same time, it is very difficult to practically obtain a rectangular speed pattern at the entrance to the channel. Most often, the diagram at the entrance is different from the rectangular one. For example, in the initial section, which is located after any local obstacle (valve, valve, flap, choke), the diagram may differ significantly from the rectangular one. To prove this statement, possible forms of velocity diagrams at the entrance to the initial section were considered (Fig. 2), which are given in [23, 24].

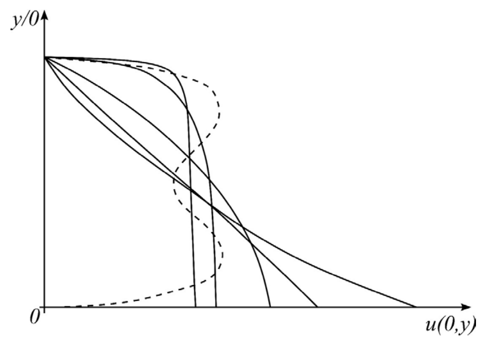

Fig. 2. Possible forms of velocity diagrams at the entrance to the initial section of the flat-slit channel

In order to take into account these conditions, it was believed that at the entrance to the initial section of the flat-slit channel, the velocity diagram was described by the following expression:

$$u(0,h)-\frac{2}{3}\frac{n+1}{n}\left(1-|\eta|^{n}\right), \tag{7}$$

where n is an indicator of the power that takes different values depending on the characteristics of the fluid flow before the initial section.

Such a representation of the diagram at the input made it possible to consider a rectangular diagram as a special case of expression (7), when $n\to\infty$. At $n=2$, formula (7) takes the form of a quadratic parabola; in this case, $L_I=0$, and the inertia forces in the flow do not work.

Formula (7) can describe velocity diagrams at the entrance to the initial section, which is located behind the local obstacle of the type of sudden narrowing. Behind the damper-type controlling element, the velocity diagram at the entrance to the initial section was most often described by an equation of the following form:

$$\upsilon_x = a_1\sin\left(my+B_1\right)+a_2\sin\left(2my+B_2\right)+\ldots+$$
$$+b_1\cos\left(my+B_1\right)+b_2\cos\left(my+B_2\right)+\ldots, \tag{8}$$

where a_1, a_2, ..., b_1, b_2, ..., m, B_1, B_2 are the parameters corresponding to the entry conditions, or by the followingdependences:

$$\upsilon_x = a_1\sin y + a_2\sin my. \tag{9}$$

There are cases when the velocity diagram in the input section of the initial section has an asymmetrical appearance or the appearance of an incompletely developed parabola, as shown in Fig. 2.

The boundary conditions corresponding to the velocity distribution at the end of the original section were expressed as follows:

$$u\left(l,\eta\right)=1-y^2, \tag{10}$$

where $l=L_I/a$.

For all speed values on the channel wall:

$$u\left(\xi,1\right)=0. \tag{11}$$

To solve differential equations, integrals were represented in terms of corresponding sums, and derivatives in the form of differences. Then equation (4) was written in finite differences through a system of algebraic equations KM by unknowns, where K, M are the number of nodes in the grid ξ and η, respectively.

Thus, equation (4) was replaced by a system of the following algebraic equations:

$$u_K^M\left[\frac{\delta^2 u_{\xi\xi}}{\Delta\eta\Delta\xi}-\frac{1}{2}\frac{\Delta\eta}{\Delta\xi^3}\sum_{K=1}^{K}\left(\delta^3 u_{\xi\xi\xi}^K+\delta^3 u_{\xi\xi\xi}^{K+1}\right)\right]+$$
$$+\left[\frac{\delta^2 u_{\eta\eta}}{\Delta\eta^2}+\frac{\delta^2 u_{\xi\xi}}{\Delta\xi^2}\right]\frac{1}{2}\frac{\Delta\eta}{\Delta\xi}\sum_{K=1}^{K}\left(\delta^2 u_{\xi\xi}^K+\delta^2 u_{\xi\xi}^{K+1}\right)=$$
$$=\frac{1}{\mathrm{Re}}\left[\frac{\delta^3 u_{\xi\xi\eta}}{\Delta\xi^2\Delta\eta}+\frac{\delta^3 u_{\eta\eta\eta}}{\Delta\eta^3}-\frac{\Delta\eta}{\Delta\xi^4}\sum_{K=1}^{K}\left(\delta^4 u_{\xi\xi\xi\xi}^K+\delta^4 u_{\xi\xi\xi\xi}^{K+1}\right)\right]. \tag{12}$$

After multiplying the left and right parts of expression (12) by $\Delta\xi^3$, and simple transformations, we built a dependence of the following form:

$$u_K^M=-\frac{B}{A}+\frac{1}{\mathrm{Re}}\frac{C}{A}, \tag{13}$$

where

$$A=\frac{\Delta\xi}{\Delta\eta^2}\delta^2 u_{\eta\xi}-\frac{1}{2}\sum_{K=1}^{K}\left(\delta^3 u_{\xi\xi\xi}^K+\delta^3 u_{\xi\xi\xi}^{K+1}\right);$$

$$B=\left(\frac{\Delta\xi}{\Delta\eta^2}\delta^2 u_{\eta\eta}+\frac{\Delta\eta}{\Delta\xi}\delta^2 u_{\xi\xi}\right)\sum_{K=1}^{K}\left(\delta^2 u_{\xi\xi}^K+\delta^2 u_{\xi\xi}^{K+1}\right);$$

$$C=2\left(\frac{\Delta\xi}{\Delta\eta}\delta^2 u_{\eta\eta}+\frac{\Delta\eta}{\Delta\xi}\delta^2 u_{\xi\xi}\right)\sum_{K=1}^{K}\left(\delta^4 u_{\xi\xi\xi\xi}^K+\delta^4 u_{\xi\xi\xi\xi}^{K+1}\right);$$

$$\delta^2 u_{\xi\eta}=u_{K+1}^{M+1}-u_{K-1}^{M+1}-u_{K-1}^{K+1}+u_{K-1}^{M-1};$$

$$\delta^2 u_{\xi\xi}=u_{K+1}^{M}-2u_K^M+u_{K-1}^{M};$$

$$\delta^2 u_{\eta\eta}=u_K^{M+1}-2u_K^M+u_K^{M-1};$$

$$\delta^3 u_{\xi\xi\eta}=u_{K+1}^{M+1}-u_{K+1}^{M}+2u\left(u_K^{M+1}-u_K^M\right)+u_{K-1}^{M+1}-u_{K-1}^{M};$$

$$\delta^3 u_{\xi\xi\xi} = u^M_{K+1} - 3u^M_{K+1} + 3u^M_K - u^M_{K-1};$$

$$\delta^3 u_{\xi\xi\xi\xi} = u^M_{K+2} - 4u^M_{K+1} + 6u^M_K - 4u^M_{K-1} + u^M_{K-2}.$$

Using the iterative method, the local value of the velocity was determined based on equation (12):

$$u^\alpha_K = \alpha\left(-\frac{B}{A} + \frac{1}{\mathrm{Re}}\frac{C}{A}\right) + (1-\alpha)u^M_K, \qquad (14)$$

where α is the relaxation parameter.

The above scheme of solutions to the equations of motion of a viscous fluid (12) to(14) and local velocities in the initial section were calculated using numerical modeling.

The initial and boundary conditions were approximated by the following expression:

$$u^M_1 = \frac{2}{3}n + \frac{1}{n}\left[1 - (M\Delta\eta)^n\right];$$

$$u^M_K = 0; \qquad (15)$$

$$u^M_K = 1 - (M\Delta\eta)^2.$$

5. 3. Comparing the effectiveness of the devised method for solving the problem with existing data and making adjustments to the calculation results

According to the results of our calculations, a picture of the flow of a viscous fluid was obtained, with local obstacles such as sudden narrowing and the type of flap; Fig. 3.

Table 1

Value of the indicator of power n in formula (7), from different sources

n	Const	Source
$n\to\infty$	0.113	[19]
$n\to\infty$	0.111	[20]
2.5	0.050	Our data
3.0	0.080	
3.5	0.102	
4.0	0.113	
4.5	0.122	
5.0	0.132	
5.5	0.133	
6.0	0.140	
7.0	0.144	
$\to\infty$ (10^6)	0.150	

For the region of change of the parameter n ($2 \leq n \leq 10^6$), the current in the flow core is accelerated and the formation of a velocity profile in the hydrodynamic stabilization region is associated with more significant pressure energy losses than those observed in the stabilized flow zone (at $n=2$). That is, the dependences for determining pressure losses along the length of the initial section need to be adjusted.

The length of the hydrodynamic initial section was determined on the basis of the following dependence:

$$\left|u^M_K - \left(1 - M\Delta\eta^2\right)\right| \leq 0.01. \qquad (16)$$

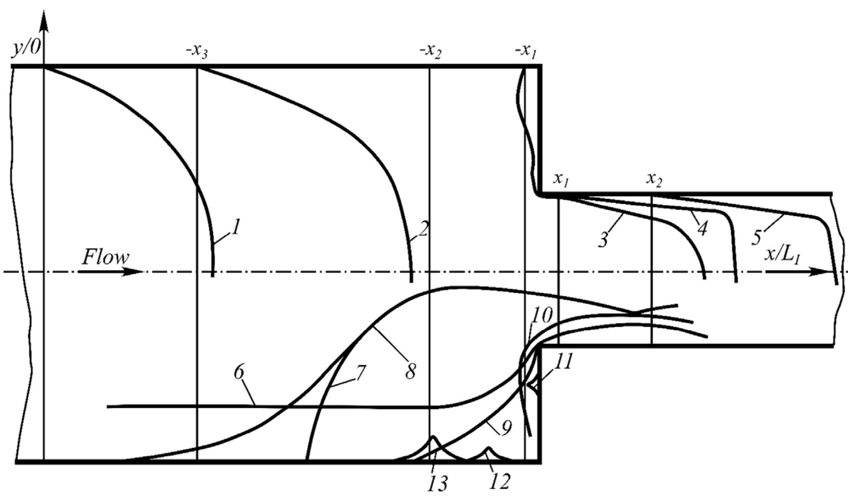

Fig. 3. Model of viscous fluid flow with a local obstacle of the type of sudden narrowing and type of damper: 1–5 – flow curves at Re=1200; 6–10 – current lines at Re=500–1500; 11–13 – areas of stagnant zones at Re=1200; x_1=0,0027; x_2=0,0171; x_1=0,0025; x_2=0,0143; x_3=0,048

Fig. 3 shows the curves of the viscous fluid flow with a local obstacle, which are directly located near the entrance to the initial section. Further, along the current, near-boundary layers develop near the walls. With distance from the input section, the horizontal velocity profile is gradually transformed into a parabola, which is formed at the end of the stabilization section as a result of the connection of the near-boundary layers. The evolution length of a velocity profile is determined from dependence (16). Figure 3 also shows that in the angular region of sudden narrowing, splitting of fluid elements occurs, one of which accelerates, merging with the main flow and falls into a channel with a smaller cross-section. The second volume of liquid loses its speed and enters the junction zone of the side and end walls of the channel, as shown in Fig. 3 (stagnant and vortex zones 11–13).

Shown in Fig. 3, design profiles correspond to different distances of cross sections from the entrance to the channel.

Our results of the structure of viscous fluid flow with a local obstacle of the type of sudden narrowing and the type of damper make it possible to construct estimation dependences for determining pressure losses in the working channels of equipment.

Table 1 gives the range of change of the indicator n in (7), which makes it possible to obtain various forms of speed profiles at the entrance to the initial section of the channel.

Calculations at numbers Re corresponding to the laminar flow regime ($50 \leq Re \leq 2\cdot10^3$) and different values of n (Table 1) showed a significant influence of the conditions of entry into the channel on the degree of manifestation of inertia forces in its initial section and the length of this section. It is shown that our research data provide fairly accurate information about the evolution of vortex structures in the region of local obstacles.

It has been established that the manifestation of the anomaly of viscosity of media significantly affects the processes of emergence and development of vortex structures.

When the flow index in the power rheological equation decreases from 0.967 to 0.652, the dimensions of the main vortex A decrease by 38–40 %. Thus, there is reason to assert that vortex stagnant zones are a negative phenomenon for extrusion processes. Circulation flows lead to stress peaks, pulsations of pressure in the flow, disruption of heat and mass transfer processes and, as a result, to the deterioration of the quality of the resulting articles. This indicates that our studies have applied importance and can be the basis for the development of a procedure for the hydrodynamic calculation of existing and designed technological equipment.

6. Discussion of results of the influence of hydrodynamic conditions for entering the initial section of the channel

When modeling an unstabilized flow in a hydrodynamic initial section, a system of differential equations describing such a flow, after appropriate transformations, reduces to one differential equation regarding the projection of the velocity vector onto the longitudinal coordinate. An important criterion of the computational scheme for studying the flows of a viscous fluid that is not compressed is the efficiency of taking into account the hydrodynamic conditions of entry into the initial section, which is located behind local obstacles of various types and regulating equipment – formula (1).

The solution to the obtained system of difference algebraic equations is derived by known iterative methods. A characteristic feature of flows in the channels after local obstacles is that inertia forces are present in the flow and the longitudinal pressure drop is not constant. In addition, at the exit from the calculated section, boundary conditions are assumed that the horizontal velocity distribution corresponds to the known Poiseil profile.

The results of our calculations of local velocities, based on the solution to equation (13), showed that in the region near the entrance to the initial section they have different velocity profiles from the rectangular shape – curves 1–5 in Fig. 3.

As expected, in the angular region of sudden narrowing, splitting of fluid elements occurs (stagnant and vortex zones 11–13). The main vortex 12 begins to form at $Re>100$, but a distinct axis of rotation of the vortex occurs at $Re>300$. The growth mode of the main vortex A occurs up to a certain value of the Reynolds number, depending on the rheological properties of the fluid, that is, the viscosity forces exceed the inertia forces in this zone.

Further, as the Reynolds criterion ($Re>1010$) increases, a decrease in the intensity of the vortex region, a decrease in its size, and some displacement of the axis of rotation in the direction of the side wall of the channel are observed. When the number Re reaches the value of 1460–1480 (for this sudden narrowing), the main vortex is disconnected and stagnant zone 13 is formed (Fig. 3). When the value of the number Re>1650 is reached, a pulsation of the vortex zone occurs relative to the longitudinal and transverse axis of the channel and the reverse flow along the bottom of the channel.

Thus, the analysis of the real flow picture in the region of sudden narrowing with narrowing coefficients of 2:1, 3:1, and 4:1 for viscous liquids give the right to assert that the pattern of formation of vortex zones (Fig. 3) have the same tendency and differ in quantitative factors.

The presence of the above zones clearly illustrates their significant influence on the formation of a high-speed field at the entrance to the initial section.

Our results of the influence of hydrodynamic conditions of entrance to the initial site are well consistent with experimental data [11, 22] and the results of analytical solutions in [23].

Numerical calculations have shown that in the approximate area from the entrance to the channel, the speed profile depends on the entry conditions, as shown in Fig. 3, and is described by dependence (7). The criterion for the length of the hydrodynamic stabilization section is the condition under which the value of the horizontal velocity is determined by dependence (10), and near-boundary layers develop near the walls and conditions (11) are valid for all velocity values on the channel wall.

The estimated length of the initial section was determined at different Reynolds numbers based on dependence (16) for Re numbers corresponding to the laminar flow regime and different values of n (Table 1). These data showed a significant influence of the conditions of entry into the channel on the degree of manifestation of inertia forces in its initial section and the length of this section.

With distance from the input section of the channel, the velocity profile is gradually transformed into a parabola, which is realized at the end of the hydrodynamic stabilization region L_I determined by formula (1). The length of the initial section was compared with similar calculations obtained by the analytical method and experimental data given in [22, 23].

In general, our results of calculations of velocity fields in the region of sudden narrowings of the cross-section of the plane channel agree well with known ideas of the flow pattern, which is observed in physical experiments and the results of analytical solutions given in [19, 23].

Our results of the influence of hydrodynamic conditions for entering the initial section of the channel can be used at the design stage of working channels for technological machines in order to develop scientifically based recommendations.

The disadvantages of our studies include the quantitative difference between the results of the dynamic behavior of the circulation zone and its interaction with the angular flow at different Weissenberg numbers, which can be explained by the choice in the above works [7–9] of the rheological model characterizing the elastic properties of the fluid.

The given method has shown its effectiveness in modeling unstabilized viscous fluid flows in the channels of technological equipment that have complex geometric shapes and has applied value, in particular in chemical engineering in the process of production of viscose fibers. The organization of input hydrodynamic conditions by profiling channels in the apparatus of precipitation and plasticization baths made it possible to increase the productivity of the technological process with reduced energy consumption.

Due to the fact that these experiments used viscous and anomalous viscous media of various concentrations and the flow index of the "power" rheological law varied between 1–0.86, in the future it is advisable to continue research, based on this method, into more complex problems in which there is an unstabilized flow of non-Newtonian fluids with viscoelastic properties.

To expand the scope of application of the proposed solutions, it is necessary to improve the mathematical model of the flow at the initial section of the channels. Under constant initial and boundary conditions adopted in this paper, an improved mathematical model should take into account the peculiarities of the flow of solutions and melts of polymers. A mathematical model should be constructed on the basis of rheological equations of state of non-Newtonian media describing different types of viscoelastic fluids.

7. Conclusions

1. Approximate solutions to the system of complete equations of inertial motion of fluid in the initial section have been obtained using the analytical-numerical method. The defined universal analog of complete equations of motion took the form of a nonlinear integral-differential equation with respect to the desired function, the value of which was found by the iterative method.

2. Based on the results of experimental studies, initial conditions were accepted that correspond to the real picture of the flow in the field of local obstacles. Using the iterative method, the values of local velocities are determined. It was established that the flow pattern in the region of local obstacles of different types has a complex flow structure with vortex regions and stagnant zones, the intensity of which depends on the shape of the velocity diagram and the Reynolds number. Given this, it can be argued that the hydrodynamic input conditions significantly affect the process of development of the velocity field in the initial section of the channel. This is manifested in additional energy losses associated with the manifestation of inertia forces in the area of hydrodynamic stabilization of the flow.

3. It was established that the assumption about the rectangular shape of the velocity diagram at the entrance to the hydrodynamic initial section does not correspond to the flow pattern in real channels of technological equipment. These assumptions are not sufficiently substantiated, which leads to significant quantitative discrepancies with experimental data.

Our results are consistent with experimental data and indicate that if hydrodynamic and entry conditions to the initial section are not taken into account, the error in determining pressure losses can be in the range of 21–42 %, depending on the Reynolds number and the rheological properties of the fluid.

It is shown that changes in the conditions of fluid entry into a narrow part of channels affect the flow structure and pressure redistribution along the length of the hydrodynamic initial section. Therefore, the estimation dependences should include the pressure drop at the stabilized flow section and additional energy losses associated with the inertial flow structure at the initial section.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

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Data availability

All data are available in the main text of the manuscript.

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