

The object of the study is to evaluate the quality of the frequency distribution of aircraft, which characterizes the effectiveness of radar surveillance of aircraft and determines the effectiveness of their control using radio signals. The frequency resolution of an aircraft is usually studied using the frequency ambiguity function for a coherent packet of radio pulses. However, there is a problem of estimating phase fluctuations, which is caused by the heterogeneity of the propagation of radio pulses, which affects the functioning of radar stations under different atmospheric conditions. A feature of the study is the development of theoretical provisions for the process of detection and radio control of single aircraft under the organized action of swarms. A normalized frequency ambiguity function is obtained, which takes into account the transformations caused by the radial motion of the aircraft. The calculations made it possible to estimate the range of changes in the frequency distribution under the condition of the additive effect of the internal noise of the radar receiver and the multiplicative effect of the cartelized phase fluctuations of the control radio signal. The statistical characteristics of phase fluctuations of radio pulses were obtained, under which their influence on the operation of radio technical control and radar systems is the most significant. Such statistical characteristics are important for the theory of radar location and of practical importance for the improvement of radio control of objects. Method is proposed for numerical evaluation of the influence of atmospheric disturbances on the frequency distribution function of aircraft during flight. This method is a convenient tool for analyzing the quality of the frequency distribution of a radar station in various conditions of radar surveillance of single aircraft in their organized swarm action

Keywords: *detection and radio control of individual aircraft, frequency uncertainty function, resolution*

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METHOD OF ASSESSMENT OF FREQUENCY RESOLUTION FOR AIRCRAFT

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1. Introduction

High-quality resolution of radar signals for reliable observation of aircraft is among the main requirements for

modern radar system [1]. In addition, greatly demanded today became the enhancement of signal resolution for the operation by aircraft. First of all, the latter concerns unmanned aerial vehicles (UAV) and especially their control

in swarms. The sufficient resolution of radar signals implies the capability of well-resolved detection and measurement of the informative parameters for signals reflected from aircraft with close coordinates and motion parameters. Because of its complexity, the classical radar mission, traditionally, has been reduced to the decision-making on the presence/absence of the reflected signal within an adopted concept [2]. However, nowadays, when new radar technologies such as Synthetic Aperture Radar (SAR), Inverse SAR (ISAR), Interferometric SAR (InSAR) have emerged, with which a 3D image of an object can be synthesized [1, 3, 4]. The advent of powerful computing tools makes it possible to expand the solution of the main problem of radar through new applications. Thus, new opportunities open up for taking into account additional important factors that were traditionally not taken into account due to their complexity of calculations. In particular, velocity measurements have become possible by exploiting the Doppler effect [5] with the advent of coherent pulse radar as well as target detection and localization measuring azimuth and elevation angles with specialized radars [4]. The emergence of new capabilities associated with advances in hardware leads to the need to develop new software concepts for radar systems. For example, machine learning as a new trend in radar technologies allows improving the performance of traditional signal processing methods [6], especially for complex, but more efficient, cognitive radar systems [7].

In turn, novel hardware and software trends demand the development of the theory of radiolocation as the basis for their application. Along with modern theoretical directions such as Radar Waveform Optimization in various domains: spectral, spatial, temporal, and polarization [8], still there are fundamental problems of purely academic character which have to be solved to improve the performance of modern radar systems. In particular, in real radar observations, detection is carried out by a series of reflected signals by processing a packet of radio pulses. Accounting for changes in the coordinates and parameters of an object in the place of its detection is one of such promising tasks that deserve to be solved and the result of which should be implemented when making a decision on the detection of an object.

Another fundamental problem of radar theory that deserves its development is related to the complexity of the object motion. On one hand, an object under detection (a target) *per se* might have a complex structure with its flexible body changing shape during the motion and thus introducing the motion component called micro-motion, thereby involving vibration, rotation, and acceleration of the structural elements such as rotating propellers of an aircraft or rotor blades of a helicopter, rotating antenna, engine-generated vibration in a ship or vehicle, but not only, extending to the motion of the organs of living bodies (human, animals, birds, etc.) [9]. On the other hand, modern aircraft as wholes are capable of changing their radial velocities within a wide range. The capability of aircraft to perform sudden and complex maneuvers implies their movement with radial accelerations and consequently imposes the need to use higher derivatives to characterize their motion. This significantly complicates the problem of resolution in terms of the radial velocity of the aircraft. The coherent packet of radio pulses is widely used in pulse radars to provide high radial velocity resolution. The radial velocity resolution of aircraft is based on the Doppler frequency resolution of radar signals and can be studied using the frequency ambiguity function for a coherent burst of radio pulses. It should be noted, that

available in the literature expressions for the frequency ambiguity function for a coherent packet of radio pulses are inconvenient for practical calculations and do not take into account the impact of the radial motion of the radar object on the transformation of the ambiguity function.

Therefore, there is a great need to derive the frequency ambiguity functions for a coherent packet of radio pulses in a handy form, taking into account the transformation due to the radial motion of the aircraft. Accounting for the additive and multiplicative influence of the real conditions affecting the quality of resolution would be yet another contribution to the theory of radiolocation and is of practical importance regarding detection and radio control of single aircraft and their swarms.

2. Literature data analysis and problem formulation

Current study on the resolution of aircraft and the interference effects affecting radar signal processing is a significant part of the literature on radar theory. Characteristics of electronic systems designed to solve the problem of resolution of aerodynamic and ballistic objects at radar observations are considered in [10]. It should be noted that mathematical expressions for the frequency ambiguity function for main types of radar signals, in particular, for a coherent packet of radio pulses usually are presented in forms that are too general. Thus, these mathematical expressions are somewhat inconvenient for calculating the frequency ambiguity and practical implementation. Function for particular cases valuable for practical application. However, the most important problem is that the available expressions for the frequency ambiguity function do not account for the transformation of the time scale of the radar signal due to the radial motion of the radar object. Therefore, there is a need to derive new relationships that take into account the given transformation of the time scale and provide a separate analysis of the frequency ambiguity function between the main and secondary maxima.

The principles of construction of appropriate processing devices for modern radars can be found in [11]. Specifics of coordinated radar signal processing during its multi-channel reception are considered in [12]. Traditionally, radar signal processing accounts for the internal noise within the receiver, assuming that it is of an additive nature [11–14]. In particular, mathematical derivations of the features of radar signal processing reduced to additive interference are considered in [13]. However, in real conditions, the radar operates under the multiplicative influence of the environment. The latter leads to correlated phase fluctuations of the radar signal, which significantly reduce the quality of the frequency resolution of the radar signals. Statistical analysis of radar signal parameters regarding the tasks of maintenance inspection and control of radio devices is considered in [14]. However, the results presented in [14] are not applicable to the task of radar resolution by the radial velocity of aircraft.

Atmospheric inhomogeneity implies fluctuations of the refractive index for radio waves, leads to phase distortions of the radar signal and significantly complicates the operation of the radar. Employment of modern systems dealing with radio waves under atmospheric disturbances is considered in [15]. According to [16] the atmospheric impact is enhanced by wind activity. Therefore, the problem of radar signal resolution should be extended to the models accounting for the atmospheric inhomogeneity. The mathematical approaches to the solution of such problems are analyzed in [17].

The task of radar resolution is of special importance for radars monitoring inconspicuous UAVs, which are capable of acting in swarms. The corresponding substantiation of the radio wave characteristics, which are important at the consideration of the resolution for high-speed aircraft, was performed in [18].

Expressions accounting for the transformation of spectral and temporal representations for single and packet radio signals reflected from an aircraft that moves with the acceleration have been obtained in [19]. Practical application of this transformation to the assessment of the quality indicators of detection for moving objects is considered in [20], taking into account the differences in time scales for oscillations and their complex envelope. The results of these studies might also be useful when solving the problem of frequency resolution of signals. However, the results presented in [20] concern the transformation of a signal in an ionized environment of radio wave propagation and are of practical interest only for space location facilities. It should also be noted that the model [20] does not take into account the influence of fluctuations on the transformation of the phase structure of a packet radio signal and is not effective for the practical application of radar systems.

The influence of phase fluctuations caused by the atmospheric instability is considered in the study of modern methods of signal processing [21] and the specifics of its implementation in Doppler measurement systems is analyzed in [22]. In such a case, a necessary requirement is to define the appropriate assumptions about the statistical characteristics of correlated phase fluctuations. Such assumptions concerning the normal distribution law and the oscillating form of the correlation function of phase fluctuations supported by their experimental confirmation are given in [23]. Numerical analysis of the accuracy of measuring the radial velocity of an aerodynamic object for cases of exponential and alternating laws of change of correlation of phase fluctuations of radio pulses of the received packet was performed in [24]. Superposition of the internal noise of receiving device and correlated phase fluctuations of radio pulses of the received packet with oscillating correlation function was considered in [25]. Calculations of root mean square errors of Doppler frequency measurement have shown that for modern coherent-pulse radars, such errors for packets of (8...16) radio pulses in the troposphere can be (67.1...95.5) Hz, and (7.8...11.3) Hz in the ionosphere. These results can be used as the starting point for further calculation of the fluctuation components of the frequency resolution measure for the coherent packet of radio pulses.

In addition to the mentioned factors, the coherence of the received radio signal might be influenced by the complexity of the shape of the aircraft as well as by the interference of direct radio waves and those reflected from the interface, thereby leading to the Doppler noise. These phenomena cause the occurrence of additional fluctuations of the phases of the radio pulses of the received packet and correspondingly the additional transformation of its ambiguity function. However, these phenomena as a rule are irregular being of unstable nature and thus have a weaker impact on the resolution of radar signals in comparison with the permanent impact of the environment on wave propagation. Though the references alluded to above consider important factors affecting the frequency ambiguity function, which have recently attracted the attention of the researchers, to our best knowledge, there are no reports in the literature considering the transformation of the frequency ambiguity function due to the radial motion of the radar object. Also, in the literature, there is no consideration of the impact

of statistical characteristics of phase fluctuations on the radar frequency resolution. The inconveniences of the available methods, alluded to above, encourage to derive expressions that will be handy for the calculations of the normalized frequency ambiguity function for a coherent pulse and account for its transformation due to the radial motion of the object. In this paper, let's also study the effect of phase fluctuations of the radio pulses of the packet on the degree of frequency resolution. In our opinion, it is also important to perform the assessment of the degree of frequency resolution in the expected ranges of variation of statistical characteristics of phase fluctuations of the radar signal, which exceeds the values acceptable for the reliable operation of coherent-pulse radars.

In work [26] a generalized signal model is presented to accommodate both narrowband and wideband signals in a multi-input multi-output (MIMO) sensor system scenarios. The proposed formulation is parameterized using the signal and channel correlation matrices to account for different waveform and sensor placement designs, thereby allowing a flexible modelling approach. However, the work does not evaluate the errors of the signal parameters, which are determined by the medium of propagation of radio waves and the shape of the aircraft (radar object). And the values of such errors can make a significant contribution to the quality of determining the flight parameters of highly maneuverable aircraft. A feature of the presented article is consideration of the issue of taking into account the transformation of the frequency function due to the radial movement of the aircraft. The relevance of taking into account the phase fluctuations of radio pulses of the beam to the measure of resolution in terms of aircraft frequency is substantiated.

In the presented work, the problem of radar resolution of signals by the Doppler frequency for a coherent packet of radar pulses, which is the probing signal of coherent-pulse radars, is considered. Typically, coherent pulsed radars, which detect and track aircraft, work in the centimeter wavelength range (3 cm to 10 cm). The pulse repetition frequency (PRF) in coherent-pulse surveillance-type radars can vary from 700 Hz to 1700 Hz. Coherent-pulse tracking radars are capable of auto-tracking by range and radial speed. Such radars use quasi-continuous probing signals with the PRF from 20 to 100 kHz [27, 28]. Regarding the origin of phase fluctuations, it should be noted that the main contributions come from the inhomogeneity of radio waves propagation, affecting the operation of radar practically at any atmospheric conditions. When the radar signal propagates in the turbulent troposphere, there are significant fluctuations in its refractive index, which, in turn, is the origin of phase fluctuations. According to [15, 23–25], the corresponding values of the dispersion of the signal phase fluctuations for radar wavelengths from 3 cm to 1 m can vary from 0.0044 rad^2 to 19.4 rad^2 . Respectively, for the centimeter-wave range, the corresponding values of the dispersion can reach values of up to 10 rad^2 or even more. The correlation interval of phase fluctuations can be as large as $\tau=(0.1...1) \text{ s}$. To analyze the effect of statistical characteristics of phase fluctuations on the degree of frequency resolution, let's calculate the root mean square error for the frequency measured for the packet of radio pulses in the presence of phase fluctuations, following [24]. Using this method, one can determine the degree of broadening of the main maximum of the normalized ambiguity frequency function and estimate the decrease in the degree of frequency resolution. The analysis of the multiplicative influence alluded to above will be performed in function of possible values of statistical characteristics of phase fluctuations

discussed in [25]. To assess the degree of frequency resolution let's follow the approach developed in [27] for the determination of the degree of angular resolution at the combined effect of random fluctuations of the received wavefront (multiplicative interference) and additive noise oscillations.

Thus, the problem of assessing the quality of radio frequency resolution for aircraft today requires a solution. An important problem is that the available expressions for the frequency ambiguity function do not take into account the transformation of the time scale of the radar signal due to the radial motion of the radar object. Also, the inhomogeneity of the atmosphere is not taken into account, which affects the fluctuations of the refractive index of radio waves and leads to phase distortions of the radar signal, which significantly complicates the operation of the radar. The identified problem of radar is relevant in the practical detection or tracking of individual aircraft in groups (for example, individual aircraft or unmanned aerial vehicles when working as part of a swarm) and needs to be addressed.

3. The aim and objects of the study

The aim of this study is to develop a method for assessing the quality of RF resolution for aircraft. This will make it possible to form theoretical provisions for the process of detection and radio control of single aircraft during their organized operation in swarms.

To achieve the purpose of the work, it is necessary to solve the following tasks:

- to perform derivation of the normalized frequency ambiguity function;
- to evaluate the decrease in the degree of frequency resolution, which occurs due to the influence of additive noise and multiplicative interference.

4. Materials and methods of research

The object of the presented study is to assess the quality of the frequency distribution of aircraft, which characterizes the effectiveness of radar surveillance of aircraft and determines the effectiveness of their control using radio signals.

When developing a method for estimating the separation power of aircraft, the inhomogeneity of the atmosphere is taken into account, which leads to fluctuations in the refractive index of radio waves and to phase distortions of the radar signal and significantly complicates the operation of the radar. During the study, an assumption was made about the absence of internal noise during the generation of a radar signal, which can introduce additional distortions into the process of separating aircraft by frequency.

A normalized frequency ambiguity function for a coherent packet of radio pulses is derived in a convenient form, taking into account the transformation of its time scale due to the radial motion of an object. In addition, the total additive effect of uncorrelated internal noise of the receiver and the multiplicative effect of correlated phase fluctuations of radio pulse packets on the decrease in the degree of frequency resolution were estimated.

The motion of an aircraft implies consideration of variations in the whole frequency spectrum of the signal. The transformation of the parameters of the signal spectrum, including the carrier frequency, is of the form:

$$f_s = \eta f, \tag{1}$$

where f_s and f are the frequencies of the received and emitted signals; respectively, and η is the time scale factor of the received signal relative to the transmitted signal. For the transformation of the scale of the radar signal caused by the radial motion of the aircraft, the parameter η is of the form [10]:

$$\eta = \frac{1 - \frac{V_r}{c}}{1 + \frac{V_r}{c}}, \tag{2}$$

where V_r is the radial speed of the aircraft; c is the speed of propagation of the electromagnetic wave.

Taking into account the relationship between the radial speed of the object under radar observation and the Doppler shift of the frequency of the received signal [10, 19, 20, 28], the transformation parameter (2) can be rewritten in the form:

$$\eta = \frac{f_0 - \Delta F_D}{f_0}, \tag{3}$$

where f_0 is the signal carrier frequency, $\Delta F_D = f_s - f_0$ is the Doppler frequency shift.

According to such frequency variation, the time parameters of the signal change as well; periods of oscillation, the duration of the signal, time intervals, law of signal modulation are among these parameters. In addition, there are changes in the amplitude of the signal, which are associated with the energy preservation due to the stretching/compression of the signal and loss (gain) of energy at the reflection from the target.

Thus, the instantaneous value of the reflected signal is of the form:

$$u_s(t) = \eta u(\eta t), \tag{4}$$

where $u_s(t)$ and $u(t)$ are received and emitted signals, respectively.

Explicitly, in its complex form, eq. (4) can be rewritten as:

$$u_s = \eta \operatorname{Re} \left[U(\eta t) e^{j2\pi\eta t} \right]. \tag{5}$$

Numerical estimates indicate that changes in signal amplitude and energy due to the object motion, as a rule, are negligible and can be ignored. For the signal with the frequency spectrum of the width W and duration τ_c , the changes in the time scale can be neglected if the signal base is small, such that $W\tau_c \approx 1$. Contrary, for signals with a large base $W\tau_c \gg 1$, the effect of changes in the time scale of the modulation law due to the radial motion of the object is significant.

For the most commonly used types of radio pulse packets of radar signals, both incoherent and coherent, the variations in the time scale of the modulation law can significantly affect the form of frequency ambiguity function.

5. Results of development the method for assessing the quality of frequency resolution for aircraft

5.1. Derivation of the normalized frequency ambiguity function

The frequency ambiguity function determines the resolution of the Doppler frequency and the accuracy of the param-

eter measurement. When deriving the ambiguity function, one can neglect changes in the amplitude and duration of a single pulse due to the Doppler effect. However, it is necessary to take into account much more significant changes in the time intervals between pulses associated with a change in the time scale during the radial motion of the aircraft.

Taking into account the transformation factor, the complex correlation integral [10, 24, 25, 27] for the i -th pulse of the packet is of the form:

$$\begin{aligned} \dot{Z}_i(\hat{\eta}, \eta) &= \\ &= \frac{1}{2} \int_{-\infty}^{\infty} \dot{U}_i(t - \hat{\eta}t_i) \dot{U}_i^*(t - \eta t_i) \exp(j2\pi f_0 \Delta\eta) dt, \end{aligned} \quad (6)$$

where $\dot{U}_i(t)$ is the complex voltage envelope of the i -th pulse of the packet; $\hat{\eta}$ is the estimation value of η ;

$$\Delta\eta = \eta_s - \eta = (\Delta F_D - \Delta \hat{F}_D) / f_0 = F / f_0,$$

is the mismatch by the transformation parameter (the difference between the transformation parameters of expected and received signals); $F = \Delta F_D - \Delta \hat{F}_D$ is the mismatch by the Doppler shift for the frequency of the expected and received signals; t_i is the shift of the center of the i -th pulse with respect to the origin of time counting.

For pulses with a rectangular envelope and amplitude U eq. (6) takes the form:

$$\dot{Z}_i = \frac{U^2}{2} \int_a^b \exp(j2\pi Ft) dt. \quad (7)$$

All possible values of the limits a and b for $t_i > 0, t_i < 0, \Delta\eta > 0, \Delta\eta < 0$ are sketched in Fig. 1. They are defined by the time limits of the shaded areas for which the product of the coefficients of the expression under the integral in eq. (6) is nonzero.

The result of integration, which is the normalized ambiguity function for the i -th pulse, can be written as:

$$\begin{aligned} \frac{\dot{Z}_i(\hat{\eta}, \eta)}{E_0} = \dot{\rho}_i(F) &= \\ &= \begin{cases} \frac{\sin \left[\pi F \tau_i \left(1 - \frac{|F \tau_i|}{f_0 \tau_i} \right) \right]}{\pi F \tau_i} \exp \left[j \pi F \tau_i (\hat{\eta} + \tau) \right] & \text{at } |F \tau_i| \leq f_0 \tau_i; \\ 0 & \text{at } |F \tau_i| > f_0 \tau_i, \end{cases} \end{aligned} \quad (8)$$

where $E_0 = U^2 \tau_i / 2$ is the energy of a single pulse of the duration τ_i .

With the assumptions alluded to above, the complex ambiguity function for the i -th pulse of a packet is of the form:

$$\dot{\Psi}_i(\hat{\eta}, \eta) = \int_a^b \dot{U}_i(t - \hat{\eta}t_i) \dot{U}_i^*(t - \eta t_i) \exp(j2\pi f_0 \Delta\eta) dt. \quad (9)$$

For a rectangular pulse, relation (9) reduces to the form:

$$\dot{\Psi}_i = U^2 \int_a^b \exp(j2\pi Ft) dt = \frac{U^2 \tau_i}{2} \frac{\sin \pi F (b-a)}{\pi F \tau_i} e^{j\pi F (b+a)}. \quad (10)$$

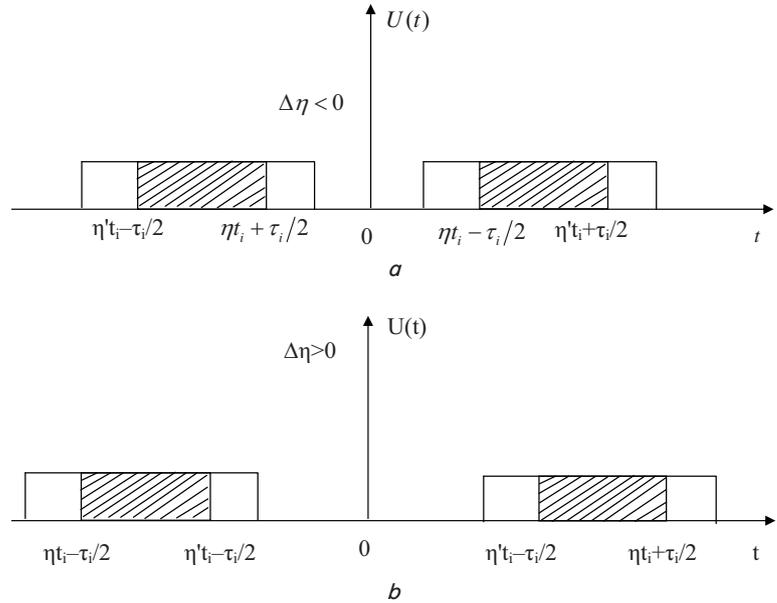


Fig. 1. Integration limits for equation (7): $a - at \Delta\eta > 0; b - \Delta\eta < 0$

For arbitrary values of t_i and $\Delta\eta$, one has $b - a = \tau_i - |\Delta\eta t_i|, b + a = (\eta + \eta_s) \tau_i$, and then eq. (10) takes the form:

$$\dot{\Psi}_i = \frac{U^2 \tau_i}{2} \frac{\sin \pi F (\tau_i - |\Delta\eta t_i|)}{\pi F \tau_i} e^{j\pi F (\eta + \hat{\eta}) t_i}. \quad (11)$$

Below let's consider the ambiguity function for a coherent packet with a rectangular packet envelope for an even number of rectangular pulses of duration τ_i , and t_i defined with respect to the center of the packet as $|\tau_i| = (2i-1)T/2$ with T being the period of the appearance of the pulses. For the purpose of approximate numerical estimates, one can assume $(\eta + \eta_s) / 2 \approx 1$.

The combination of two ambiguity functions, which are symmetric with respect to the time origin, followed by the substitution $\eta + \eta_s \approx 2$ and normalization, gives:

$$\dot{\rho}_i(F) = \begin{cases} \frac{\sin \pi F \tau_i \left(1 - \frac{|F| T}{f_0 \tau_i} \frac{2i-1}{2} \right)}{\pi F \tau_i} \times \\ \times \cos \left[\pi F T (2i-1) \right] & \text{at } 0 \leq \frac{|F| T}{f_0 \tau_i} \frac{2i-1}{2} \leq 1; \\ 0 & \text{at } \frac{|F| T}{f_0 \tau_i} \frac{2i-1}{2} > 1. \end{cases} \quad (12)$$

For a coherent packet of n pulses, the frequency ambiguity function is of the form:

$$\rho(F) = \frac{1}{n} \sum_{i=1}^n \dot{\rho}_i(F). \quad (13)$$

For an even number of pulses $n=2m$ and the time counted from the center of the packet as $\tau_i = (2i-1)T/2$, the frequency ambiguity function of the coherent pulse packet is described by the system of equations:

$$\rho(F) = \frac{1}{m} \begin{cases} \left| \sum_{i=1}^m \frac{\sin \pi F \tau_i \left(1 - \frac{|F|T}{f_0 \tau_i} \frac{2i-1}{2}\right)}{\pi F \tau_i} \right| \times \cos[\pi FT(2i-1)] \\ \text{at } 0 \leq \frac{|F|T}{f_0 \tau_i} \leq \frac{2}{2m-1}; \\ \left| \sum_{i=1}^{m-1} \frac{\sin \pi F \tau_i \left(1 - \frac{|F|T}{f_0 \tau_i} \frac{2i-1}{2}\right)}{\pi F \tau_i} \right| \times \cos[\pi FT(2i-1)] \\ \text{at } \frac{2}{2m-1} < \frac{|F|T}{f_0 \tau_i} \leq \frac{2}{2m-3}; \\ \left| \sum_{i=1}^{m-k} \frac{\sin \pi F \tau_i \left(1 - \frac{|F|T}{f_0 \tau_i} \frac{2i-1}{2}\right)}{\pi F \tau_i} \right| \times \cos[\pi FT(2i-1)] \\ \text{at } \frac{2}{2m-(2k-1)} < \frac{|F|T}{f_0 \tau_i} \leq \frac{2}{2m-(2k+1)}; \\ \left| \frac{\sin \pi F \tau_i \left(1 - \frac{|F|T}{f_0 \tau_i} \frac{2i-1}{2}\right)}{\pi F \tau_i} \right| \times \cos[\pi FT(2i-1)] \\ \text{at } \frac{2}{3} < \frac{|F|T}{f_0 \tau_i} \leq 2, \end{cases}$$

where m is the number of pairs of pulses of a packet, symmetrical with respect to its center.

Taking into account that $\pi F \tau_i \gg 1$, the above expression for $\rho(F)$ can be written as follows:

$$\rho(F) = \frac{1}{m} \begin{cases} \left| \sum_{i=1}^m \left(1 - \frac{2i-1}{2f_0 \tau_i} |F|T\right) \cos[\pi(2i-1)|F|T] \right| \\ \text{at } 0 \leq \frac{|F|T}{f_0 \tau_i} \leq \frac{2}{2m-1}; \\ \left| \sum_{i=1}^{m-1} \left(1 - \frac{2i-1}{2f_0 \tau_i} |F|T\right) \cos[\pi(2i-1)|F|T] \right| \\ \text{at } \frac{2}{2m-1} < \frac{|F|T}{f_0 \tau_i} \leq \frac{2}{2m-3}; \\ \left| \sum_{i=1}^{m-k} \left(1 - \frac{2i-1}{2f_0 \tau_i} |F|T\right) \cos[\pi(2i-1)|F|T] \right| \\ \text{at } \frac{2}{2m-(2k-1)} < \frac{|F|T}{f_0 \tau_i} \leq \frac{2}{2m-(2k+1)}; \\ \left| \left(1 - \frac{2i-1}{2f_0 \tau_i} |F|T\right) \cos[\pi(2i-1)|F|T] \right| \\ \text{at } \frac{2}{3} < \frac{|F|T}{f_0 \tau_i} \leq 2. \end{cases} \quad (14)$$

Neglecting the changes in the time scale due to the Doppler effect one has $\Delta\eta=0$ and consequently, eq. (11) reduces to the form:

$$\psi_i = \frac{U^2 \tau_i}{2} \frac{\sin \pi F \tau_i}{\pi F \tau_i} e^{j\pi F(\eta+\hat{\eta})\tau_i}. \quad (15)$$

In such a case, the normalized ambiguity function, eq. (12) is of the form:

$$\dot{\rho}_i(F) = \begin{cases} \frac{\sin \pi F \tau_i}{\pi F \tau_i} \cos[\pi FT(2i-1)] & \text{at } 0 \leq \frac{|F|T}{f_0 \tau_i} \frac{2i-1}{1} \leq 1; \\ 0 & \text{at } \frac{|F|T}{f_0 \tau_i} \frac{2i-1}{1} > 1. \end{cases} \quad (16)$$

Thus, in the case of neglecting the transformation of the time scale of a coherent packet of n radio pulses, eq. (14) takes the form:

$$\rho(F) = \begin{cases} \left| \sum_{i=1}^m \cos[\pi FT(2i-1)] \right| \\ \text{at } 0 \leq \frac{|F|T}{f_0 \tau_i} \leq \frac{2}{2m-1}; \\ \left| \sum_{i=1}^{m-1} \cos[\pi FT(2i-1)] \right| \\ \text{at } \frac{2}{2m-1} < \frac{|F|T}{f_0 \tau_i} \leq \frac{2}{2m-3}; \\ \left| \sum_{i=1}^{m-k} \cos[\pi FT(2i-1)] \right| \\ \text{at } \frac{2}{2m-(2k-1)} < \frac{|F|T}{f_0 \tau_i} \leq \frac{2}{2m-(2k+1)}; \\ \left| \cos[\pi FT(2i-1)] \right| \\ \text{at } \frac{2}{3} < \frac{|F|T}{f_0 \tau_i} \leq 2. \end{cases} \quad (17)$$

The effect of time scale transformation on the frequency ambiguity function is illustrated in Fig. 2, where it is shown the ambiguity functions $\rho(|F|T)$ calculated using eq. (14), which accounts (curves 1) and using eq. (17) which does not account (curves 2) for the transformation of its time scale for three different numbers of pulses in the packet: $n=8$ (Fig. 2, a), $n=12$ (Fig. 2, b) and $n=16$ (Fig. 2, c), widely used in modern coherent-pulse survey radars and radars for the tracking of aircraft. It is clearly seen from Fig. 2 that the account for the time scale transformation significantly reduces the ambiguity function values, thereby considerably improving the quality of frequency resolution. In other words, due to the considerable change in the time scale of the packet on increasing frequency mismatch F , the envelope of the single peaks of the ambiguity function decreases much faster than in the traditional case, i.e. when the transformation of the time scale is not taken into account, and, consequently, the peaks broaden.

In practical cases, due to the small difference in the amplitude of the peaks at $F=0$ and nonzero $|F|T=1$, one can significantly reduce the restrictions for the values of the pulse period T , and in some cases it allows one to remove them at all. The latter becomes possible if the condition for the unambiguous Doppler frequency measurement holds.

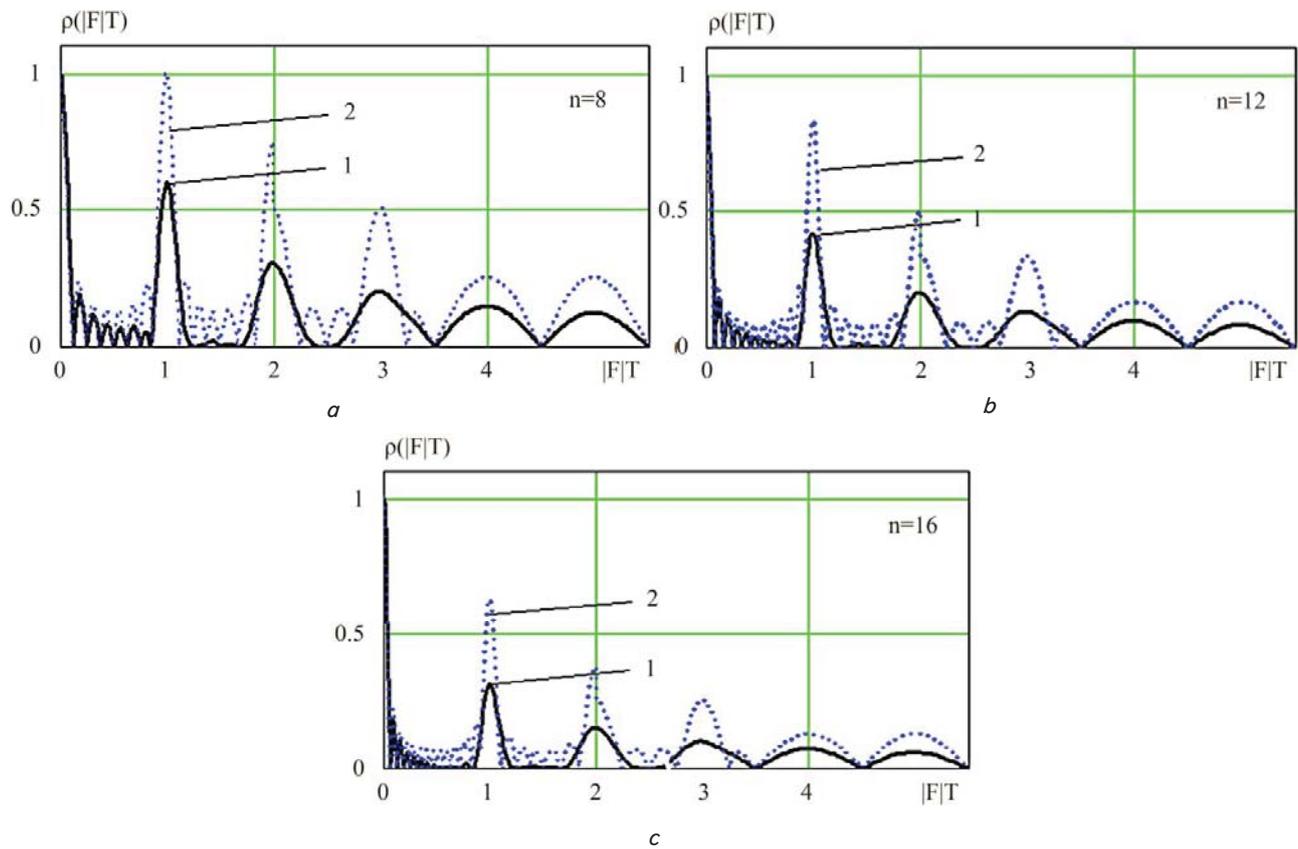


Fig. 2. The normalized ambiguity functions $\rho(|F|T)$ calculated using eq. (14), which accounts (curves 1) and using eq. (17) which does not account (curves 2) for the transformation of its time scale for the values: $a - n=8$; $b - n=12$; $c - n=16$

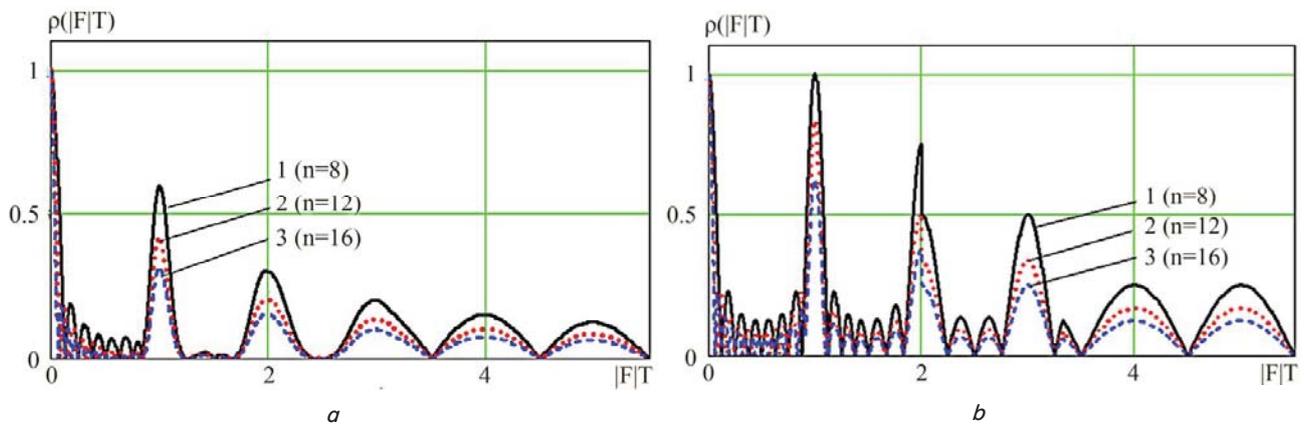


Fig. 3. Normalized frequency ambiguity function, built for three different numbers of pulses in the burst $n=8$, $n=12$ and $n=16$: $a -$ according to equation (14); $b -$ according to equation (17)

To illustrate the decrease of the ambiguity function on increasing the number of pulses in the packet, in Fig. 3 let's plot the ambiguity function for $n=8$, $n=12$ and $n=16$ separately for the case, when the time scale transformation is accounted using eq. (14), Fig. 3, a and is not accounted using eq. (17), Fig. 3, b . For the sake of presentation clarity of plots within the same scale, for data shown in Fig. 2, 3 let's choose $f_0\tau_r=5$; even though this is not a typical value it does not affect the generality of the consideration. The corresponding calculations at the values typical for coherent-pulse radars are plotted in Fig. 4 and will be discussed later in this section.

Comparison of the plots for different values of n in Fig. 3, a, b reveals that at small values $|F|T < 1$, the corresponding ambiguity functions are almost indistinguishable, while the difference between them considerably grows on increasing $|F|T > 1$.

It is worth noting that increasing the number n of radio pulses in the packet narrows the normalized frequency ambiguity function and reduces the height of its lateral maxima, which thereby increases the degree of Doppler resolution and improves the conditions of its unambiguous measurement. In Fig. 3, this effect shows up at $|F|T > 1$ for $n=8, 12, 16$. However, without the account for the transformation of the time

scale of the studied ambiguity function, these positive effects weaken, especially, as it is seen in Fig. 3b, the latter concerns the conditions of unambiguous Doppler frequency measurement, which hold for $n=12$ at $|F|T>2$, and for $n=8$ at $|F|T>3$. The jumps in amplitude observed at $|F|T=2$ (Fig. 3, b) are due to the frequency mismatch, which leads to the halving of the pulse period and consequently to the overlapping of the received pulse signal and the subsequent one, when the time scale transformation is not accounted. The sharpening of peaks observed for all values of n in Fig. 3, b is of the same origin. Remarkably, both these features (the jump and peak sharpening) that are present in Fig. 3, b, neglecting the time scale transformation, eq. (17), disappear (Fig. 3, a), when the time scale transformation is accounted, eq. (14).

As announced above, now let's consider the results obtained for parameters of radio pulses packets, which are typical for coherent-pulse radars. Typical parameters for such radars are in the following ranges: a few microseconds for the duration of the radio pulse; centimeter range for the wavelength of the electromagnetic emission; from tens of microseconds to several milliseconds for the period of repetition of radio pulses [25]. In such cases, the root mean square errors at the Doppler frequency measurement, which are due to the motion of aircraft, can be from tens to hundreds of Hz [28, 29].

Fig. 4 shows functions $\rho(F)$, obtained using eq. (14), for the case of coherent packets of 8 (curve 1), 12 (curve 2), and 16 (curve 3) pulses at the following typical parameters for the coherent-pulse radar: carrier frequency $f_0=3$ GHz (corresponding wavelength $\lambda=10$), pulse duration $\tau_i=3$ μ s, pulse repetition period $T=1$ ms.

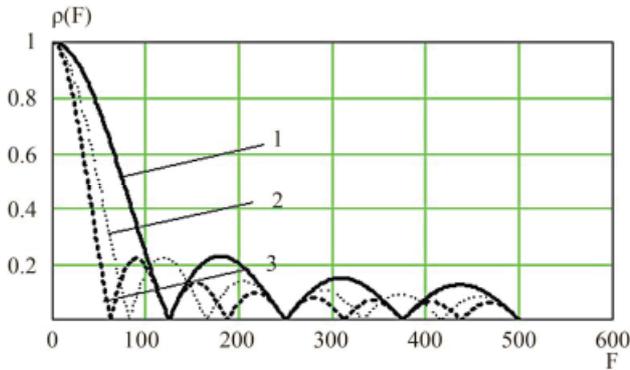


Fig. 4. Normalized ambiguity functions plotted using eq. (14) for coherent packets of different numbers of pulses: $n=8$ (curve 1), $n=12$ (curve 2), and $n=16$ (curve 3)

Fig. 4 illustrates that for these parameters of the coherent packet of radio pulses, due to the accounted time scale transformation, the condition of unambiguous Doppler frequency measurement indeed holds. On increasing the frequency mismatch F , in the range of its typical values, the envelope of the peaks of the ambiguity function decreases considerably in comparison with the central peak.

The degree of frequency resolution for radar signals can be determined directly from the expression for the normalized frequency ambiguity function. If $\rho(F)$ is a twice differentiable function, then the measure of frequency resolution of the level of 0.5 can be determined as following [10, 17]:

$$\Delta F_{0.5} = \frac{1}{\sqrt{\rho''(0)}}, \quad (18)$$

where $\rho''(0)$ is the value of the second derivative of the normalized ambiguity function corresponding to the central peak.

Double differentiability of eq. (14) at the central peak allows one to obtain the following result:

$$\rho''(0) = -\frac{\pi^2 T^2}{m} \sum_{i=1}^m (2i-1)^2. \quad (19)$$

Substitution of eq. in eq. gives:

$$\Delta F_{0.5} = \frac{1}{\pi T \sqrt{\frac{1}{m} \sum_{i=1}^m (2i-1)^2}}. \quad (20)$$

The degree of frequency resolution determined according to eq. (20) can be considered as a maximally possible value since it corresponds to the case of absence of influence of external and internal interference.

The values of $\Delta F_{0.5}$ calculated at $T=1$ ms for the packets of 8, 12, and 16 pulses considered above are presented in Table 1.

Table 1

Potential measure of frequency resolution

n	8	12	16
$\Delta F_{0.5}$ (Hz)	70	46	35

Radar observation of aircraft at real conditions is associated with the occurrence of fluctuations in the propagation of the radar signal, which in turn causes fluctuations in the normalized frequency ambiguity function. Therefore, based on the obtained results, in the next 3.2 section, let's evaluate the decrease in the degree of frequency resolution, which occurs due to the influence of additive noise and multiplicative interference.

5. 2. Evaluation of the total additive effect of internal noise in the receiving device on the lowering of the frequency resolution

By the additive noise in the literature on radars, it is meant the internal noise of the radar receiver, which can be considered as being Gaussian [10, 17, 28]. The inhomogeneity of the medium in which the radio wave propagates is considered to be the main source of the multiplicative noise. It is the influence of multiplicative interferences that causes the fluctuations of the initial phases of the radio pulses of the packet radio signal, which, in turn, leads to the lowering of its coherence and distortion of the normalized frequency ambiguity function.

Theoretical and experimental studies suggest that the distribution law for phase fluctuations of the radar signal is close to being normal, and their correlation function can be approximated by an oscillating dependence [14, 23–25]. The dispersion of the measurement error for the frequency of the packet of radio pulses due to the additive effect of the internal noise of the receiving device is of the form [24, 30, 31]:

$$D_F = \frac{12}{q^2 T^2 (4m^2 - 1)}, \quad (21)$$

where q^2 is the power signal-to-noise ratio. For the multiplicative effect of correlated phase fluctuations described by the oscillating correlation function the corresponding expression is of the form [25, 29]:

$$D_{fl} = \frac{18\sigma_\phi^2}{m^2(4m^2-1)^2T^2} \times \left[\sum_{j=1}^m (2j-1)^2 \left\{ 1 - e^{-\frac{T}{\tau}(2j-1)} \times \left[\cos[(2j-1)vT] \right] \right\} + 2 \sum_{k=1}^{m-j} e^{-\frac{T}{\tau}k} \times \right. \\ \left. \times \sum_{j=1}^{m-1} (2j-1)(2j+2k-1) \times \left[\cos(jvT) - e^{-\frac{T}{\tau}(2j-1)} \cos[(2j+k-1)vT] \right] \right\} \right], \quad (22)$$

where σ_ϕ^2 is the dispersion of phase fluctuations; τ is phase correlation interval of fluctuations; $v=2\pi/T_{fl}$ is the frequency of oscillations of the correlation coefficient of phase fluctuations with T_{fl} being the period of oscillations of the correlation coefficient of phase fluctuations.

The total dispersion of the error in measuring the frequency of the packet of radio pulses, which is due to the influence of both these factors, is:

$$D = D_f + D_{fl}. \quad (23)$$

According to [27], in the presence of random distortion of the wavefront of the radar signal, the degree of angular resolution $\Delta\theta$, can be determined as:

$$\Delta\theta = q\sigma_\theta, \quad (24)$$

where q is the signal-to-noise voltage ratio at the output of the processing device; σ_θ is the root mean square error of measuring the angle of arrival of the wave.

The measure of frequency resolution under the condition of the presence of the additive effect of internal noise and the multiplicative effect of correlated phase fluctuations can be determined as:

$$\Delta F_{0.5} = q\sqrt{D}, \quad (25)$$

in which the dispersion D of the measurement error for the frequency of the packet of radio pulses is determined according to eq. (21)–(23).

According to the data [15, 23–25], the influence of tropospheric inhomogeneity leads to the phase fluctuations with the dispersion $\sigma_\phi^2 = (0.01...10) \text{ rad}^2$ and the correlation interval $\tau = (0.1...1) \text{ s}$. The evaluation is worthwhile for cases when the resolution coefficient $K_r = 10 \lg[q^2/2]$ takes values from 17 to 27 dB, which are typical for radars of the surveillance and tracking type according to the estimates of detection quality indicators provided in [10, 28].

The results of numerical evaluation of the measure of frequency resolution obtained from eq. (25) taking into account eq. (21)–(23) for packets of 8, 12, and 16 radio pulses at $T=1 \text{ ms}$, are presented in Table 2; the period of oscillations of the correlation coefficient of phase fluctuations was chosen to satisfy the condition $T_{fl}/\tau=3$, experimentally confirmed in [23].

The calculations were performed for the values of the coefficient $K_r=17 \text{ dB}$ and 27 dB and two cases of the values of statistical characteristics of the phase fluctuations:

1. $\sigma_\phi^2 = 0.01 \text{ rad}^2$ and $\tau=1 \text{ s}$ corresponding to the predominant additive effect of internal noise; the multiplicative effect of phase fluctuations is almost absent.

2. $\sigma_\phi^2 = 10 \text{ rad}^2$ and $\tau=1 \text{ s}$ corresponding to the multiplicative effect of phase fluctuations being significantly predominant in comparison with the additive effect of internal noise.

Table 2

The measure of frequency resolution in the presence of additive effects of internal noise and multiplicative effects of correlated phase fluctuations

N	8	12	16
$K_r=17 \text{ dB}, \sigma_\phi^2 = 0.01 \text{ rad}^2, \tau=1 \text{ s}$			
$\Delta F_{0.5} \text{ (Hz)}$	437	290	217
$K_r=27 \text{ dB}, \sigma_\phi^2 = 0.01 \text{ rad}^2, \tau=1 \text{ s}$			
$\Delta F_{0.5} \text{ (Hz)}$	439	293	220
$K_r=17 \text{ dB}, \sigma_\phi^2 = 10 \text{ rad}^2, \tau=1 \text{ s}$			
$\Delta F_{0.5} \text{ (Hz)}$	1889	1540	1343
$K_r=27 \text{ dB}, \sigma_\phi^2 = 10 \text{ rad}^2, \tau=1 \text{ s}$			
$\Delta F_{0.5} \text{ (Hz)}$	5714	4700	4115

Comparison of the values of $\Delta F_{0.5} \text{ (Hz)}$ presented in Tables 1, 2 indicates that the additive effect of the internal noise of the receiving device can lead to the broadening of the normalized frequency ambiguity function and worsening of the frequency resolution by 6 times. The multiplicative effect of correlated phase fluctuations can lead to the enhancement of this effect from 27 to 118 times.

Fig. 5 shows the dependence of the frequency resolution measure $\Delta F_{0.5}$ on the dispersion of phase fluctuations σ_ϕ^2 , obtained according to eq. (25) taking into account eq. (21)–(23), for packets with the number of radio pulses $n=8, 12$, and 16 . The plots were obtained at $K_r=27 \text{ dB}$ for two values of the correlation interval of phase fluctuations $\tau=0.1 \text{ s}$ for $n=8$ (curve 1); $n=12$ (curve 2); $n=16$ (curve 3) and $\tau=1 \text{ s}$ for $n=8$ (curve 1’), $n=12$ (curve 2’); $n=16$ (curve 3’).

Fig. 6 shows the dependence of the measure of frequency resolution $\Delta F_{0.5}$ on the correlation interval of phase fluctuations τ for the same packets of radio pulses. The plots were obtained at $K_r=27 \text{ dB}$ for two values of the dispersion of phase fluctuations $\sigma_\phi^2 = 0.1 \text{ rad}^2$ for $n=8$ (curve 1), $n=12$ (curve 2), $n=16$ (curve 3) and $\sigma_\phi^2 = 10 \text{ rad}^2$ for $n=8$ (curve 1’), $n=12$ (curve 2’), $n=16$ (curve 3’).

6. Discussion of research results

The object of the presented study is to assess the quality of the frequency distribution of aircraft, which characterizes the effectiveness of radar surveillance of aircraft and determines the effectiveness of their control using radio signals.

When developing a method for estimating the resolution of aircraft, the inhomogeneity of the atmosphere is taken into account, which leads to fluctuations in the refractive index of radio waves and to phase distortions of the radar signal and significantly complicates the operation of the radar. During the study, an assumption was made about the absence of internal noise during the generation of a radar

signal, which can introduce additional distortions into the process of separating aircraft by frequency.

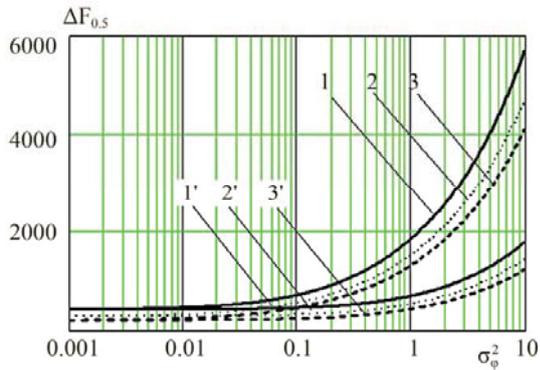


Fig. 5. Dependences of the measure of frequency resolution on the dispersion of phase fluctuations for the coherent packets of 8, 12, and 16 radio pulses

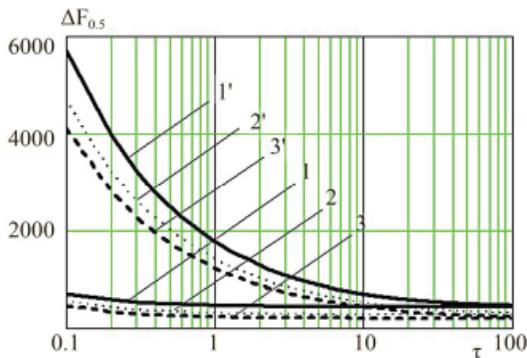


Fig. 6. Dependences of the measure of frequency resolution on the correlation interval of phase fluctuations for the coherent packets of 8, 12, and 16 radio pulses

Under the condition of insignificant multiplicative influence of correlated phase fluctuations ($\sigma_\phi^2 = 0.01 \text{ rad}^2$ and $\tau = 1 \text{ s}$), on increasing the number of radio pulses in the packet there is an obvious tendency of proportional improvement of the frequency resolution. Thus, doubling the number of radio pulses from 8 to 16 radio pulses improves the degree of frequency resolution (reducing the width of the normalized frequency ambiguity function) almost twice (by 100%). However, due to the multiplicative effect of phase fluctuations, and due to the partial loss of coherence of the packet, the improvement is significantly reduced.

At the predominant multiplicative effect of correlated phase fluctuations ($\sigma_\phi^2 = 10 \text{ rad}^2$ and $\tau = 1 \text{ s}$), the increase of the number of radio pulses from 8 to 16 leads to the improvement in the frequency resolution by approximately 1.4 times (40%). In other words, the effect of improving the quality of resolution is reduced by 60%.

From Fig. 5 it is seen that in the case of almost complete correlations of phase fluctuations at $\tau = 1 \text{ s}$ (curves 1', 2', 3'), there is an insignificant lowering of $\Delta F_{0.5}$ in comparison with the case of solely the internal noise of the receiving device, even at high values of phase fluctuations. However, at $\tau = 0.1 \text{ s}$ (curves 1, 2, 3) one observes the effect of a sharp broadening of the normalized frequency ambiguity function consequently lowering the frequency resolution.

At the values of the dispersion of phase fluctuations $\sigma_\phi^2 < 0.1 \text{ rad}^2$, their multiplicative effect on the degree of frequency resolution almost vanishes and the value of the dispersion is solely due to the influence of the internal noise of the receiving device. However, when the dispersion of phase fluctuations is in the range of values from 0.1 rad^2 to 10 rad^2 , which can take place at real conditions of radar tasks, the contribution of the multiplicative effect of phase fluctuations in the lowering of the frequency resolution increases significantly and can exceed dozens of times the influence of internal noise.

As follows from Fig. 6, the excess of the multiplicative influence of phase fluctuations over the additive influence of internal noise begins to appear at values of the correlation interval of phase fluctuations less than 7 s and reaches the maximum at $\tau = 0.1 \text{ s}$, which indeed can occur in the perturbed troposphere. Thus, the obtained results directly determine the conditions under which the quality of frequency resolution for radar signals depends rather on the statistical characteristics of phase fluctuations than on the signal-to-noise ratio, which allows one to predict the possible effectiveness of radar observation of aircraft at real weather conditions.

A feature of this study is the derivation of the functions of the frequency ambiguity of a coherent packet of radio pulses, taking into account the transformation due to the radial motion of the aircraft. The use of the proposed functions made it possible to evaluate the effect of phase fluctuations of the packet radio pulses on the degree of frequency resolution.

In [28], a generalized signal model is presented to accommodate both narrowband and wideband signals in multi-input and multi-output sensor system scenarios. However, this paper does not evaluate the errors of the signal parameters, which are due to the propagation of radio waves and the shape of the aircraft (radar object). The value of such errors can make a significant contribution to determining the flight parameters of highly maneuverable aircraft. A feature of the presented article is the consideration of the issue of taking into account the transformation of the frequency function of the mismatch. A study of the influence of phase fluctuations of radio pulses of a burst with frequency resolution was carried out.

When conducting the study, a limitation was introduced: the structure of the construction of the radar station and the influence of its characteristics on the propagation of the proposed signals were not taken into account. Due to the accepted limitation, the presented study has some practical drawbacks. Thus, the presented ratios and the resulting graphs do not take into account the design features of the construction of radar stations. For example, the level of internal noise during the generation of a radar signal can introduce additional distortions into the process of separating aircraft by frequency. However, this shortcoming will be resolved in future studies and presented in future publications. The difficulties of the presented research for today and for the future are the practical implementation of the developed methods. When conducting an experimental confirmation of the results obtained, it is necessary to be aware of an experimental radar station and use a swarm of aircraft. A test site and related support is required. But, the results of the conducted physical experiment will make it possible to make adequate changes to the obtained theoretical results and models. It should be noted that, on the one hand, the results of computer simulation confirm the adequacy of the

presented results. But, on the other hand, the difficulties in conducting a physical experiment leave questions for experimental confirmation of the results obtained.

7. Conclusions

1. A frequency ambiguity function is proposed that takes into account some of the distorting effects of radar signal propagation. An approach has been developed to determine the normalized frequency ambiguity function for a coherent packet of radio pulses. The proposed approach takes into account the transformation of the time scale due to the radial movement of the aircraft during radar observation. Based on the relations obtained, for the frequency ambiguity function, it is established:

– at small frequency detunings $F < 1/T$ the corresponding contribution is very small;

– at $F > 1/T$ the contribution increases and is expressed in a significant increase in the amplitude and expansion of the peaks of the diversity function. This allows to draw a conclusion about the removal of frequency ambiguity in some, special, cases.

2. The simultaneous additive effect of uncorrelated internal noise of the receiver and the multiplicative effect of correlated phase fluctuations of burst radio pulses on the decrease in the degree of frequency resolution are estimated. It has been established that the multiplicative effect of correlated phase fluctuations arises due to the real conditions of observation of aircraft with a coherent-pulse radar. Such an

influence causes a broadening of the normalized frequency ambiguity function and leads to a decrease in the frequency resolution by a hundred or even more times. The developed approach for numerical evaluation of the effect of atmospheric disturbances on the frequency ambiguity function will become a convenient tool for analyzing the quality of radar frequency resolution under ideal, real, and complex conditions of radar surveillance of aircraft.

The results obtained in the study are proposed to be used in the construction of radar systems for detecting or tracking individual aircraft in groups (for example, individual unmanned aerial vehicles when operating as part of a swarm).

Conflict of interest

The authors declare that there is no conflict of interest regarding this study, including financial, personal nature, authorship or other nature that could affect the research and its results presented in this article.

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Data availability

The manuscript has no associated data.

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